

The study developed in this thesis is focused on numerical modeling of dynamics behavior of dry soils in the framework of small and large deformation theory and of dynamics behavior of saturated soils in the framework of small deformation theory. A numerical code has been developed to analyze such behaviors and presents three versions:

- The Runge-Kutta Taylor-SPH for analysis of dry soils behavior in dynamic (small deformation theory)
- The new Runge-Kutta Taylor-SPH for analysis of dry soils behavior in dynamic (large deformation theory)
- The  $v - p_w$  Runge-Kutta Taylor-SPH for analysis of saturated soils behavior in dynamic (small deformation theory)

Both of them are based on a mathematical model which describes the dynamic of viscoplastic solids. This mathematical is based on the  $u - p_w$  model of Zienkiewicz and his team. The basis of the models used in this work are:

- The momentum balance equation which are formulated in terms of stress,  $\sigma$  and velocity,  $v$ . This formulation has been chosen in order to assure the good propagation of short wavelengths and to prevent a possible volumetric locking as it occurs in finite element methods. Volumetric locking is responsible of an overestimation of the limit loads and wrong mechanisms of failure.
- The constitutive model chosen is the viscoplastic model of Perzyna. This model avoids the ill-posing of the mathematical model. Indeed when using an elastoplastic model velocities of propagation can become imaginary and are in contradiction with the boundary conditions of soil dynamics problems. In the present thesis the Perzyna's model is completed by the Von Mises yield criterion and the yield surface of the modified Cam-Clay model. Both of them indicate where viscoplastic deformations occur.
- The kinetic relation gives us the relation between the rate of deformation and the gradient of velocity.

The set of the momentum balance equations, the constitutive model and the kinetic model constitutes the basis of the mathematical model used for describing the dynamics behavior of dry soil in small and large deformation theory framework (Chapter 2 and Chapter 5).

In this model the Jauman stress rate has to be introduced in order to take into account the large deformation and the rotation in the material (Chapter 5).

When considering the coupled behavior of saturated soil, the variable of the mathematical model are the effective stress, the velocity and the pore pressure. The mathematical model to represent the coupled behavior is based on the mass balance and momentum balance equations

for the solid particles, for the pore water and for the mixture (Chapter 6). In this mathematical model the set of the mass momentum equation for the mixture, the kinetic equation and the constitutive relation is completed by the introduction of the following equation:

$$-\operatorname{div}(k_w \operatorname{grad} p_w) + S_w \operatorname{div} v_s + \frac{1}{Q^*} \frac{D^{(s)} p_w}{Dt} = 0$$

This equation indicates us how the water pore pressure varies in time and in the material.

In the present thesis, three mathematical models have been presented. Both of them are based on the same constitutive equation, the Perzyna's model. Differences have been introduced to have possibility to represent the behaviors described in the objectives of this work:

- Dynamics behavior of dry soils in small deformation theory
- Dynamics behavior of dry soils in large deformation theory
- Dynamics behavior of saturated soils in small deformation theory.

These mathematical models consist on a system of hyperbolic equations.

These equations are discretized by a new proposed numerical model which is based on:

- The two-step Taylor-Galerkin method: this method has been chosen to prevent to the spurious node-to-node oscillations which can appear when solving hyperbolic problems with classical finite elements methods. The two-step Taylor-Galerkin algorithm proposed a discretization in time of the equations before their discretization in space. The discretization in time is done in two steps and is based on a forward-time Taylor series expansion. In the Taylor-Galerkin method the discretization in space is done with finite elements.
- The Runge-Kutta Taylor-Galerkin method has been used in order to propose a more accurate solution of the problems when the partial differential equations are dominated by the source term (viscoplastic deformations and gravity forces). This algorithm introduces a splitting scheme where the hyperbolic equation is split into an ordinary differential equation and a pure advection problem.
- The numerical method to discretize in space the equations of the Runge-Kutta Taylor-Galerkin is the Smoothed Particle Hydrodynamics (SPH) method. Being a meshless method, it is well-adapted to large deformation problems. However the method presents some pitfalls: i) the deficiency boundary problem solved using the Corrective Smoothed Particle Hydrodynamics (CSPH) ; ii) the SPH tensile instability.

In the present thesis the main contribution is the proposition of a method to stabilize SPH algorithms and to avoid the SPH tensile instability. The use of a SPH grid with a staggered arrangement is the proposed solution here. Two set of SPH particle are used in the calculation:

- The SPH nodes where the variable are calculated at time  $t = n$
- The SPH elements where the variable are calculated at time  $t = n + \frac{1}{2}$ .

The use of the double set of particles associated with the Runge-Kutta Taylor-Galerkin method stabilizes the SPH method to solve problems which cannot be solved by the classical SPH method.

The association of the Runge-Kutta Taylor-Galerkin method with the double set of particles has been used to discretize the equation of the mathematical model in order to analyze dynamics behavior of dry soils. We called the proposed model: The Runge-Kutta Taylor-SPH model.

This numerical model has been improved to discretize the equations of the mathematical models for dry soils in large deformation theory and for saturated soil in small deformation theory.

The improvements to obtain a numerical model for large deformation are:

- The second step of the Runge-Kutta Taylor-SPH algorithm is replaced by a simple scheme in order to calculate the velocity at time  $t = n + 1$ . The simple scheme avoids the zero-energy mode problems and the hourglass deformation disappears.
- A subroutine to detect the free-surface has been developed. The algorithm is able to recognize the SPH nodes where traction-free boundary conditions have to be applied. Although the algorithm of free-surface detection is based on existing work, some improvements have been done order to get more accurate results in the definition of the normal to the free-surface.
- The boundary to the free-surface are traction free, i.e.  $\sigma_n = 0$  and  $\tau = 0$ . In order to apply these boundary conditions, a subroutine has been written to pass from the reference coordinate system to the new coordinate system related to the normal of the free-surface.
- As the position of the SPH nodes is update, at each time step, the neighbors of each particle have to been identified. The algorithm of nearest neighboring particles searching (NNPS) is based on the linked-list method where the problem domain is divided in square regions in order to fasten the searching processes and to save computational time.

The introduction of all these improvement into the first version of the Runge-Kutta Taylor-SPH comes to a new version of the model able to analyze the dynamics behaviors of dry soils in the large deformation theory (Chapter 5). This version of the model has been called The new Runge-Kutta Taylor-SPH model.

On another side, improvements have been done to the Runge-Kutta Taylor-SPH in order to discretize the equations of the mathematical model for dynamics behaviors of saturated soils:

- The fractional step method is used to discretize the equations satisfying the Babuska-Brezzi conditions. This method introduces an intermediate velocity,  $v^*$  and allows us calculating the pore pressure at time  $t = n + 1$  from  $v^*$  and the pore pressure  $p_w^n$ .
- In the fractional step method, the laplacian of the pore pressure has to be calculated. The method proposed by Schwaiger has been chosen and has given acceptable accurate results for our staggered arrangement of particles (Schwaiger 2008).
- In the fractional step algorithm, the system of equation  $K_{Laplacian} \cdot p_w^{n+1} = RHS$  is solved with the preconditioned conjugate gradient. The method of the preconditioned conjugate gradient has been preferred to the classical conjugate gradient in order to save computational time.

The implementation of these improvements and of the fractional step method has led to a new version of the numerical model able to solve coupled dynamic problems of saturated soil (Chapter 6). The version has been called the  $v - p_w$  Runge-Kutta-Taylor-SPH.

From the different mathematical models, three numerical algorithms have been written in FORTRAN 90 in order to obtain only one numerical program.

In order to valid the numerical program, a large number of case studies have been done. Different types of case studies have been designed in order to assess the accuracy of our numerical models and the capacity of the models to analyze dynamics behavior of geomaterials. All the case studies are summarized in the following table:

Name of the case study	Dimension		Material property			Analytical solution		Localized Failure	Small / Large deformations		Dry / saturated material	
	1D	2D	El.	V-VM	V-CC	A	V		S	L	Dry	Sat.
1D elastic bar: Test of stability	X		X			X	X		X		X	
1D viscoplastic bar	X			X		No		X	X		X	
Localization in 2D soil sample		X			X	X	X	X	X		X	
Elastic vertical cut		X	X			X	X		X		X	
Viscoplastic vertical cut without softening		X		X		X	X	X	X		X	
Viscoplastic vertical cut with softening		X		X		No		X	X		X	
Elastic one-dimensional bar	X		X			No			X		X	
Viscoplastic bidimensional bar		X		X		No		X	X		X	
Low cohesive vertical slope		X		X		No			X		X	
Cohesive vertical slope		X		X		No		X	X		X	
Vertical cut under loading		X		X		X	X	X	X		X	
Footing bearing capacity test		X		X		X	X	X	X		X	
Consolidation of saturated soil column	X		X			X	X		X			X
Harmonic loading on saturated soil column	X		X			X	X		X			X
Strip foundation	X		X			No			X			X
Strain localization in saturated soil		X			X	No		X	X			X
Saturated vertical cut		X		X		No		X	X			X
Total	6	11	6	9	2	8	8	10	11	6	12	5

EL: Elastic ; V-VM: Viscoplastic (Von Mises) ; V-CC: Viscoplastic Cam-Clay  
A: Available ; V: validated ; S: Small deformation ; L: Large deformation  
Dry: dry soils ; Sat. : Saturated soils

In total 17 case studies have validated our numerical models. In this set of examples, we proposed:

- 6 one-dimensional examples and 11 bi-dimensional examples
- 6 cases with elastic material and 11 with viscoplastic behavior
- 6 cases for validating the Runge-Kutta Taylor-SPH model (Chapter 4)
- 6 cases for validating the new Runge-Kutta Taylor-SPH model (Chapter 5)
- 5 cases for validating the  $v - p_w$  Runge-Kutta Taylor-SPH model (Chapter 6)

When it was possible, the results obtained with our proposed numerical models have been compared to analytical solutions.