Sand-production and sand internal erosion: Continuum modeling

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Basic Concepts from Geomechanics of bi-phasic media
Partial velocities: interstitial fluid flow during consolidation
Total stress, effective stress and pore-water pressure
Forces acting on spherical particle submerged in a moving fluid

1.1 Buoyancy force on a sphere

Figure E3.3.1. Buoyancy force on a sphere

- Buoyancy: \( F_B = \frac{4\pi}{3} \rho_f g R^3 \)
1.2 Lift and drag forces

We notice that irrotational flow around a sphere$^2$ generates neither lift nor drag forces.
- **Surface drag** due to shear stress: $F_D = 4\pi \mu_f RU_\infty$
- **Form drag** due to dynamic pressure: $F_D = 2\pi \mu_f RU_\infty$
- Total drag: $F_D = 6\pi \mu_f RU_\infty$
1.3 Distributed volume forces

- Number of particles: \( N = \frac{V_s}{V_g}, \quad V_g = \frac{4}{3} \pi R^3 \)

- Distributed (volume) force: \( f = \frac{NF}{V} = \frac{V_s}{V} \frac{F}{V_g} = (1 - \phi) \frac{F}{V_g} \)

\[
\frac{4\pi}{3} \rho_f g R^3
\]

- Buoyancy body force: \( f_B = (1 - \phi) \frac{4\pi}{3} \frac{4}{\pi R^3} = (1 - \phi) \rho_f g \)

- Drag body force: \( f_D = (1 - \phi) \frac{6\pi \mu_f R U_\infty}{4 \frac{4}{\pi R^3}} = (1 - \phi) \frac{\mu_f}{2 \frac{9}{R^2}} U_\infty \)

- fluid velocity: \( U_\infty = \frac{q}{\phi} \)

- grain diameter: \( D_g = 2R \)

\[
f_D = 18 \frac{1 - \phi}{\phi} \frac{\mu_f}{D_g^2} q
\]
The dimensionless drag force

solid packing ($c_1=180$)
fluidized bed ($n=4$)
rarefied suspension

\[ f_D^* \]

\[ \phi \]

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forces acting on grains

1. gravity forces: \((1-n)\rho_s g\).
2. buoyancy forces: \(-(1-n)\rho_w g\).
3. seepage forces \(\vec{f}\)
2. Continuum Interpretations

Equilibrium of forces acting on the fluid phase yields is expressed by

\[- \frac{\partial p}{\partial z} - \rho_f g - \frac{1}{\phi} f_z = 0\]

By setting

\[f_D = f\phi q\]

above equilibrium condition is interpreted as Darcy's law (cf. Vardoulakis & Sulem, 1995).
Groundwater flow induces a drag force on the grains

\[ q^{(2)} \approx \phi v^{(2)} = -k_f \frac{\partial p}{\partial x} \]

\[ k_f = \frac{1}{f} = \frac{k}{\mu_f} \quad \text{(Darcy’s law)} \]
The continuum sand production model
Related publications


The continuum erosion model

This is based on a 3 phase continuum mixtures theory. If compared to ordinary poro-elasto-plasticity, this theory introduces two additional unknowns:

• the concentration of the fluidized solids in the pore-fluid

• The velocity of the fluidized solids
Background

The sand erosion model was originally proposed in the mid 90’s (Vardoulakis et al. 1995). It has been applied since to volumetric sand production prediction by many researchers worldwide, who are working in this field.

During the elapsed decade new and efficient, coupled F.E. codes are proposed to model sand production.

The open questions refer to the appropriate constitutive model of sand production.
The 3 phase continuum model

\[ v_i^{(1)} = 0 \]

\[ v_i^{(3)} = \chi v_i^{(2)} \quad (0 < \chi < 1) \]

\[ dV_v = dV_f + dV_{ts} \]

\[ \phi = \frac{dV_v}{dV} \]

\[ c = \frac{dV_{ts}}{dV_f + dV_{ts}} \]
Definitions

Porosity:

\[ \phi = \frac{dV_v}{dV} \]

Fluidized-particles concentration:

\[ c = \frac{dV_{fs}}{dV_{ff} + dV_{fs}} \]

\[ dV_v = dV_{ff} + dV_{fs} \]
Partial densities

\[ \rho_1 = (1 - \phi) \rho_s \]

\[ \rho_2 = (1 - c) \phi \rho_f \]

\[ \rho_3 = c \phi \rho_s \]

Density of the “mixture”

\[ \rho = \sum_{\alpha=1}^{3} \rho_{\alpha} = (1 - \phi) \rho_s + \phi((1 - c) \rho_f + c \rho_s) \]
We introduce partial velocities for each one of the three phases,

\[ v_i^{(\alpha)} \quad \alpha = 1,2,3 \]

In particular we will assume here that the velocity of the solid-skeleton particles is small as compared to the velocity of the fluid

\[ |v_i^{(1)}| << |v_i^{(2)}| \quad v_i^{(1)} = 0 \]

The velocity of the fluidized particles is in general a fraction of the velocity of the carrier fluid.
Mass- and volume fluxes

\[ \dot{m}_i^{(2)} = \rho_2 v_i^{(2)} = (1 - c)\phi \rho_f v_i^{(2)} = \rho_f q_i^{(2)} \quad , \quad q_i^{(2)} = (1 - c)\phi v_i^{(2)} \]

\[ \dot{m}_i^{(3)} = \rho_3 v_i^{(3)} = c\phi \rho_s v_i^{(3)} = \rho_s q_i^{(3)} \quad , \quad q_i^{(3)} = c\phi v_i^{(3)} \]

\[ q_i = q_i^{(2)} + q_i^{(3)} = \phi \left( (1 - c)v_i^{(2)} + cv_i^{(3)} \right) \]
Partial stresses

$$\sigma_{ij}^{(2)} = -(1 - c) \phi p \delta_{ij}$$

$$\sigma_{ij}^{(3)} = -c \phi p \delta_{ij}$$
Mass balance for the **solid phase**

\[
\frac{\partial}{\partial t} \rho^{(1)} + \text{div}(\rho^{(1)} v^{(1)}_i) = j^{(1)}
\]

rigidity \( v^{(1)}_i = 0 \)

\[-\frac{\partial}{\partial t} ((1 + c)\phi) = \frac{j^{(1)}}{\rho_s}\]
Mass balance for the **fluid phase**

\[
\frac{\partial}{\partial t} \rho^{(2)} + \text{div}(\rho^{(2)} \mathbf{v}_i^{(2)}) = 0 \quad , \quad \rho_f = \text{const.} \quad \Rightarrow
\]

\[
\frac{\partial \phi}{\partial t} + \text{div}(q_i) = 0
\]
Mass balance for the **fluidized solid phase**

\[
\frac{\partial}{\partial t} \rho^{(3)} + \text{div}(\rho^{(3)} v^{(3)}_i) = j^{(3)} , \quad \rho_s = \text{const.} \quad \Rightarrow
\]

\[
\frac{\partial}{\partial t} (c\phi) + \text{div}(c\phi v^{(3)}_i) = \frac{j^{(3)}}{\rho_s}
\]

Since all mass eroded stems from the solid skeleton, we assume that

\[
 j^{(3)} = - j^{(1)}
\]
All porosity changes are due to erosion:

\[ \frac{\partial \phi}{\partial t} = \frac{\dot{m}}{\rho_s} \]

Transport equation:

\[ \frac{\partial \phi}{\partial t} = \frac{\partial}{\partial x_i} (cq_i) \]

Continuity of flow

\[ \frac{\partial}{\partial x_i} ((1 + c)q_i) = 0 \]
The “Einstein-Sakthivadivel” erosion model (Vardoulakis et al. 1995)

The rate of eroded mass is driven by the discharge of the transported solids with respect to the stationary solids:

\[ \dot{m} = \Lambda \rho_s (1 - \varphi) c \varphi |v_i^{(3)} - v_i^{(1)}| \]

\[ v_i^{(3)} = \chi v_i^{(2)} \quad (0 < \chi < 1) \]

Den Adel & Bakker (1993) estimated that in filtration processes \( \chi = 0.5 \)
Modified models

\[ \dot{m} = \Lambda \rho_s (1 - \varphi) c \varphi |v_i^{(3)} - v_i^{(1)}| \]

\[ \Lambda' = \Lambda \rho_s \approx \text{const.} \]

Concerning the value of the parameter $\tilde{\Lambda}$ we remark that its inverse product with the characteristic velocity of the transported particles can be replaced by the characteristic time that governs such a process. The relatively large value of this characteristic time is reflecting the low probability that has a fluidized particle to escape from its position, because it is inhibited by the stationary solids and because during its jittery motion it does not find often a large enough pore which will allow it to escape.

$$\dot{m} \approx \rho_s \tilde{\Lambda} \frac{<q - q_{cr}>^2}{\int_0^t \int_0^t <q - q_{cr}>^2 d\tau}$$
Related publications


The volumetric sand-production experiment (cavity test) is run on hollow cylinder specimens of sandstone. The specimen is characterized by internal (cavity) radius $r$, the external radius $R_{ext}$ and its height $h$.

**Discharges monitored with time:**

a) **the total gas out-flow** $Q_g$ in [lt/min],

\[ q_{R_i} = \frac{Q_g}{A} \quad \text{in} \quad [m/\text{sec}] \quad \text{(specific gas discharge)} \]

b) **the cumulative sand-mass, produced in the cavity**, $M_s$ in [g],

\[ m_s = \frac{M_s}{A} \quad \text{in} \quad [\text{gr/m}^2] \quad \text{(specific mass discharge)} \]
mass balance \( \frac{\partial \phi}{\partial t} = \text{div} q_{fs} \)
The volumetric sand-production experiment (SINTEF cavity test) is run on hollow cylinder specimens of sandstone.
The cavity test

X-ray CT scans of tested Synthetic sandstone specimen showing a vertical (left) and a horizontal (right) cross section
Cumulative sand over time produced under constant external stress at different fluid fluxes at the cavity during a volumetric sand production test on a Synthetic sandstone.

FE simulations

The hydro mechanical sand production model, proposed by Vardoulakis, was investigated to predict transient solid mass flow rate of debris transported to wellbore.

OTRC PROJECT STATUS REPORT, Serguei Jourine (http://otrc.tamu.edu/Pages/wellblowout.htm)

Fig. 1. Hollow cylinder failure related to wellbore stability within soft weak layer bounded by strong rigid layers
a) Calculated failure criterion for homogeneous rock: green – high failure probability, red – low failure probability, b) Density (X-Ray cross-section) of tested sample (sandstone).
1- zones with highest failure probability at the hole wall, 2 - zones with highest failure probability at the contact, 3 – circular failure at the wall with fine debris, 4 – secondary fractures, 5 - potential coarse debris (larger than hole diameter), 6 – hole.
Hence, a continuum mechanics approach is still a powerful alternative modelling framework in which mass balance is applied to a three-phase system comprised of solid, fluid and fluidized solid within homogenization mixture theory, as was proposed by Vardoulakis et al., 1996 [1]. This approach results in solving a set of coupled non-linear time-dependent equations with fluidized solid concentration, fluid pressure and porosity as main variables.
Fig. 8. Mobilized sand concentration at selected times: (a) $t=0.07$ min; (b) $t=0.1$ min; (c) $t=0.1$ min (3D representation).
Erosion of material (sand production) from oil bore hole perforation tunnel (Abaqus)


\[ V_e = \lambda (1 - n) c v_w, \]

Total volume of the sand produced in m³ after four days.

Shape of the perforation tunnel after four days of erosion.
1.1.21 Erosion of material (sand production) from oil bore hole perforation tunnel

**Product:** ABAQUS/Standard

This example demonstrates the use of adaptive meshing and adaptive mesh constraints in ABAQUS/Standard to model the large-scale erosion of material such as sand production in an oil well during oil extraction occurring at external surfaces according to local solution-dependent criteria. In ABAQUS/Standard the erosion of material at the external surface is modeled by declaring the surface to be part of an adaptive mesh domain and by prescribing surface mesh motions that recede into the material. ABAQUS/Standard will then remesh the adaptive mesh domain using the same mesh topology but the new location of the surface. All the material point and node point quantities will be advected to their new locations.
3D Bore (E. Papamichos, SINTEF)

**Fig. 2.** FE simulation of onset of borehole failure, serving as conceptual model for Bagnold flow model.
Non axisymmetric erosion patterns: cusping
Slot erosion
DEM Simulations

Fig. 5. Comparison of failure pattern as simulated in PFC2D for different simulated materials, again resembling patterns obtained in the laboratory.
The Porosity diffusion model
(Papamichos & Vardoulakis, 2005)

Based on research done in the previous and the current phase of the project, we have proposed a simple volume erosion model that essentially eliminates the unknowns, introduced by the original volume-erosion model:

Assumptions:

1. The fluidized-particle concentration is small

2. The particles are fine enough to share the velocity of the carrier fluid.
Simplified mass balance

\[ \frac{\partial \phi}{\partial t} = \frac{\partial q_i^{(3)}}{\partial x_i} , \quad q_i^{(3)} = c \varphi v_i^{(3)} \quad (0 < c \ll 1) \]
The “nuzzling” force

Piping erosion: \[ q^{(3)}_i = \lambda \frac{\partial \phi}{\partial x_i} \]
The physical meaning of this assumption lies in the recognition of the fact that fluidized particles experience an additional force, which is due to the nuzzling effect of increasing porosity (cf. Papalexandris 2004, Vardoulakis and Alevizos 2005). This nuzzling force acting on the grains is the reaction of the component in flow-direction of the pressure force acting on the boundaries of an opening flow-tube.

\[ v^{(3)} = v^{(2)} \quad \Rightarrow \]

\[ q^{(3)} = \frac{1}{18\pi^2} \frac{\phi^2}{1-\phi} \frac{D_g^2\rho_f}{\mu_f} \frac{\partial \phi}{\partial x} \]
• The piping erosion law enforces the flow lines of the eroded particles to follow the porosity gradient.

\[ q_{i}^{(3)} = \lambda \frac{\partial \phi}{\partial x_i} \]

• Combination of continuity- and piping erosion eq. yields a **porosity diffusion law** and to a sqrt-time sand-production equation:

\[ \frac{\partial \phi}{\partial t} = \frac{\partial q_{i}^{(3)}}{\partial x_i} \]

\[ \frac{\partial \phi}{\partial t} = \lambda \frac{\partial \phi}{\partial x_i} \left( \frac{\partial \phi}{\partial x_i} \right) \Rightarrow m_{\text{sand}} \propto \sqrt{t} \]

Cavity test simulation results for the porosity, based on a porosity diffusion model (Papamichos & Vardoulakis, 2005)

Piping erosion law which enforces the flow lines of the eroded particles to follow the porosity gradient.

\[ q^{(3)}_i = \lambda \nabla_i \phi \]

which for small particle concentrations results in a porosity diffusion law

\[ \frac{\partial \phi}{\partial t} = \nabla (\lambda \nabla \phi) \]
Hydrodynamically “unstable” granular media: Piping erosion


Internal mobility in bimodal granular media

\[ d_p,\text{min} = (\sqrt{2} - 1) \quad D = 0.4142 \quad D \]
\[ D/d_p,\text{min} = 2.414 \]
\[ n = 0.4767 \]

\[ d_p,\text{min} = \left( \frac{2}{3} \sqrt{3} - 1 \right) \quad D = 0.1547 \quad D \]
\[ D/d_p,\text{min} = 6.464 \]
\[ n = 0.2595 \]
The Tarbela Dam: The largest earth-filled dam in the world (469 feet high and 2,264 feet thick at the base)
An impervious upstream blanket connected to a sloping impervious core was placed during the construction of Tarbela Dam on the Indus River in Pakistan (Lowe 1978). The blanket material consisted of sandy silt mixed with a sandy silt angular boulder gravel. The blanket lay over an alluvium of cobble gravel choked with fine sand. The blanket, which was to increase the length of seepage path and not necessarily to reduce seepage quantities, met the piping criteria, \( D_{15} \) (alluvium) < \( 5D_{85} \) (blanket). As the reservoir emptied after first filling, several sinkholes and cracks were noted in the blanket. Sinkholes ranged from 1 to 15 ft in diameter and 4 to 6 ft in depth. It was felt that uneven settlement during the first reservoir filling caused tension and compression cracks in the blanket which allowed considerable seepage into the underlying sand-choked gravel. In areas where the sand was less dense, the seepage moved the sand down to form a layer in the lower part of the gravel. This created open work gravel just beneath the blanket, and fines from the blanket moved into and through this open layer forming the sink-holes. Sinkholes were filled with filter material and mounded over with blanket material. Typically, the blanket mounds were approximately 15 ft high and extended 30 to 35 ft beyond the sinkhole edge. After filling of the reservoir, sinkholes were located by side-scan sonar and filled with a mixture of filter material and silt from self-propelled bottom dump barges. Each sinkhole generally received 50 barge loads of material. Sinkholes continued to be discovered and covered over another 3-4 years after the initial remedial action. Siltation on the reservoir blanket and filling of sinkholes have reduced seepage about one half.

www.usace.army.mil/inet/usace-docs/ eng-manuals
Erosion equation
\[ \frac{d \phi}{dt} = \Lambda'(1 - \phi)q \]

Darcy's law
\[ q = \frac{k(\phi)}{\mu_f} J \]

Kozeny-Carman
\[ k = \frac{1}{c_1 (1 - \phi)^2} \frac{\phi^3}{D_g^2} \]
\[
\frac{d\phi}{d\tau} = \left(\frac{\phi}{1 - \phi}\right)^2
\]

\[
\tau = C_1 t, \quad C_1 = \frac{1}{c_1} \Lambda' D_{eq}^2 \frac{J}{\mu_f}
\]

\[
\tau = \phi - \phi_0 + \left(\frac{1}{\phi_0} - \frac{1}{\phi}\right) - \ln \left(\frac{\phi}{\phi_0}\right)^2
\]

\[
\tau_F = 1 - \phi_0 + \left(\frac{1}{\phi_0} - 1\right) - \ln \left(\frac{1}{\phi_0}\right)^2
\]
$\Phi$ vs. $(T - T_0)$

$t_{\text{Failure}} = \frac{1}{J}$
segregation piping erosion
column tests on gap-graded soils
(Skempton & Brogan, 1994)
well-graded sands: increasing hydraulic resistivity with flow-rate

\[ r_f \text{ [s/mm]} \]

non-fluidized bed

fluidized bed

\[ q \text{ [mm/s]} \]

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gap-graded sandy gravels:
        decreasing hydraulic resistivity with flow rate

\[ r_f \approx \frac{1}{\sqrt{q_{\text{ref}} q}} \]
Bimodal materials (mixed soils)
piping erosion
(Skempton & Brogan, 1994)

Fig. 8. Sample A test results
We assume that the gravel constitutes a rigid fabric and that the sand is filling only a part of the void space of the gravel-pack. Thus we assume that the sand particles have mobility. If the applied hydraulic gradient is above a threshold value, then, depending on its value, fine particles with sizes less than a critical size become fluidized. During this phase the fine fraction is expanding. During expansion the fines are unable to travel large distances and move far away from their initial positions, since there are still large enough particle sizes locked in place which block their way. In that case the mean velocity of the fluidized particles is zero and their expansion is attributed to rapid velocity fluctuations.
Craters and pipes in fluidized bed
1964, Nigata earthquake, Japan

Postquake sandboils due to in depth loose-sand liquefaction.
The effect of nozzling terms*

\[ f_i = f_i^{(\text{neq})} + f_i^{(\text{eq})} \]

\[ f_i^{(\text{neq})} = -p_w \frac{\partial \phi}{\partial x_i} \]

\[-(1 - \phi) \frac{\partial p_s}{\partial x_k} + (p_s - p_w) \frac{\partial \phi}{\partial x_i} + \rho_1 g_i + \phi f_q = \rho_1 \left( \frac{\partial}{\partial t} v_i^{(1)} + v_k^{(1)} \frac{\partial}{\partial x_k} v_i^{(1)} \right) \]

The dynamic equation for the granular phase loses its conservation character

*Alevizos, Vardoulakis & Steeb, (2005), in preparation*
Numerical simulation of detonations in mixtures of gases and solid particles

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According to the third assumption, the effects of the nozzling terms are ignored. These terms appear in the momentum and energy equation for each phase and they have the form $p_i \nabla \phi_s$ and $p_i u_i \nabla \phi_s$. In these expressions, $p_i$ and $u_i$ represent interfacial pressure and velocity, respectively, and they are constructed in a non-unique manner. Noozling terms are non-conservative. As a result, the evolution of gasdynamic discontinuities does not follow the classical jump conditions. Such terms are included in some two-phase models (Baer & Nunziato 1996; Bdzil et al. 1999; Saurel & Lemetayer 2001) and are excluded in others (Butler & Krier 1986; Powers et al. 1990a).
The appearance of the **nozzling terms** in connection with evidence of **porosity shocks** leads to a serious mathematical problems

\[-(1-\phi) \frac{\partial p_s}{\partial x_k} + (p_s - p_w) \frac{\partial \phi}{\partial x_i} + \rho_1 g_i + \phi f q_i = \rho_1 \left( \frac{\partial}{\partial t} v_i^{(1)} + v_k^{(1)} \frac{\partial}{\partial x_k} v_i^{(1)} \right)\]