Generalized Plasticity modelling of soils: the role of dilatancy

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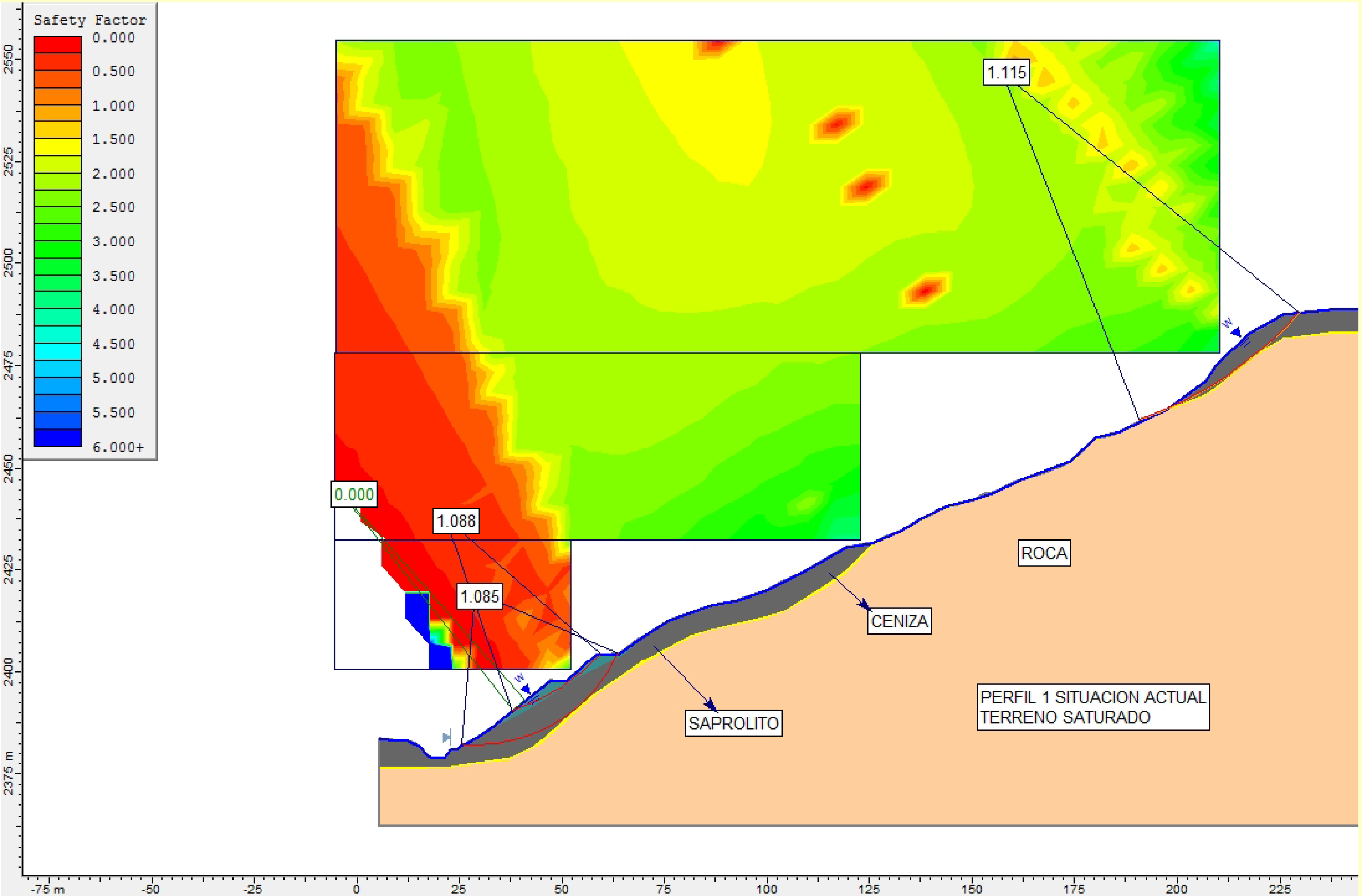
I Introduction

Simplified methods of analysis (slide ©)



Simplified methods of analysis (slide ©)

Assumptions

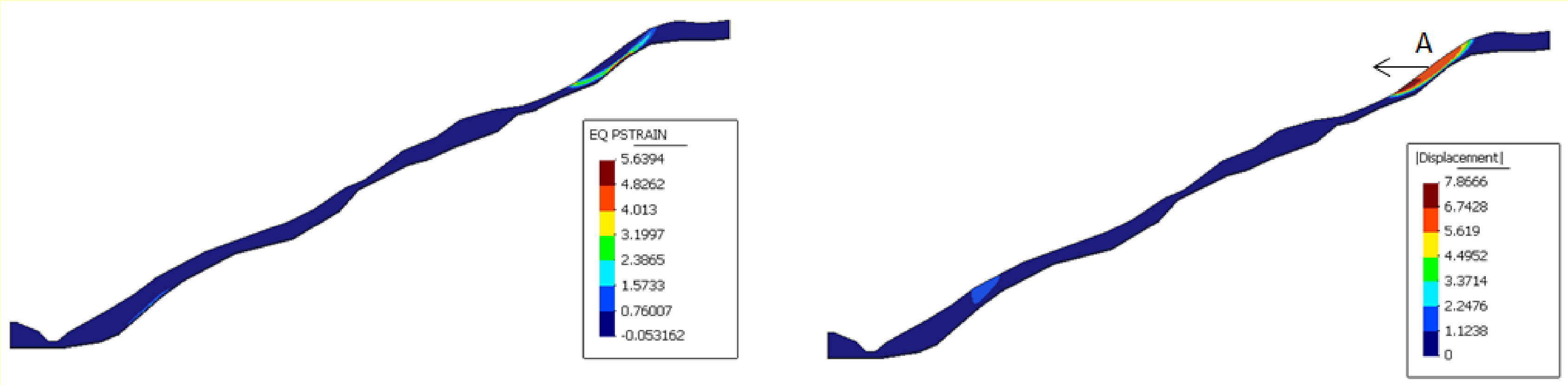
- Failure is of localized type
- Stress at failure surface

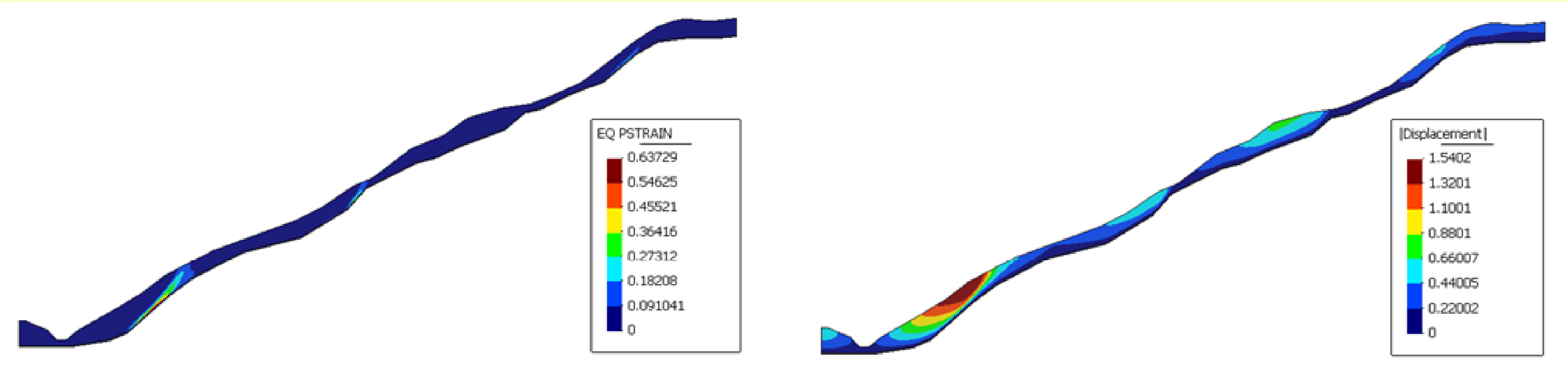
But…

- What if failure is of diffuse type?
- Stress inside failure surface

So?

Finite elements + classical plasticity (Mohr Coulomb)





But…

- Data from Lizcano, Herrera & Santamarina (2007)

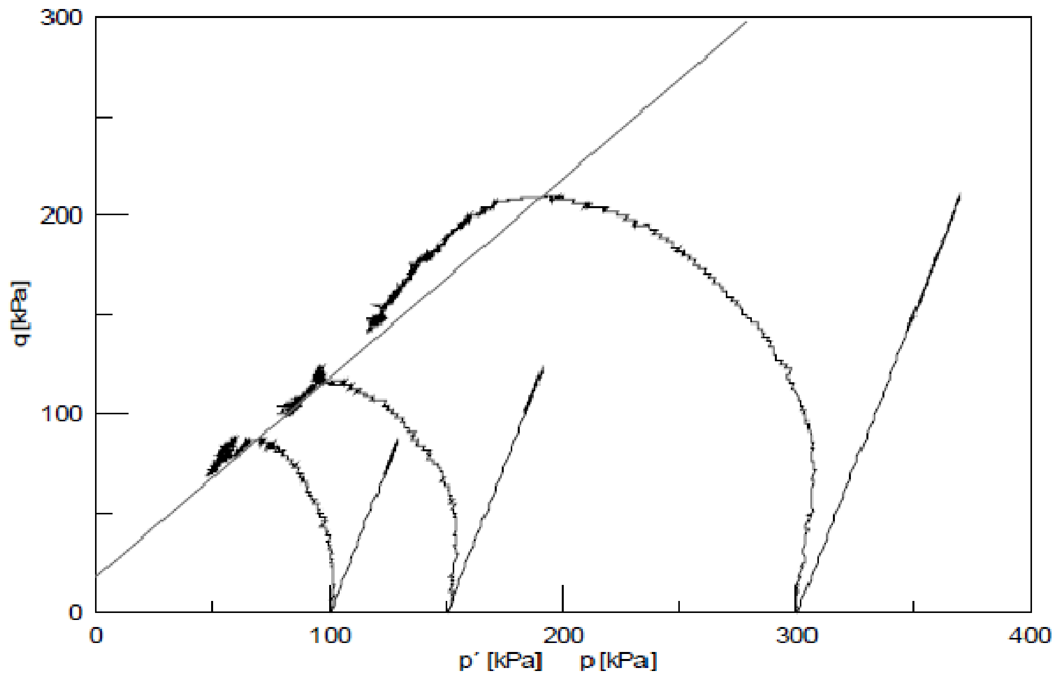
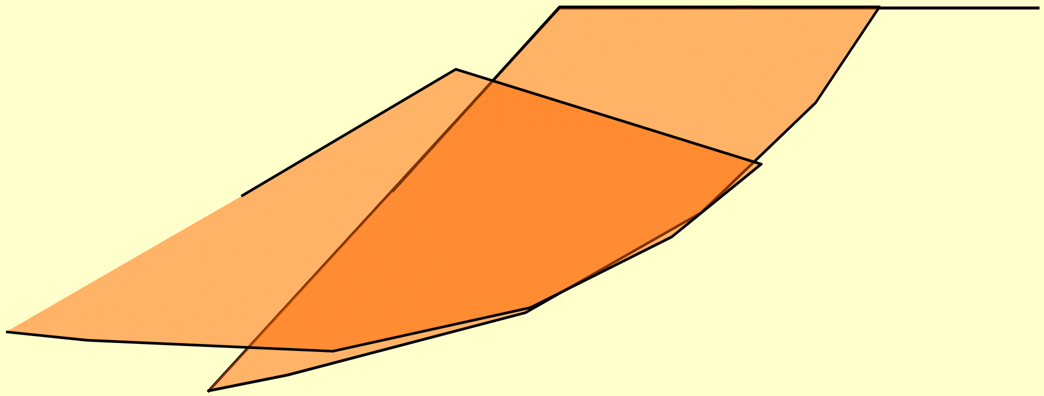


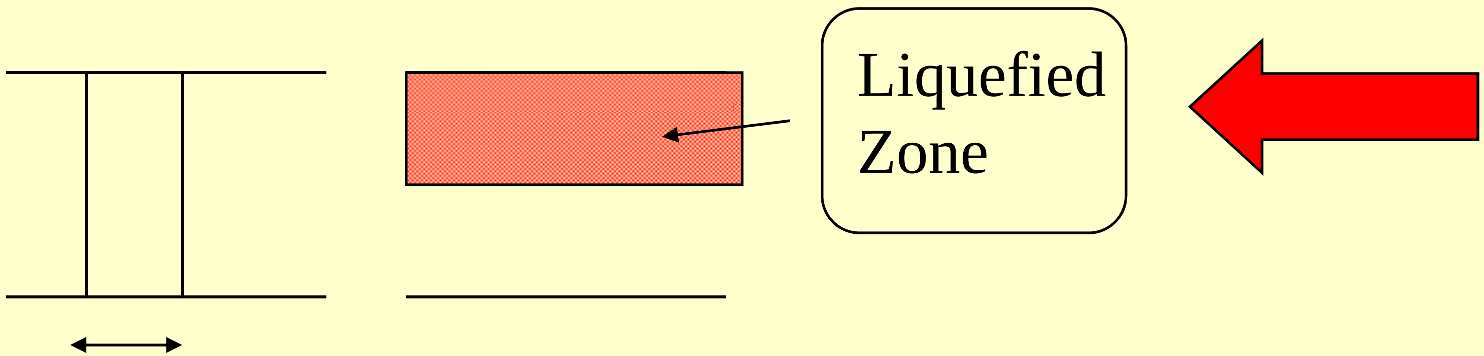
Figura 15: Datos de ensayo de compresión triaxial CU – Muestras saturadas (Localización: Manizales. Profundidad: 7.0 m).

Types of Failure

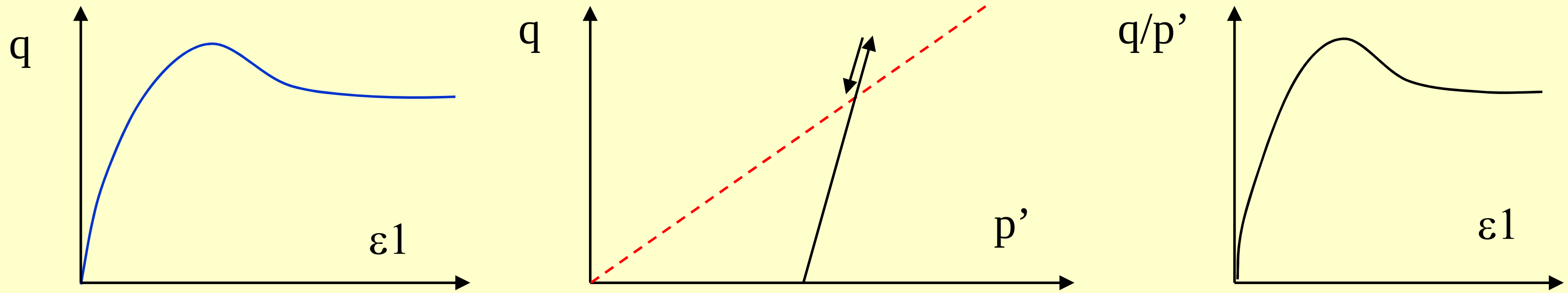
- Slides: Localized



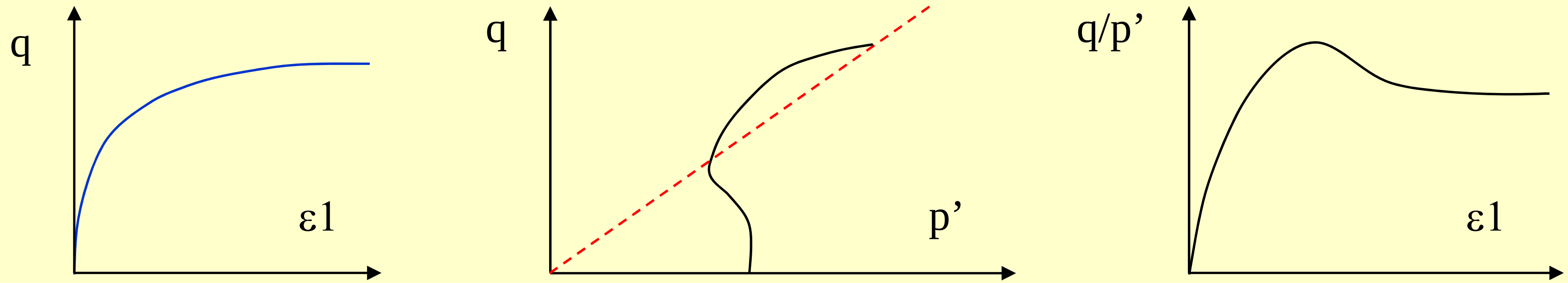
- Diffuse



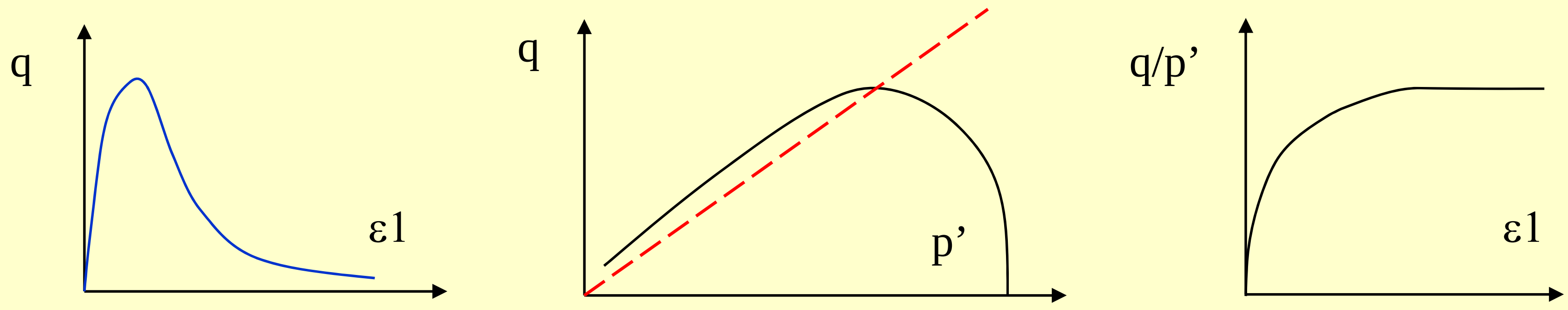
Softening (dense sand, drained)



Softening (OC clay, undrained)



Liquefaction (very loose sand, undrained)



IIa. Constitutive Generalized Plasticity: Intro

Introduction

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Tensor representation | Vector representation

$$\sigma = \begin{pmatrix} \sigma_{xx} & \sigma_{xy} & \sigma_{xz} \\ \sigma_{yx} & \sigma_{yy} & \sigma_{yz} \\ \sigma_{zx} & \sigma_{zy} & \sigma_{zz} \end{pmatrix} \qquad \sigma = \left(\sigma_{xx}, \sigma_{yy}, \sigma_{zz}, \sigma_{xy}, \sigma_{xz}, \sigma_{yz}\right)$$

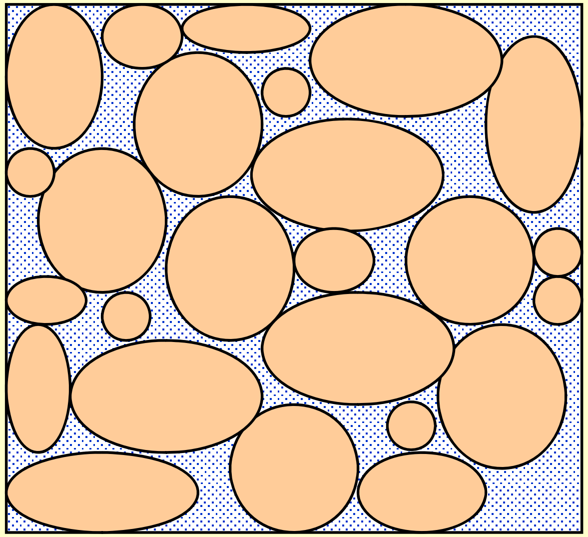
$$I = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \qquad m^T = \left(1\ 1\ 1\ 0\ 0\ 0\right)$$

$$\sigma : d\varepsilon \quad \Rightarrow \quad \sigma^T . d\varepsilon$$

$$D_{ijkl} \quad \Rightarrow \quad D_{ij}$$

$$D_{ijkl} d\varepsilon_{kl} \quad \Rightarrow \quad D_{ij} d\varepsilon_j$$

Effective Stresses



$$\sigma_{ij} = \sigma'_{ij} - p_w \delta_{ij}$$

$$\sigma = \sigma' - p_w I$$

$$\sigma = \sigma' - m\, p_w$$

- Correction

$$\sigma = \sigma'' - \alpha m\, p_w$$

K_S Vol. Stiffness particles

$$\alpha = 1 - \frac{K_T}{K_s}$$

K_T Vol. Stiffness skeleton

$\boxed{\alpha \approx 1 \quad \text{soils}}$

- **Total Stress Tensor**

$$\sigma$$

- **Effective Stress Tensor**

$$\sigma = p_w I + \sigma'$$

p_w Pore Water Pressure

- **Hydrostatic and deviatoric components**

$$\sigma' = p' I + s$$

s Deviatoric stress tensor

p' Effective Hydrostatic stress

Invariants

- Stress tensor σ_{ij}
- Eigenvalues and eigenvectors

$$\sigma . n = \lambda n \qquad \det\left(\sigma - \lambda I\right) = 0$$

- Characteristic Polynomial

$$\lambda^3 - P_1 \lambda^2 + P_2 \lambda - P_3 = 0 \qquad \sigma^3 - P_1 \sigma^2 + P_2 \sigma - P_3 = 0$$

$$P_1 = tr\left(\sigma_{ij}\right)$$

$$P_2 = \sigma_{xx}\sigma_{yy} + \sigma_{yy}\sigma_{zz} + \sigma_{zz}\sigma_{xx} - \sigma_{xy}{}^2 - \sigma_{yz}{}^2 - \sigma_{zx}{}^2$$

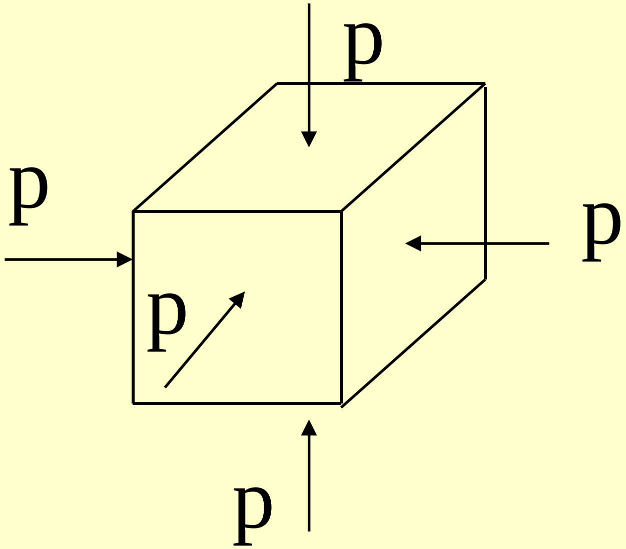
$$P_3 = \det\left(\sigma_{ij}\right)$$

P_1, P_2, P_3 Do not depend on reference system

- **Alternative I**

$$I_1 = tr(\sigma) \quad I_2 = \frac{1}{2} tr(\sigma^2) \quad I_3 = \frac{1}{3} tr(\sigma^3)$$

$$P_1 = I_1 \qquad P_2 = \frac{1}{2}\left(I_1^{\,2} - 2I_2\right) \quad P_3 = \frac{1}{6} I_1^{\,3} - I_1 I_2 + I_3$$



$$\sigma = \begin{pmatrix} p & 0 & 0 \\ 0 & p & 0 \\ 0 & 0 & p \end{pmatrix} \qquad I_1 = 3p$$

Hydrostatic and Deviatoric Components

$$\sigma = s + pI \qquad \sigma_{ij} = s_{ij} + p\delta_{ij}$$

- **Alternative II**

$$J_1 = tr(s) = 0 \quad J_2 = \frac{1}{2}tr\left(s^2\right) \quad J_3 = \frac{1}{3}tr\left(s^3\right)$$

$$I_1 = tr(s + pI) = tr(s) + 3p = 3p$$

$$J_2 = \frac{1}{6}\left\{\left(\sigma_{xx} - \sigma_{yy}\right)^2 + \left(\sigma_{yy} - \sigma_{zz}\right)^2 + \left(\sigma_{zz} - \sigma_{xx}\right)^2\right\}$$

$$+ \ \sigma^2{}_{xy} + \sigma^2{}_{yz} + \sigma^2{}_{zx}$$

$$J_2 = \frac{1}{6}\left\{\left(\sigma_1 - \sigma_2\right)^2 + \left(\sigma_2 - \sigma_3\right)^2 + \left(\sigma_3 - \sigma_1\right)^2\right\}$$

- **Uniaxial test**

$$J_2 = \frac{1}{6}\left\{(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2\right\}$$

$$= \frac{1}{6}\left\{(\sigma_1 - 0)^2 + (0 - 0)^2 + (0 - \sigma_1)^2\right\} = \frac{1}{3}\sigma_1{}^2$$

$$\sigma_1 = \sqrt{3J_2}$$

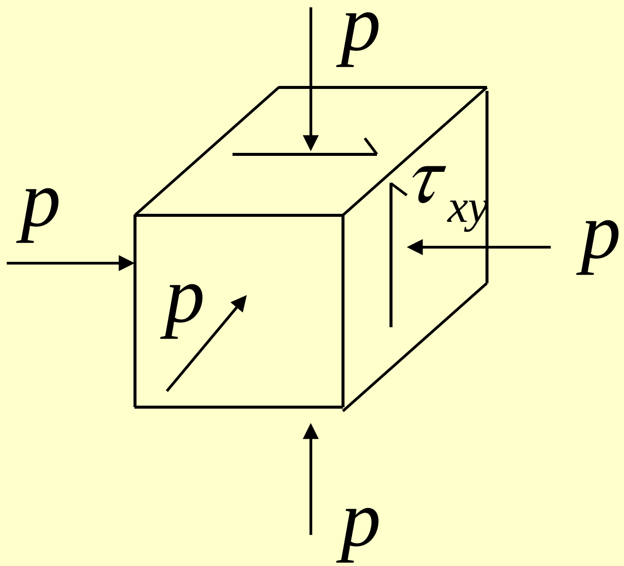
- **Triaxial test**

$$J_2 = \frac{1}{6}\left\{(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2\right\}$$

$$= \frac{1}{6}\left\{(\sigma_1 - \sigma_3)^2 + (\sigma_3 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2\right\}$$

$$= \frac{1}{3}(\sigma_1 - \sigma_3)^2 \Rightarrow \boxed{\begin{aligned}(\sigma_1 - \sigma_3) &= \sqrt{3J_2} \\ &= q\end{aligned}}$$

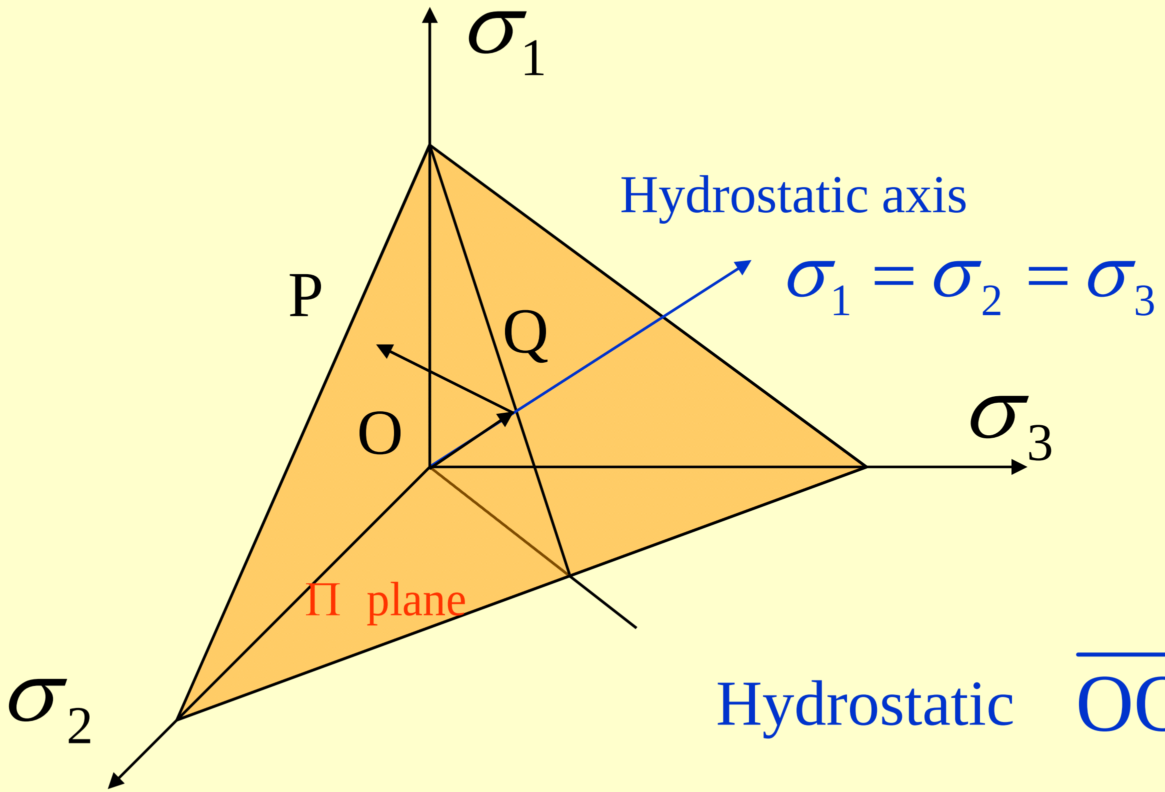
- Shear



$$J_2 = \frac{1}{6}\left\{\left(\sigma_{xx} - \sigma_{yy}\right)^2 + \left(\sigma_{yy} - \sigma_{zz}\right)^2 + \left(\sigma_{zz} - \sigma_{xx}\right)^2\right\}$$

$$+ \ \sigma^2{}_{xy} + \sigma^2{}_{yz} + \sigma^2{}_{zx}$$

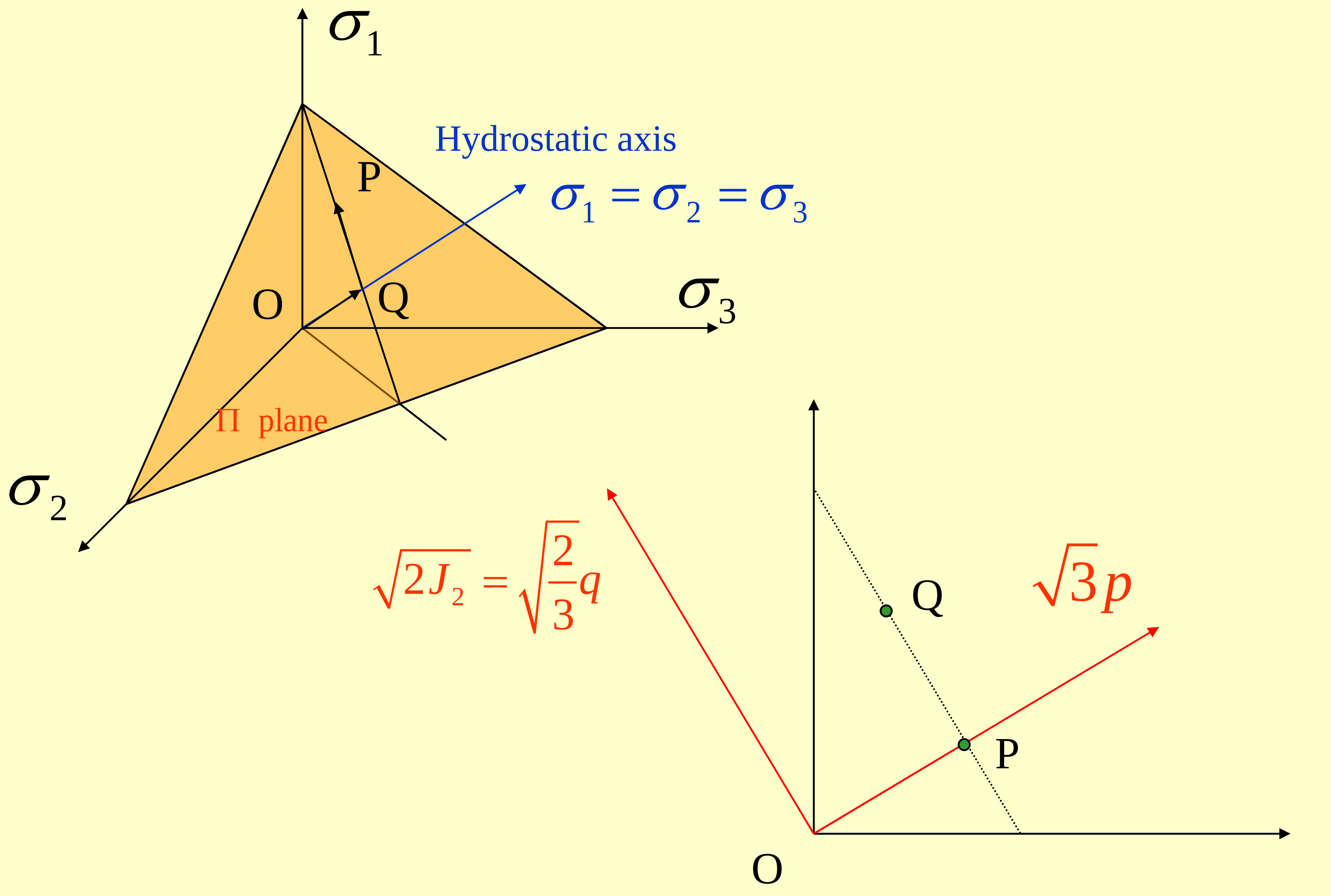
$$\boxed{J_2 = \frac{1}{6}\tau_{xy}{}^2}$$



Hydrostatic $\overline{\mathrm{OQ}} = \dfrac{I_1}{\sqrt{3}} = \sqrt{3}\,p$

Deviatoric $\overline{\mathrm{QP}} = \left(s^T.s\right)^{1/2} = \sqrt{2J_2}$

$$\sqrt{2J_2} = \sqrt{\frac{2}{3}}\sqrt{3J_2} = \sqrt{\frac{2}{3}}q$$



σ_1
Hydrostatic axis
P
$\sigma_1 = \sigma_2 = \sigma_3$
O
Q
σ_3
Π plane
σ_2
$\sqrt{2J_2} = \sqrt{\frac{2}{3}}q$
Q
$\sqrt{3}p$
P
O

Plane p'-q

$$p' = \frac{1}{3}\mathrm{tr}\left(\sigma\right) \Rightarrow \left(\sigma'_1 - \sigma_3'\right)$$

$$q = \sqrt{3J_2'}$$

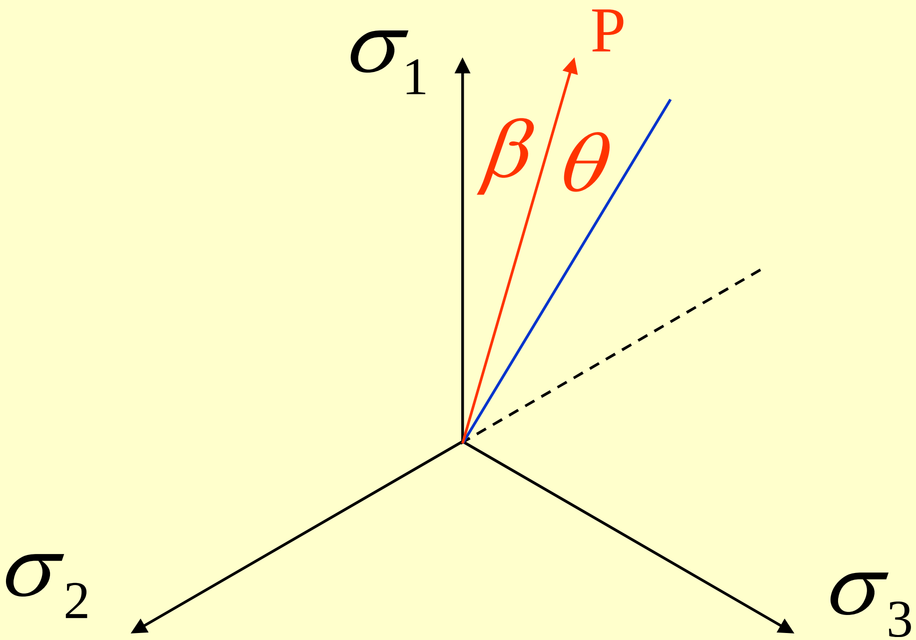
Work $$\delta W = \sigma' : d\varepsilon = p' d\varepsilon_v + q d\varepsilon_s$$

$$\varepsilon_v = \mathrm{tr}\left(\varepsilon\right) \Rightarrow \left(\varepsilon_1 + 2\varepsilon_3\right)$$

$$\varepsilon_s = \frac{2}{3}\sqrt{3J'_{2\varepsilon}} \Rightarrow \frac{2}{3}\left(\varepsilon_1 - \varepsilon_3\right)$$

Represented on same plane

Lode's Angle



$$\cos 3\beta = \frac{3\sqrt{3}}{2}\frac{J_3}{J_2^{3/2}}$$

Failure surfaces

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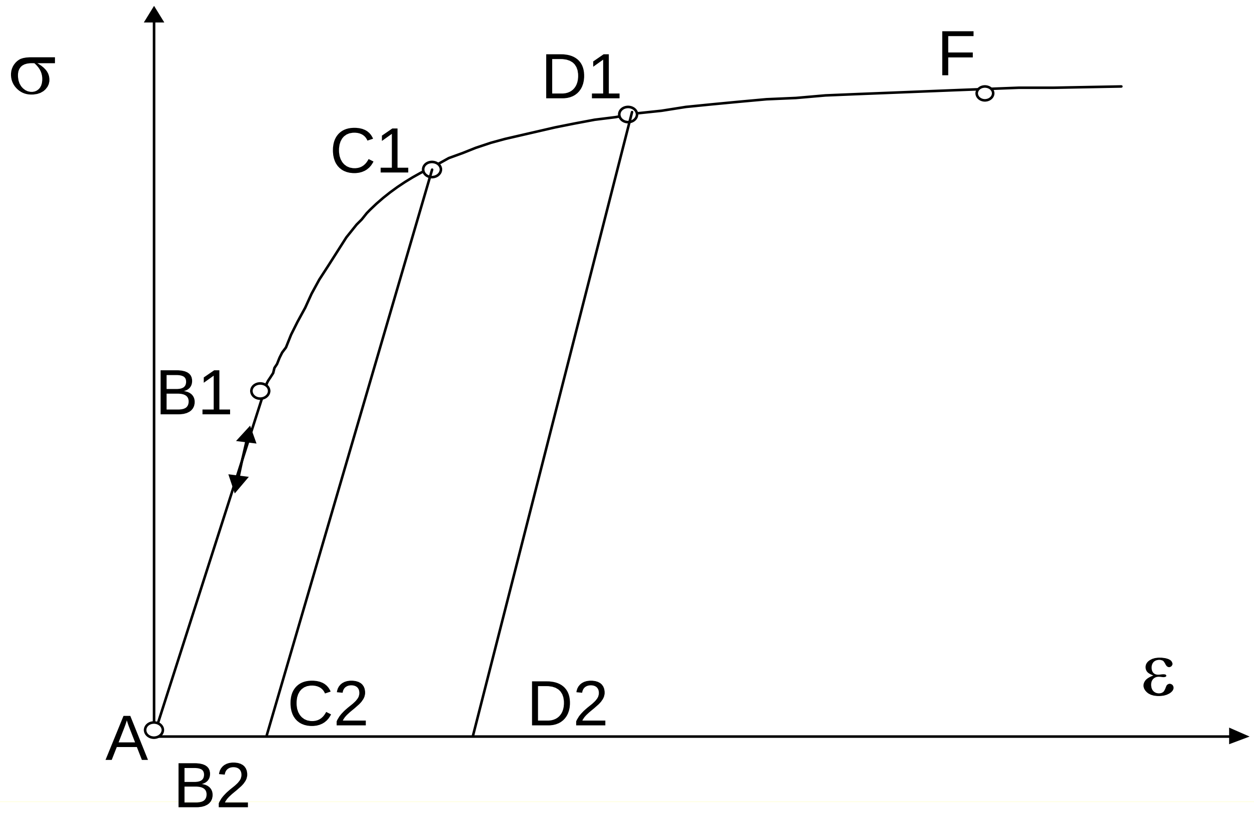
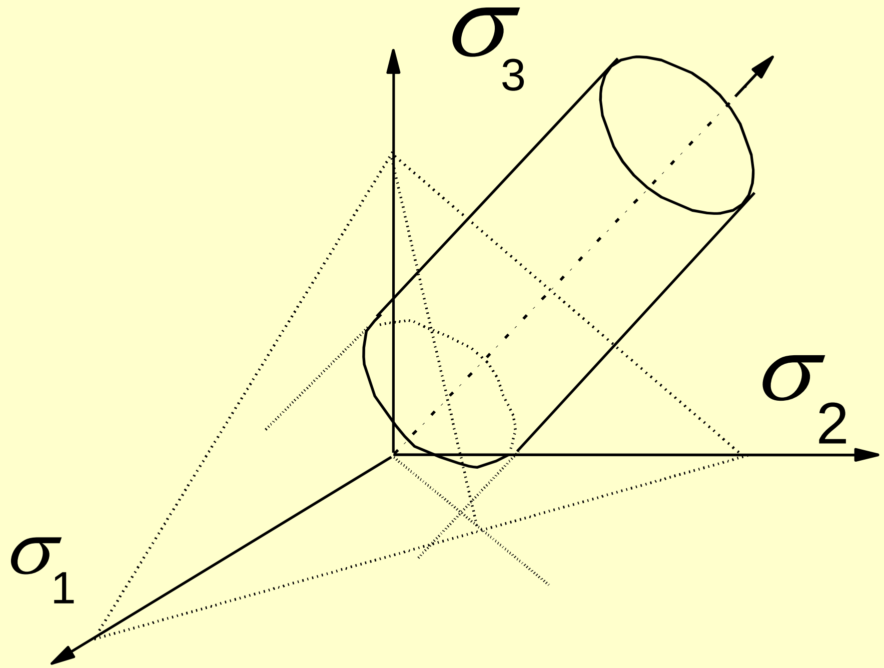
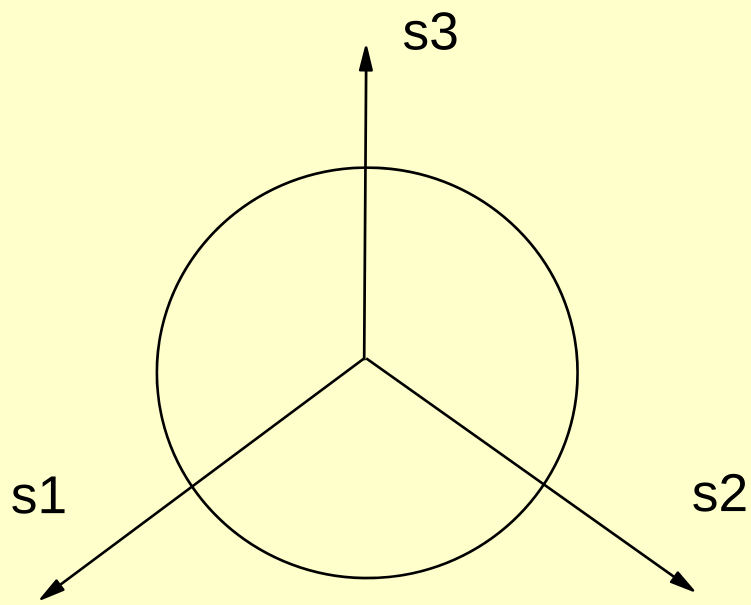
Uniaxial Behaviour



- Yield Stress: B1
- Yield Surface: B1 C1 D1 F
- Failure Surface: F

Von Mises Surface





Failure Surface

$$F\left(\sigma_{ij}\right)=0$$

- **Example: Von Mises**

$$F \equiv \left(\sigma_{xx}-\sigma_{yy}\right)^2+\left(\sigma_{yy}-\sigma_{zz}\right)^2+\left(\sigma_{zz}-\sigma_{xx}\right)^2$$

$$+\,6\sigma^2{}_{xy}+6\sigma^2{}_{yz}+6\sigma^2{}_{zx}-6Y^2=0$$

- **Isotropic Materials**

$$F\left(\sigma_{ij}\right)=0 \rightarrow F\left(\sigma_1,\sigma_2,\sigma_3\right)=0 \quad \rightarrow F\left(I_1,J_2,J_3\right)=0$$

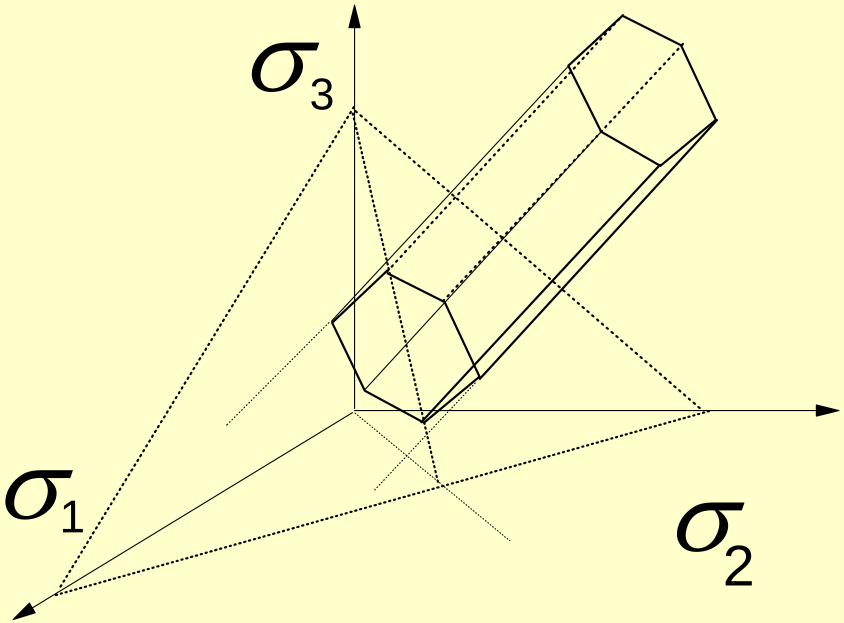
$$F \equiv \left(\sigma_1-\sigma_2\right)^2+\left(\sigma_2-\sigma_3\right)^2+\left(\sigma_3-\sigma_1\right)^2$$

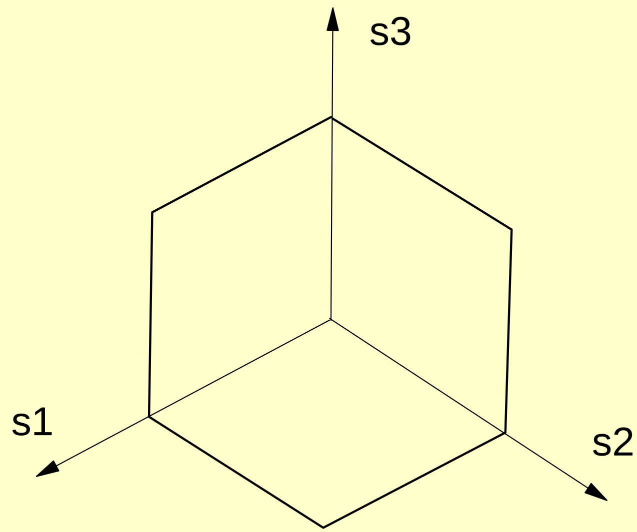
$$-6Y^2$$

$$F \equiv J_2-Y^2=0$$

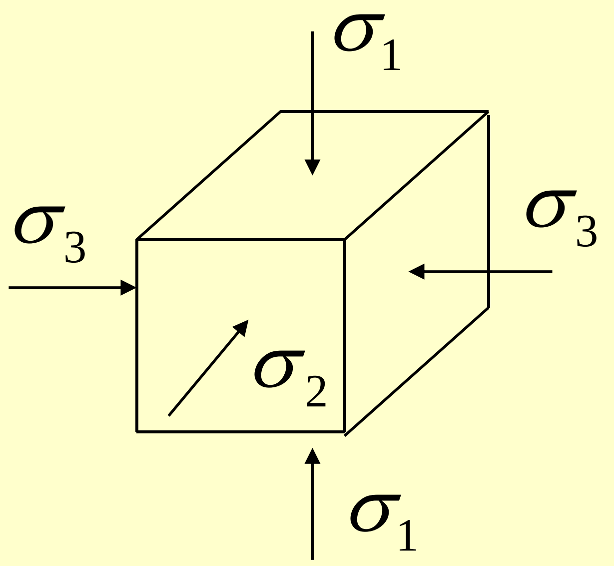
Several Alternative Sets

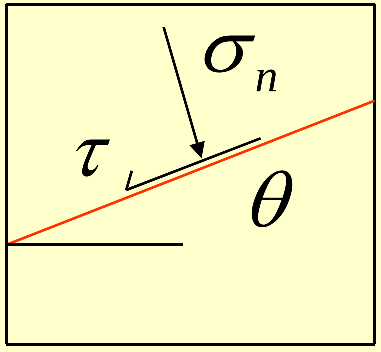
Tresca Surface

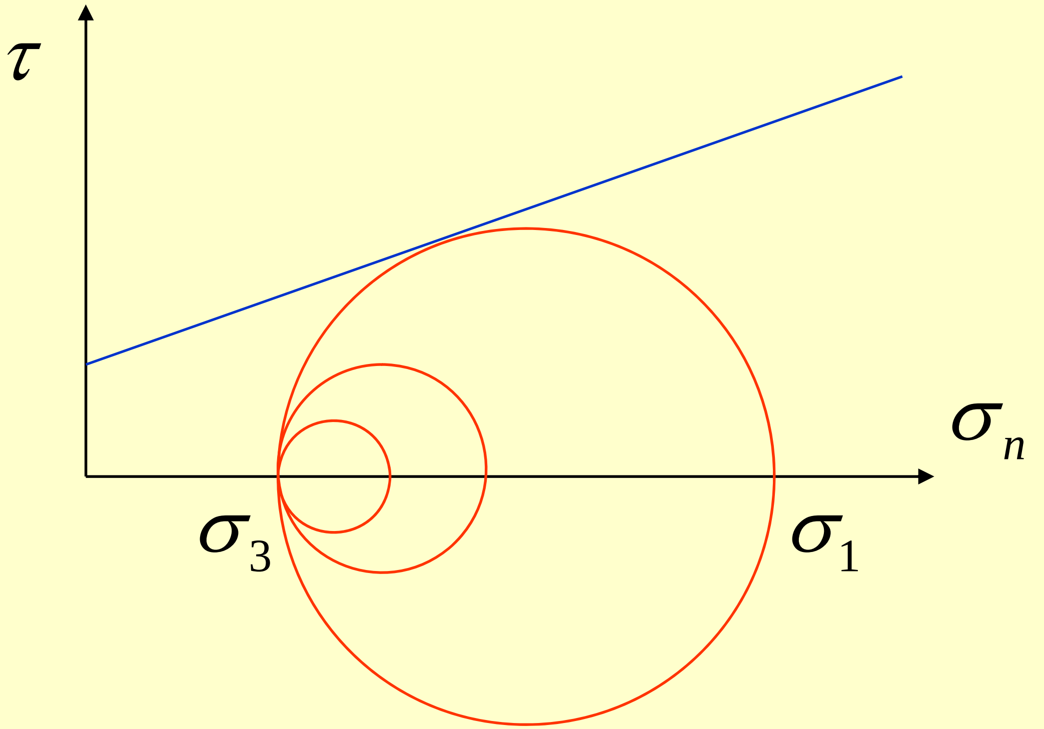




Mohr-Coulomb

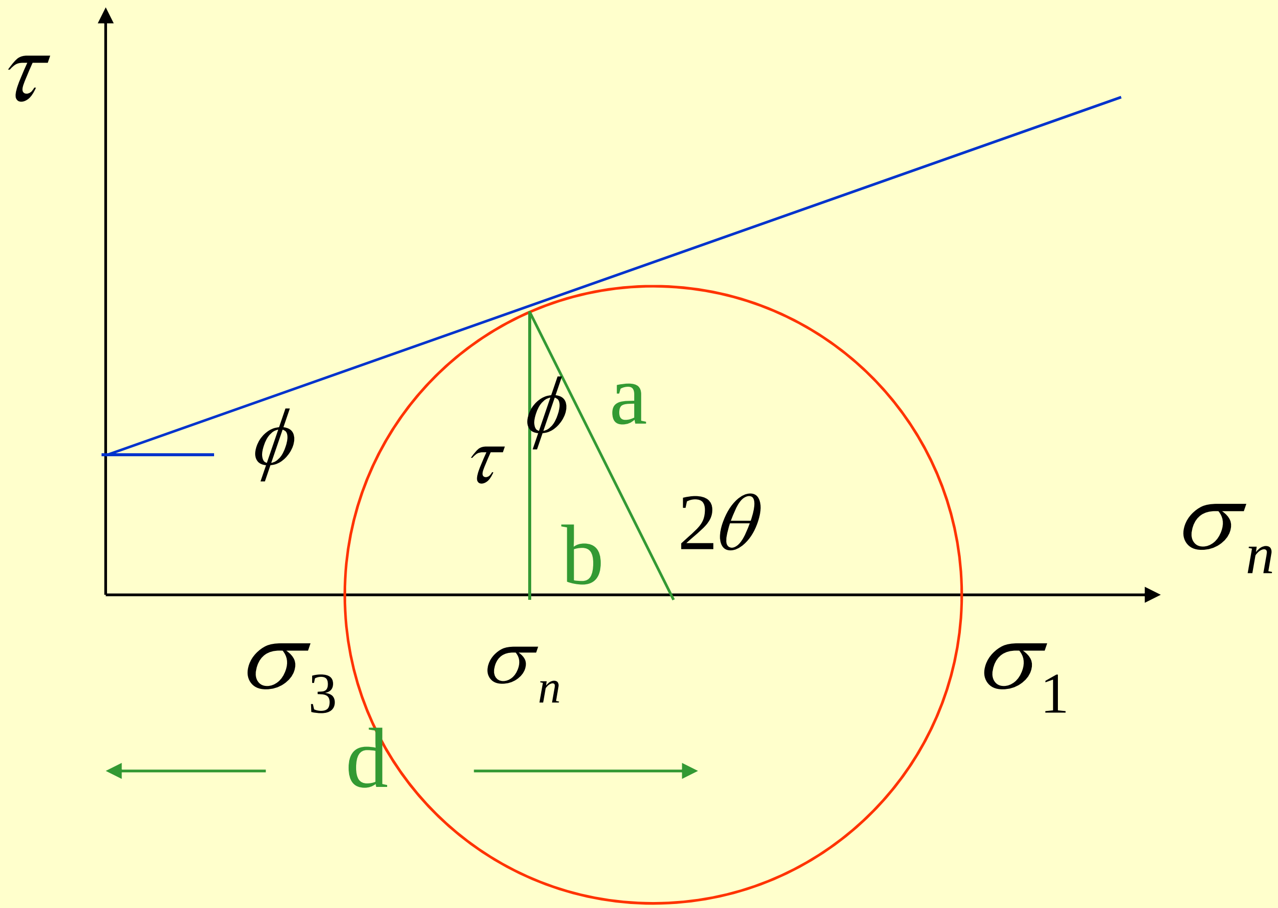






$$\tau = c + \sigma_n \tan\phi$$

$$\tau = c + \sigma_n \tan\phi$$



$$a = \frac{\sigma_1 - \sigma_3}{2}$$

$$b = \frac{\sigma_1 - \sigma_3}{2}\sin\phi$$

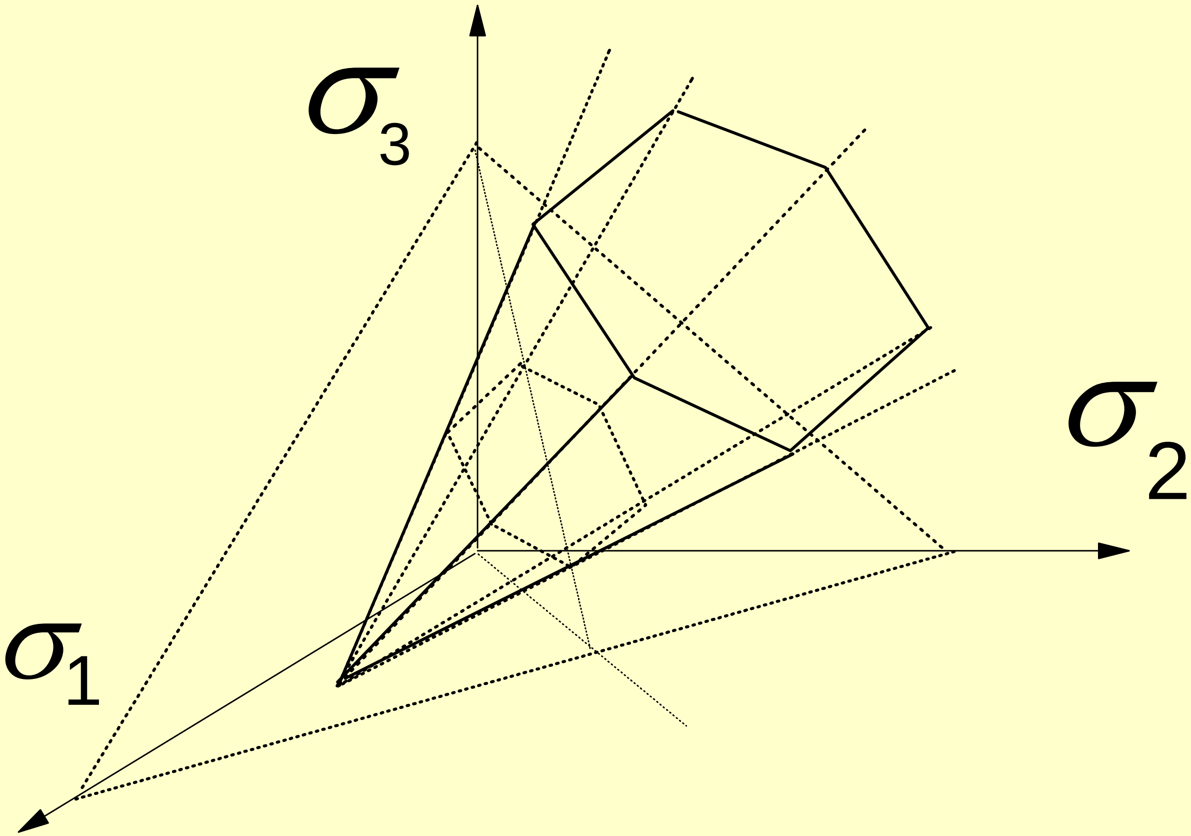
$$d = \frac{\sigma_1 + \sigma_3}{2}$$

$$\tau = \frac{\sigma_1 - \sigma_3}{2}\cos\phi$$

$$\sigma_n = \left(\frac{\sigma_1 + \sigma_3}{2} - \frac{\sigma_1 - \sigma_3}{2}\sin\phi\right)$$

$$\frac{\sigma_1 - \sigma_3}{2}\cos\phi = c + \left(\frac{\sigma_1 + \sigma_3}{2} - \frac{\sigma_1 - \sigma_3}{2}\sin\phi\right)\tan\phi$$

$$\boxed{\left(\sigma_1 - \sigma_3\right) = 2c\cos\phi + \left(\sigma_1 + \sigma_3\right)\sin\phi}$$

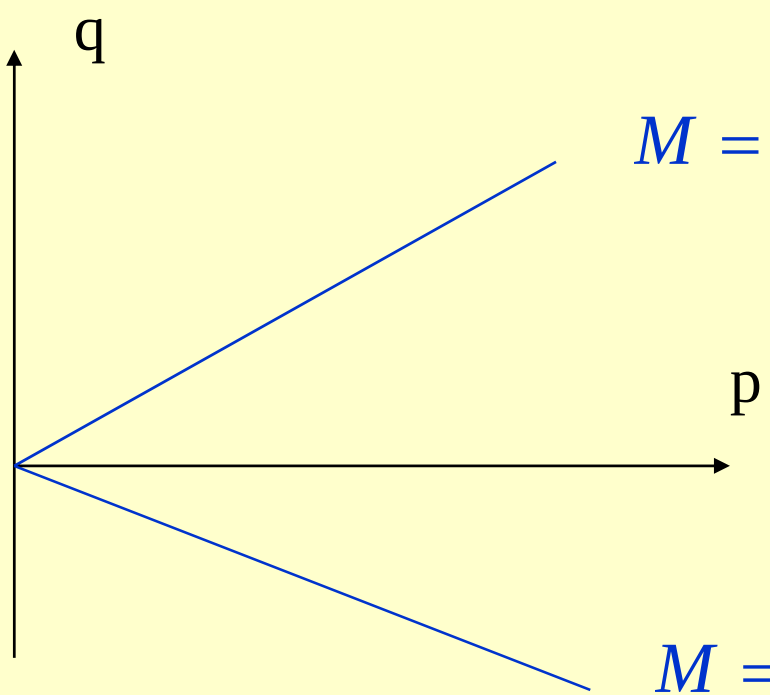


$$(\sigma_1 - \sigma_3) = 2c\cos\phi + (\sigma_1 + \sigma_3)\sin\phi$$

$$p = \frac{1}{3}(\sigma_1 + 2\sigma_3) \qquad q = \sigma_1 - \sigma_3$$

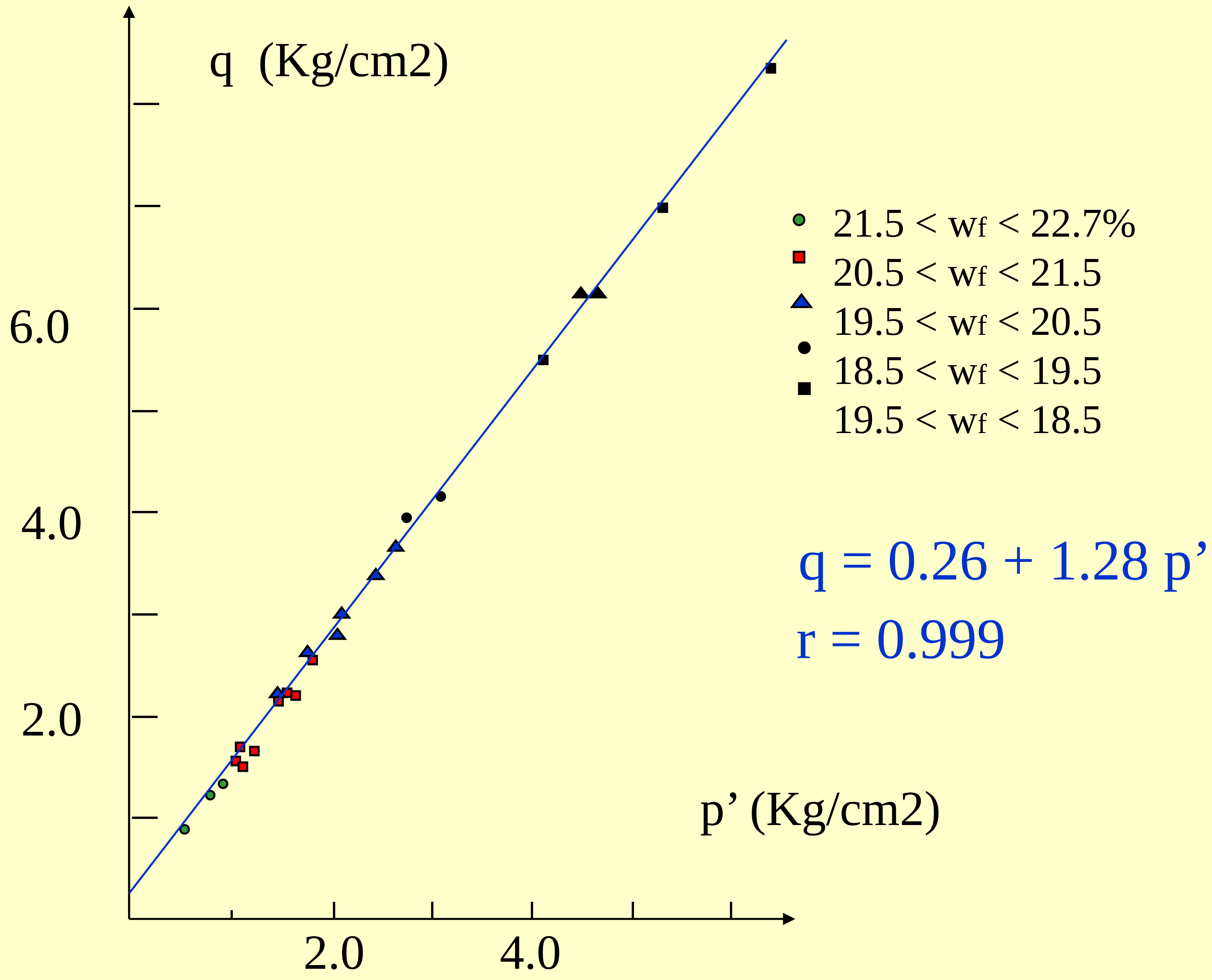
$$q = \frac{6c\cos\phi}{3-\sin\phi} + \frac{6\sin\phi}{3-\sin\phi}\,p$$

$$\boxed{q = c^* + Mp}$$



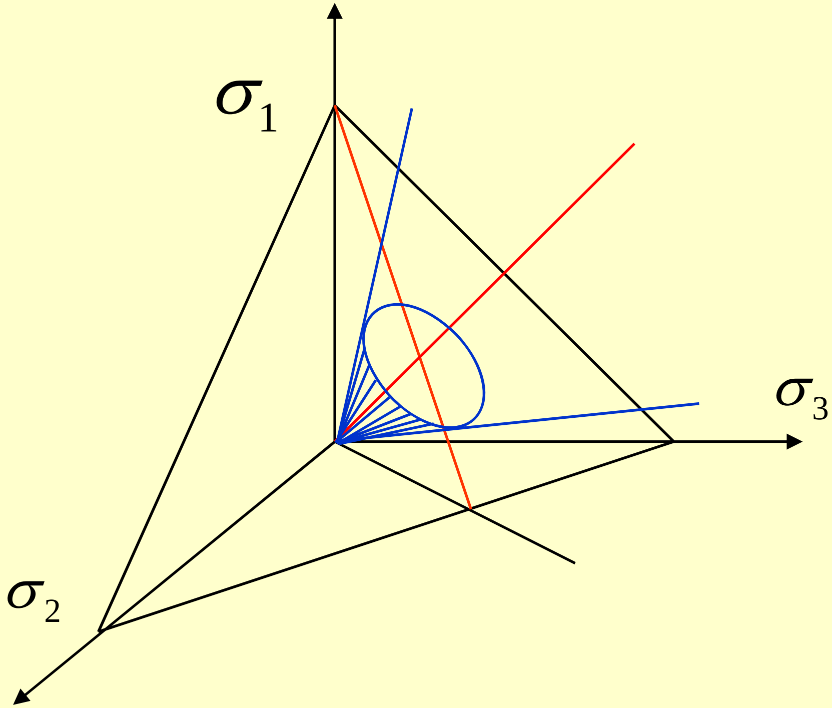
$$M = \frac{6\sin\phi}{3-\sin\phi}$$

$$M = \frac{6\sin\phi}{3+\sin\phi}$$

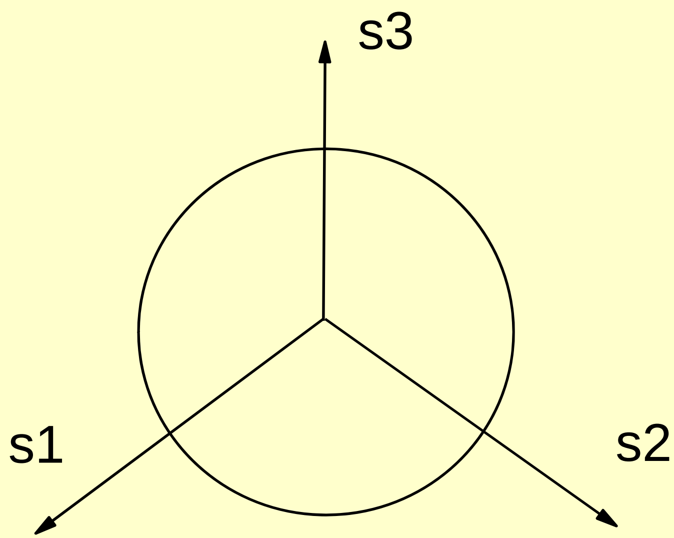


q (Kg/cm2)
6.0
4.0
2.0
2.0
4.0
p’ (Kg/cm2)
21.5 < wf < 22.7%
20.5 < wf < 21.5
19.5 < wf < 20.5
18.5 < wf < 19.5
19.5 < wf < 18.5
q = 0.26 + 1.28 p’
r = 0.999

Drucker-Prager Surface



$$J_2 - \alpha I_1 - Y = 0$$



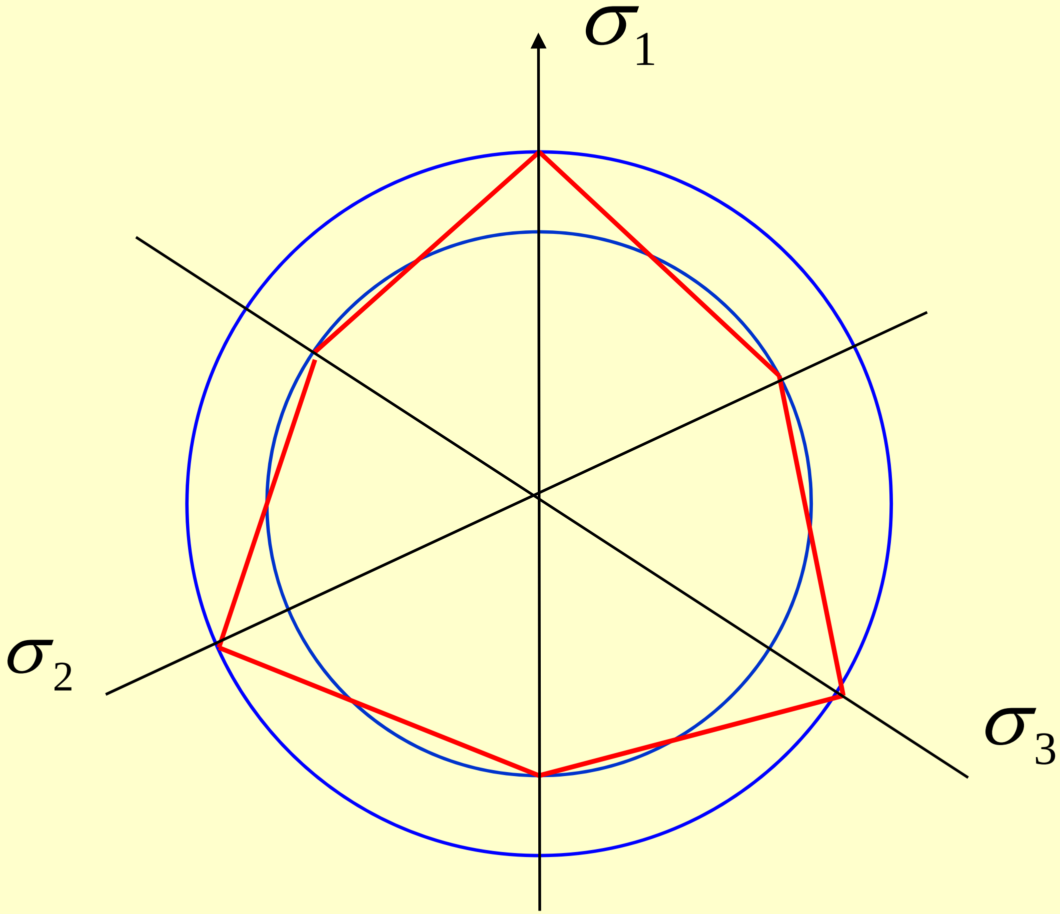
$$\alpha = \frac{\frac{1}{3}\sin\phi}{\cos\theta - \frac{1}{\sqrt{3}}\sin\theta\sin\phi}$$

$$Y = \frac{c\cos\phi}{\cos\theta - \frac{1}{\sqrt{3}}\sin\theta\sin\phi}$$

- $\phi_{compresion} \neq \phi_{extension}$

$$\sin\phi_{compression} = \frac{3\sqrt{3}\alpha}{2+\sqrt{3}\alpha} \qquad \sin\phi_{extension} = \frac{3\sqrt{3}\alpha}{2-\sqrt{3}\alpha}$$

$$\frac{\sin\phi_{extension}}{\sin\phi_{compression}} = \frac{2+\sqrt{3}\alpha}{2-\sqrt{3}\alpha}$$



σ_1
σ_2
σ_3

Improvements

- Matsuoka-Nakai

$$F \equiv \frac{I_1 I_2}{I_3} - Y = 0$$

- Zienkiewicz-Pande

$$M = \frac{6\sin\phi}{3 - \sin\phi \sin 3\theta}$$

- Lade

$$\left(\frac{I_1^{\,3}}{I_3} - 27\right)\left(\frac{I_1}{p_{atm}}\right)^{m} - \eta_1 = 0$$

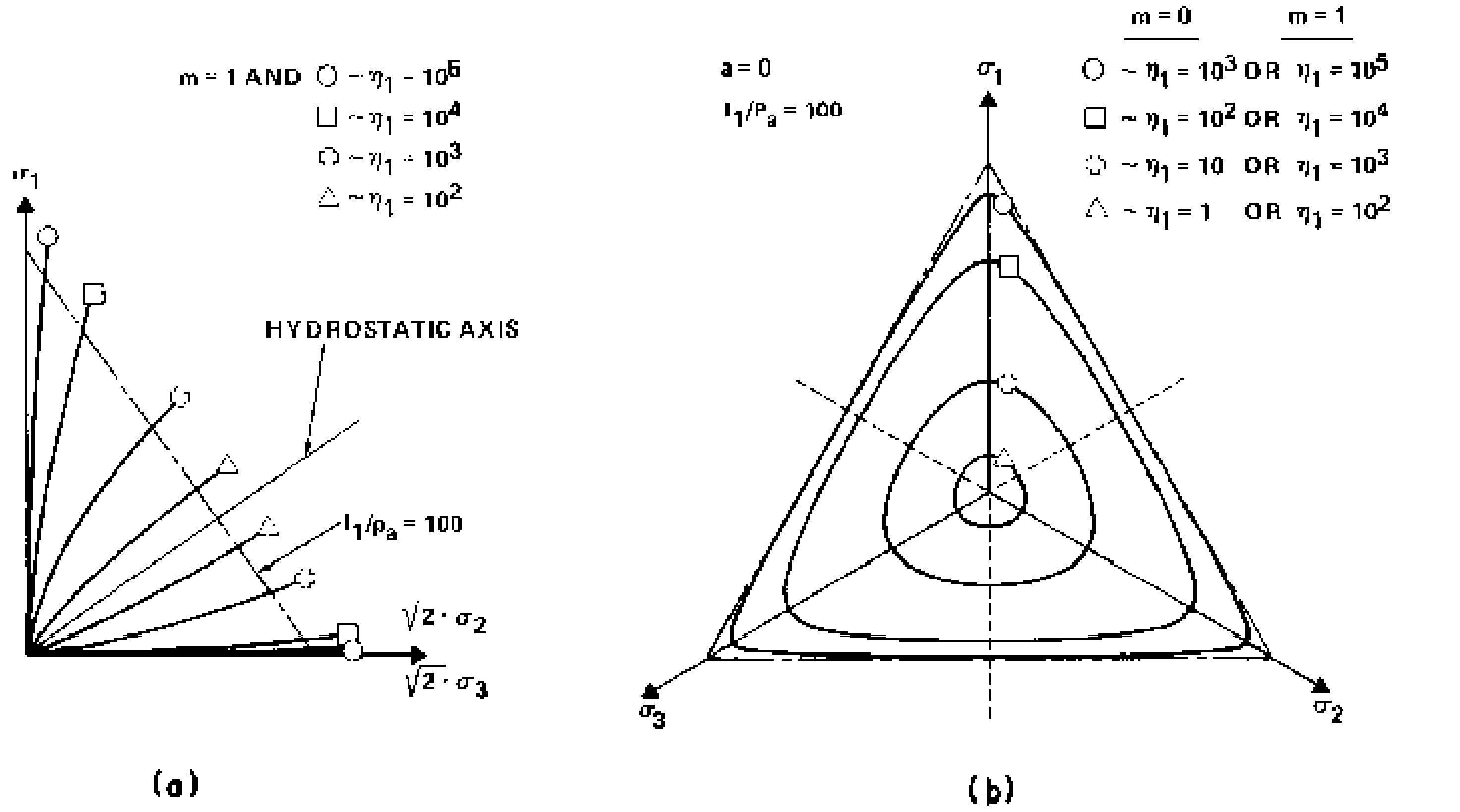


Figure 5. Characteristics of failure surfaces shown in principal stress space. Traces of failure surfaces shown in (a) triaxial plane and in (b) octahedral plane

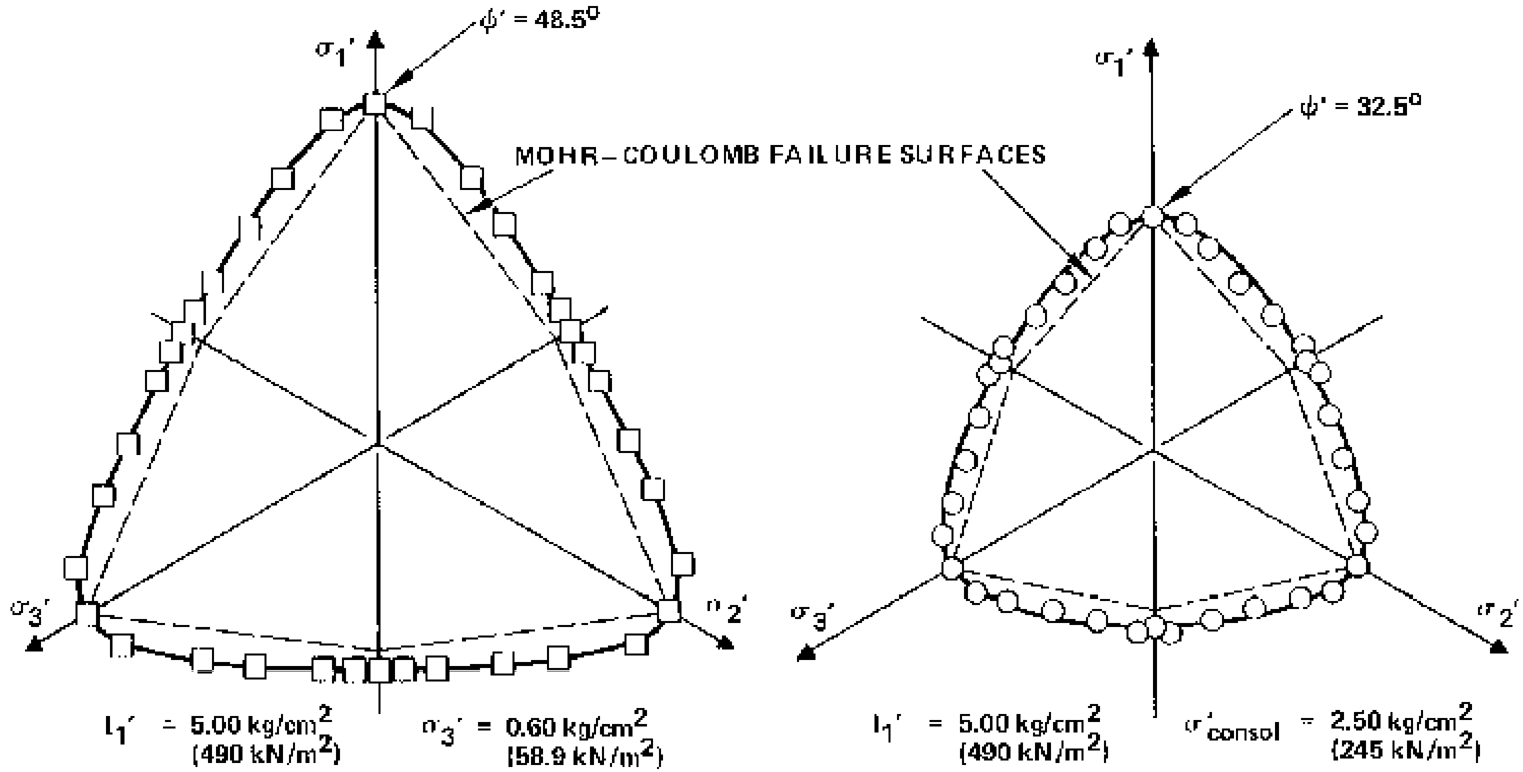


Figure 9. Comparison of failure criterion with experimental results of cubical triaxial tests projected on common octahedral planes for (a) dense Monterey No. 0 sand by Lade,[10] and for (b) normally consolidated, remolded Edgar plastic kaolinite by Lade and Musante[18]

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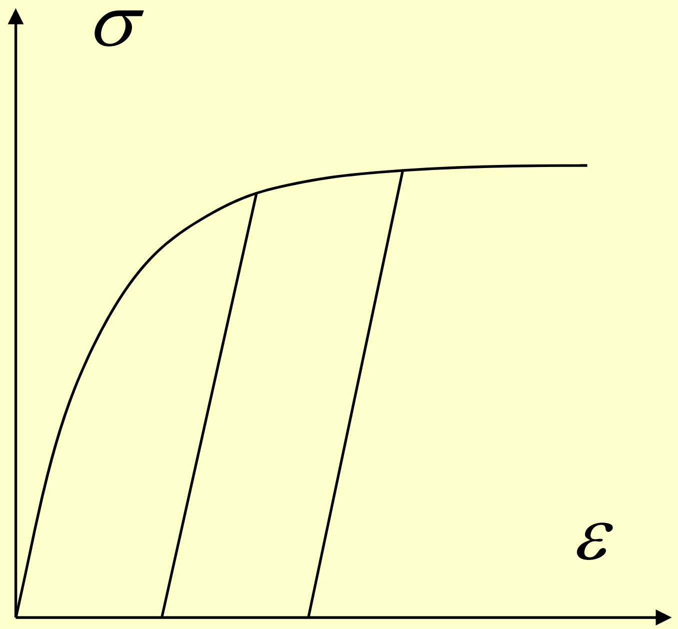
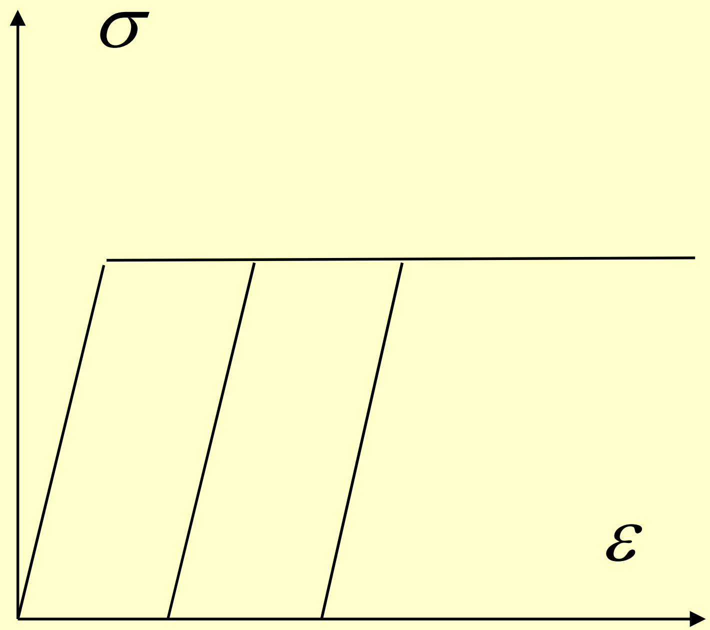
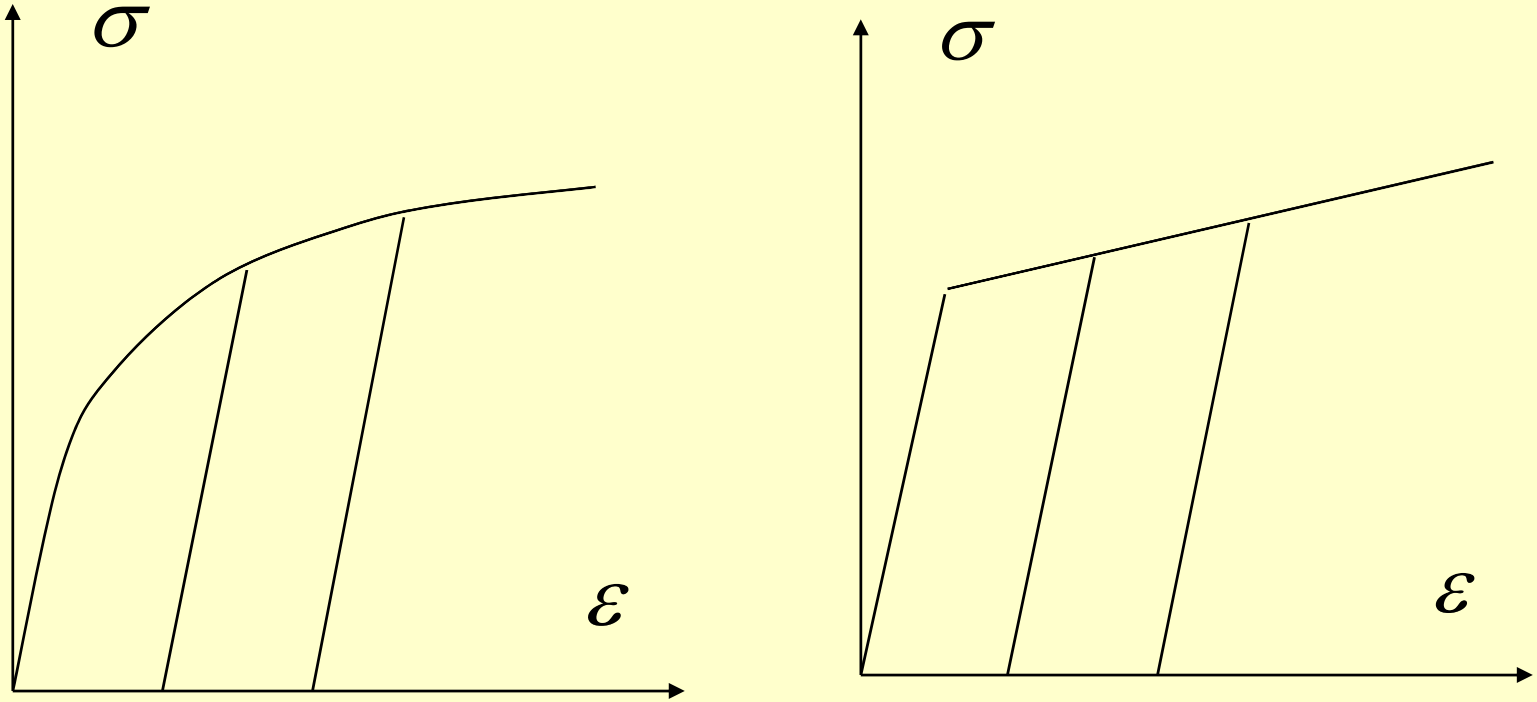
Classical Plasticity

Perfect plasticity



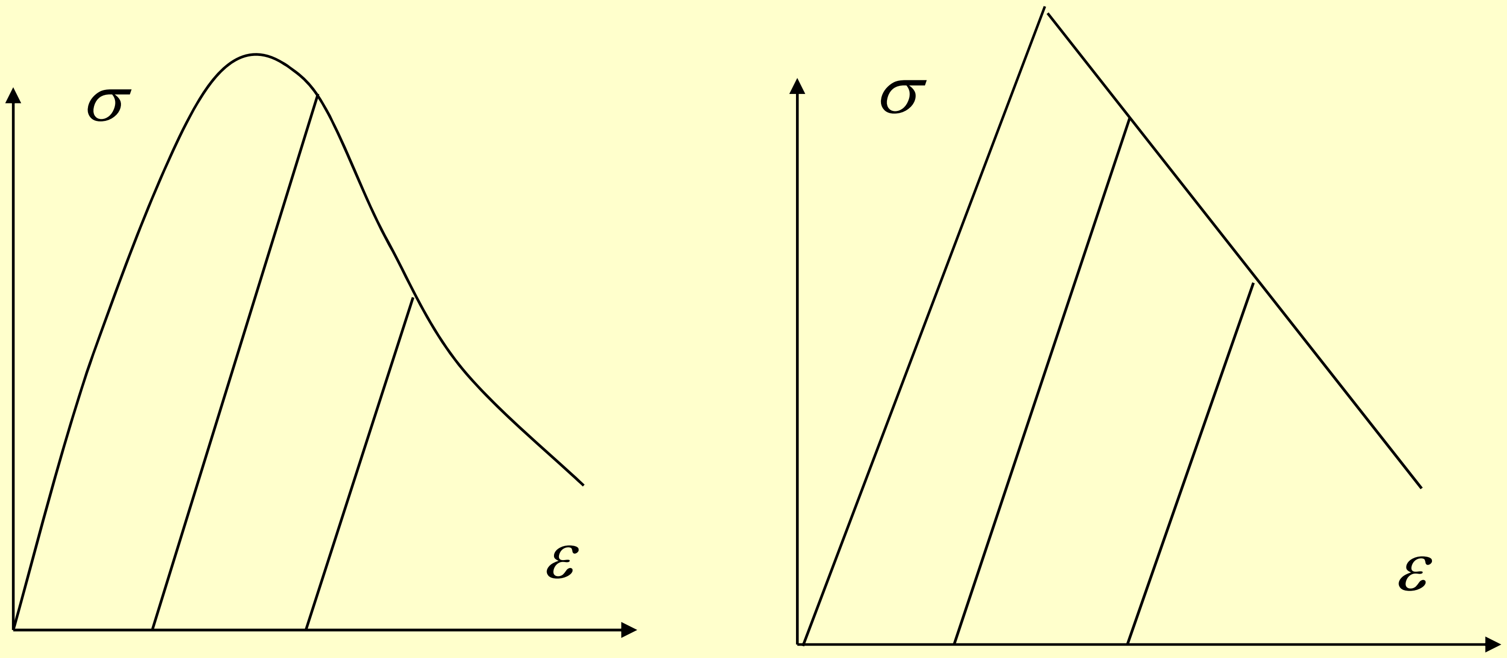


Classical Plasticity



Hardening

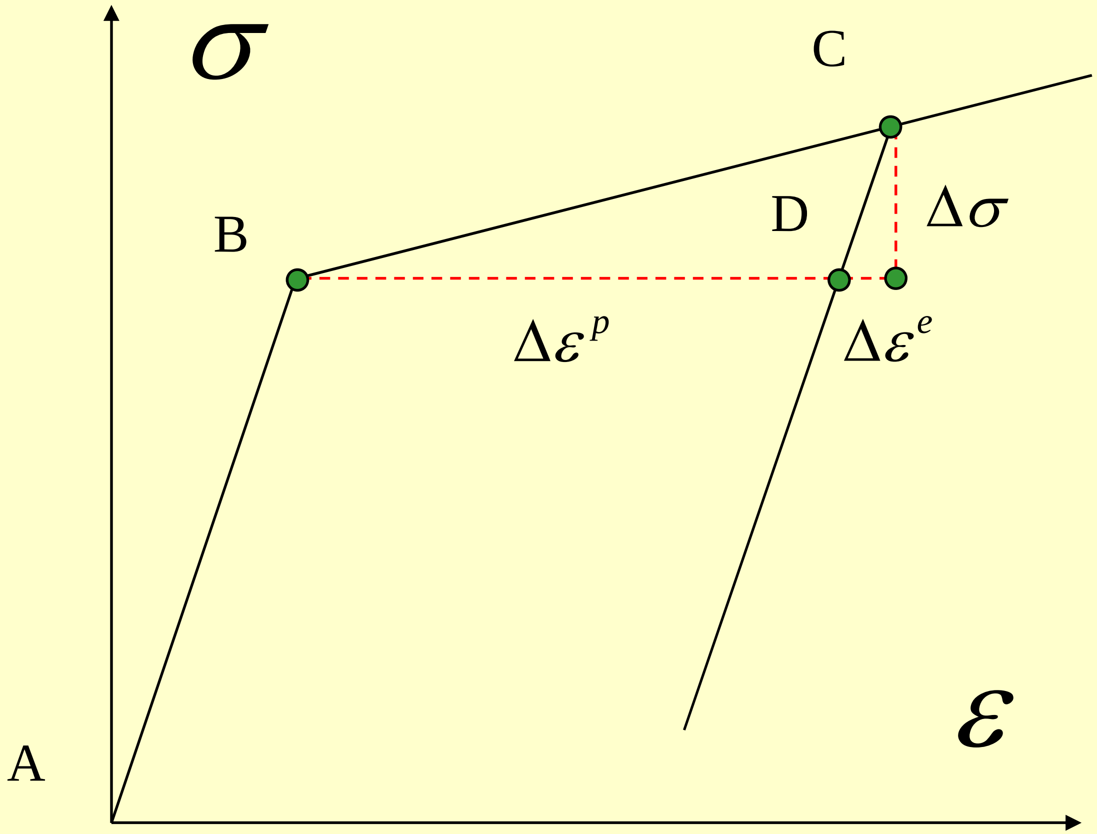
Classical Plasticity



Softening

Classical Plasticity

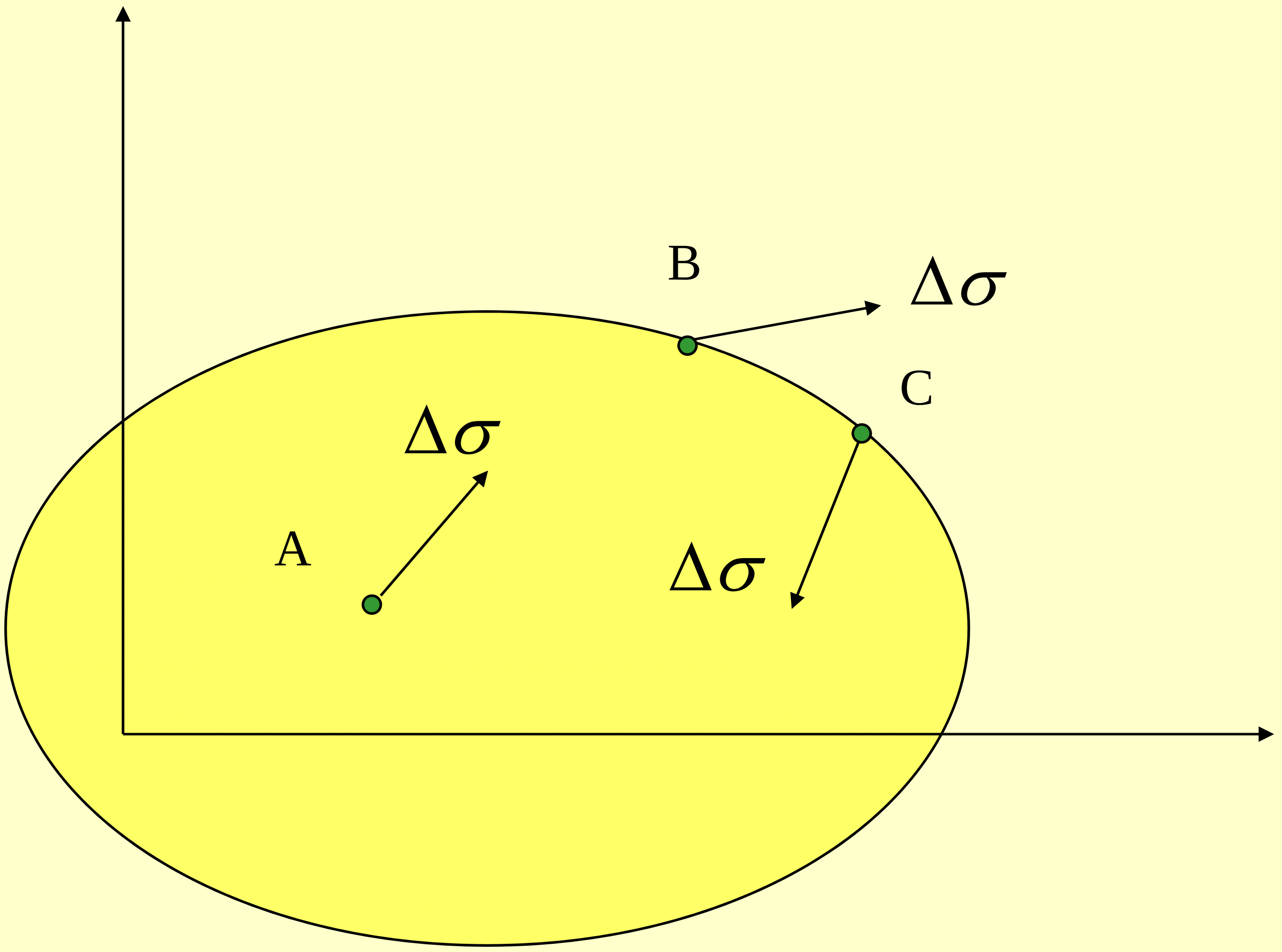
I : Decomposition of strain increment



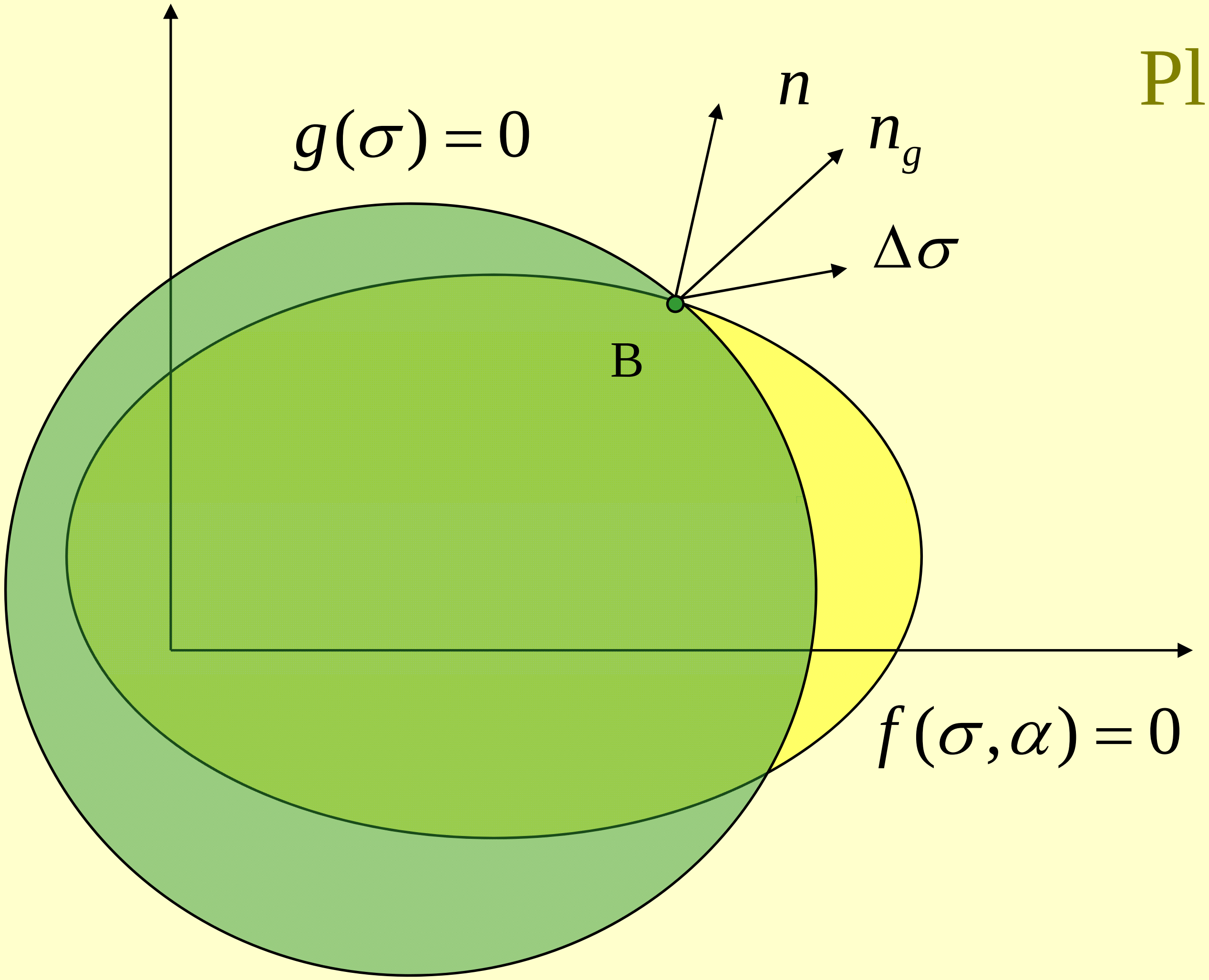
$$\Delta\varepsilon = \Delta\varepsilon^p + \Delta\varepsilon^e$$

Classical Plasticity

II : Yield surface in the stress space



Plastic Potential



$$d\varepsilon^p = \frac{1}{h} n_g \left(d\sigma : n\right) \quad if \quad \left(d\sigma : n\right) > 0$$

$$(loading)$$

Necessary elements

$$d\varepsilon^p = \frac{1}{h} n_g \left(d\sigma : n \right)$$

$$d\varepsilon^p = \frac{1}{H} \frac{\partial g}{\partial \sigma} \left(d\sigma : \frac{\partial f}{\partial \sigma} \right)$$

- **Yield Surface**

$$f(\sigma, \alpha) = 0$$

$$n = \frac{\partial f / \partial \sigma}{\left| \partial f / \partial \sigma \right|}$$

- **Plastic Potential**

$$n_g = \frac{\partial g / \partial \sigma}{\left| \partial g / \partial \sigma \right|}$$

- **Plastic Modulus** h

Plastic Modulus

- Consistency Condition

$$f(\sigma,\alpha)=0$$

$$df=0$$

$$\frac{\partial f}{\partial \sigma}:d\sigma+\frac{\partial f}{\partial \alpha}d\alpha=0$$

$$\boxed{H=-\frac{\partial f}{\partial \alpha}\left(\frac{\partial \alpha}{\partial \varepsilon^p}:\frac{\partial g}{\partial \sigma}\right)}$$

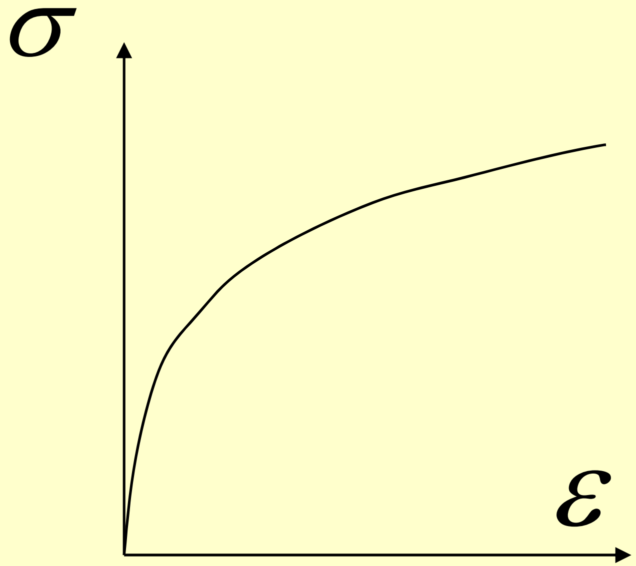
- Strain Hardening

$$\frac{\partial f}{\partial \sigma}:d\sigma+\frac{\partial f}{\partial \alpha}\left(\frac{\partial \alpha}{\partial \varepsilon^p}:d\varepsilon^p\right)=0$$

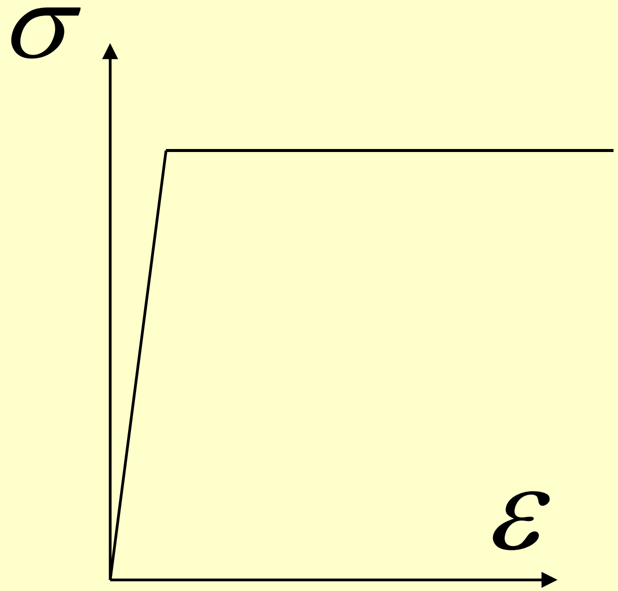
$$\frac{\partial f}{\partial \sigma}:d\sigma+\frac{\partial f}{\partial \alpha}\left(\frac{\partial \alpha}{\partial \varepsilon^p}:\frac{1}{H}\frac{\partial g}{\partial \sigma}\left(\frac{\partial f}{\partial \sigma}:d\sigma\right)\right)=0$$

Hunting of the Yield Surface

- (I) Assume f = F

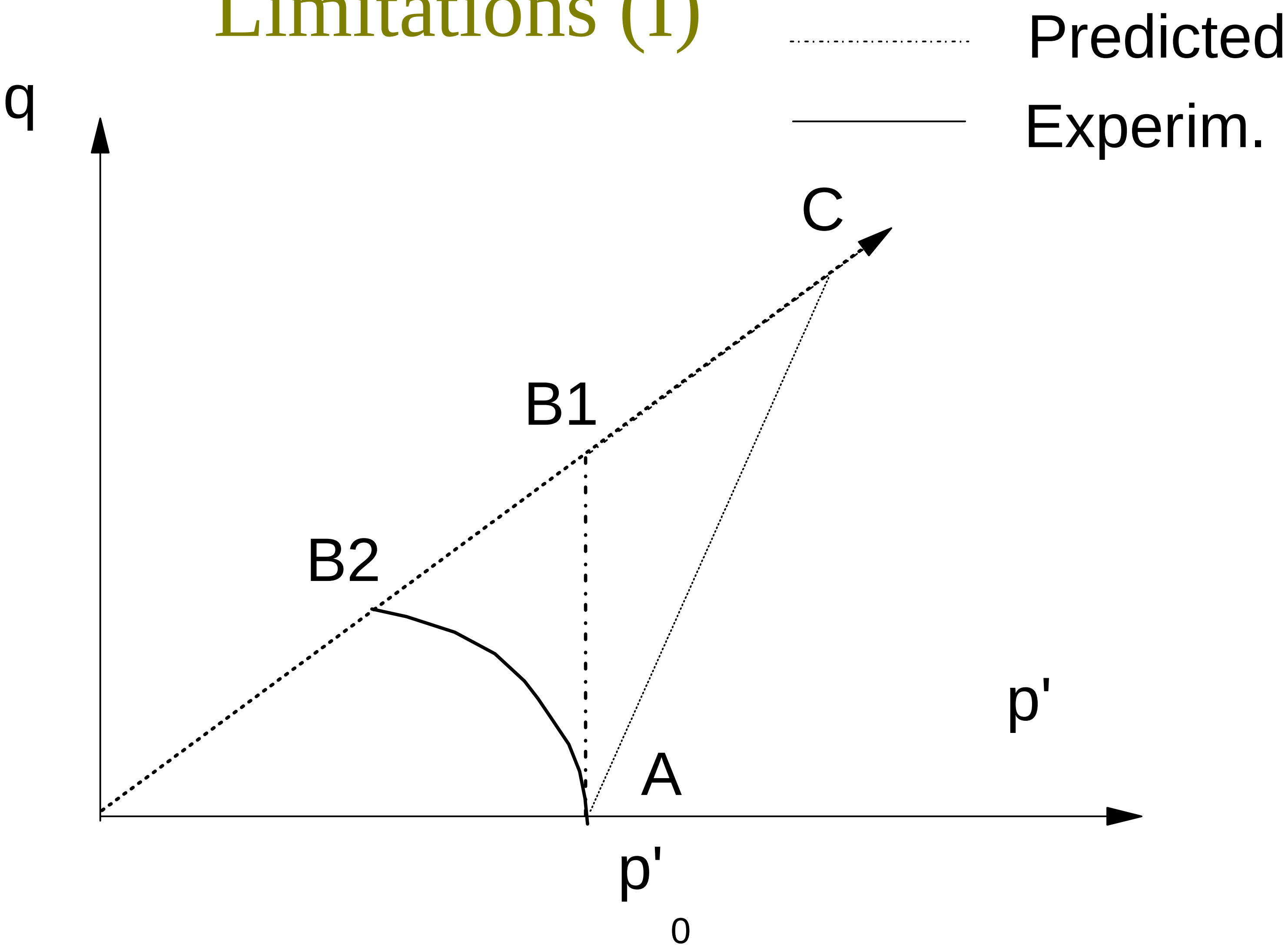


Experimental

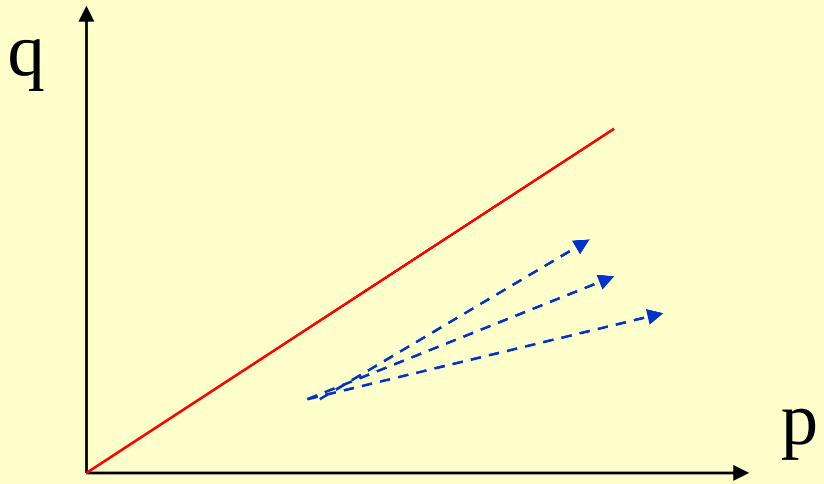


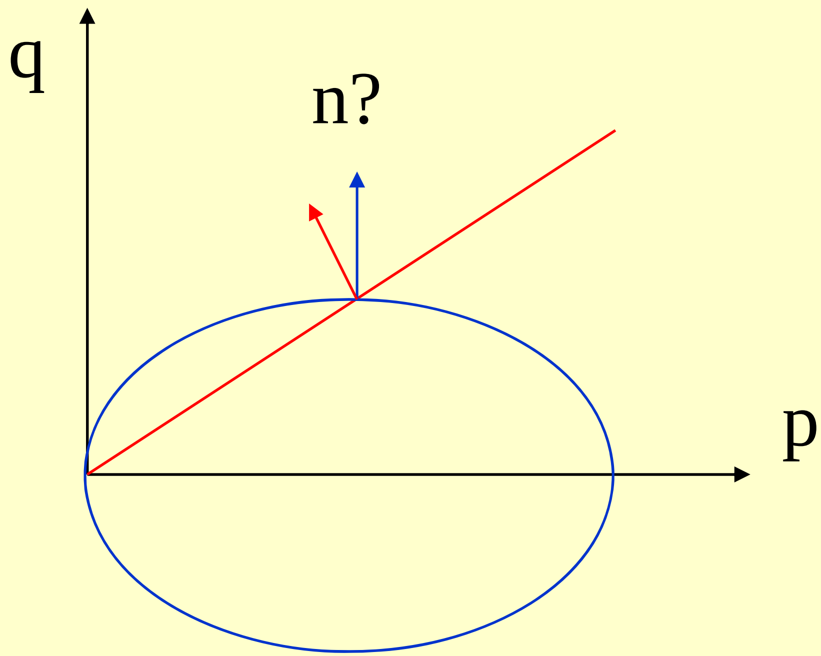
Model

Limitations (I)

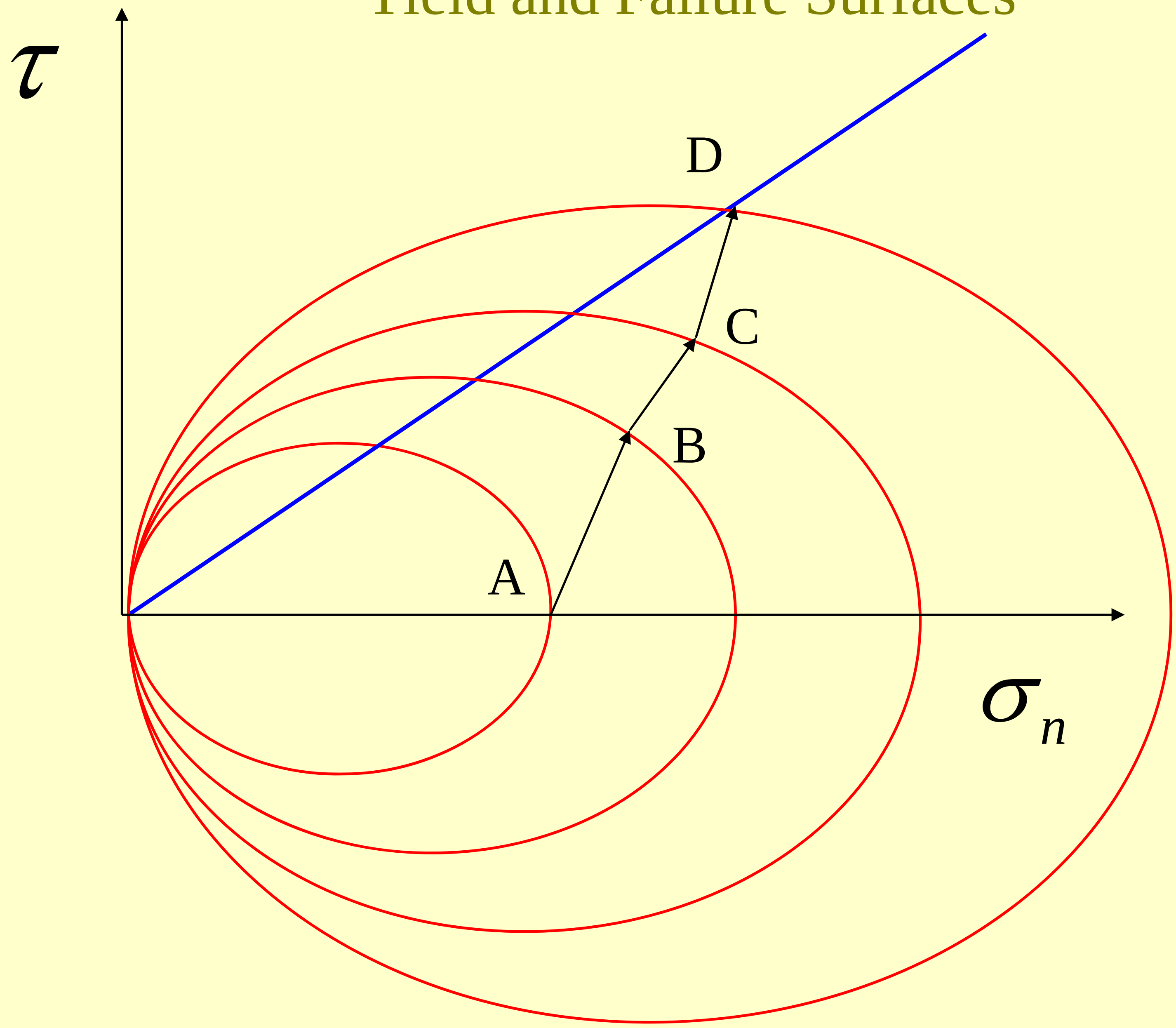


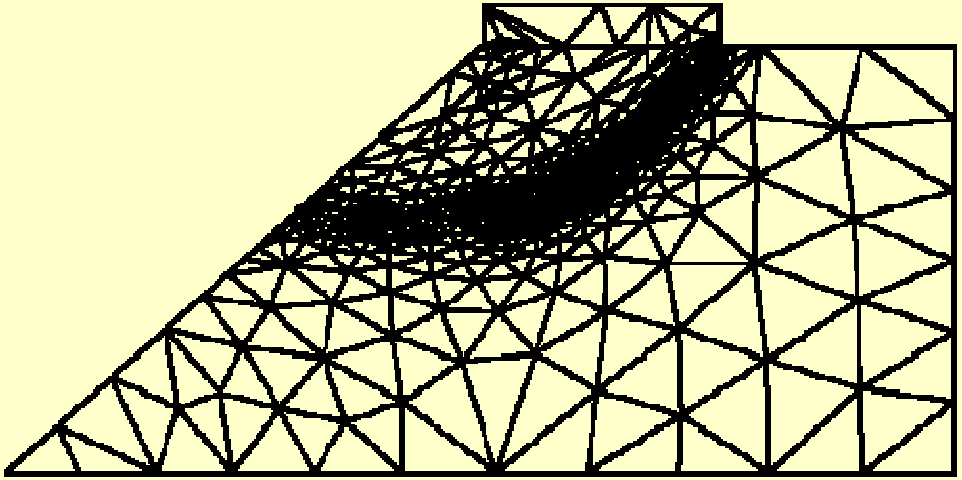
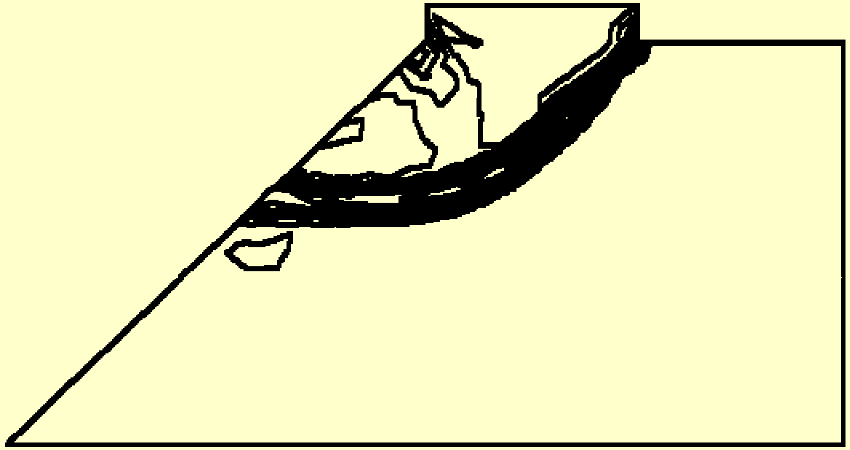
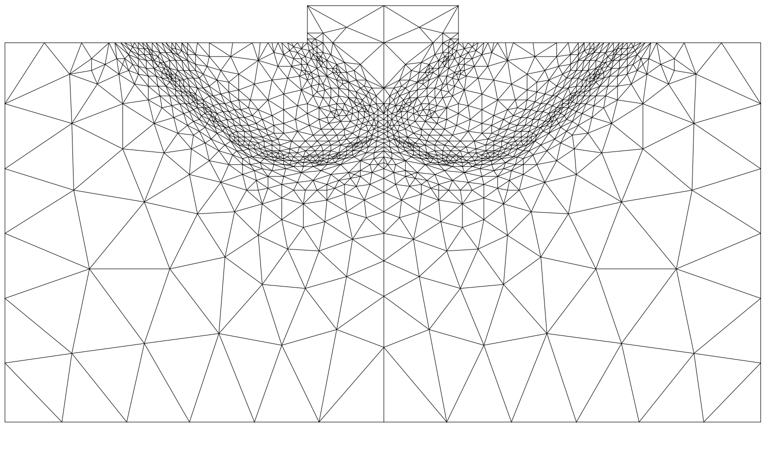
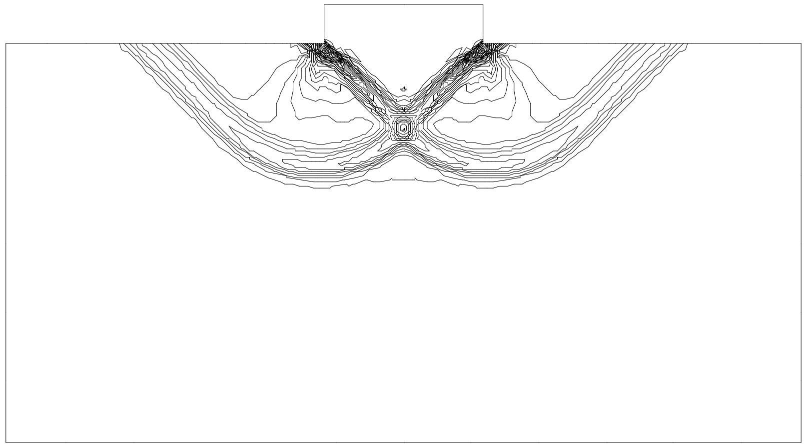
Limitations (II)



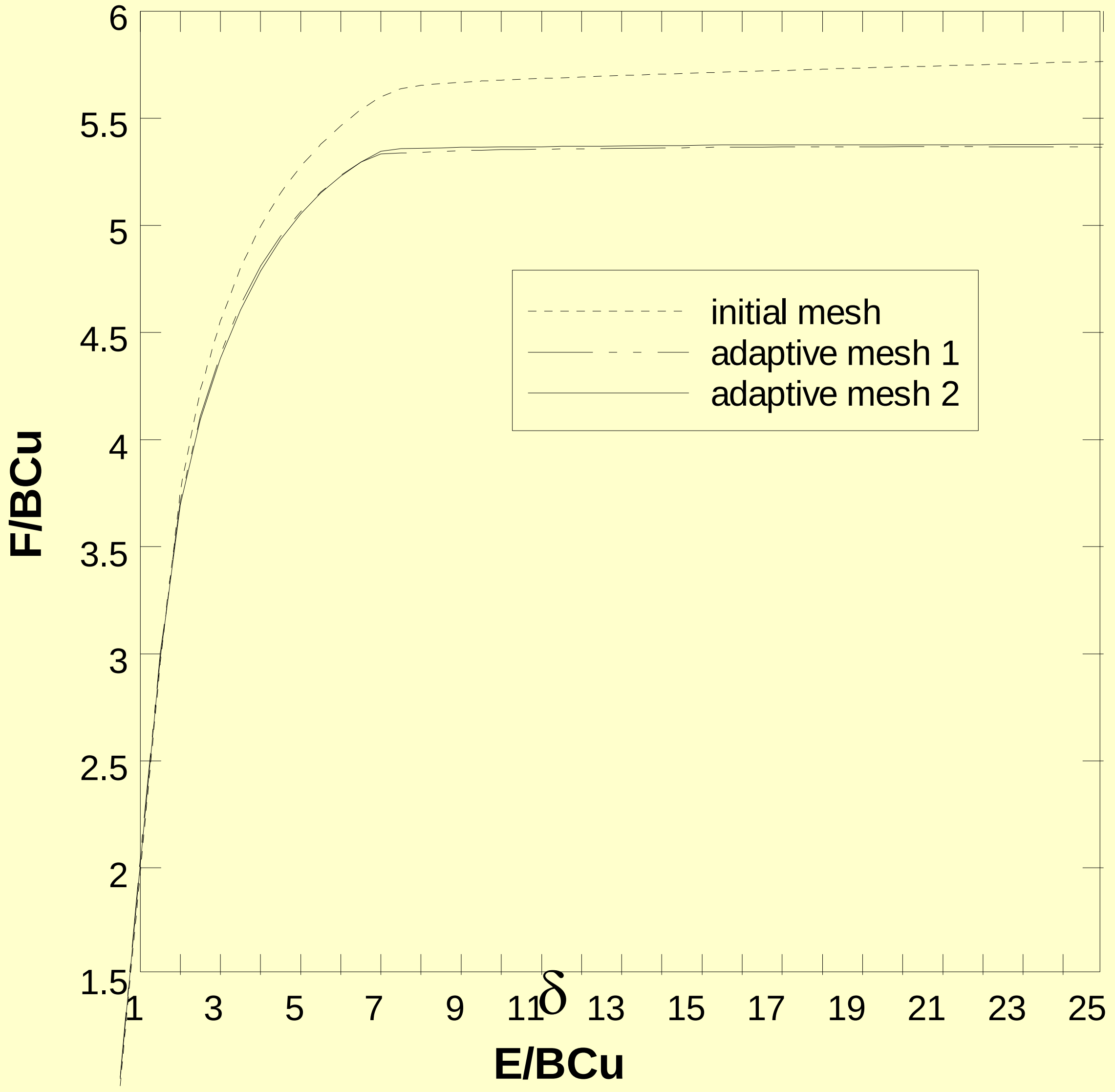


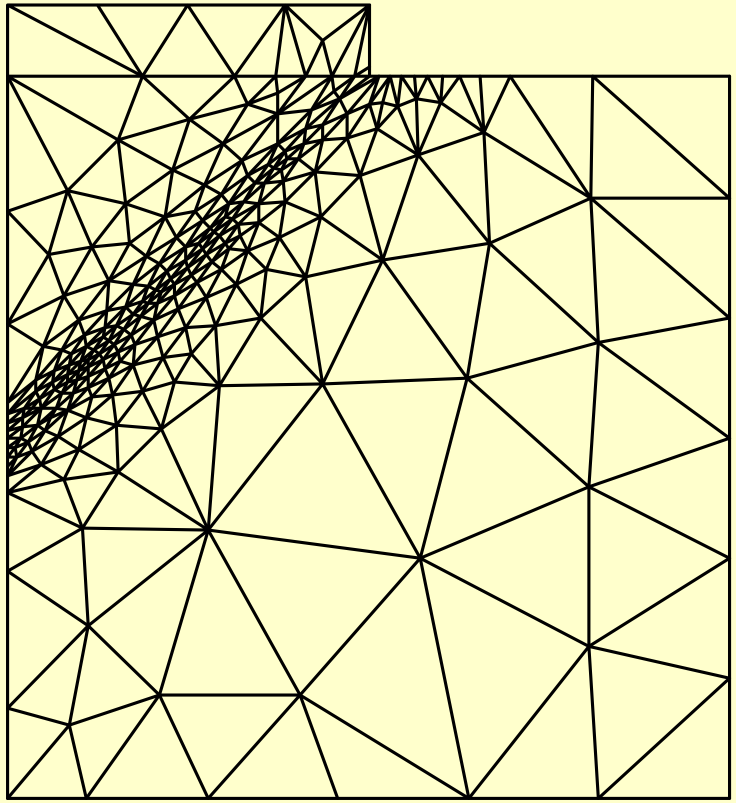
Yield and Failure Surfaces



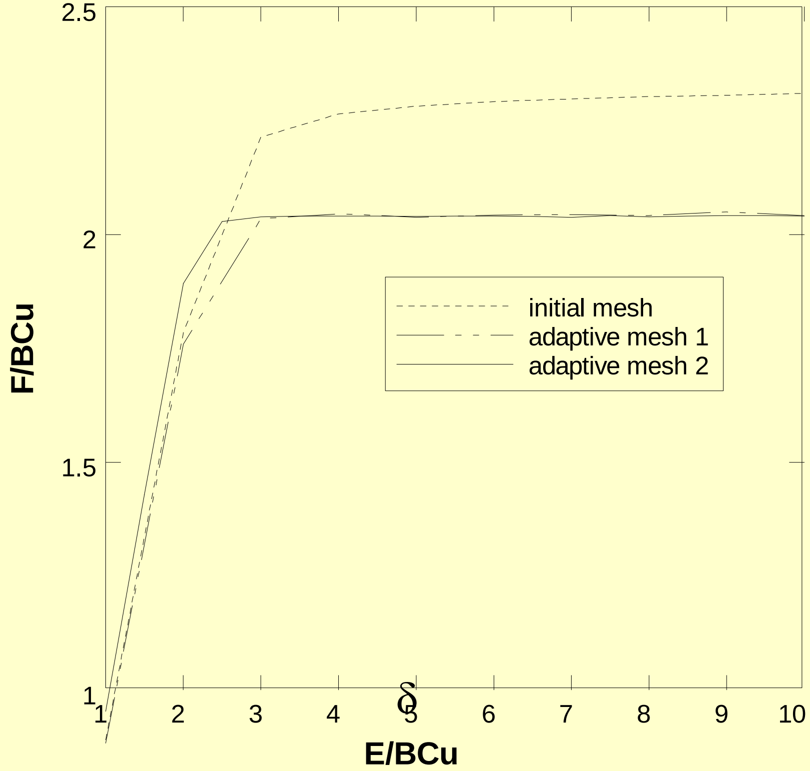





Limiting loads by different meshes for foundation problem





Limiting loads by different meshs for vertical cut problem



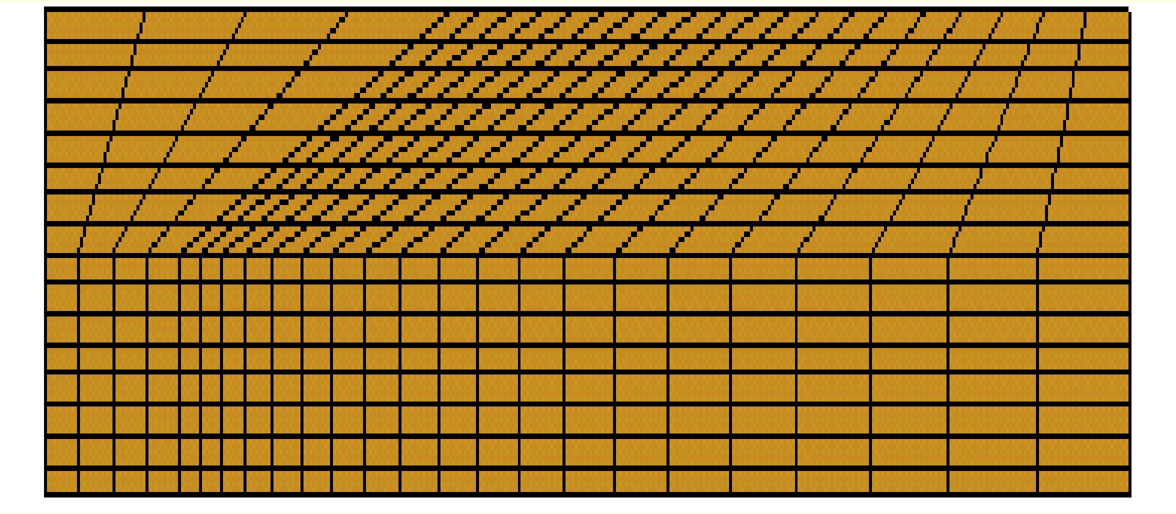
GeHoMadrid

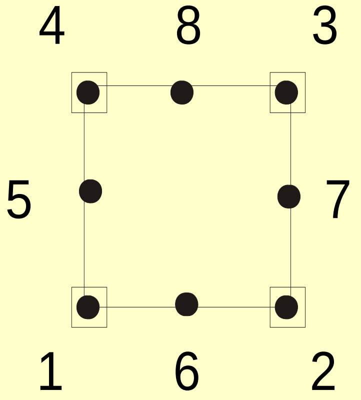
- **Constitutive Models**
  - Elastoplastic Models
  - Generalized Plasticity
  - Cam-Clay
  - Nova
  - Concrete

Special Techniques

Adaptive remeshing
Adaptive Timestepping
Backwards Euler Integration
Consistent Stifness
Stabilization ...

Slope Stability: Finite Element Model





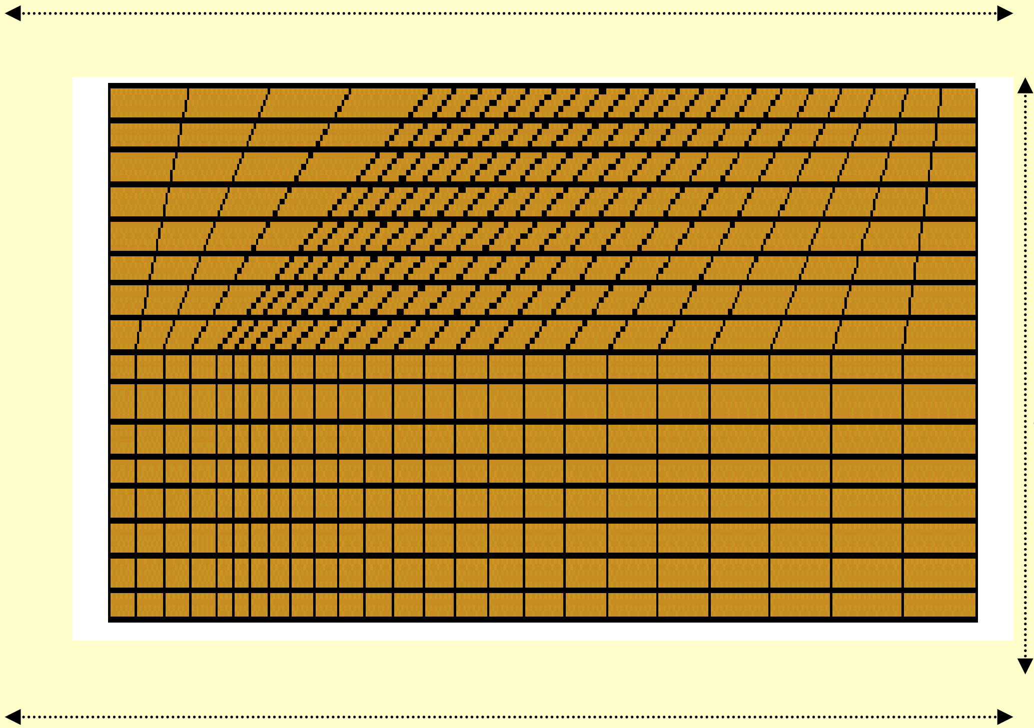
8 nodes for displacements
4 nodes for pressures

- Boundary Conditions

Pw = 0 kPa

Ux = 0

$$\frac{\partial p_w}{\partial n} = 0$$



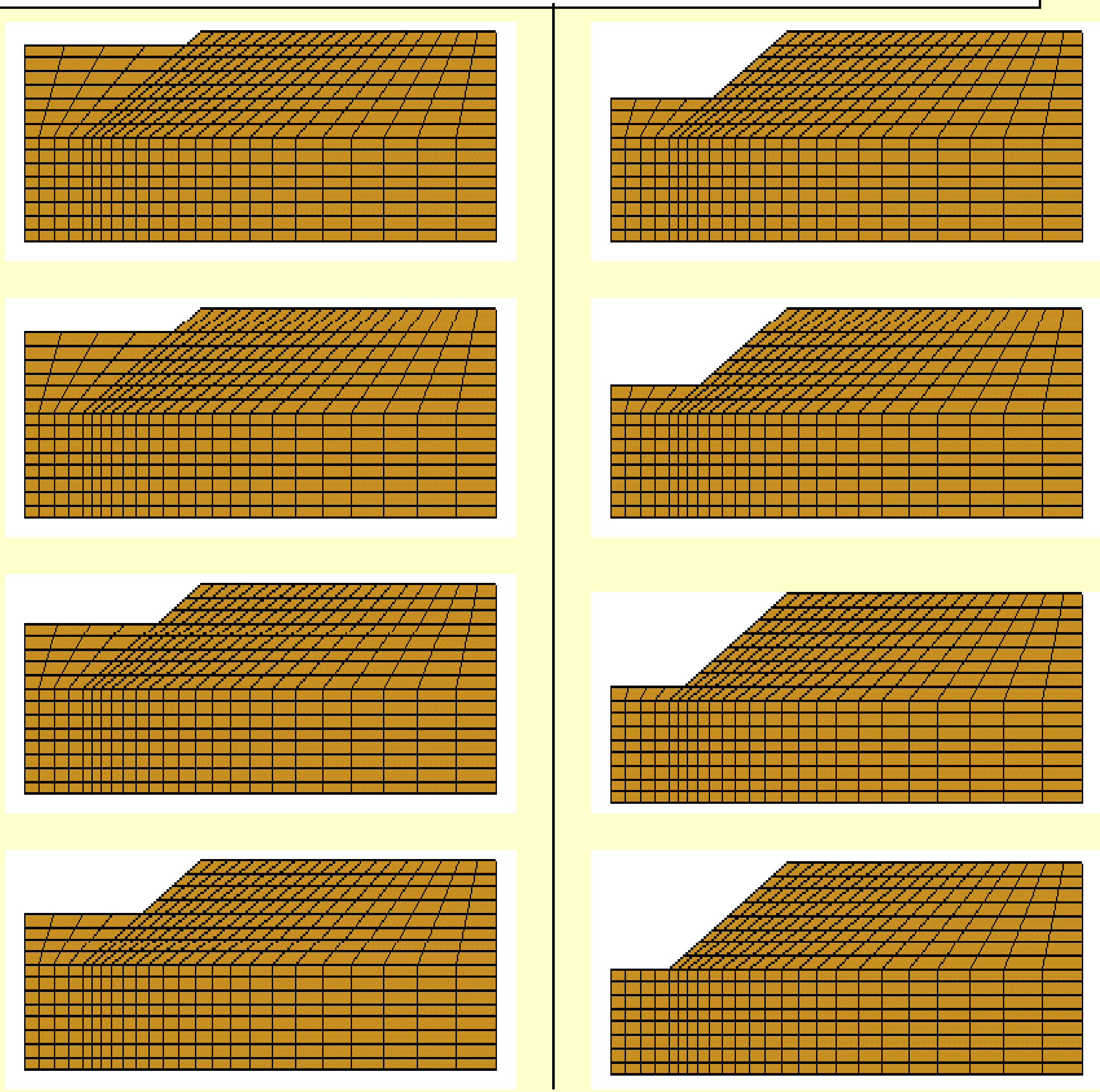
$$\frac{\partial p_w}{\partial n} = 0$$

Ux = 0

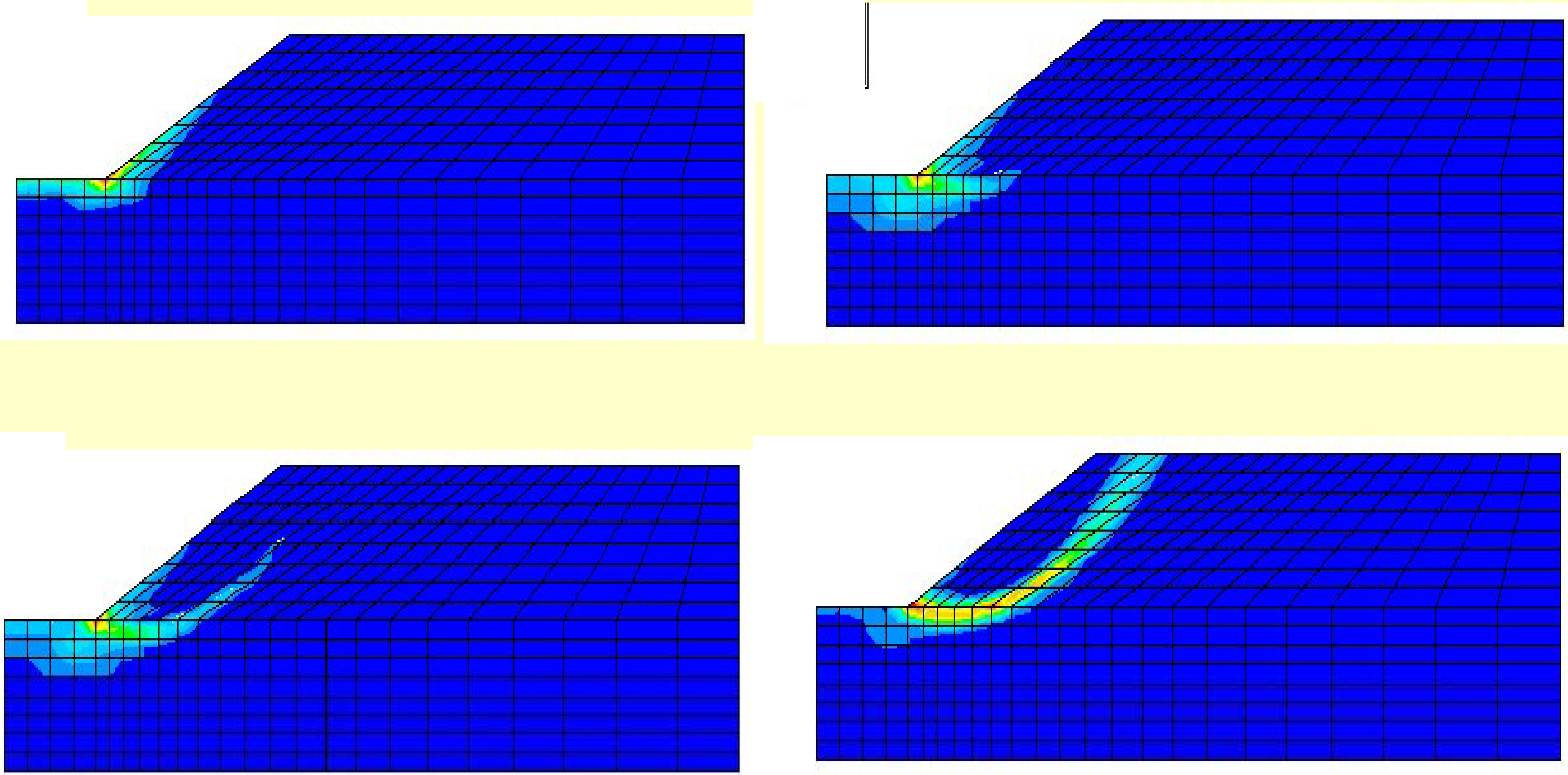
Ux = Uy = 0

$$\frac{\partial p_w}{\partial n} = 0$$

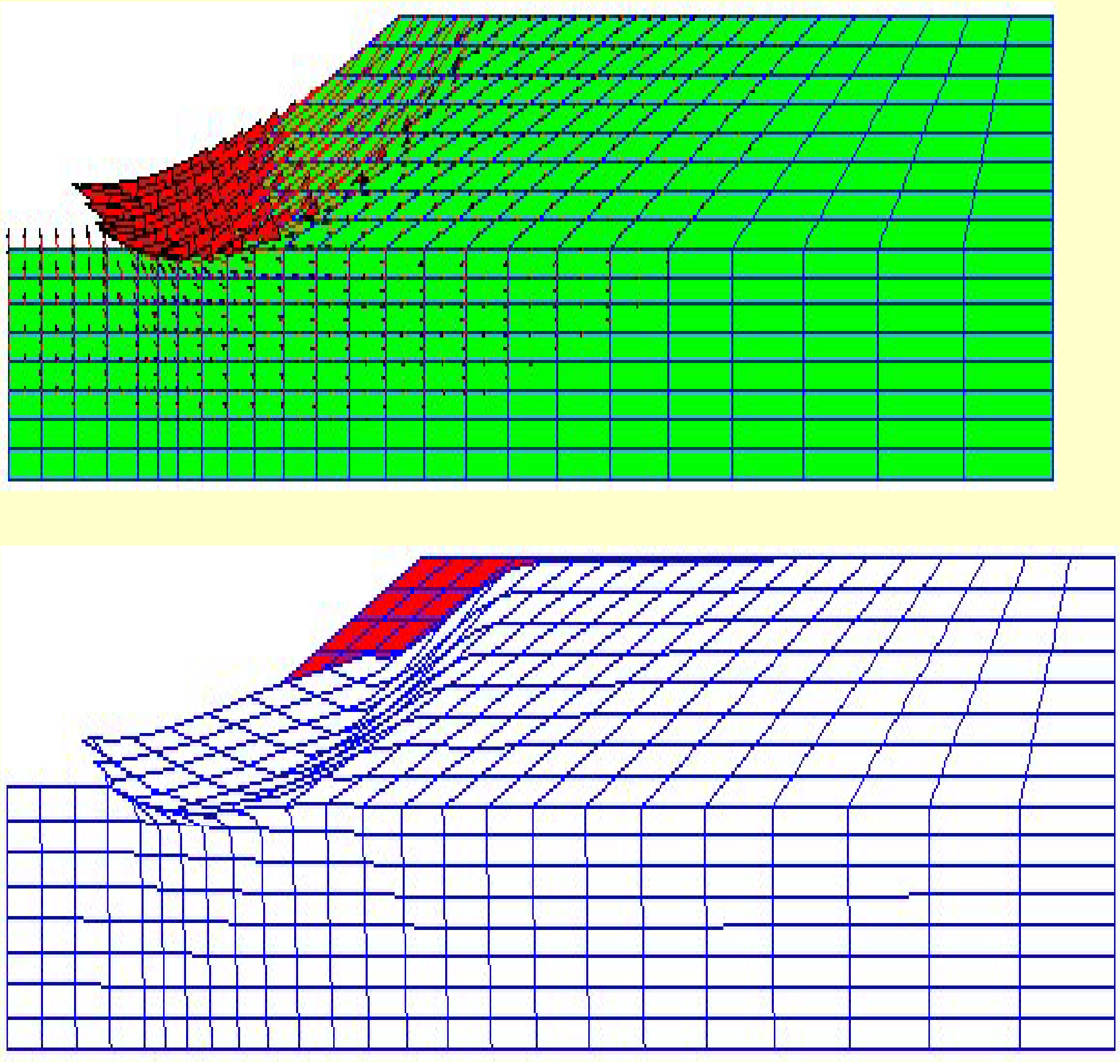
Excavation (3 months)

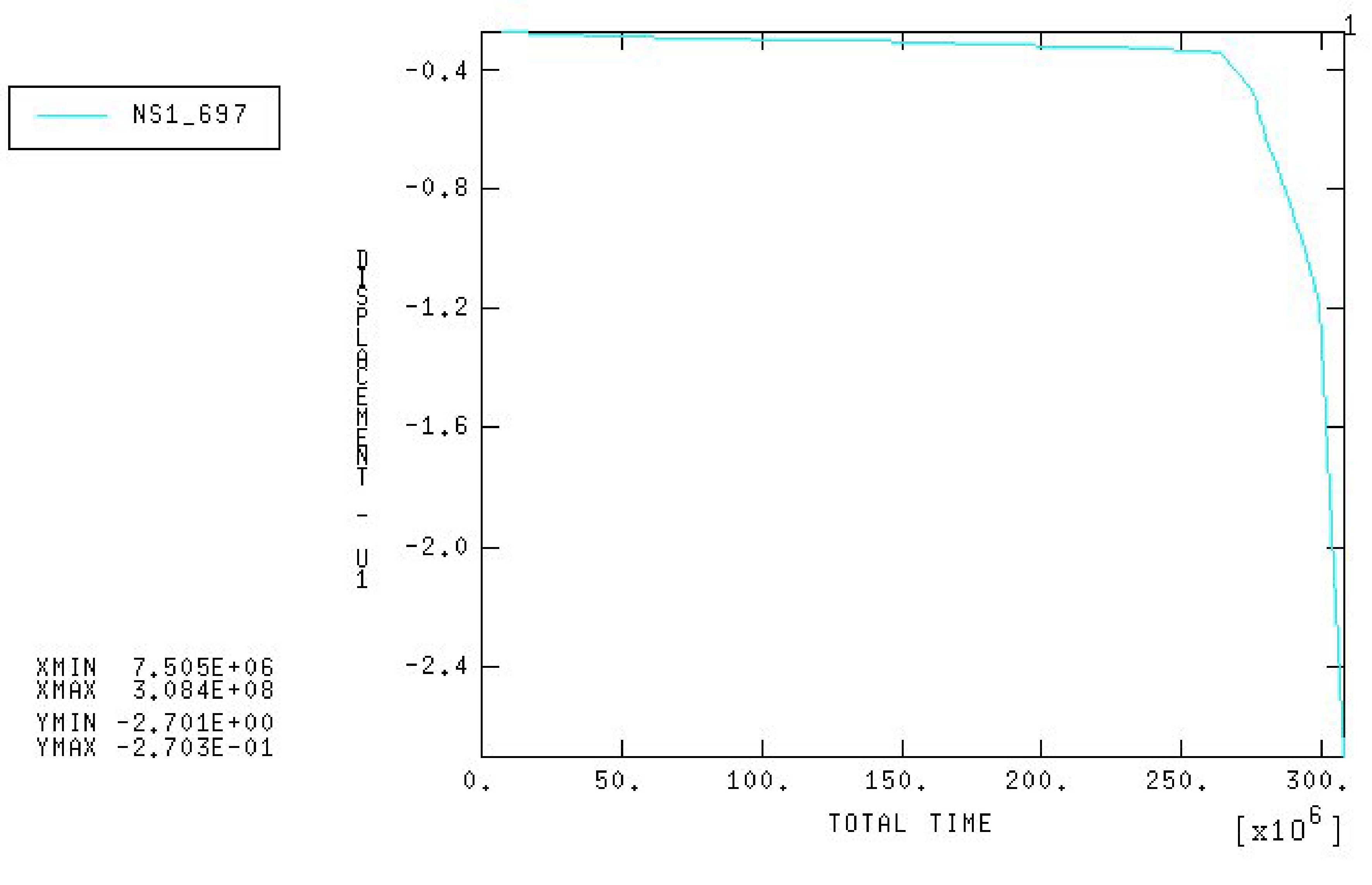


ξ_{ps}



Failure





Slow consolidation

→ Fast failure

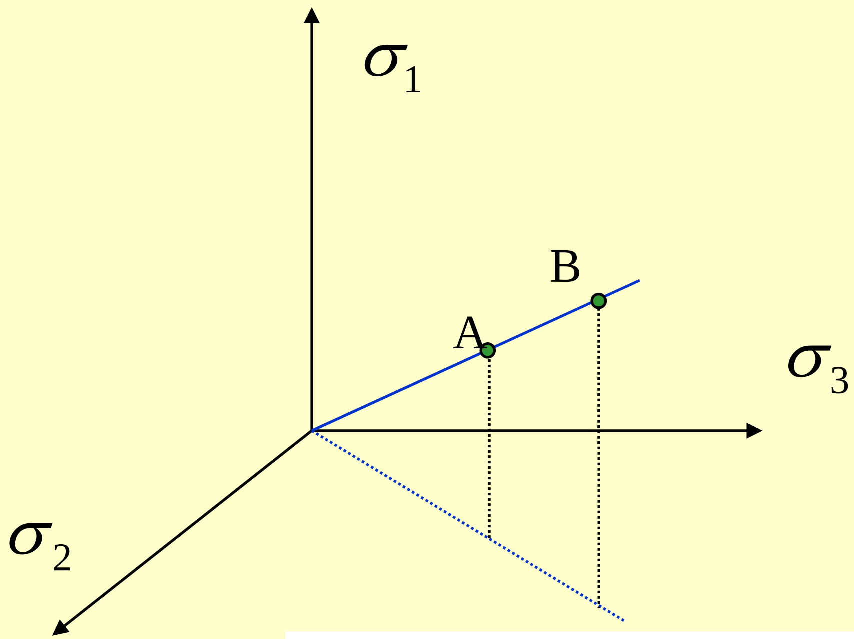
Critical state based models

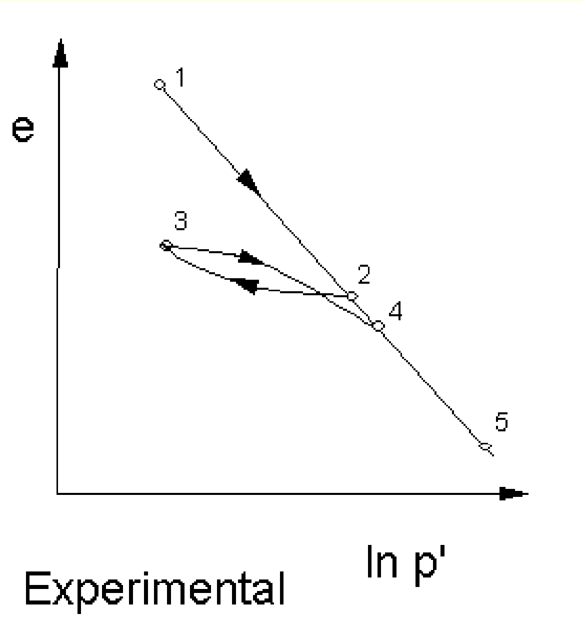
Contents

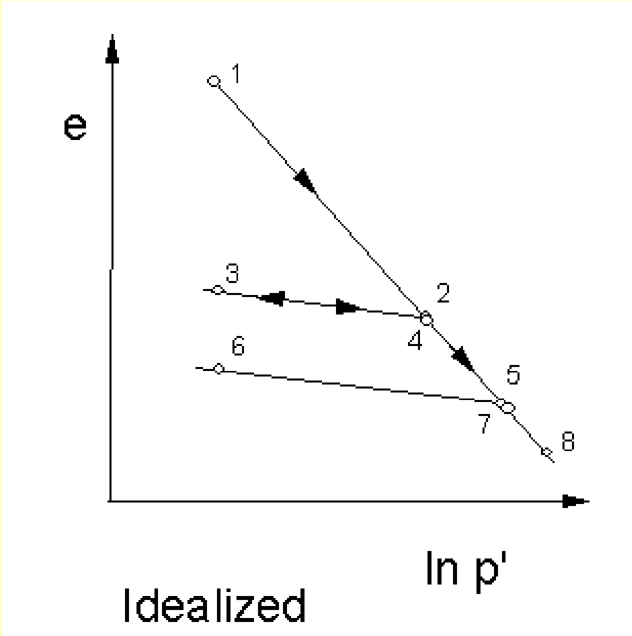
Critical State Models

- Hydrostatic Compression
  - Gives Hardening Rule
- Triaxial tests
  - Plastic Potential
  - Yield Surface
- Improvements
  - Overconsolidated Clays
  - Granular Soils

Hydrostatic Compression

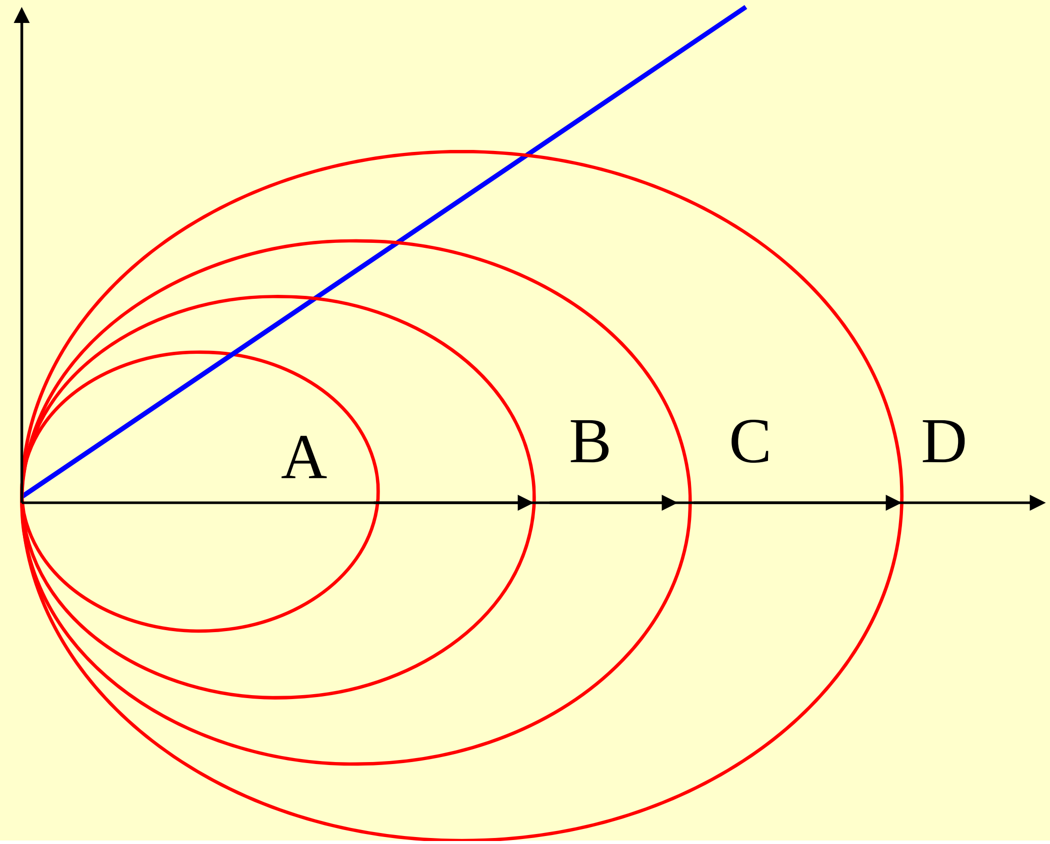




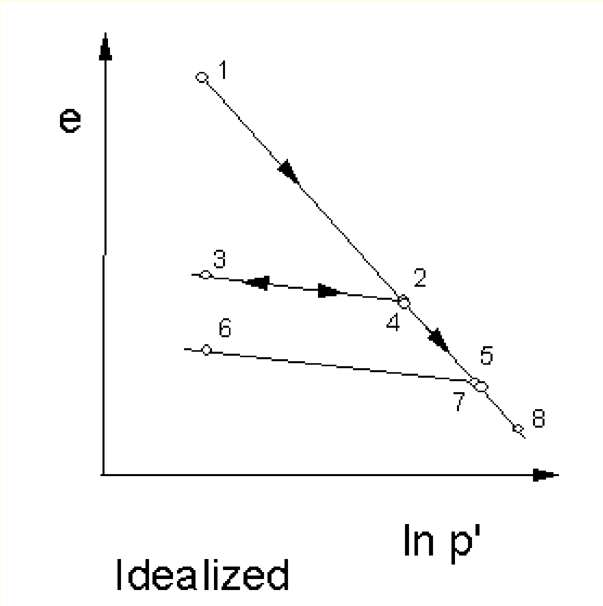


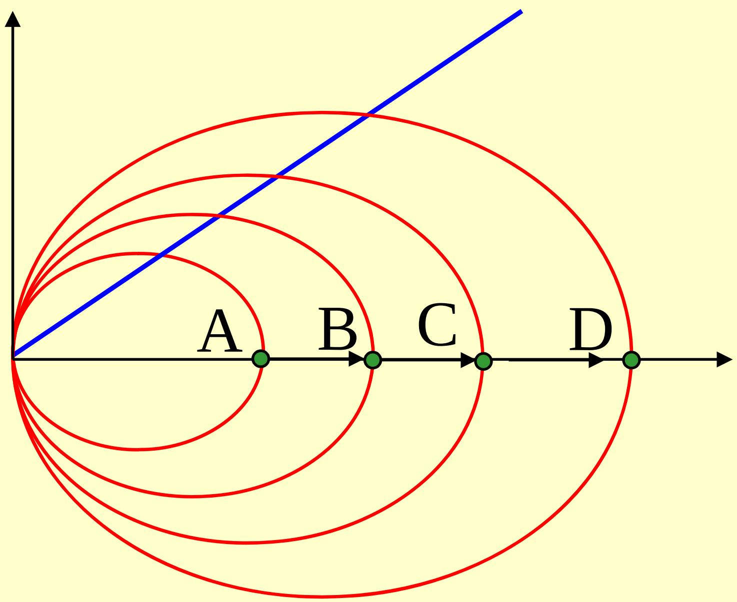
Conclusions (so far)

- Plastic strains during
    - Hydrostatic Compression
    - Oedometer
- Yield surfaces MUST be closed



Hardening Rule





$$d\varepsilon_v = -\frac{de}{1+e}$$

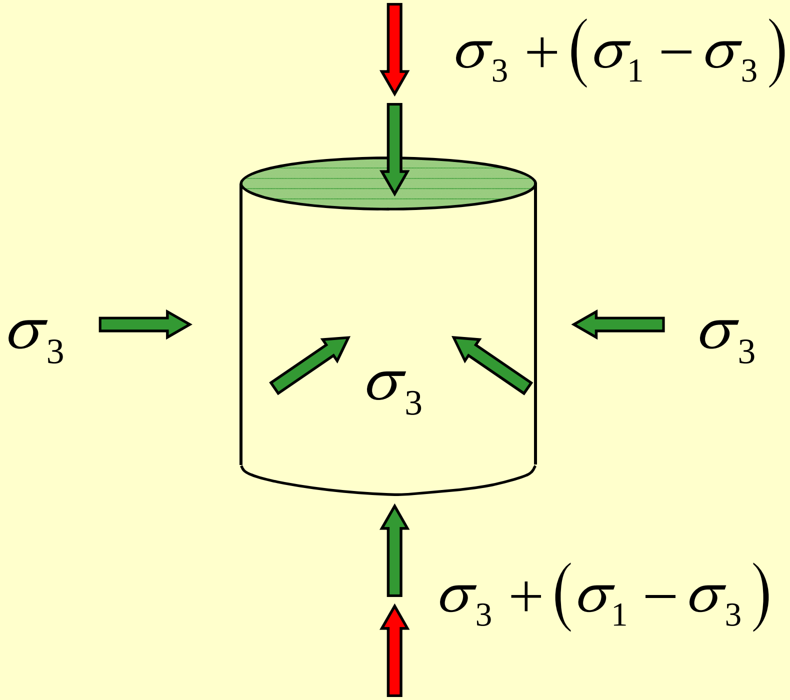
$$de = -\lambda \frac{dp}{p} \qquad de = -\kappa \frac{dp}{p}$$

$$de^p = -(\lambda - \kappa)\frac{dp}{p}$$

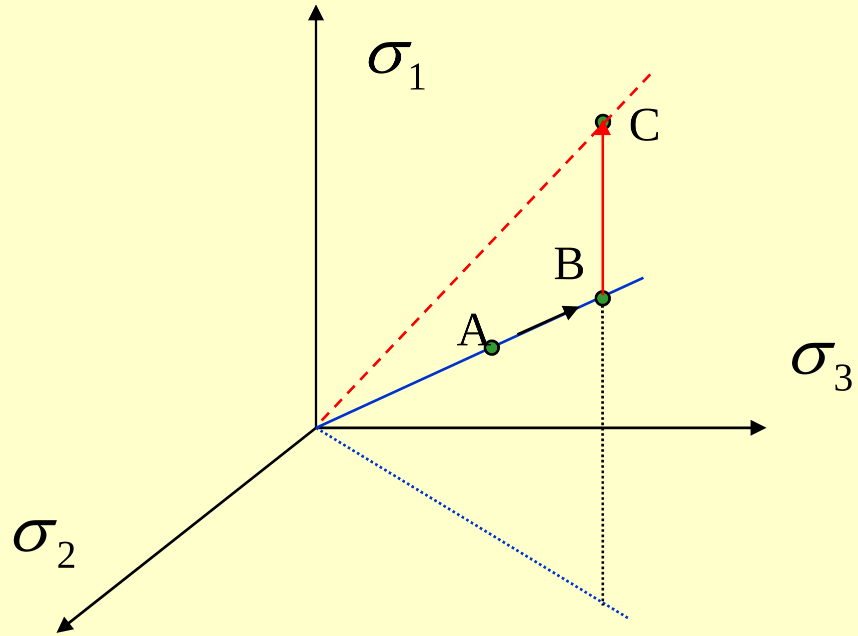
$$d\varepsilon_v{}^p = \frac{(\lambda - \kappa)}{1+e}\frac{dp}{p}$$

$$\boxed{\frac{\partial p_c}{\partial \varepsilon_v{}^p} = \frac{1+e}{\lambda - \kappa} p_c}$$

Triaxial



$\sigma_3 + (\sigma_1 - \sigma_3)$
σ_3
σ_3
σ_3
$\sigma_3 + (\sigma_1 - \sigma_3)$

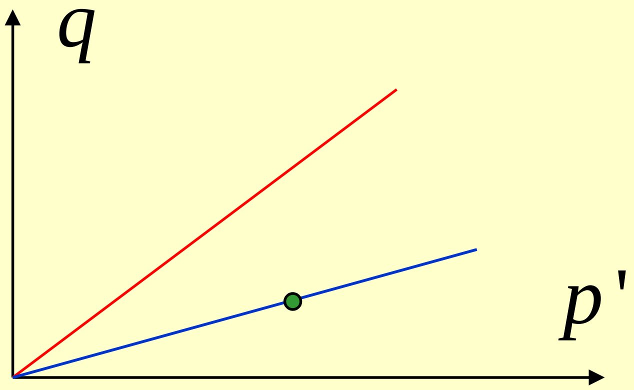


σ_1
C
B
A
σ_3
σ_2

$$p' = \frac{1}{3} tr(I'_1) = \left(\sigma'_1 + 2\sigma'_3\right)$$

$$\sigma' = \sigma - p_w \mathbf{I}$$

$$q = \sqrt{3J'_2} = \left(\sigma'_1 - \sigma'_3\right)$$

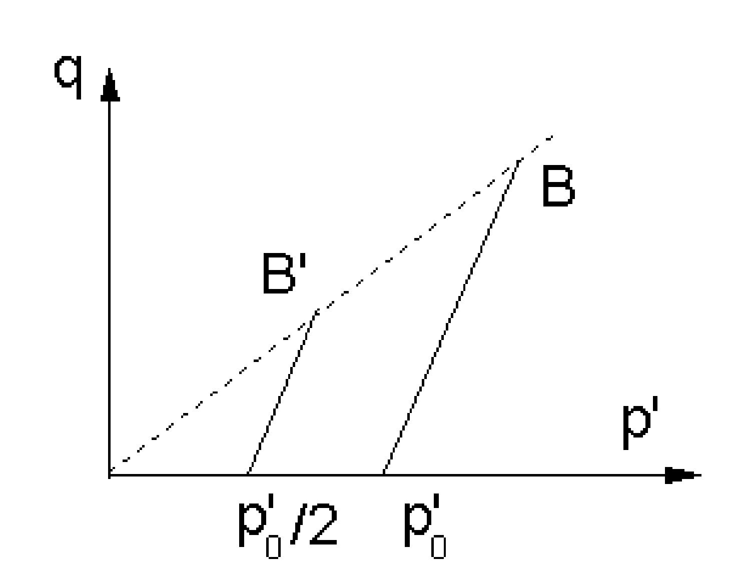


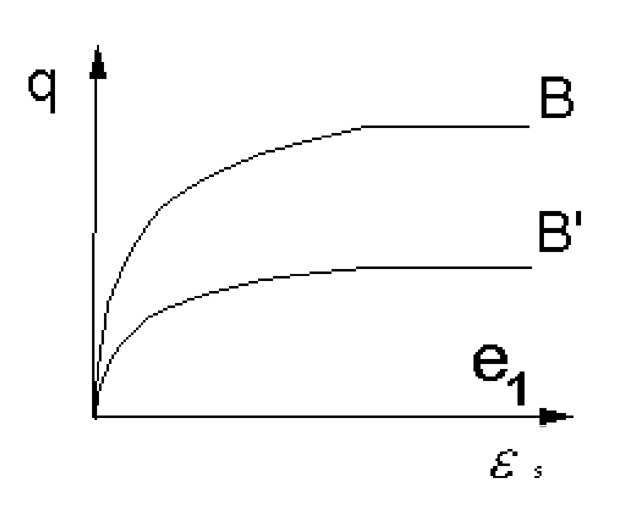
$$\eta = \frac{q}{p'}$$

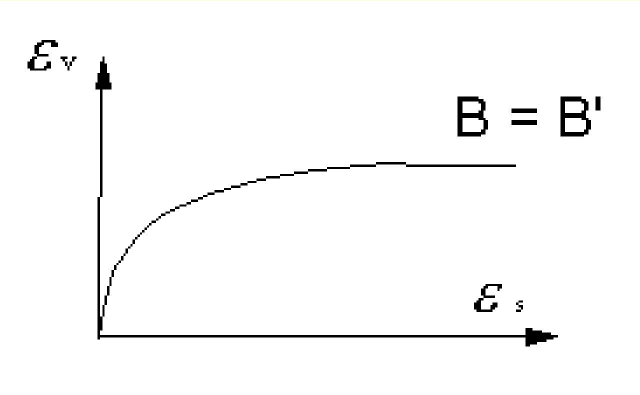
$$\varepsilon_v = tr(\varepsilon) = \varepsilon_1 + 2\varepsilon_3$$

$$\varepsilon_s = \frac{2}{3}\left(\varepsilon_1 - \varepsilon_3\right)$$

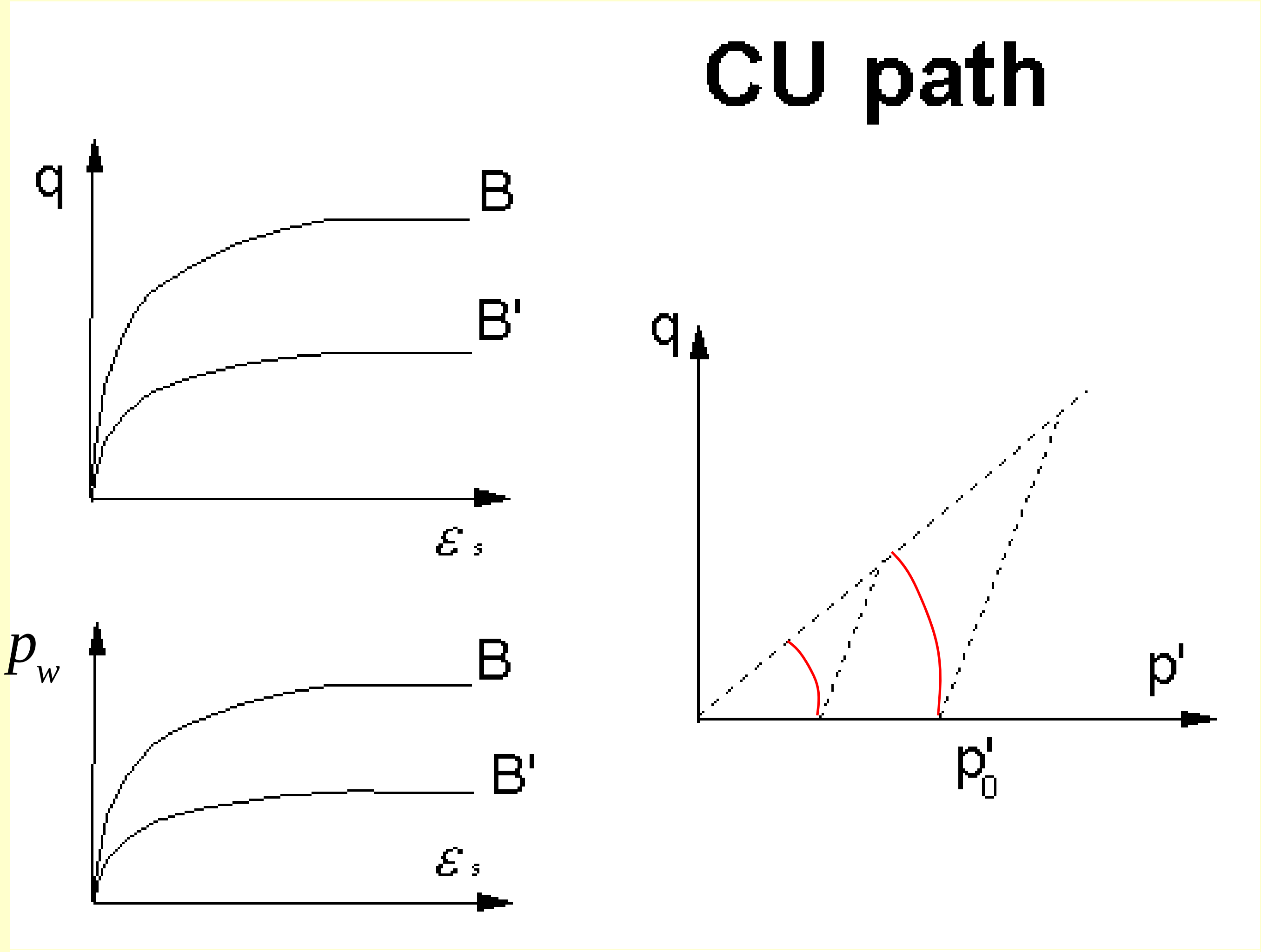
Consolidated Drained





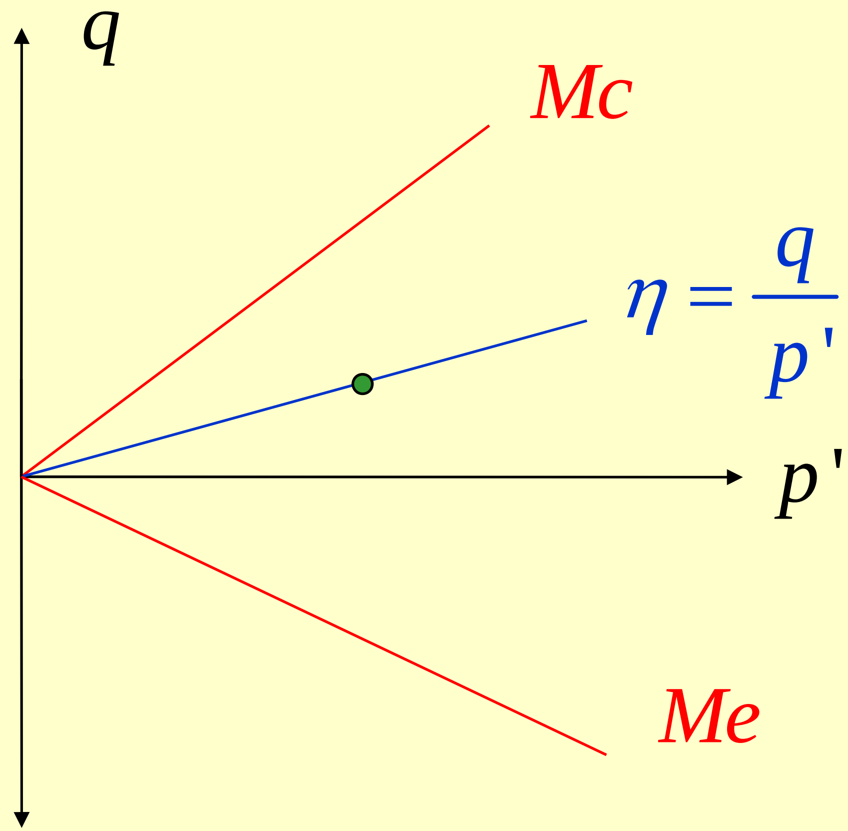


Consolidated Undrained



No volume change at failure

- **Compression and extension**



$$M = \frac{6\sin\phi}{3-\sin 3\theta \sin\phi}$$

$$M_c = \frac{6\sin\phi}{3-\sin\phi}$$

$$M_e = \frac{6\sin\phi}{3+\sin\phi}$$

$$\boxed{\frac{M_c}{M_e} = \frac{3+\sin\phi}{3-\sin\phi} > 1}$$

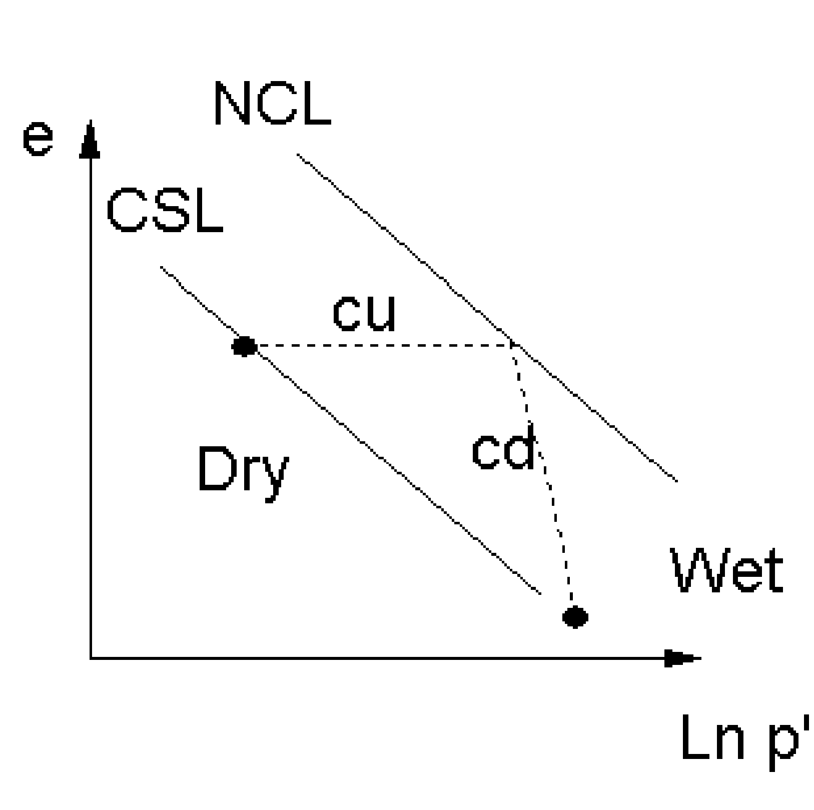
- **Conclusions CD**

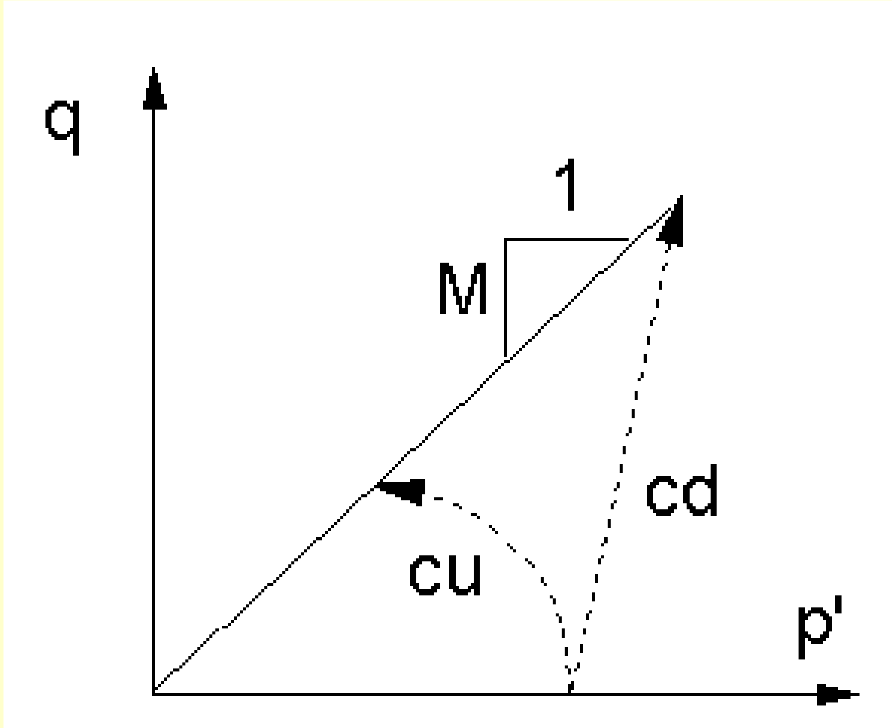
  - Material tends to compact
  - Failure at q/p=M
  - Strength increases with confining pressure
  - Failure takes place at constant volume

- **Conclusions CU**

  - PWP Increases
  - Failure at q/p=M
  - Strength increases with confining pressure
  - strength smaller than in CD
  - Failure takes place at constant Pw

Normal Consolidation and Critical State Lines





Critical State Models

Origin : work of:

- Drucker and Prager 1952
- Drucker, Gibson and Henkel 1957
- Cambridge group

Ingredients

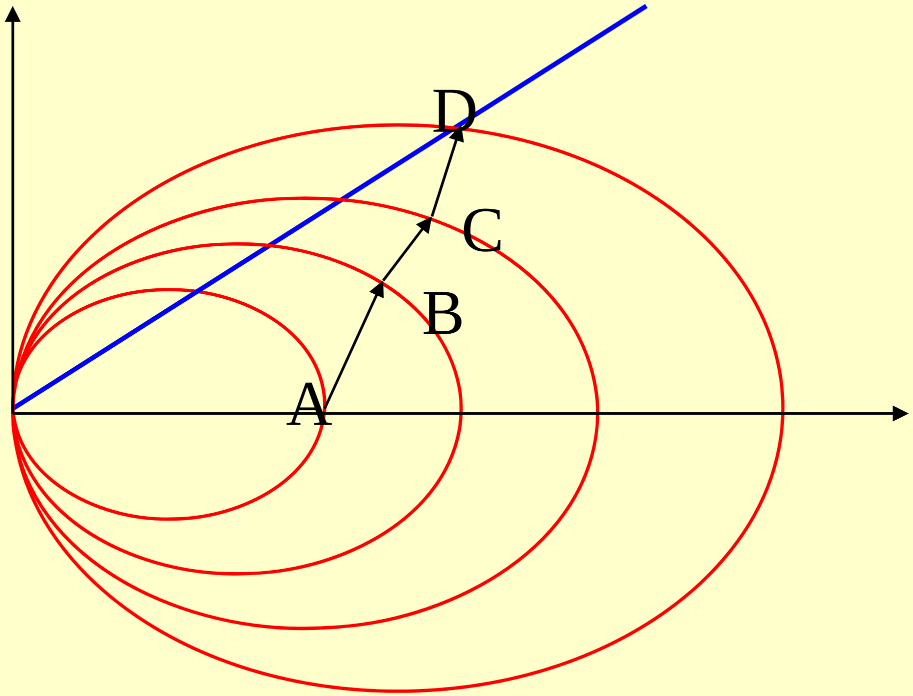
- Normal Consolidation and Critical State Lines
- Failure takes place at Critical State Line
- Hardening depends on volumetric plastic strain
- Plastic Potential and Yield surfaces

....

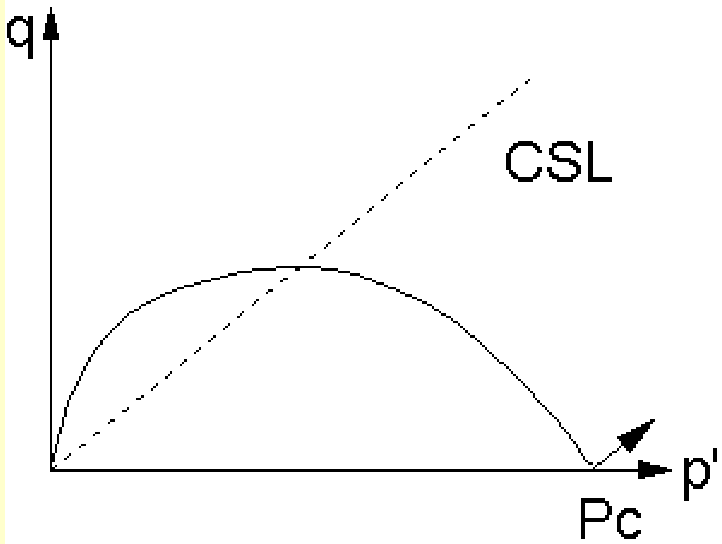
$$d\varepsilon^p = \frac{1}{h}\frac{\partial g}{\partial \sigma}\left(\frac{\partial f}{\partial \sigma}:d\sigma\right) \qquad h = -\frac{\partial f}{\partial p_c}\left(\frac{\partial p_c}{\partial \varepsilon^p}:\frac{\partial g}{\partial \sigma}\right)$$

$$h = -\frac{\partial f}{\partial p_c}\left(\frac{\partial p_c}{\partial \varepsilon_v{}^p}\frac{\partial g}{\partial p'} + \cancel{\frac{\partial p_c}{\partial \varepsilon_s{}^p}\frac{\partial g}{\partial q}}\right)$$

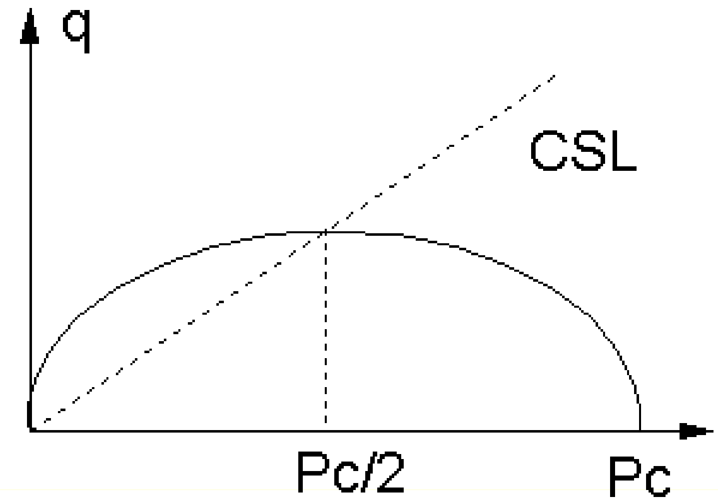
$$h = 0 \Rightarrow \begin{cases} \dfrac{\partial f}{\partial p_c} = 0 \\ \boxed{\dfrac{\partial g}{\partial p'} = 0} \end{cases}$$



$$f \equiv g = q + Mp'\ln\left(\frac{p'}{p_c}\right)$$



$$f \equiv g = q^2 + M^2 p'\left(p' - p_c\right)$$



Critical State Models

- Hardening Law

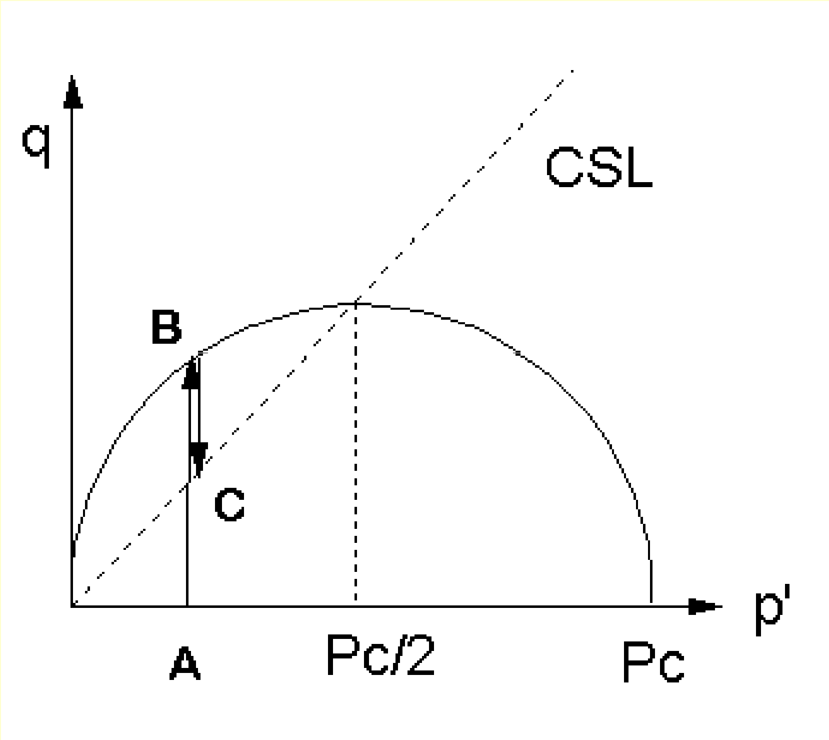
$$\boxed{\frac{\partial p_c}{\partial \varepsilon_v{}^p} = \frac{1+e}{\lambda-\kappa} p_c}$$

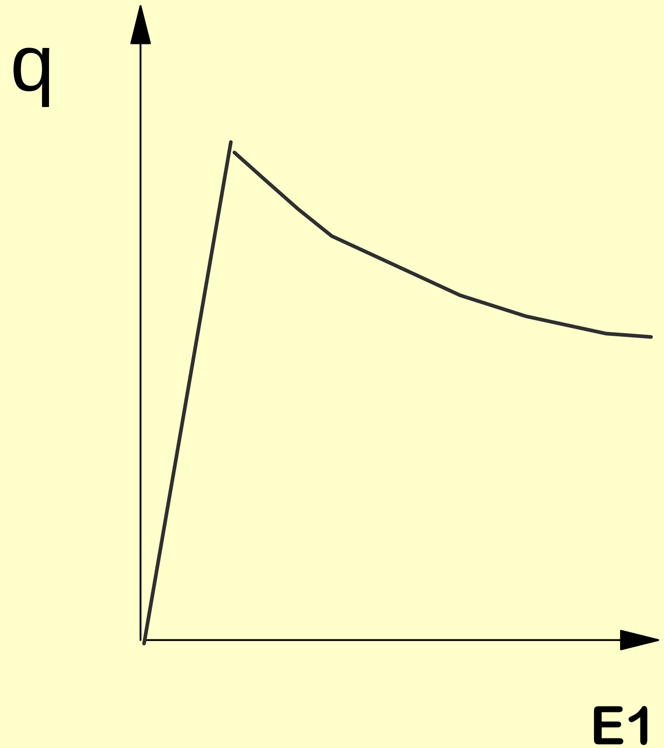
- Plastic Potential

$$\boxed{\frac{\partial g}{\partial p'} = 0} \quad \text{at} \quad \eta = M$$

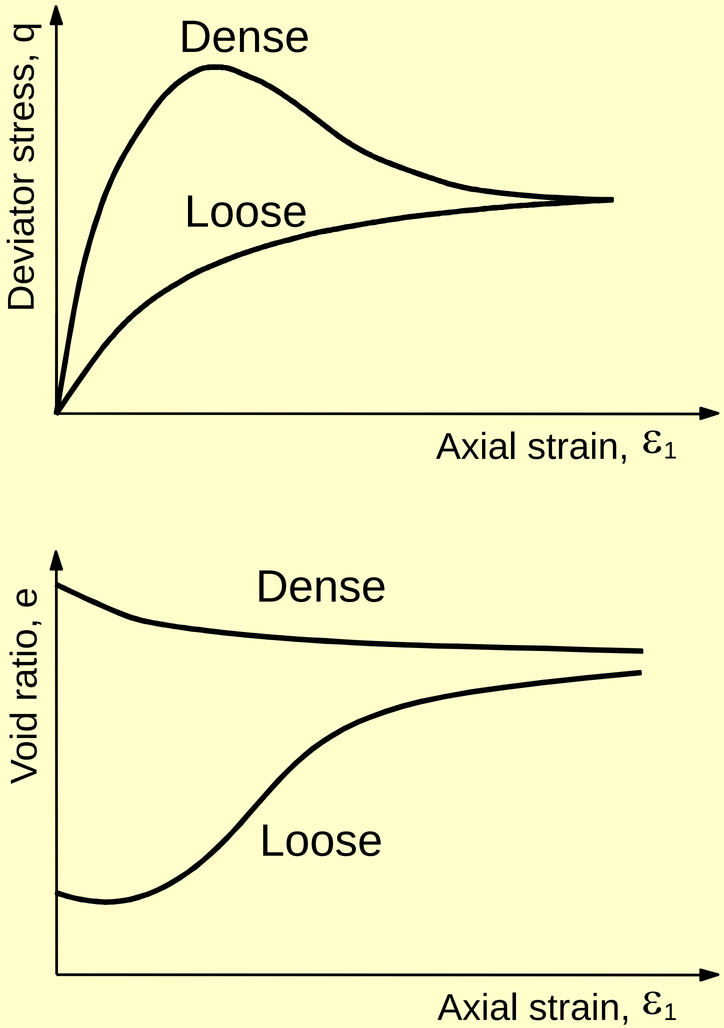
- Failure surfaces of Frictional Type

OverConsolidated Clays

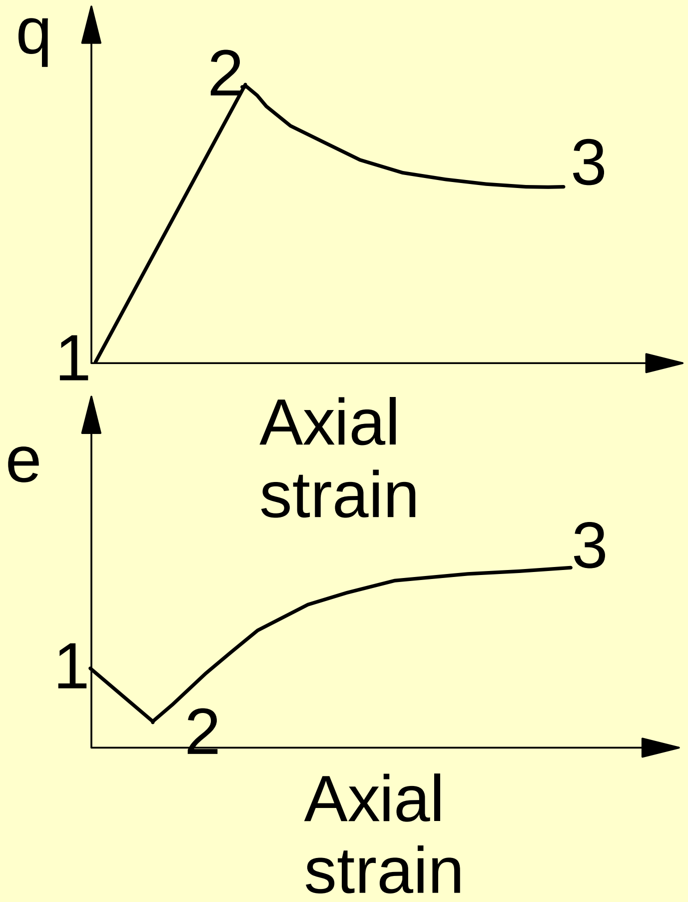


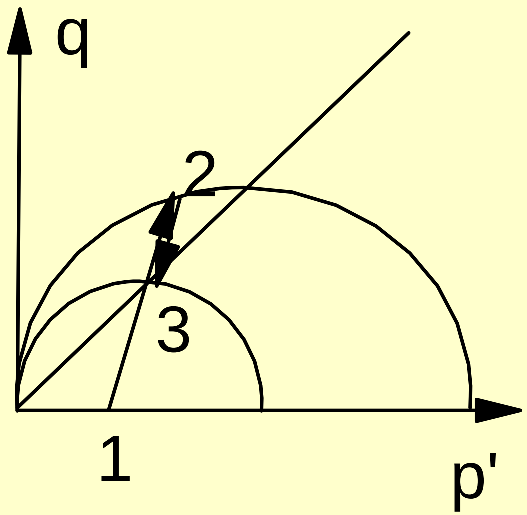


Behaviour of sands (CD)
Influence of Density

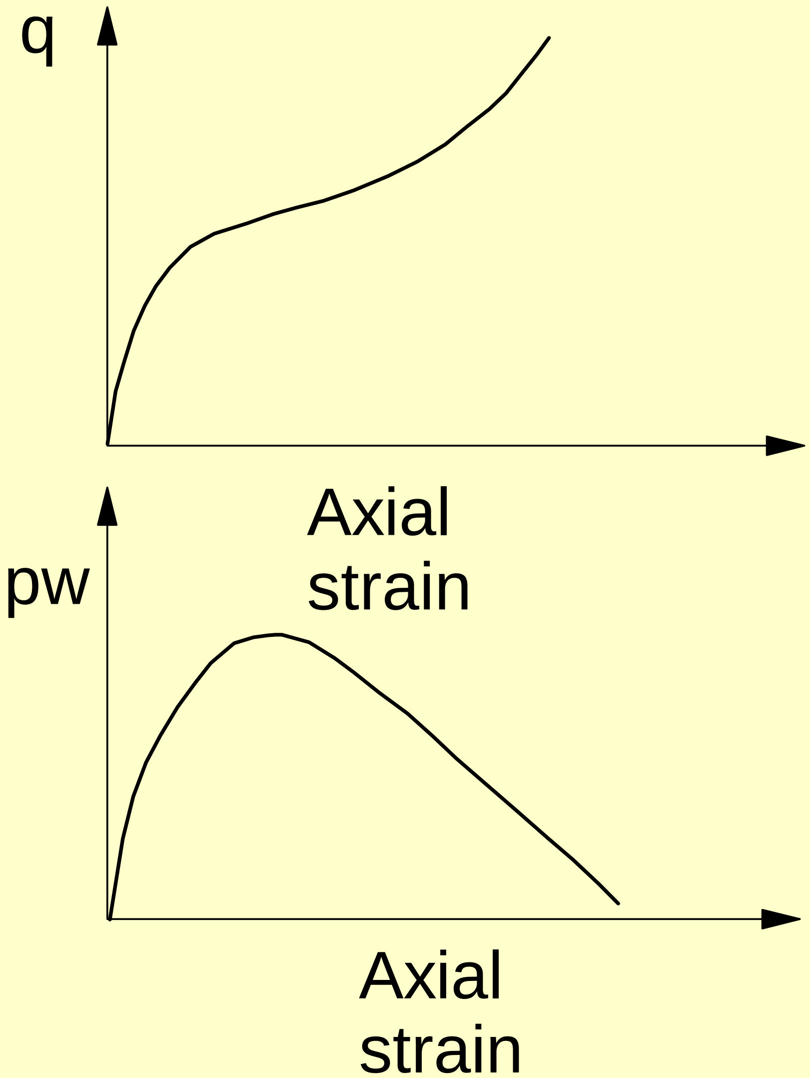


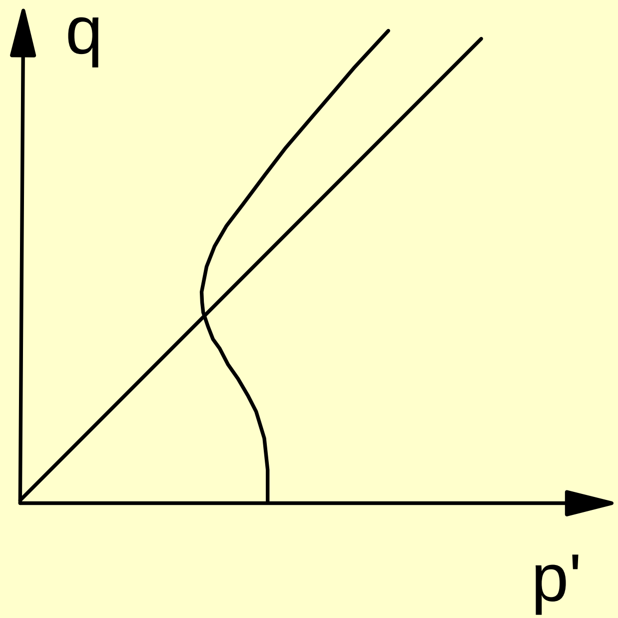
Sand Behaviour : CD
(Predictions)



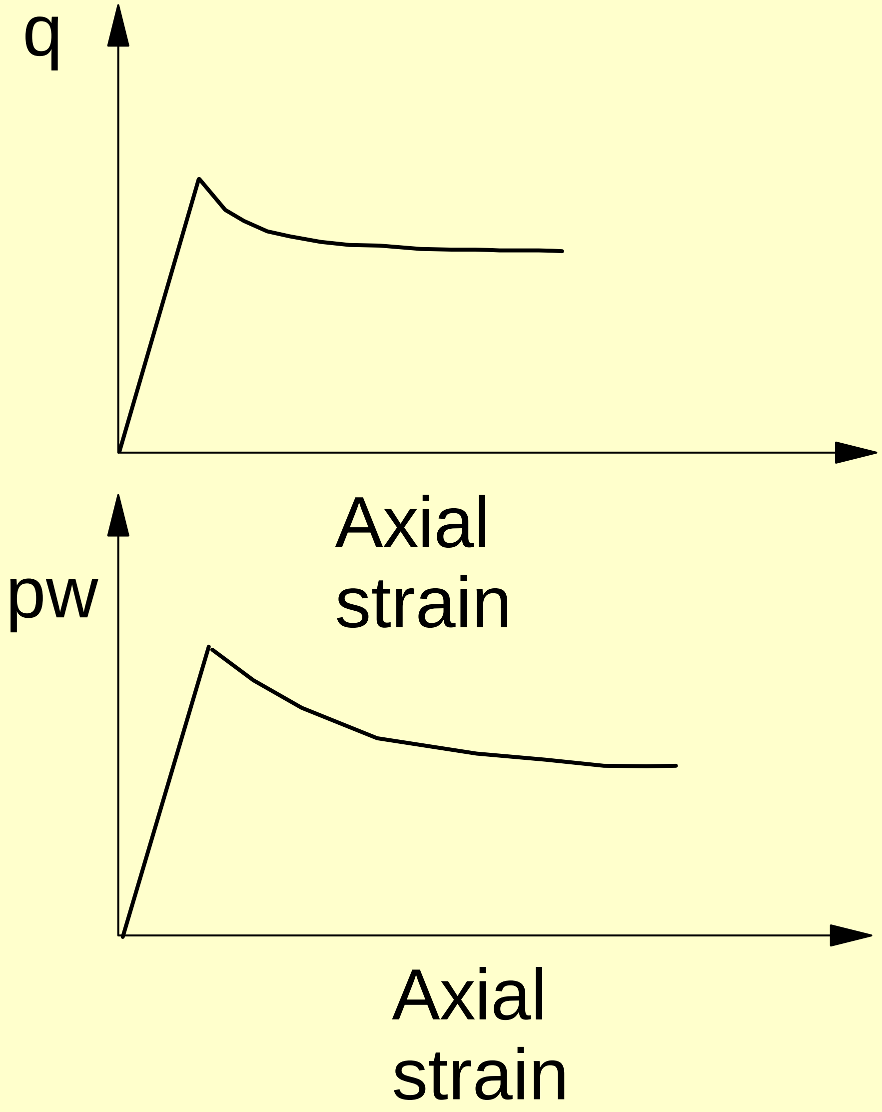


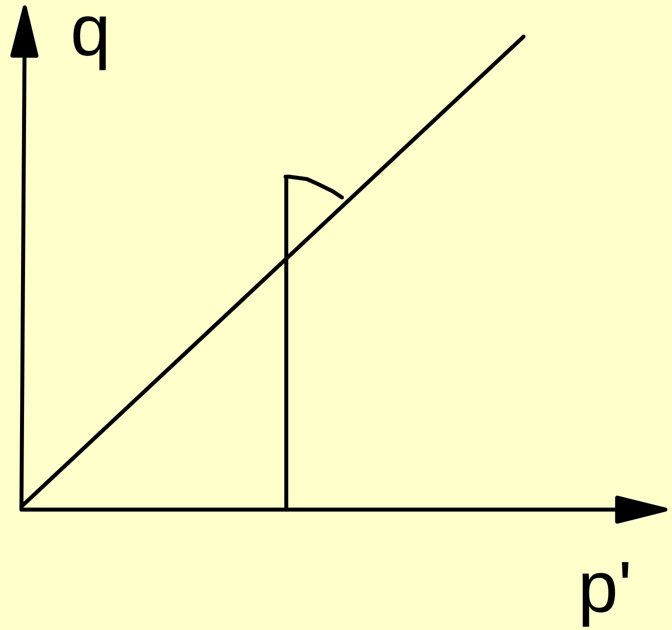
Sand Behaviour : CU



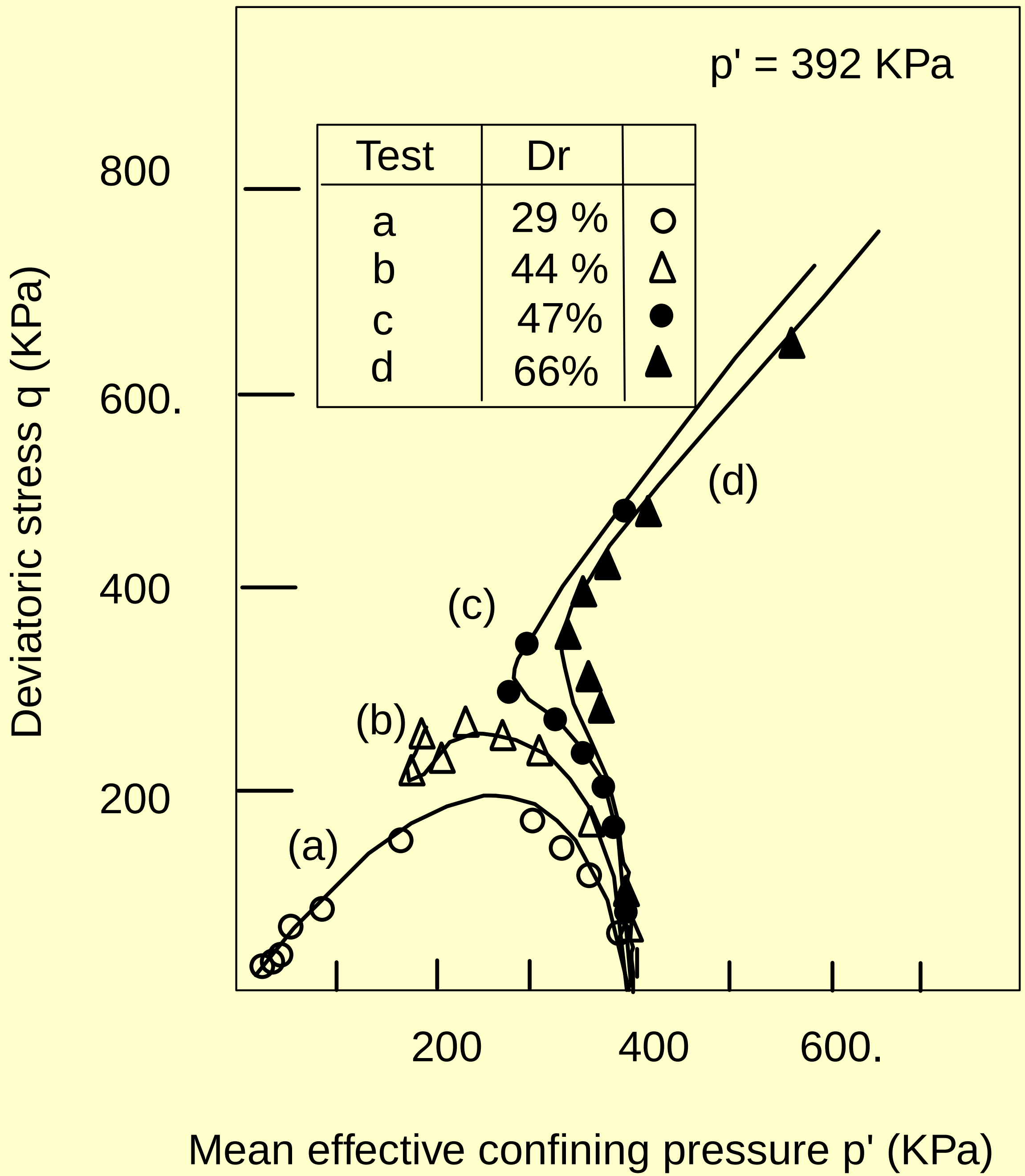


Sand Behaviour : CU (Predictions)

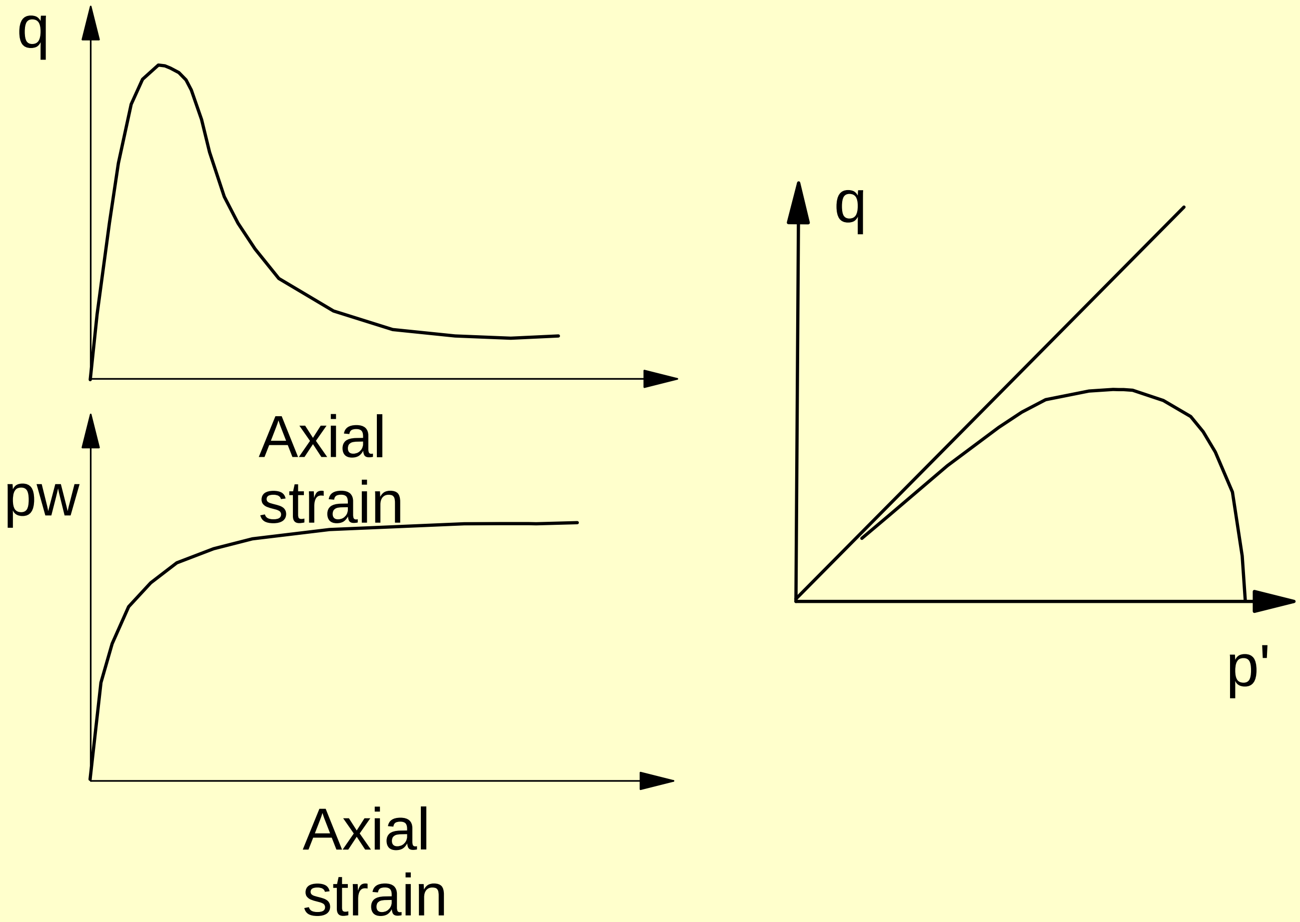




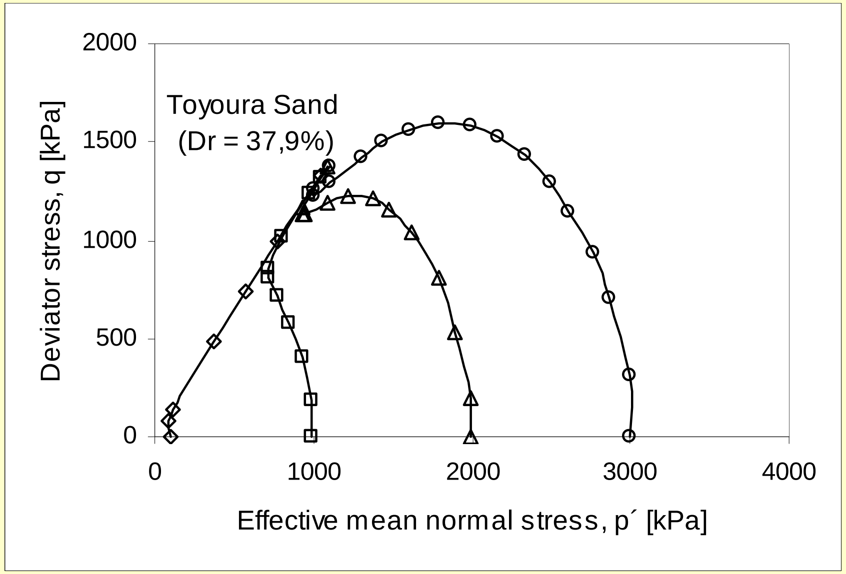
CU Tests
(Castro 69)

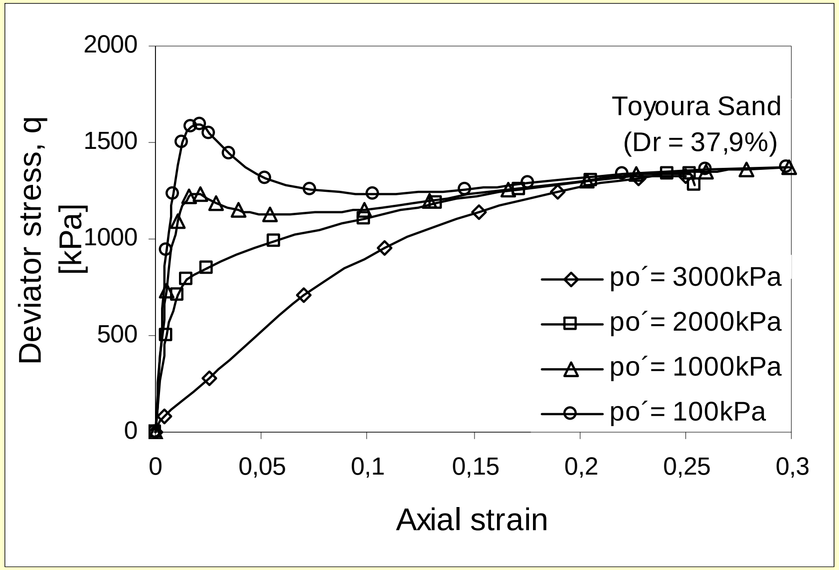


Liquefaction of very loose sands (CU)



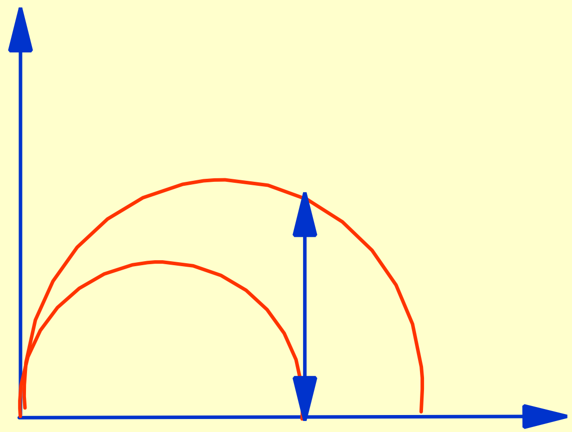
Behaviour of sands (CU)
Influence of Confining Pressure

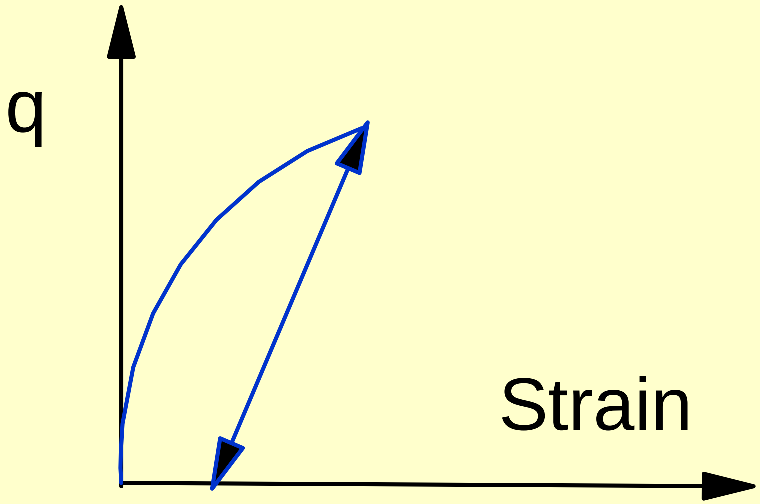


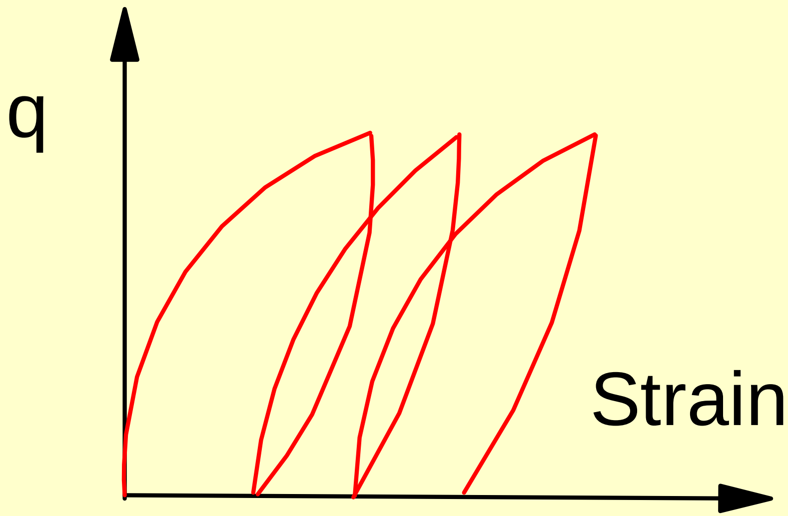


Limitations of CS models

(Drained)

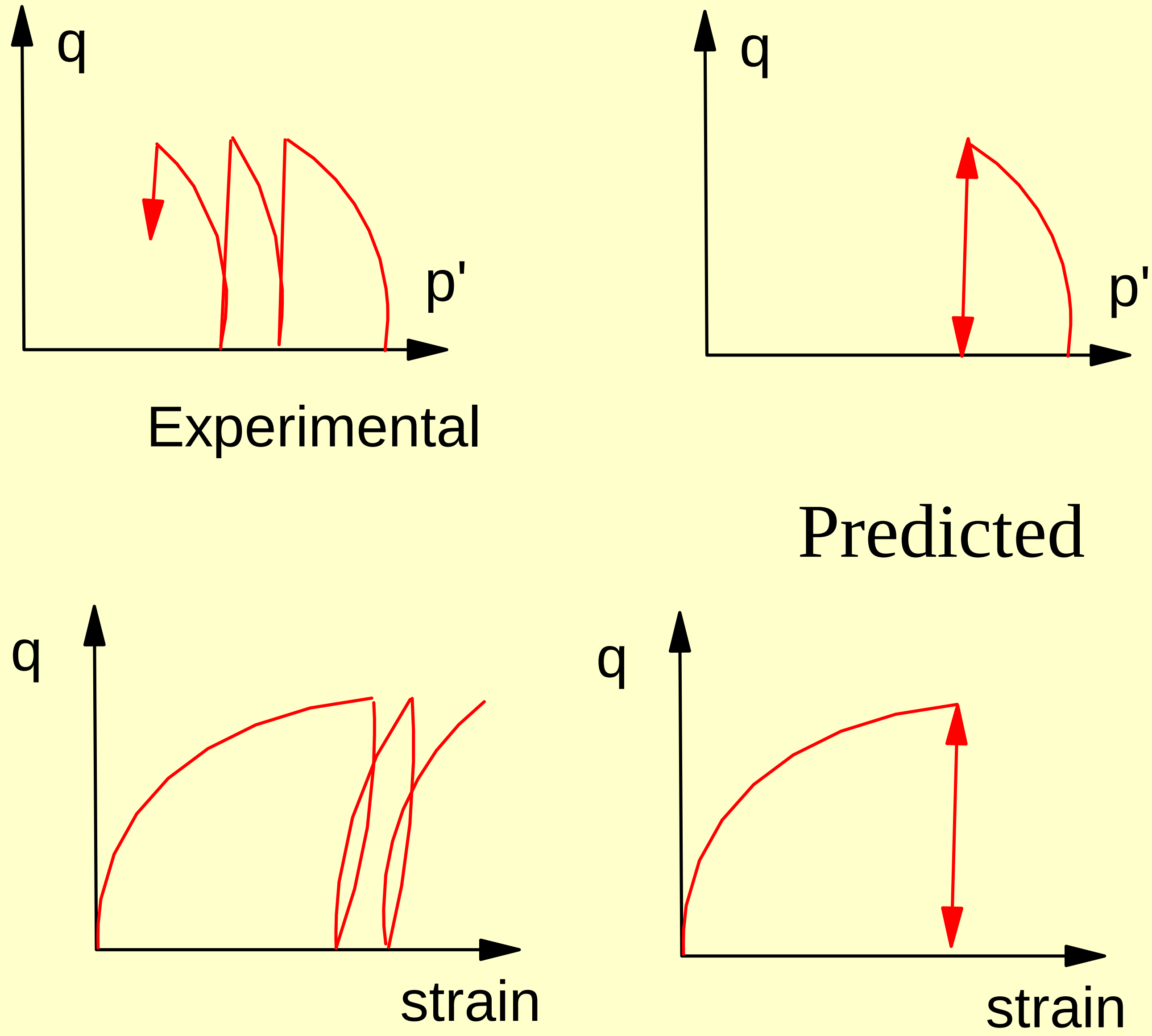






Limitations of CS models

(Undrained)

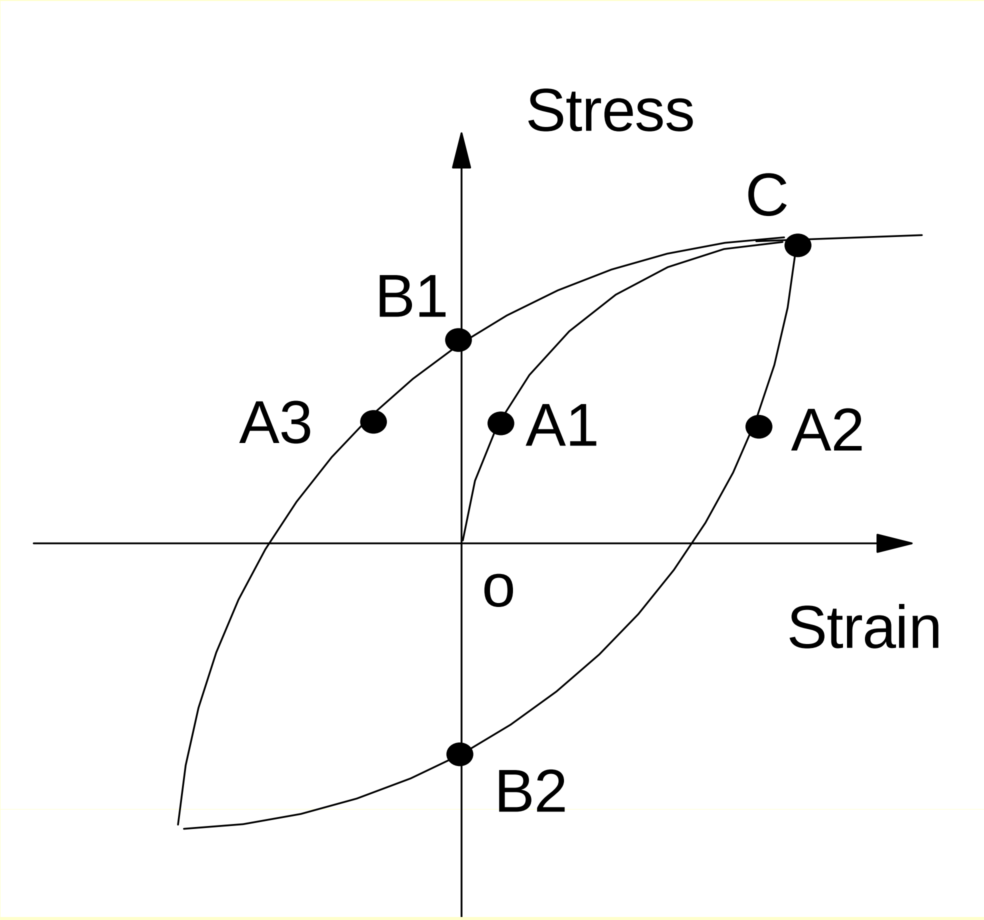


Generalized Plasticity

Contents

- Introduction
- Classical and Critical State Plasticity
  - Failure surfaces
  - Classical EPlasticity
  - Critical State Plasticity
- Generalized Plasticity
  - Basic Model
  - Bounded materials
  - State Parameter
  - Unsaturated
- Fluidized geomaterials
  - Rheology
  - Dilatancy
  - A Perzyna viscoplasticity approach

Generalized plasticity

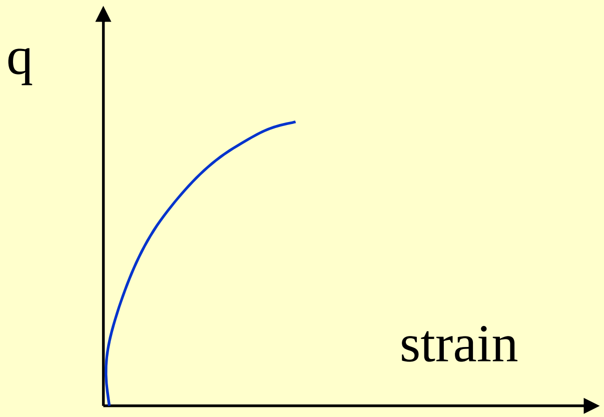


$$d\varepsilon = C : d\sigma$$

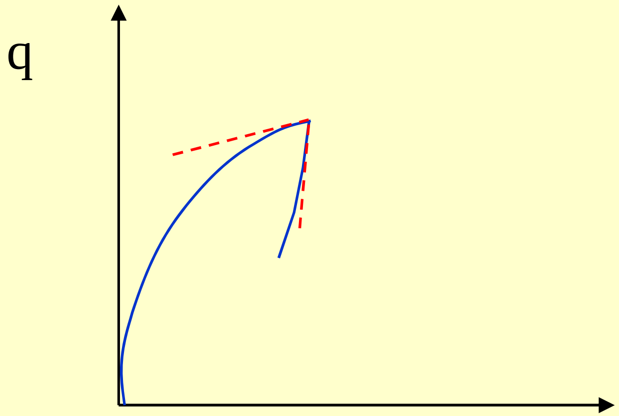
C depends on:

- stress level
- history
- direction of stress increment

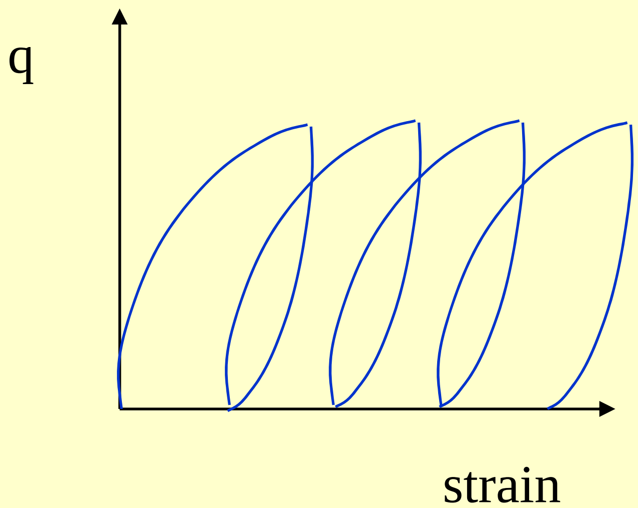
- Level of stress

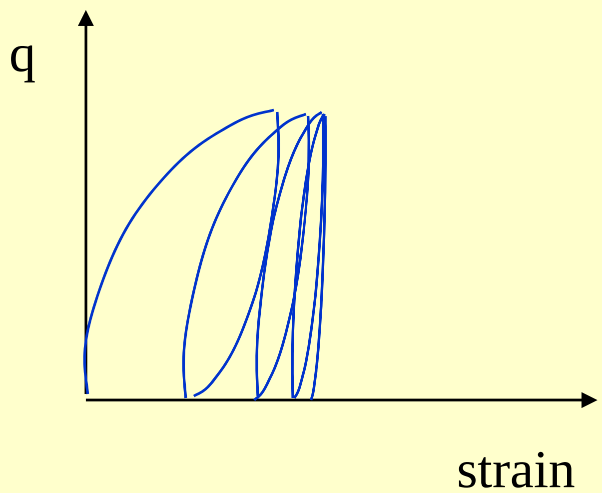


- Direction of stress increment



- history





- Direction of stress increment

  ➡ Introduce a direction n such that

$$d\varepsilon = C_L : d\sigma \quad \text{for} \quad n : d\sigma > 0$$

$$d\varepsilon = C_U : d\sigma \quad \text{for} \quad n : d\sigma > 0$$

$n : d\sigma > 0$ Loading

$n : d\sigma = 0$ Neutral loading

$n : d\sigma < 0$ Unloading

  ➡ Continuity between L-U

$$\boxed{\begin{aligned} C_{\mathrm{L}} &= C^e + \frac{1}{H_L} n_{gL} \otimes n \\ C_{\mathrm{U}} &= C^e + \frac{1}{H_U} n_{gU} \otimes n \end{aligned}}$$

Decomposition of strain increment

$$d\varepsilon = d\varepsilon^{e} + d\varepsilon^{p}$$

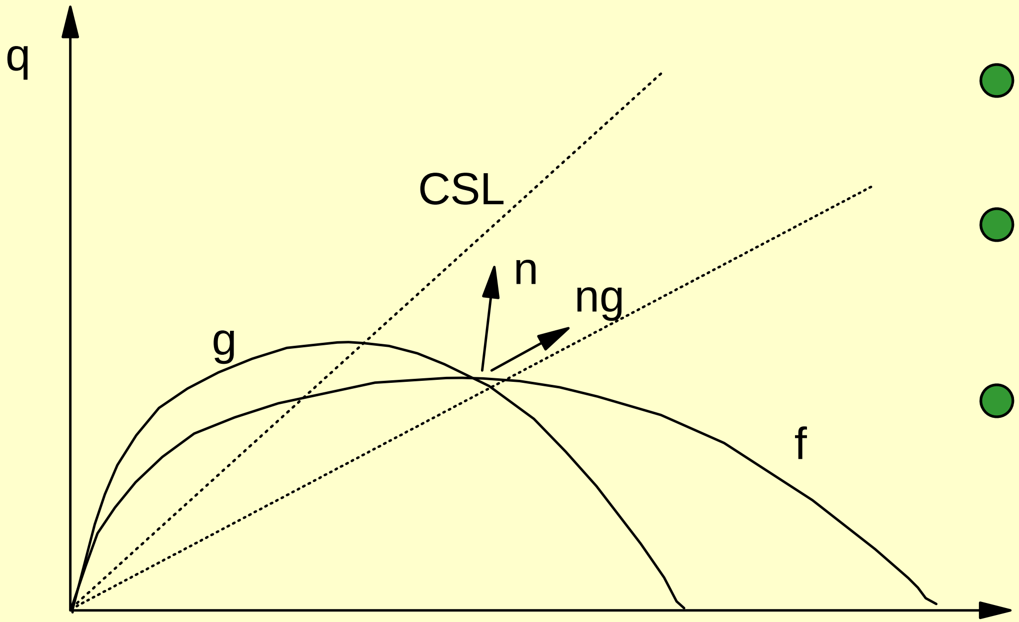
$$d\varepsilon^{e} = C^{e} : d\sigma$$

$$d\varepsilon^{p} = \left(\frac{1}{H_{L/U}} n_{gL/U} \otimes n \right) : d\sigma$$

Neccesary items

- Loading-Unloading discriminating direction n
- Direction of Plastic flow $n_{gL/U}$
- Plastic Modulus $H_{L/U}$
- Elastic constants

Classical Plasticity



- n given by normal to f
- n_g given by normal to g
- H given by consistency condition (unloading is elastic)

A Generalized Plasticity Model for sand (1991)

- ng

$$n_g = \left(n_{gv}, n_{gs}\right)$$

$$d_g = \left(1+\alpha\right)\left(M_g - \eta\right)$$

$$n_{gv} = d_g /(1+d_g^{\ 2})^{1/2}$$

$$n_{gs} = 1/(1+d_g^{\ 2})^{1/2}$$

- n

$$n = \left(n_v, n_s\right)$$

$$d = \left(1+\alpha\right)\left(M - \eta\right)$$

$$n_v = d /(1+d^{\ 2})^{1/2}$$

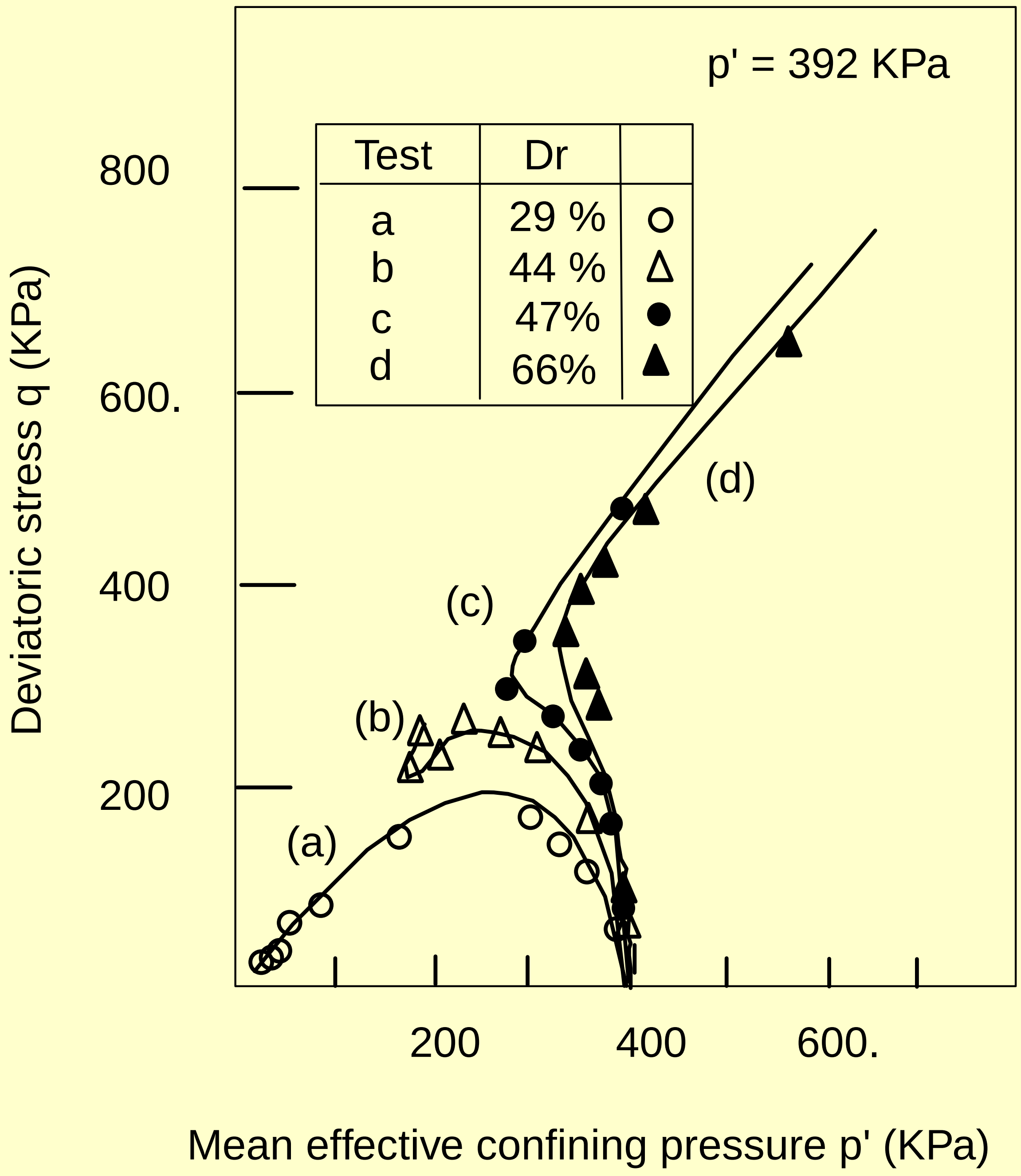
$$n_s = 1/(1+d^{\ 2})^{1/2}$$

$$\boxed{H_L = H_0 p' H_f \left\{H_v + H_s\right\}}$$

$$H_f = \left(1 - \frac{\eta}{\eta_f}\right)^4 \qquad \eta_f = \left(1 + \frac{1}{\alpha}\right) M_f \qquad H_v = \left(1 - \frac{\eta}{M_g}\right)$$

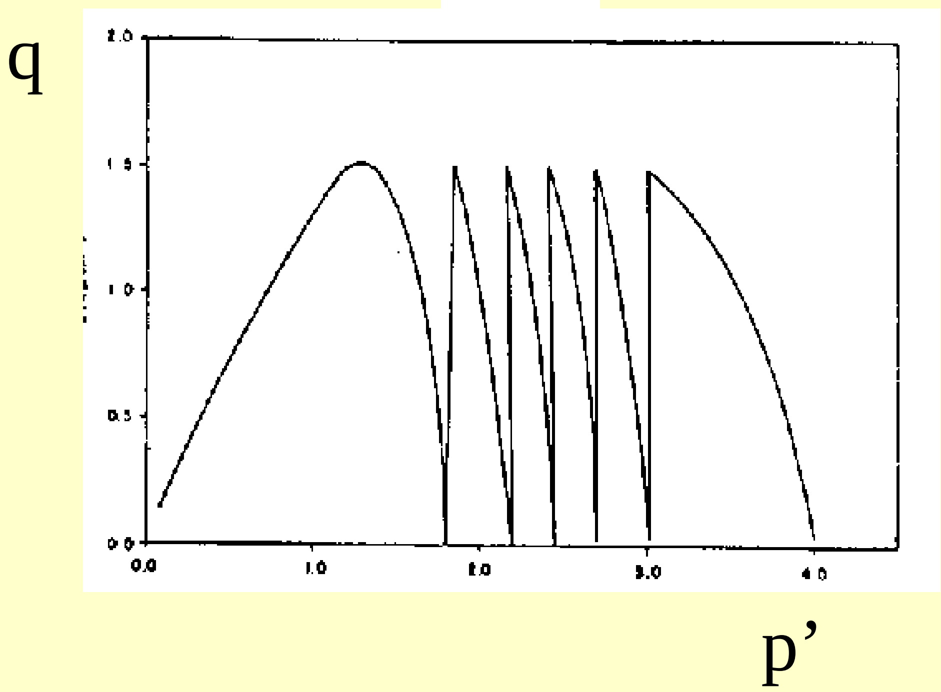
$$H_s = \beta_0 \beta_1 \exp(-\beta_0 \xi)$$

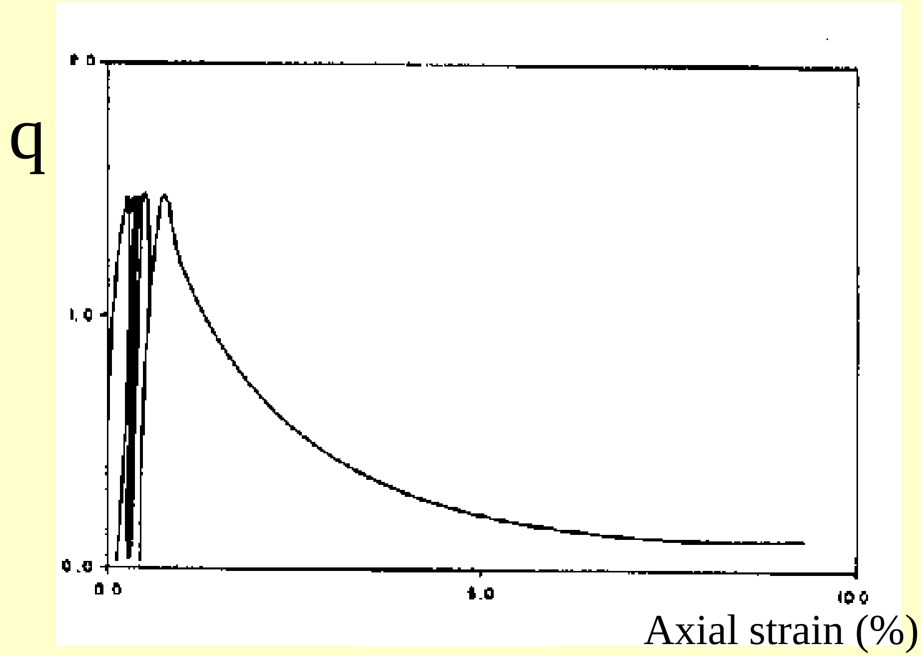
CU Tests
(Castro 69)



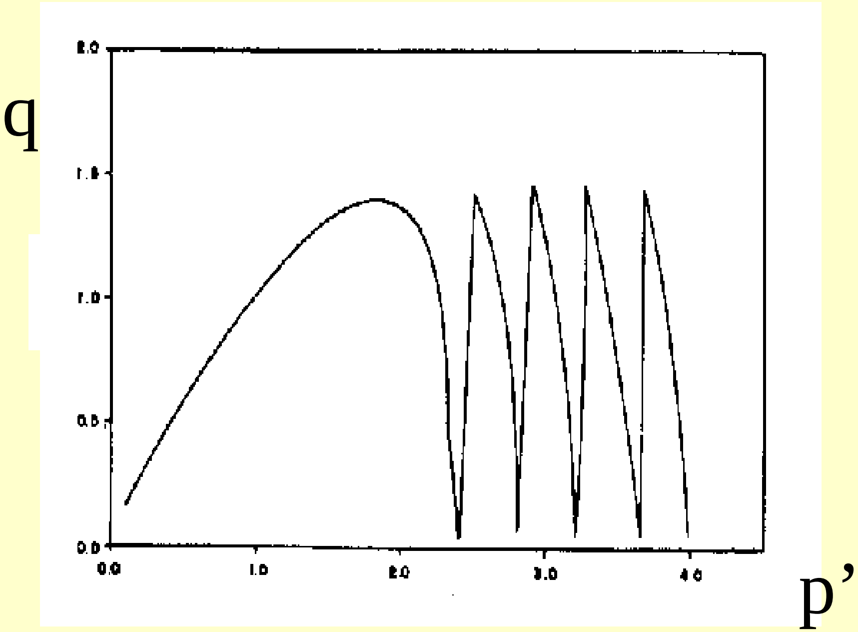
Liquefaction of very loose sands

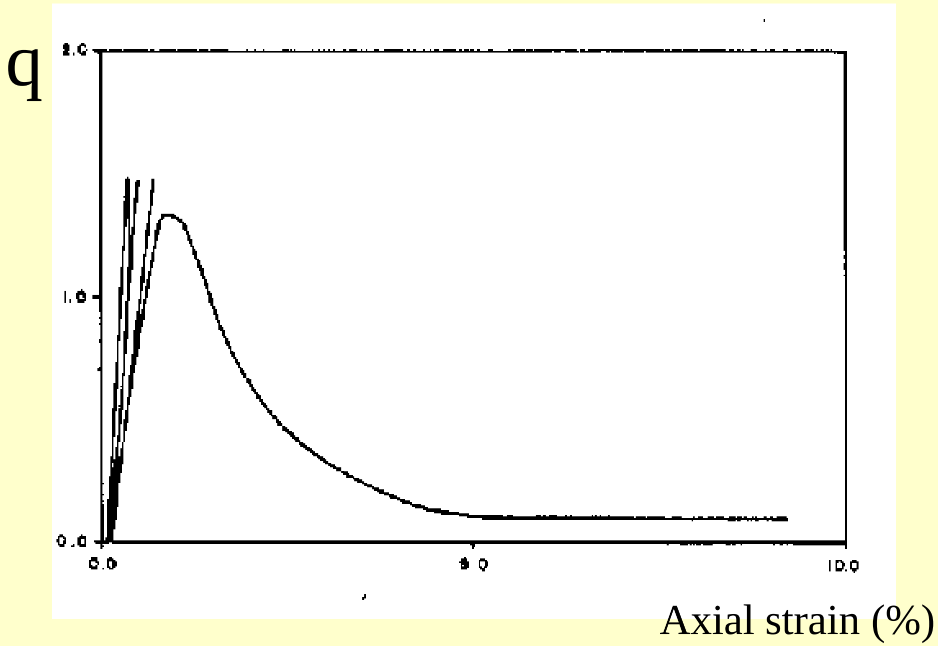
Experimental





Predicted





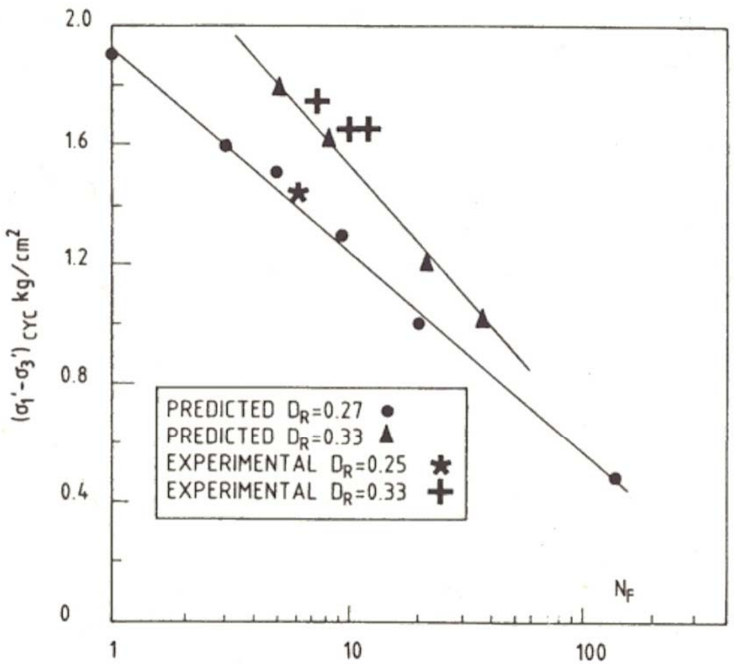


Figure 15. Influence of cyclic deviatoric stress on the number of cycles to cause liquefaction

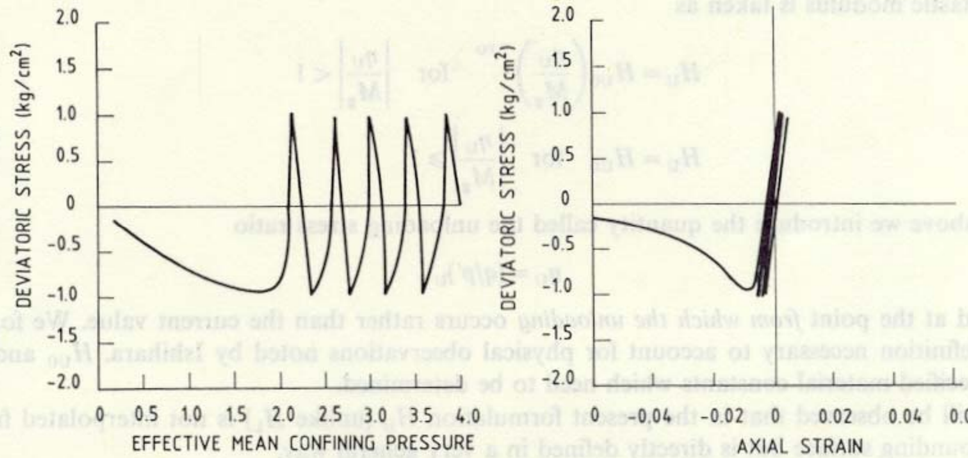


Figure 13. Liquefaction of Banding sand under two-way cyclic loading (predicted)

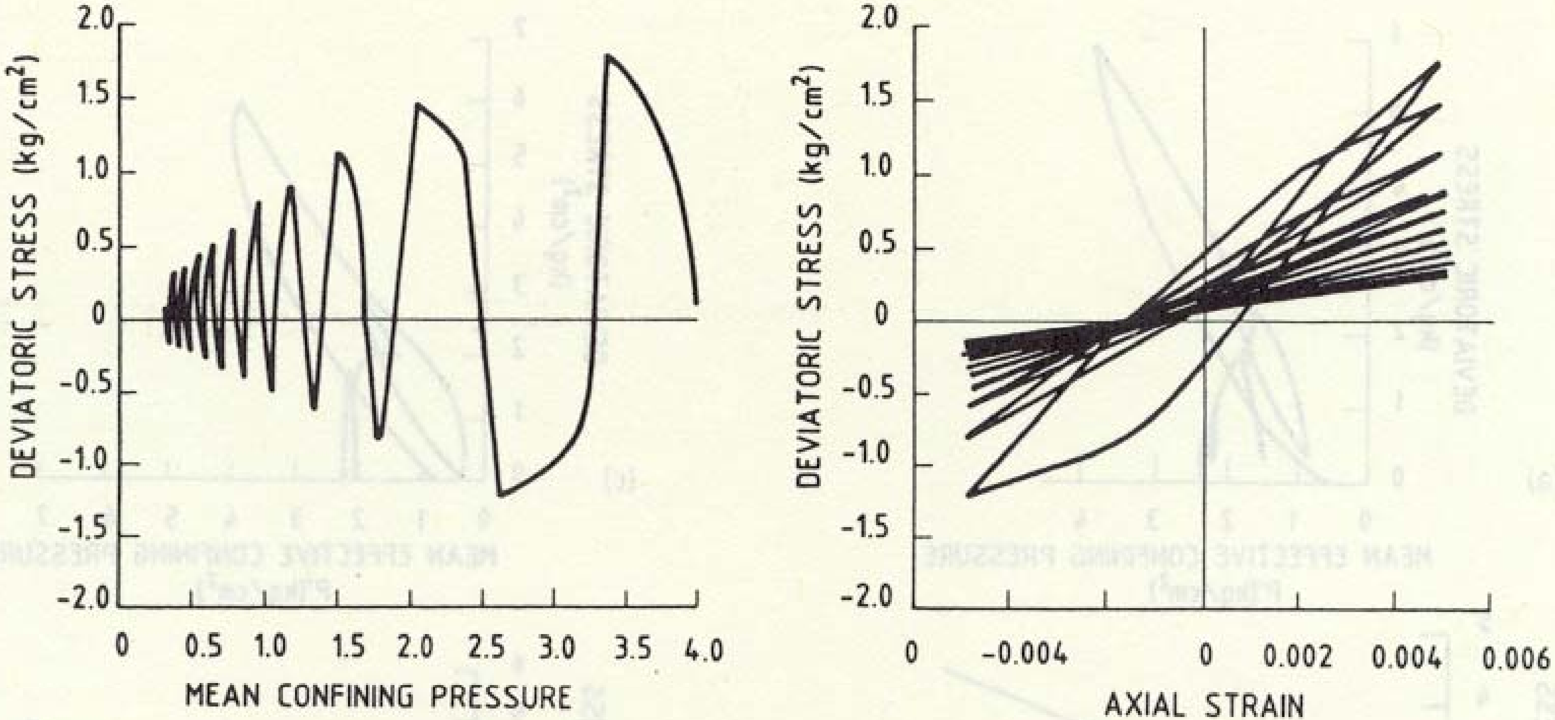
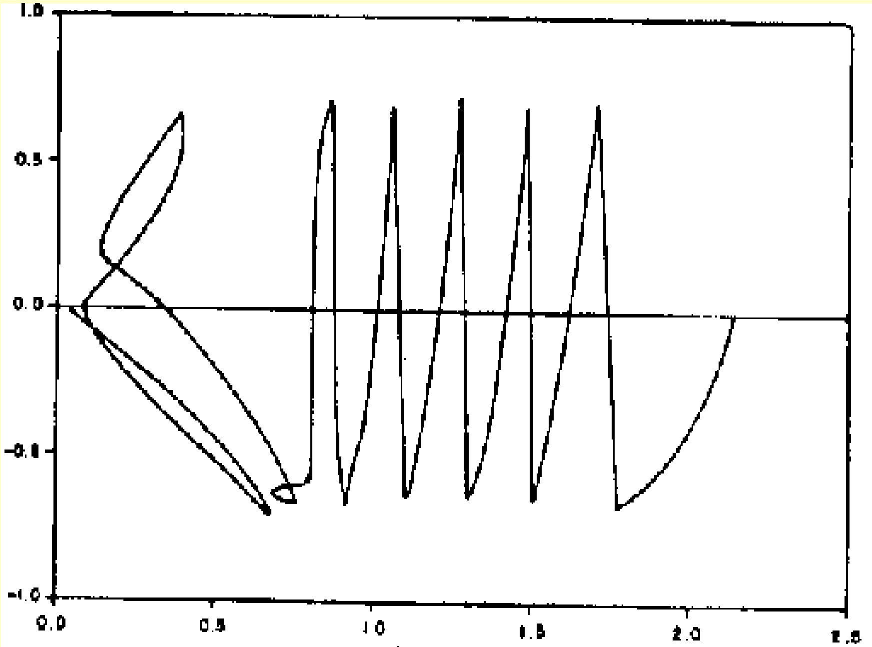
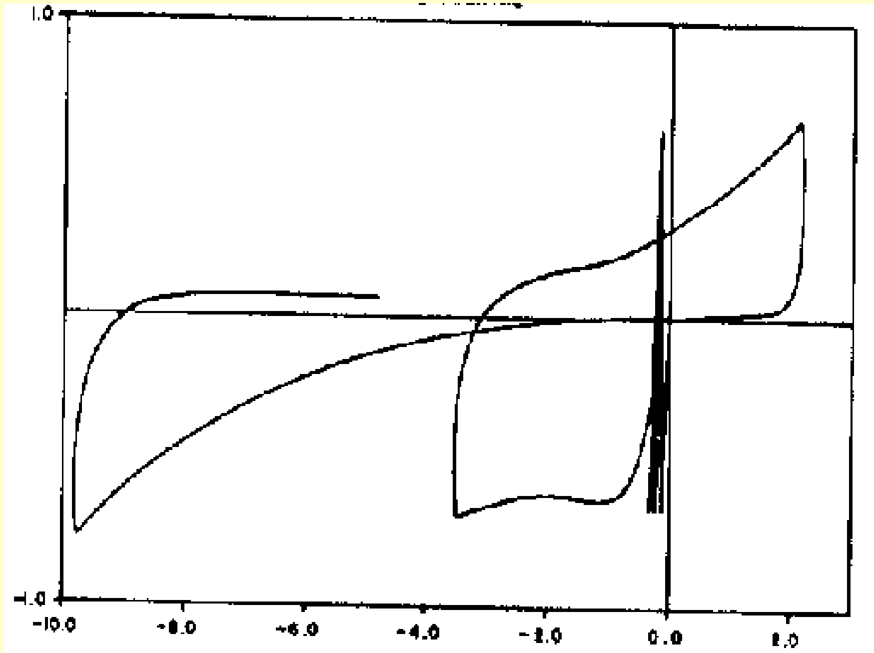


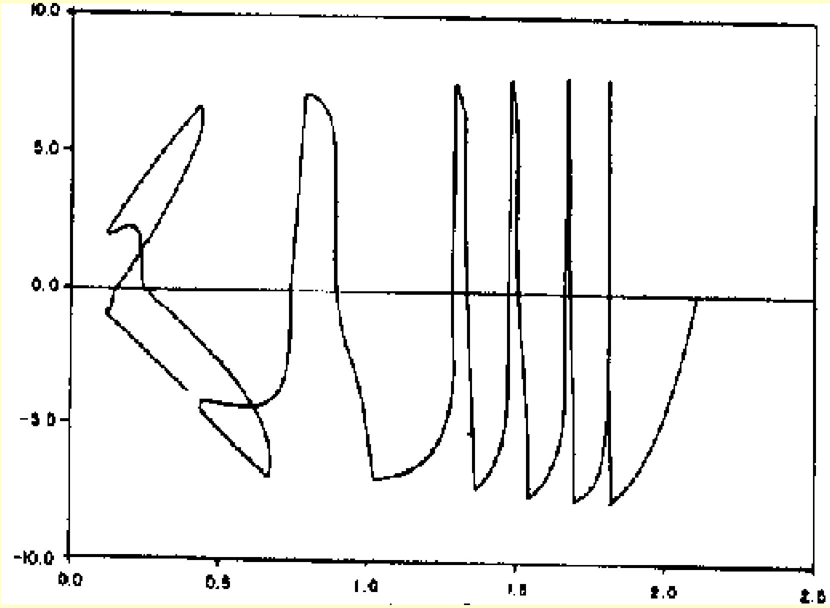
Figure 14. Strain-controlled cyclic triaxial test on loose Banding sand (predicted)

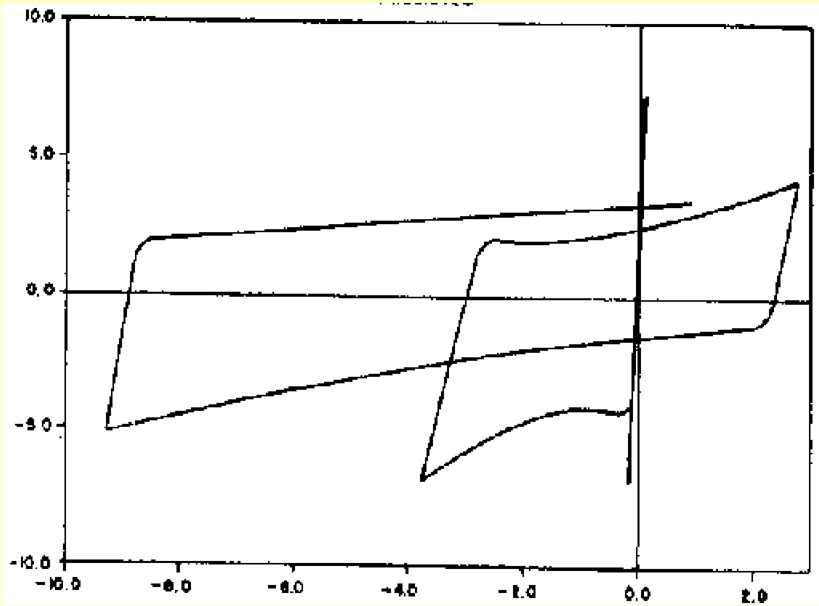
Cyclic Mobility (Experiments)



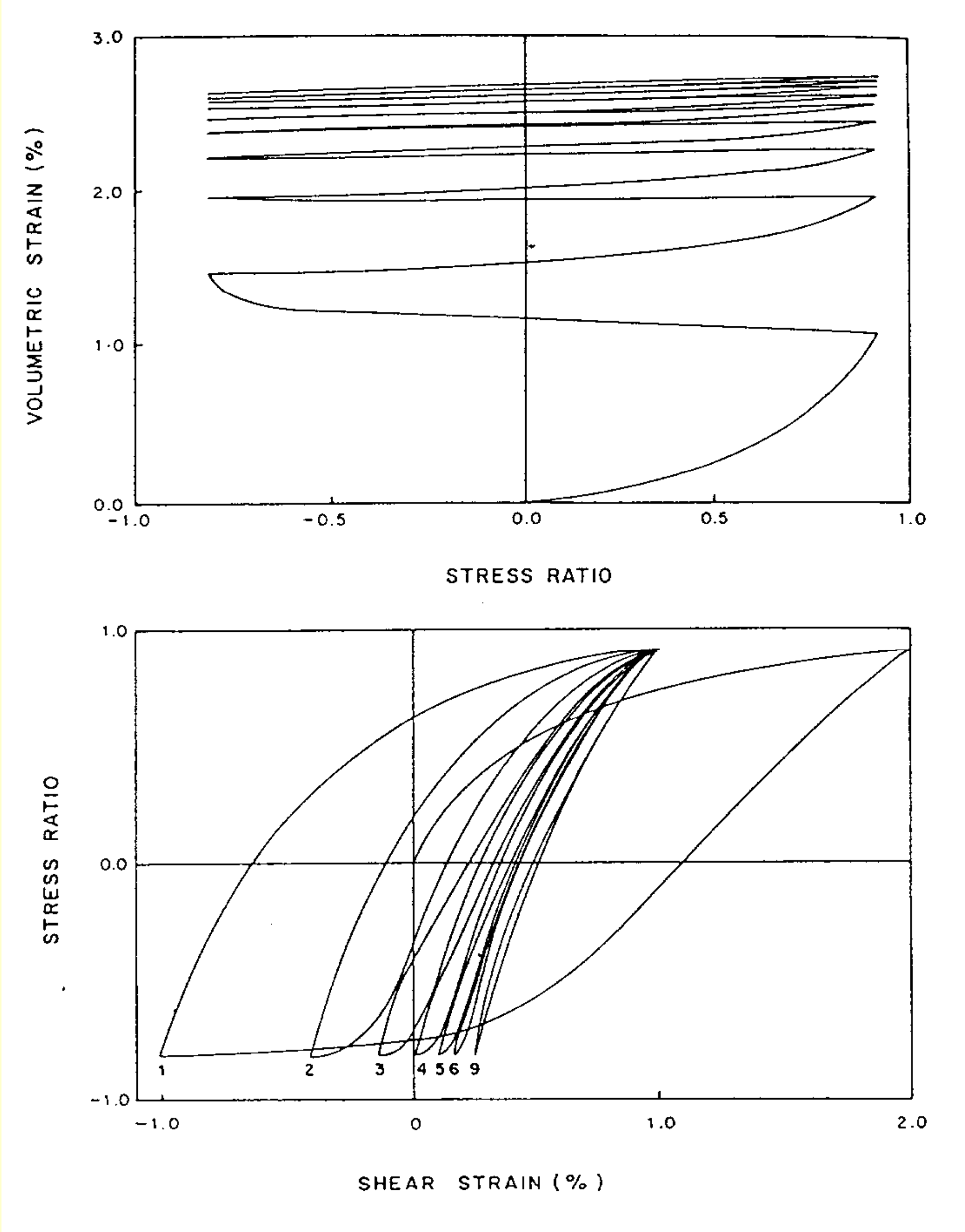


Cyclic Mobility (Predicted)





$$H_D = \exp\left(-\gamma_d \varepsilon_v{}^p\right)$$



3a2 Debonding

Contents

- Introduction
- Classical and Critical State Plasticity
  - Failure surfaces
  - Classical EPlasticity
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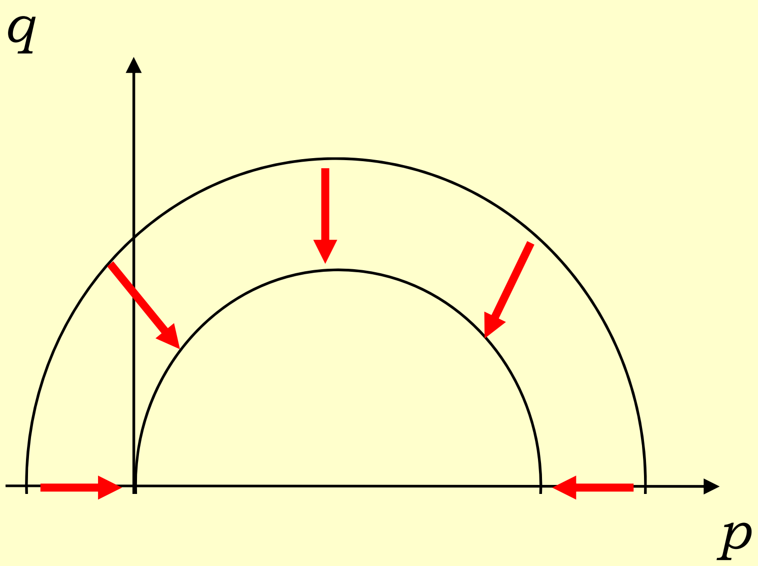
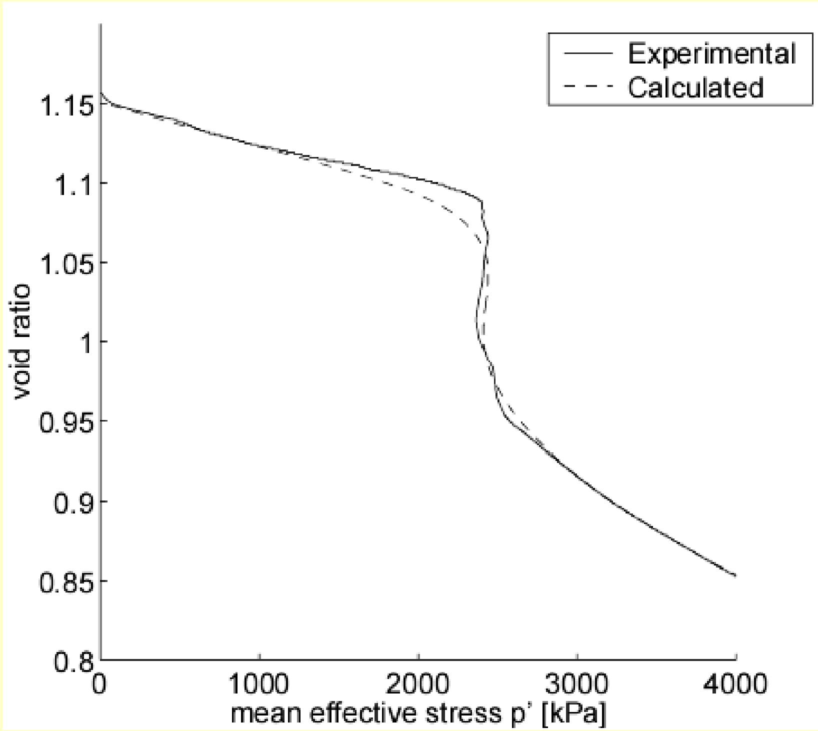
  - State Parameter
  - Unsaturated
- Fluidized geomaterials
  - Rheology
  - Dilatancy
  - A Perzyna viscoplasticity approach

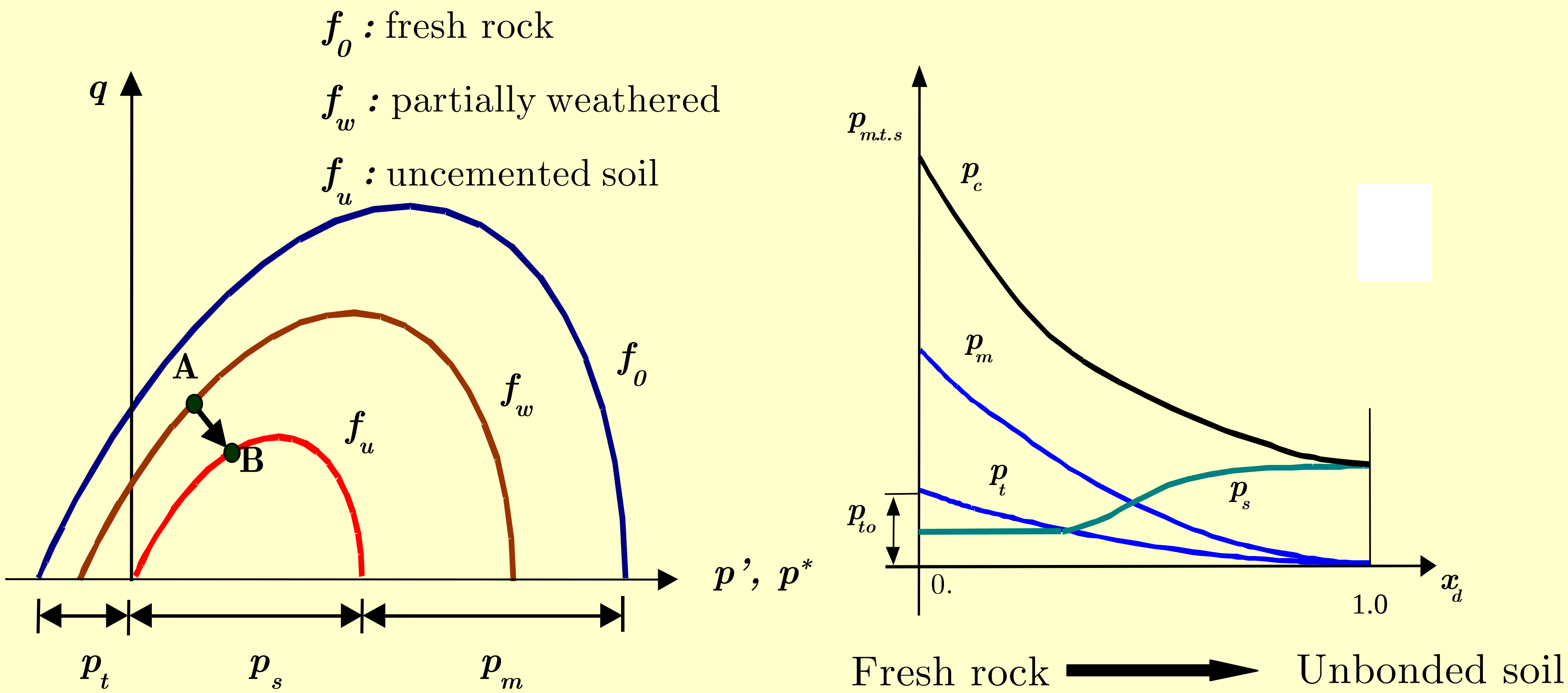
Debonding

Reduction of yield surface size



Lagioia & Nova, 1995



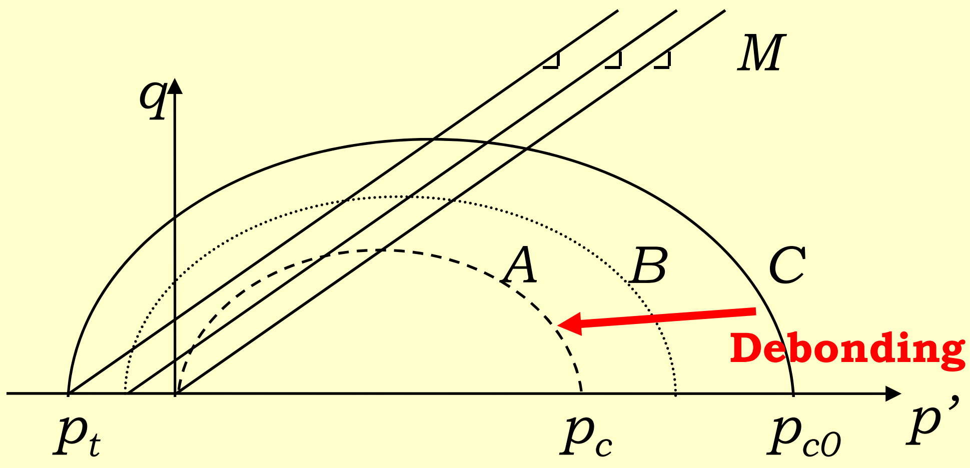


f_0 : fresh rock
f_w : partially weathered
f_u : uncemented soil
q
A
B
f_0
f_w
f_u
p', p^*
p_t
p_s
p_m
$p_{m.t.s}$
p_c
p_m
p_t
p_s
p_{to}
0.
1.0
x_d
Fresh rock
Unbonded soil

GPM for bonded geomaterials

Gens & Nova, 1993

Lagioia & Nova, **1995**



Introduce $p^* = p' + \boxed{p_t}$

$\eta^* = q / p^*$

$$\boxed{p_t = p_{t0} \exp\left(-\rho_t \varepsilon\right)}$$

$$H_L = \left(H_0 p^* - \boxed{H_b}\right) H_f^* \left(H_v^* + H_s\right) H_{DM}^*$$

$$\boxed{H_b = b_1 \varepsilon_v^p \exp(-b_2 \varepsilon_v^p)}$$

$$H_f^* = \left(1 - \frac{\eta^*}{\eta_f}\right) \qquad H_v^* = \left(1 - \frac{\eta^*}{M_g}\right) \qquad H_{DM}^* = \left(\underline{\varsigma_{\max}^*} / \varsigma\right)$$

A generalized plasticity model for debonding
(JA Fernández Merodo et al 2003)

Introduce $p^* = p' + p_t$ with $p_t = p_{t0} \exp\left(-\rho_t \varepsilon\right)$

$$\eta^* = q / p^*$$

$$H_L = \left(H_0 p^* \boxed{- H_b}\right) H_f^* \left(H_v^* + H_s\right) H_{DM}^*$$

$$H_f^* = \left(1 - \frac{\eta^*}{\eta_f}\right) \qquad H_{DM} = \left(\varsigma_{\max}^* / \varsigma\right)$$

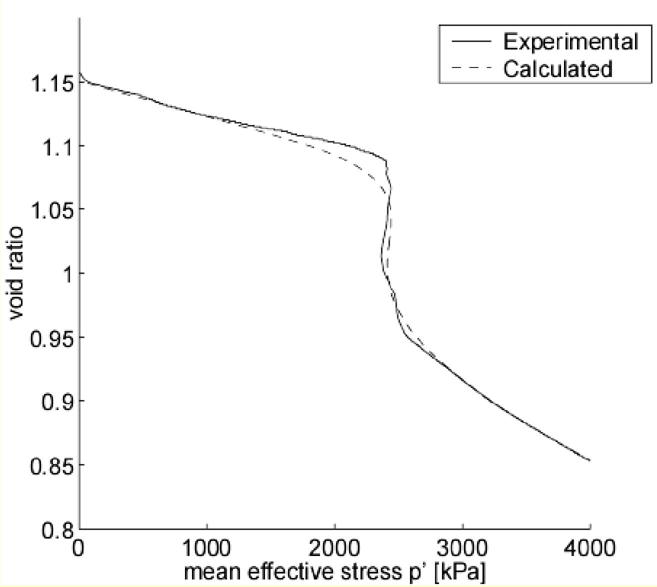
$$H_v^* = \left(1 - \frac{\eta^*}{M_g}\right)$$

$$\boxed{H_b = b_1 \varepsilon_v^p \exp(-b_2 \varepsilon_v^p)}$$

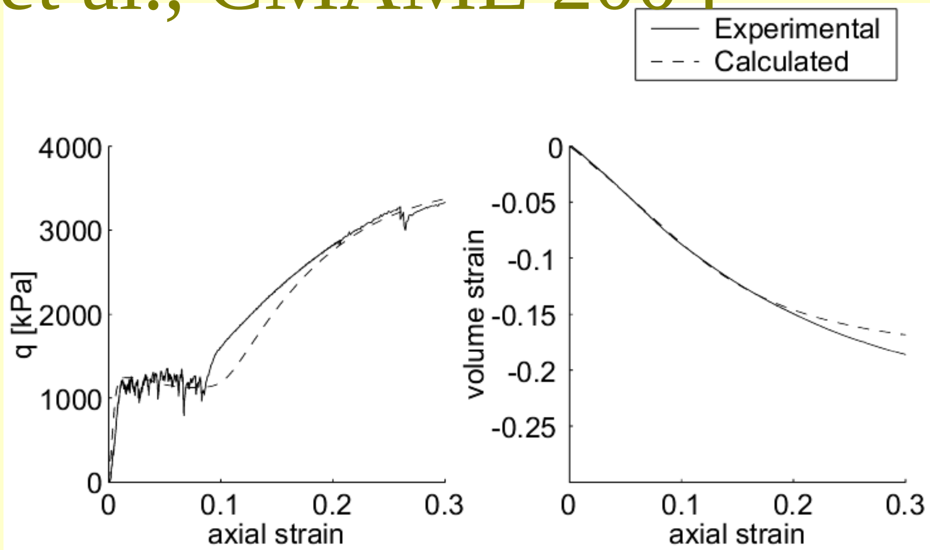
GPM for bonded geomaterials
Fernández-Merodo et al., CMAME 2004

Validation

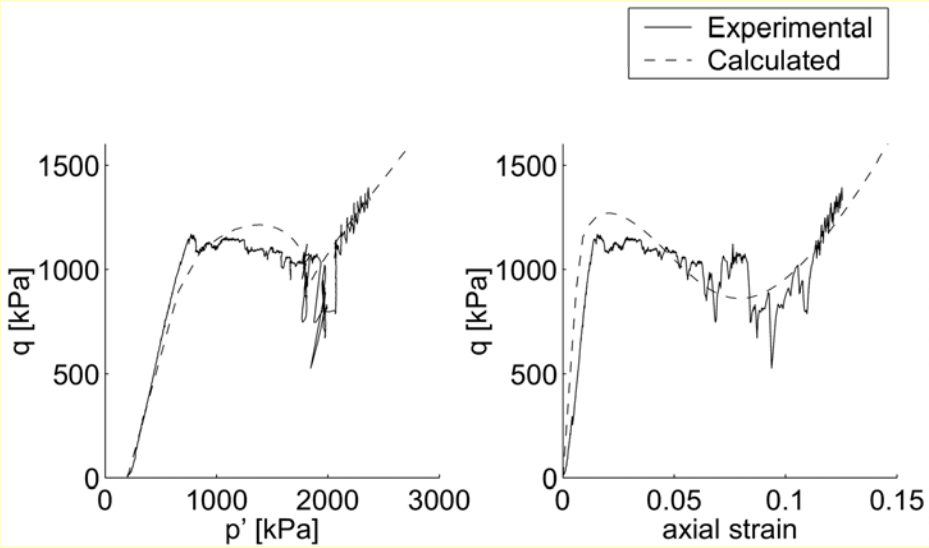
Lagioia & Nova, 1995



Isotropic compression test



CD test at cell pressure 1300 kPa



Oedometric test

F.E. modelling of Las Colinas landslide u-pw model

- Las Colinas landslide in Santa Tecla, San Salvador
  El Salvador Earthquake, 13-01-2001

Figure 2.11 Bird's eyes view of the Las Colinas landslide

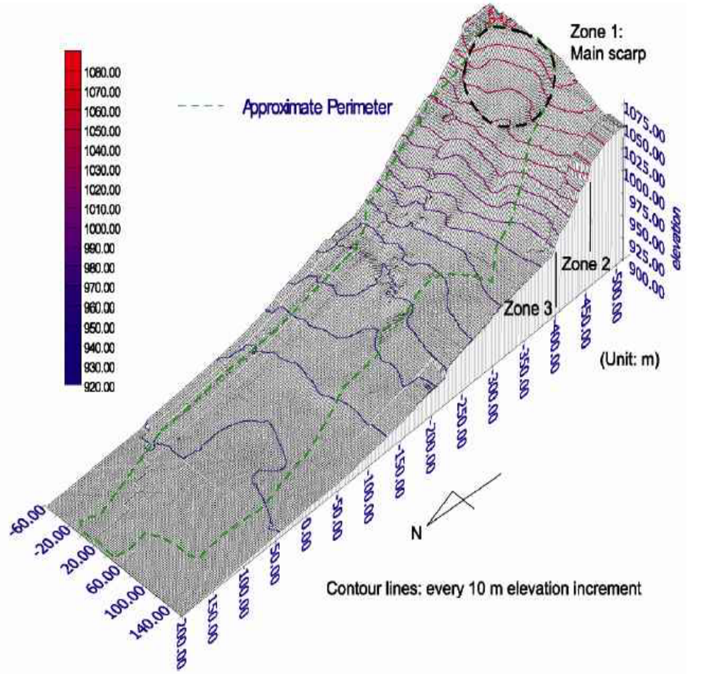


Figure 2.13 Surveyed Slope plotted with surfer with green dashed perimeter.

F.E. modelling of Las Colinas landslide

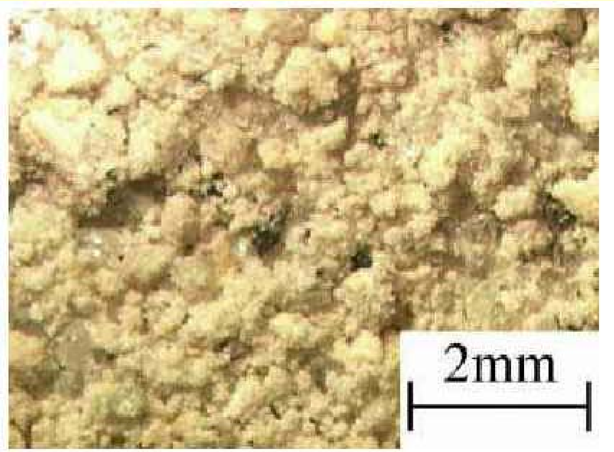
u-pw model

- Material: Tierra Blanca – pumice ash: loose cemented soil

Imperial Collège (*Bommer & al.*)

before static triaxal test

after static triaxal test



interior of the shear band

exterior of the shear band

F.E. modelling of Las Colinas landslide

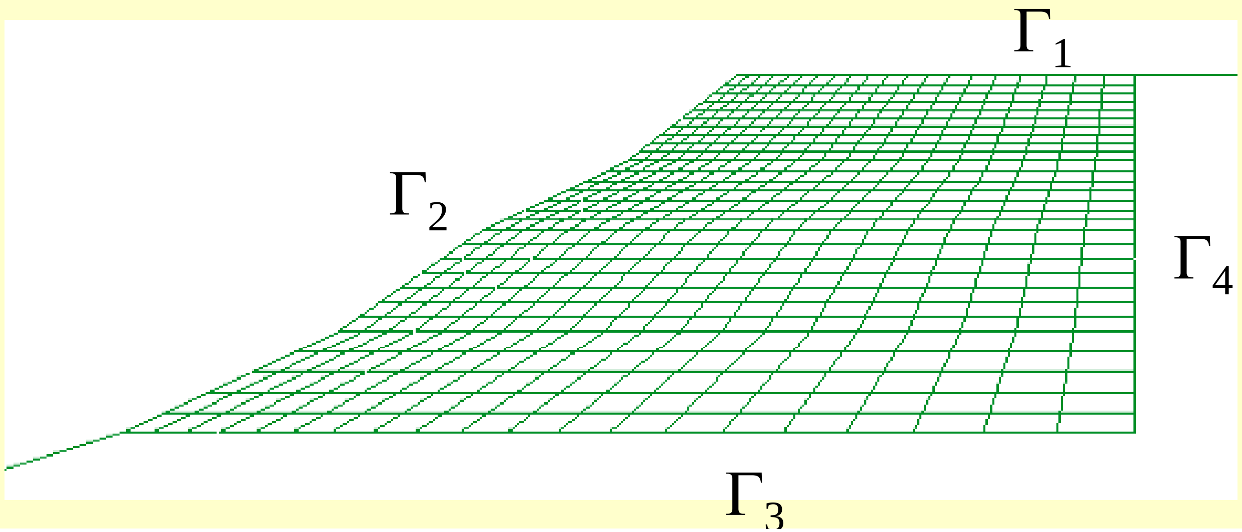
u-pw model

- Geommetry
- Finite element model: Coupled formulation **(pore fluid is air)**
- Boundary conditions :

  Γ_1, Γ_2 free stress boundaries, $p_a=0$

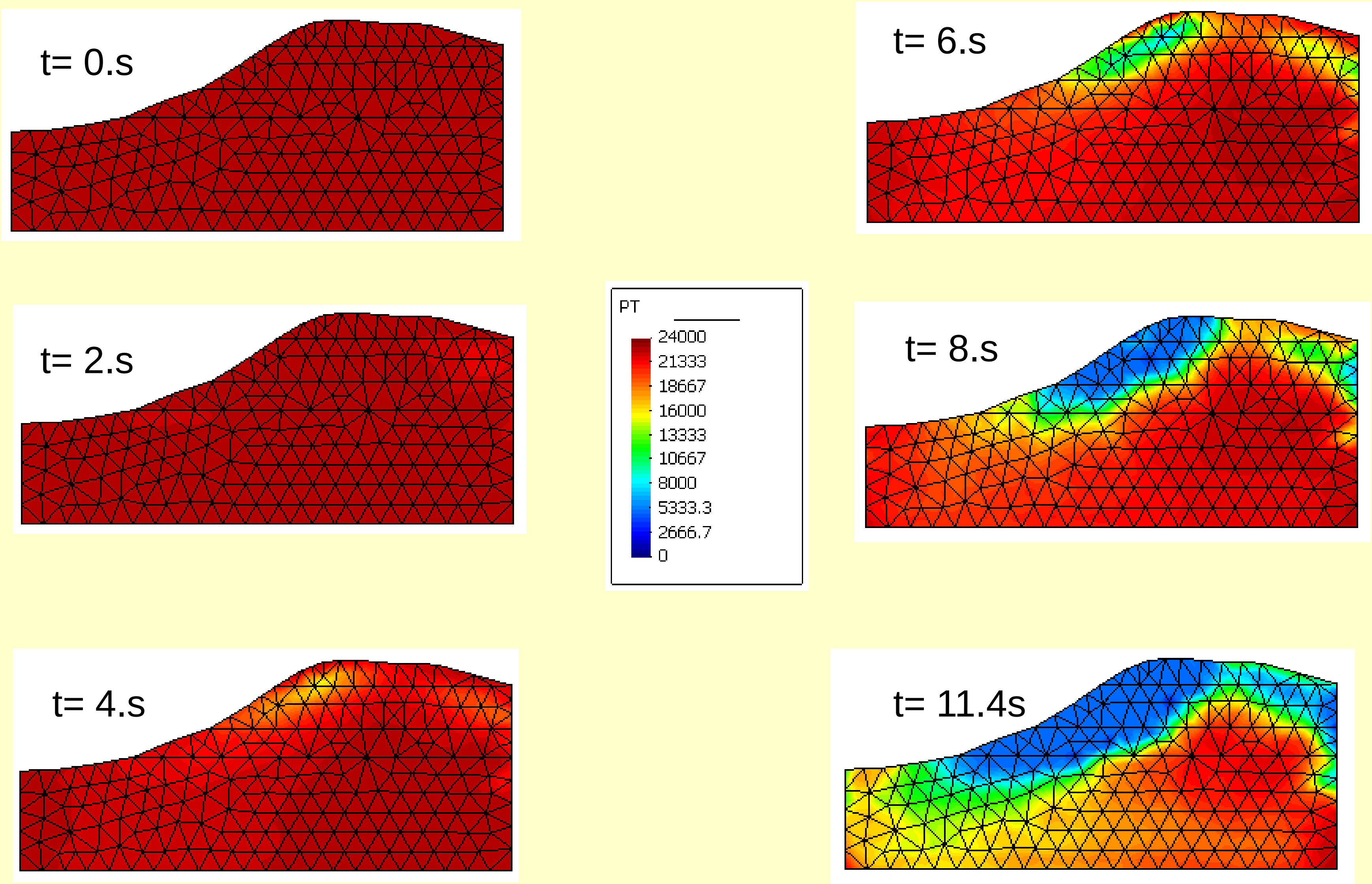
  Γ_3, Γ_4 : a) Base motion+absorbing boundary

  b) Base motion + infinite stratum condition



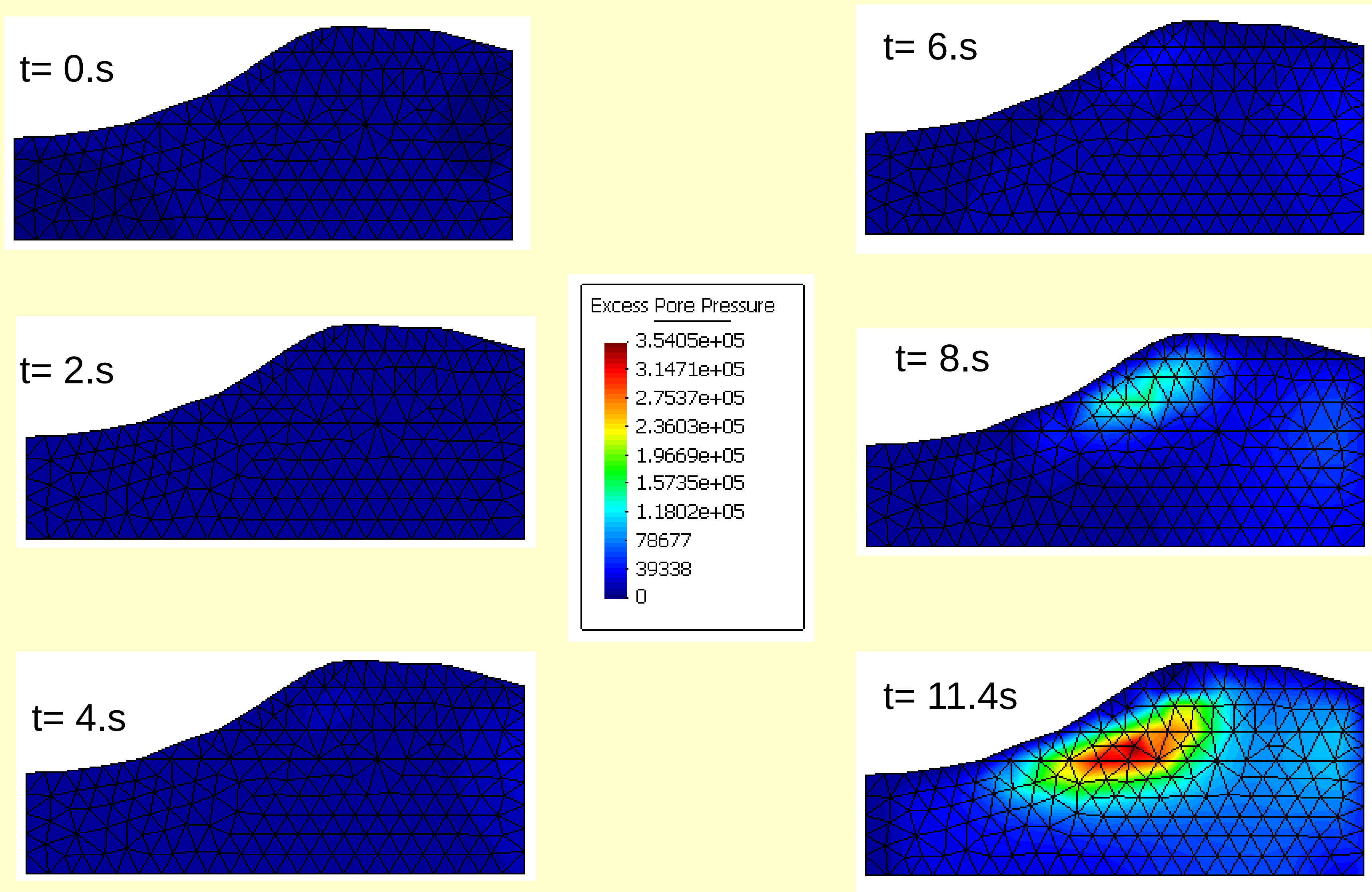
F.E. modelling of Las Colinas landslide

u-pw model



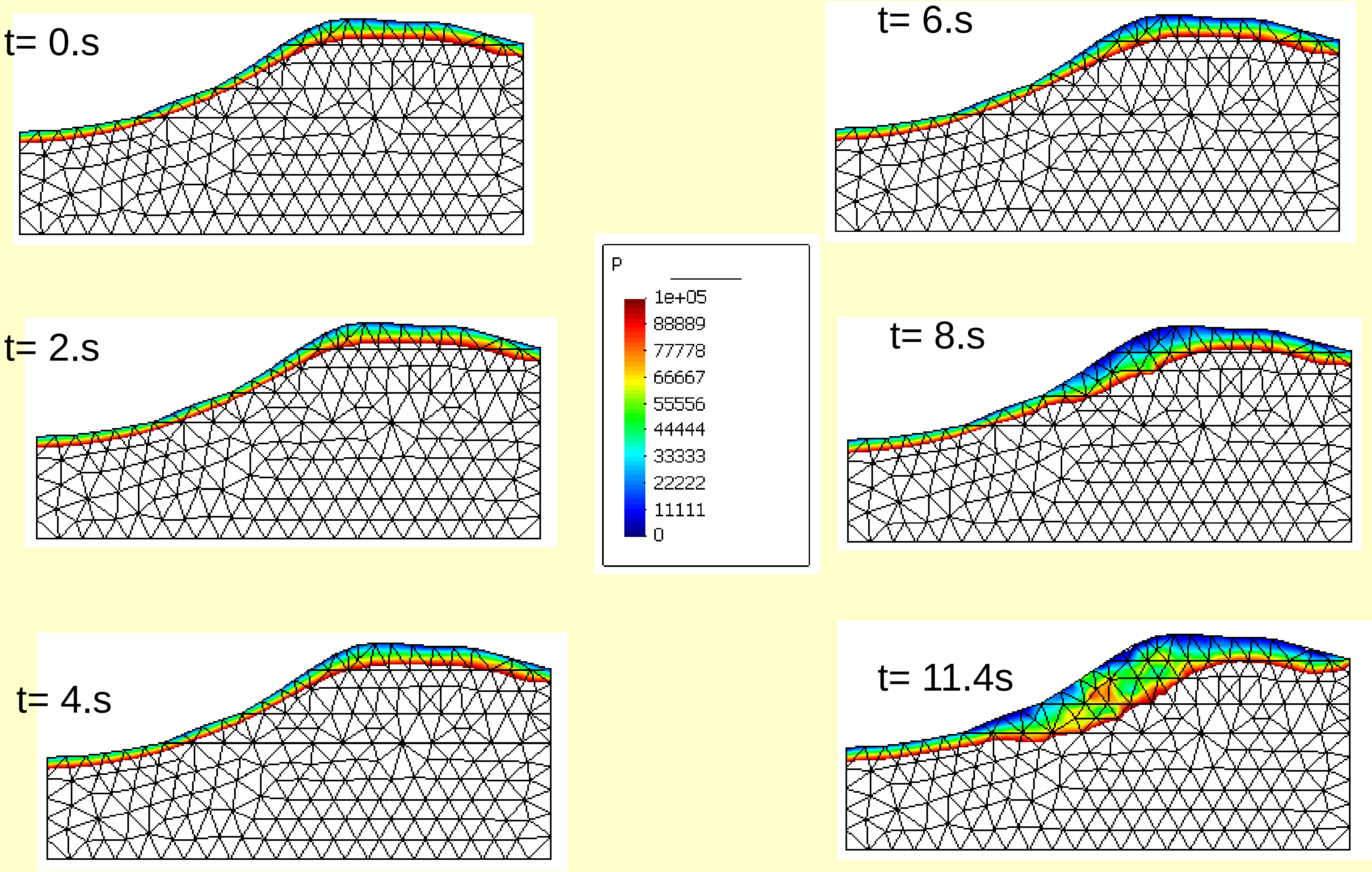
F.E. modelling of Las Colinas landslide

u-pw model



u-pw model

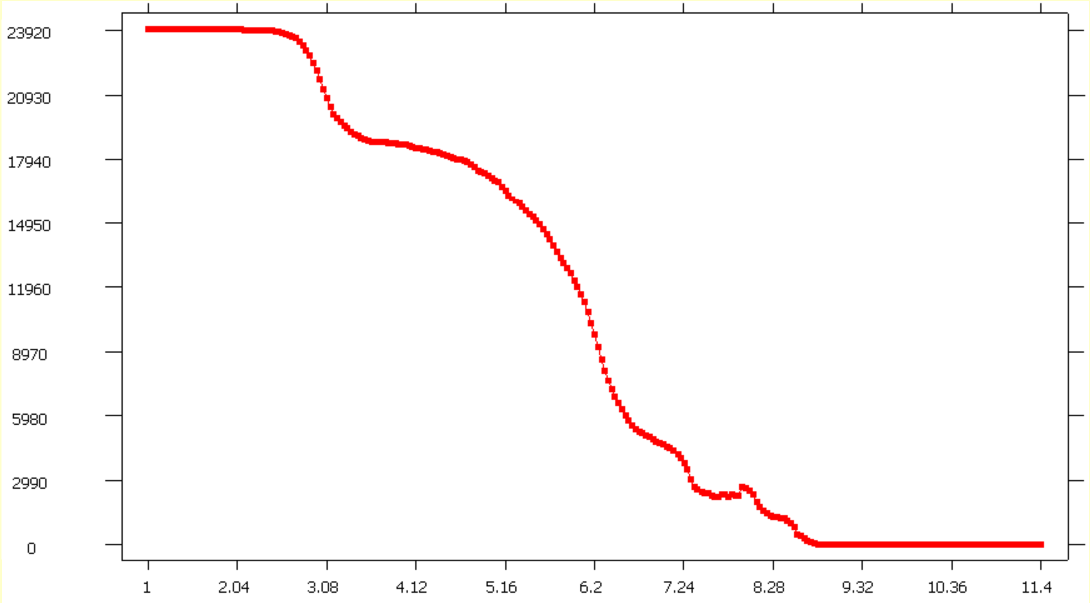
F.E. modelling of Las Colinas landslide



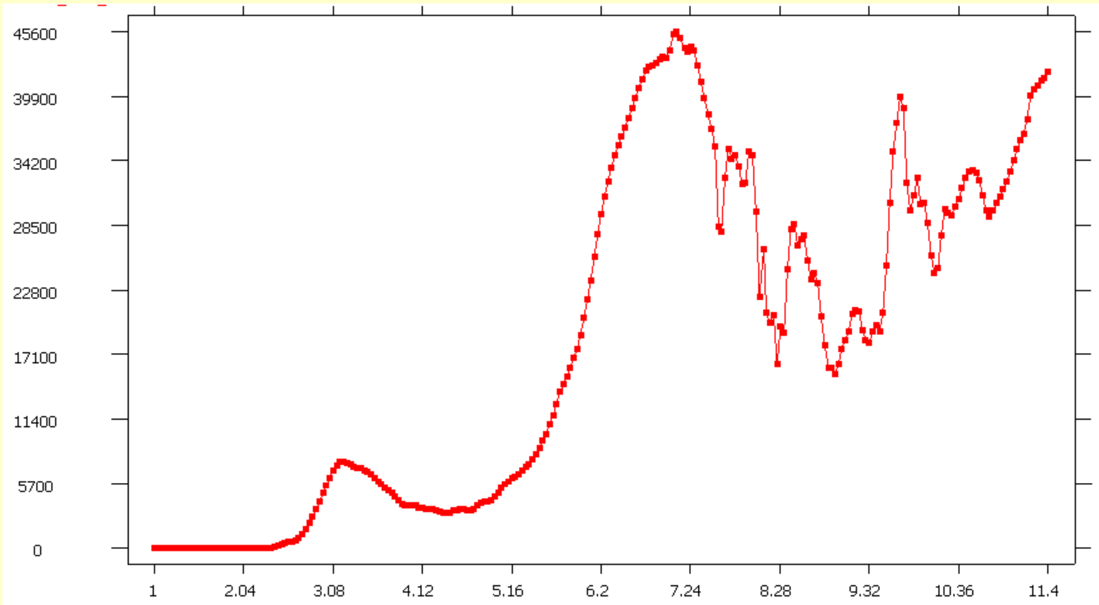
F.E. modelling of Las Colinas landslide

u-pw model

Pt (Pa)



Excess
pore pressure
(Pa)

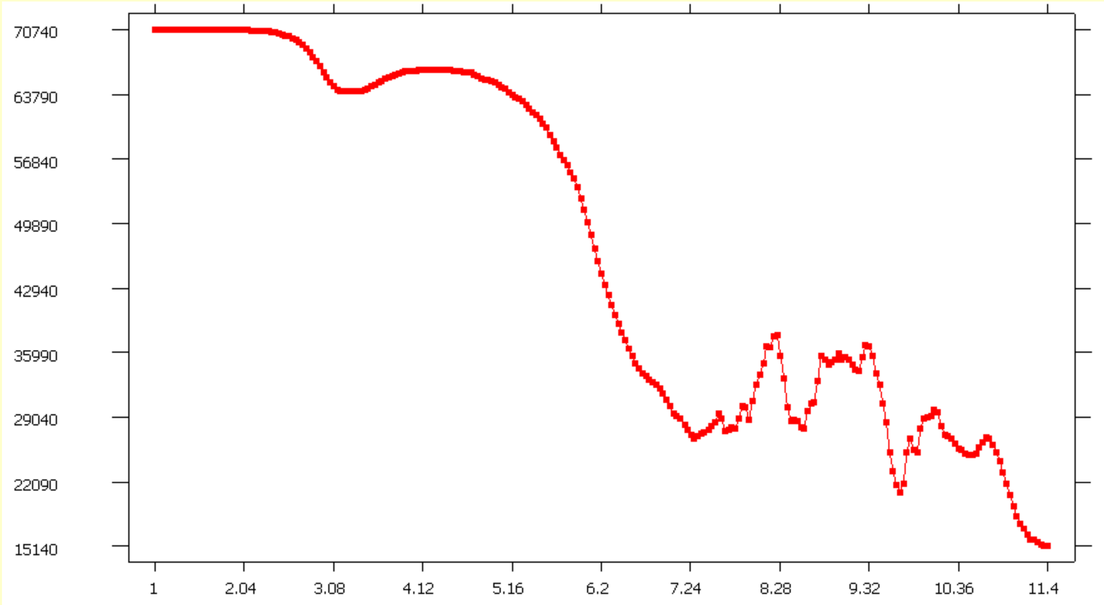


Time (s)

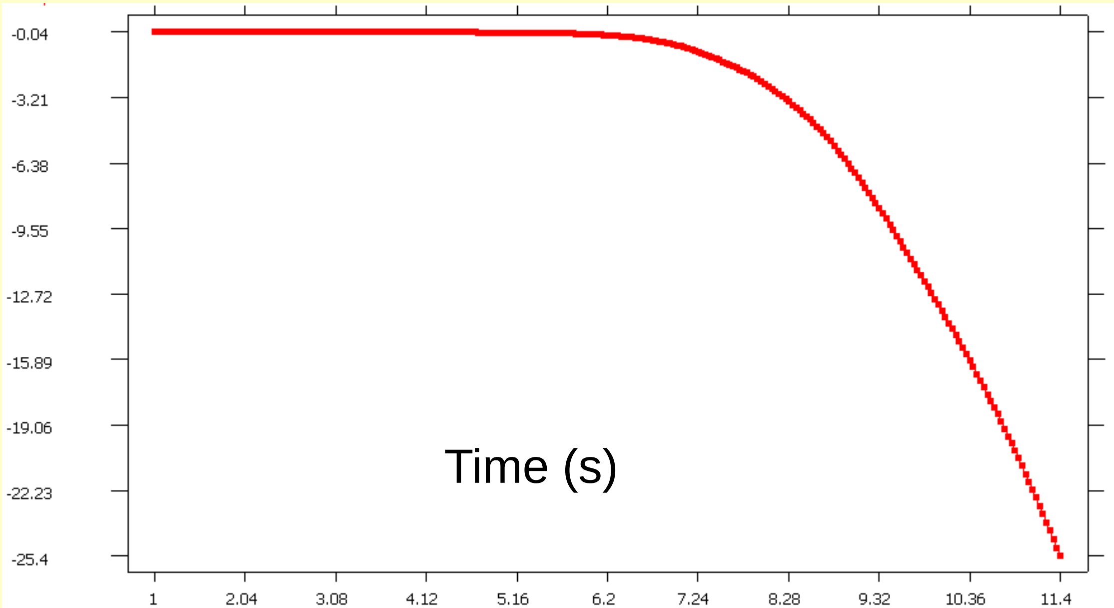
F.E. modelling of Las Colinas landslide

u-pw model

Mean effective stress (Pa)

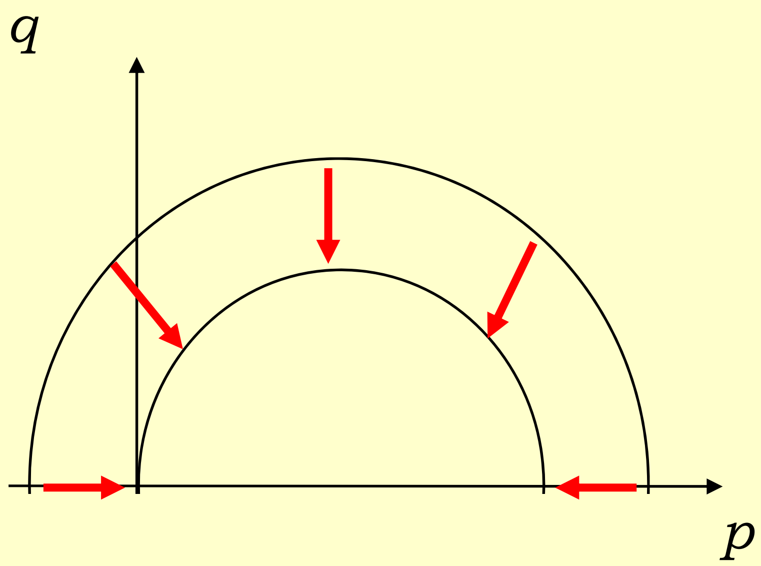


Ux (m)

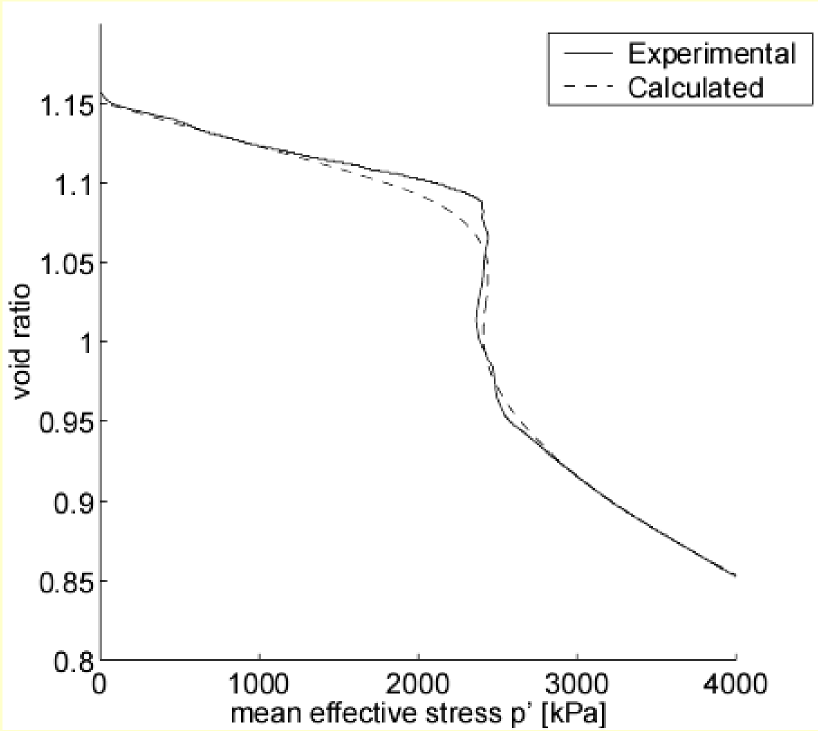


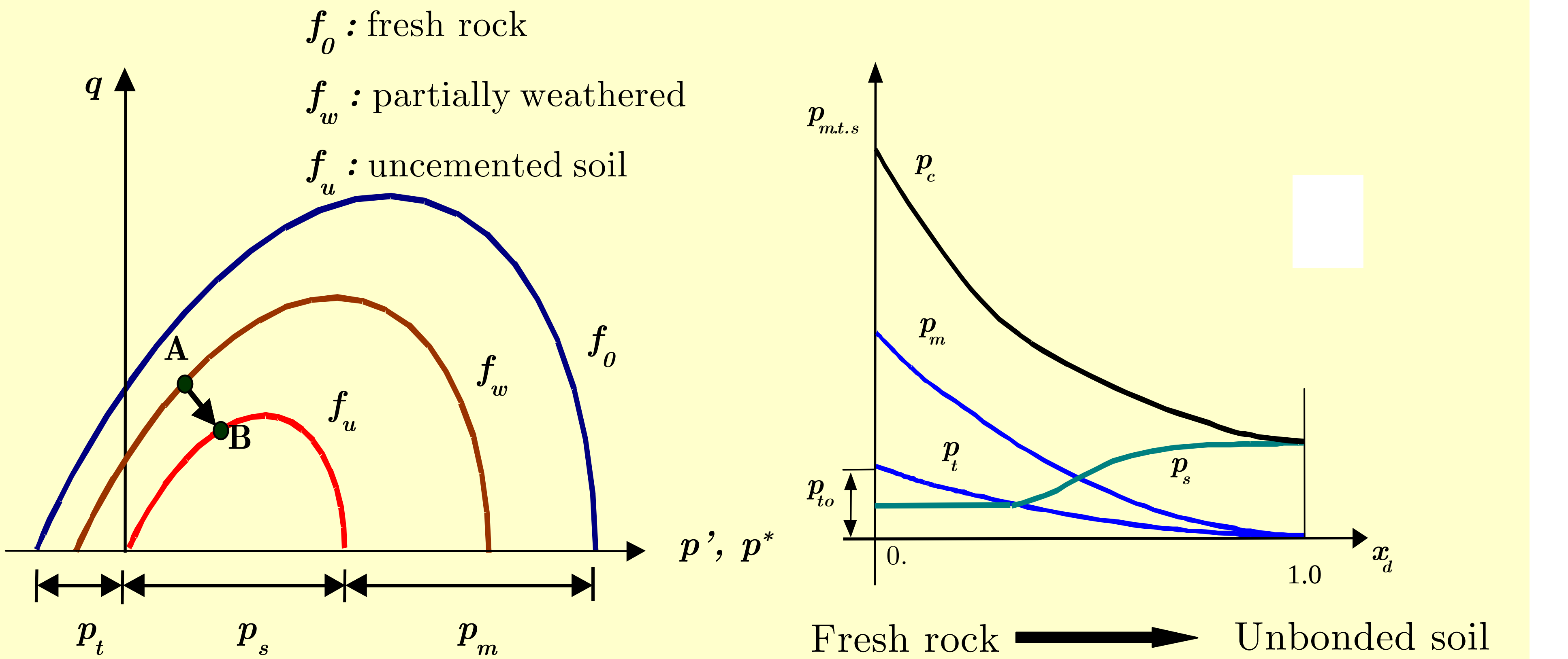
Debonding

Reduction of yield surface size



Lagioia & Nova, 1995





f0 : fresh rock
fw : partially weathered
fu : uncemented soil
q
A
B
f0
fw
fu
p', p*
pt
ps
pm
pm.t.s
pc
pm
pt
ps
pto
0.
1.0
xd
Fresh rock
Unbonded soil

3a3 State Parameter

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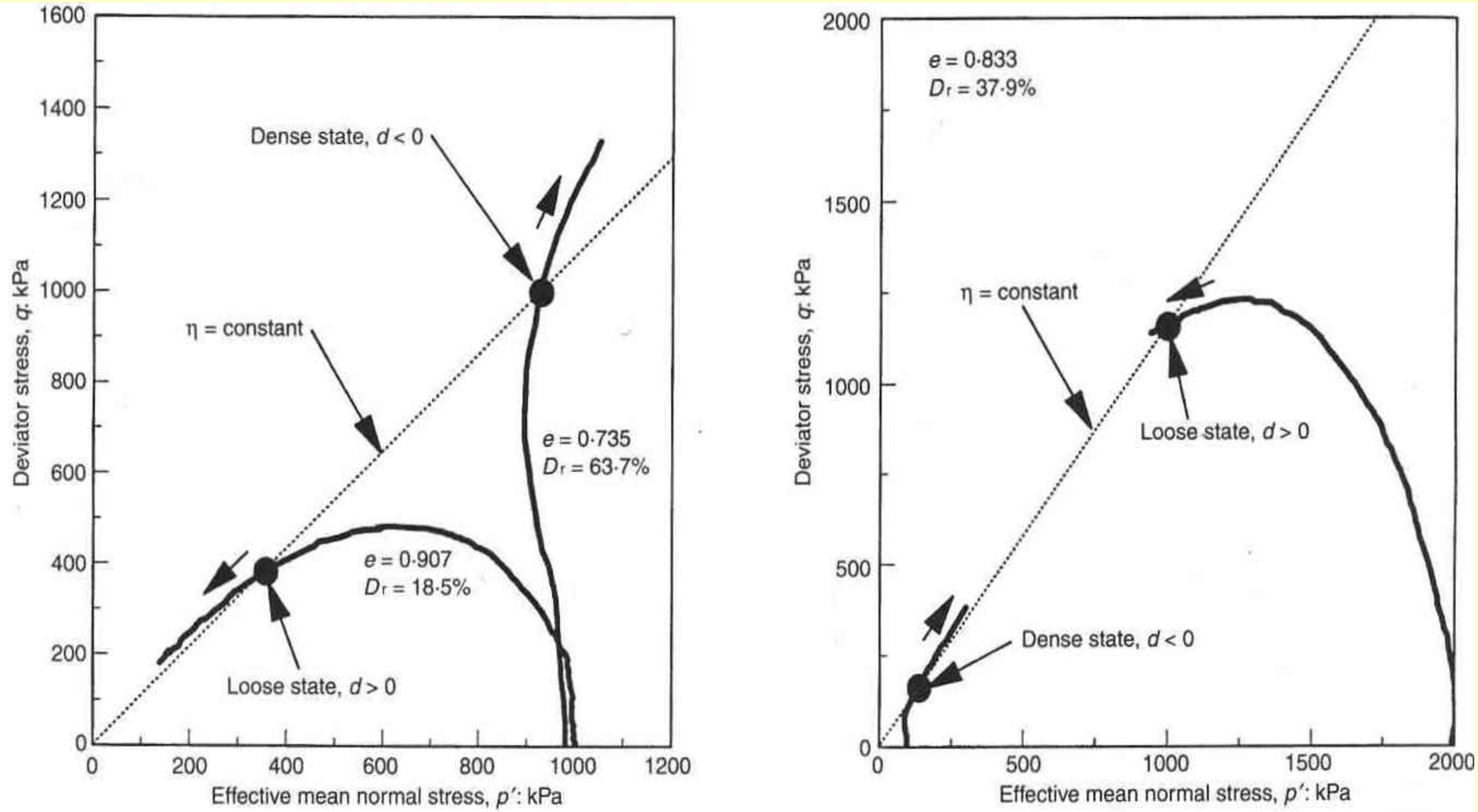
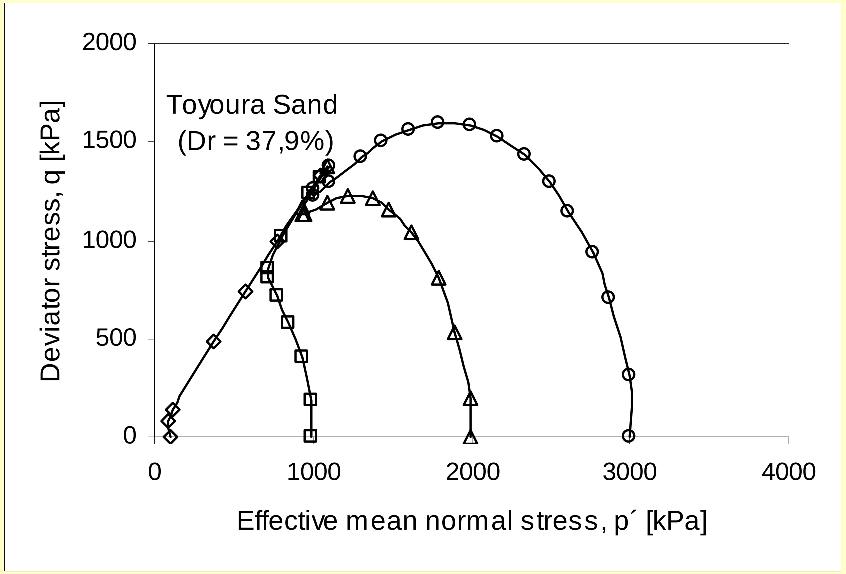
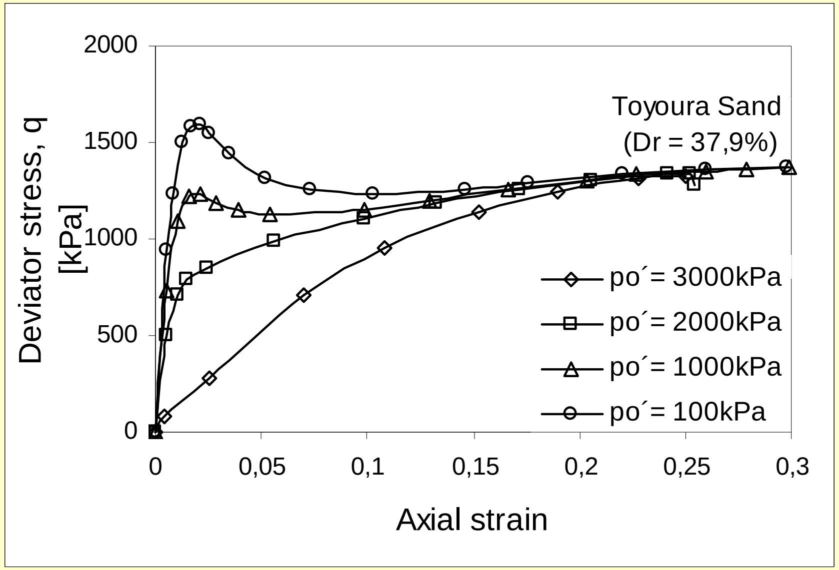


Undrained Triaxial Test (from Li & Dafalias, 2000)

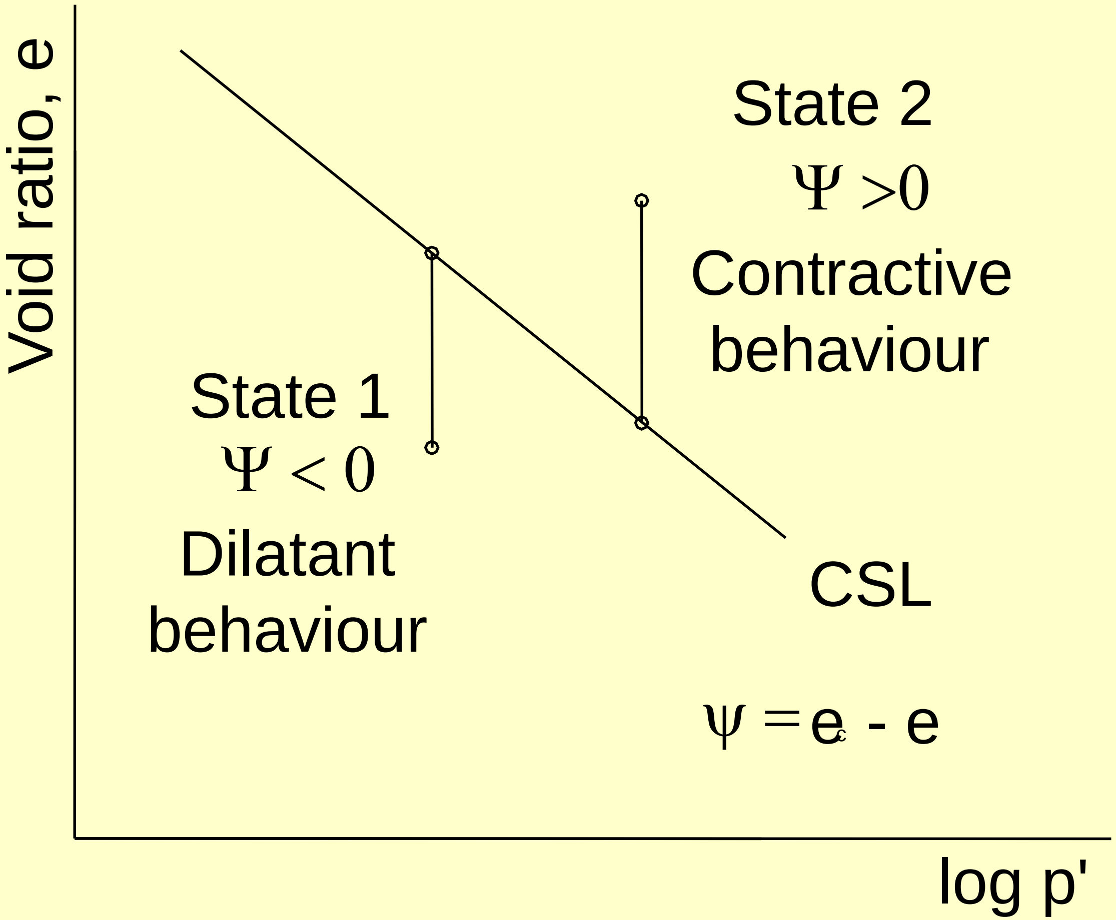
Behaviour of sands (CU)
Influence of Confining Pressure





Been & Jefferies (1985)

$$\psi = e - e_c$$



Modified flow rule (Li & Dafalias 2000)

$$d_g = \frac{d_0}{M_g} \cdot \left[M_g \cdot \mathrm{Exp}\boxed{\left(m\psi \right)} - \eta \right]$$

Direction of plastic flow

$$\mathbf{n}_g = \left(n_{gv} ; n_{gs} \right)^T \Rightarrow$$

$$n_{gv} = \frac{d_g}{\sqrt{1 + d_g^2}}$$

$$n_{gs} = \frac{1}{\sqrt{1 + d_g^2}}$$

$$d_g = \frac{d_0}{M_g} \cdot \left[M_g \cdot \mathrm{Exp}(m\psi) - \eta \right]$$

At CSL $\quad \psi = 0 \quad \eta = M_g \quad \Longrightarrow \quad d = 0$

$$\eta = M^d = M_g \exp(m\psi) \quad \Longrightarrow \quad d = 0$$

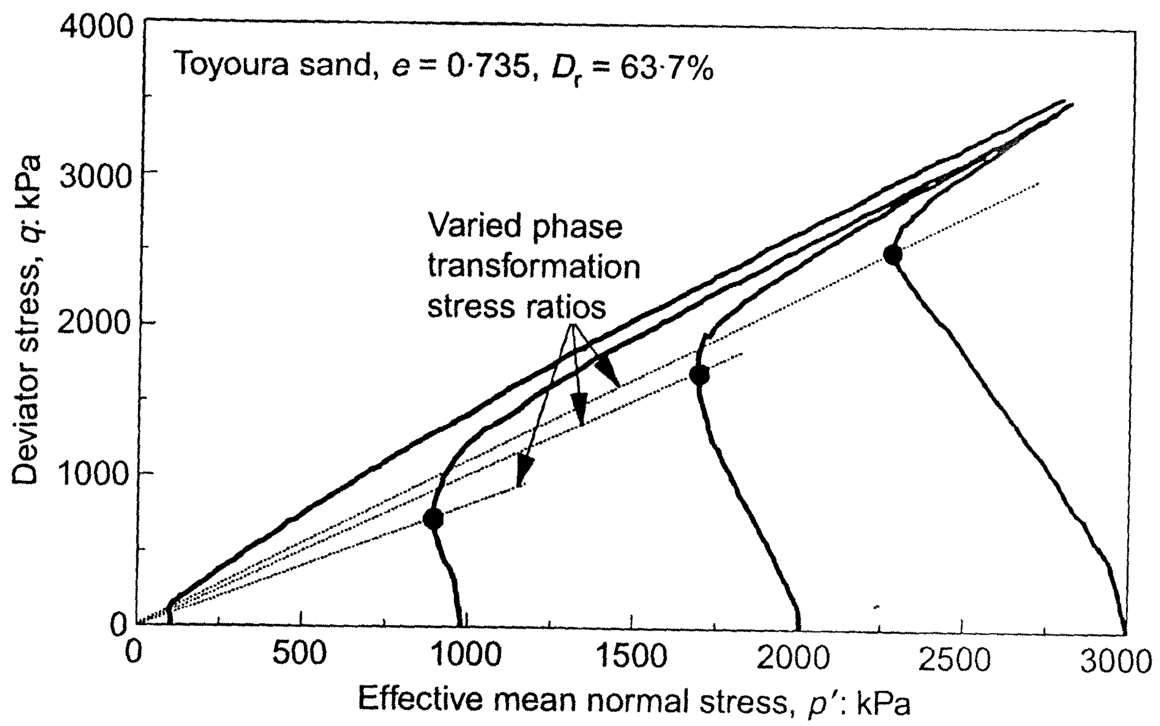
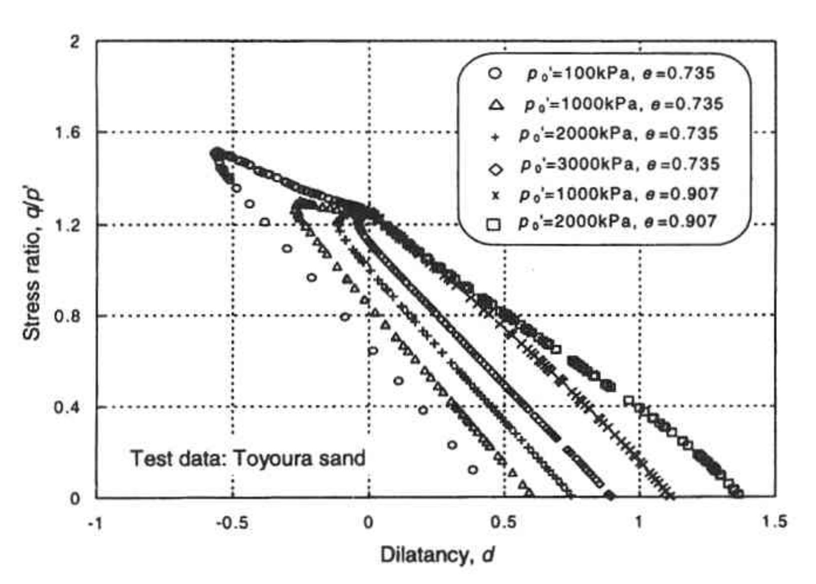
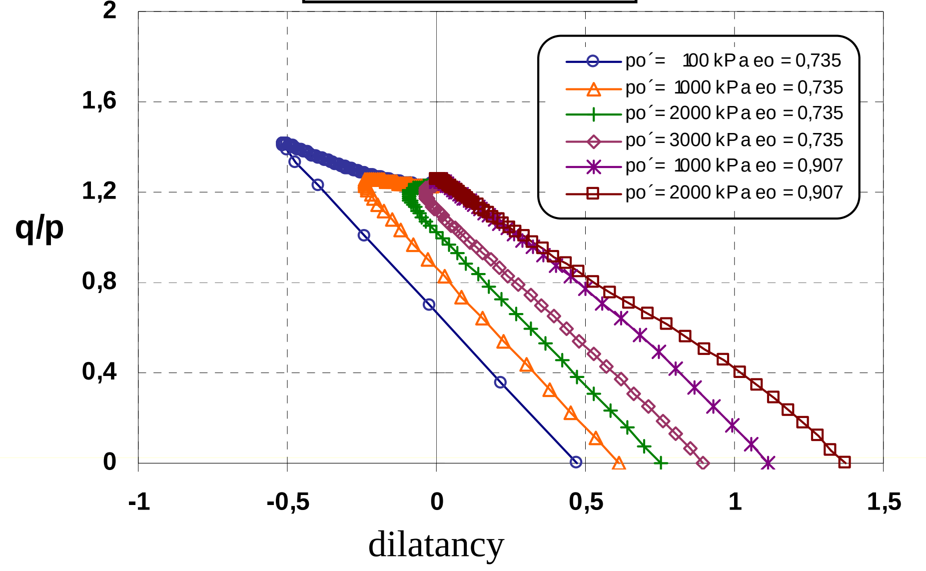


Fig. 5. Variation in the phase transformation stress ratio with material state (data from Verdugo & Ishihara (1996))

Experiments (Yang and Li 2004)



Model predictions



Loading - Unloading discriminating direction

$$n_v = \frac{d_f}{\sqrt{1+d_f^2}} \qquad n_s = \frac{1}{\sqrt{1+d_f^2}}$$

$$d_f = \frac{d_0}{M_f} \cdot \left(M_f \cdot \mathrm{Exp}\left(m\psi \right) - \eta \right)$$

$$\boxed{\frac{M_f}{M_g} = h_1 - h_2 \cdot e}$$

Modified Plastic Modulus

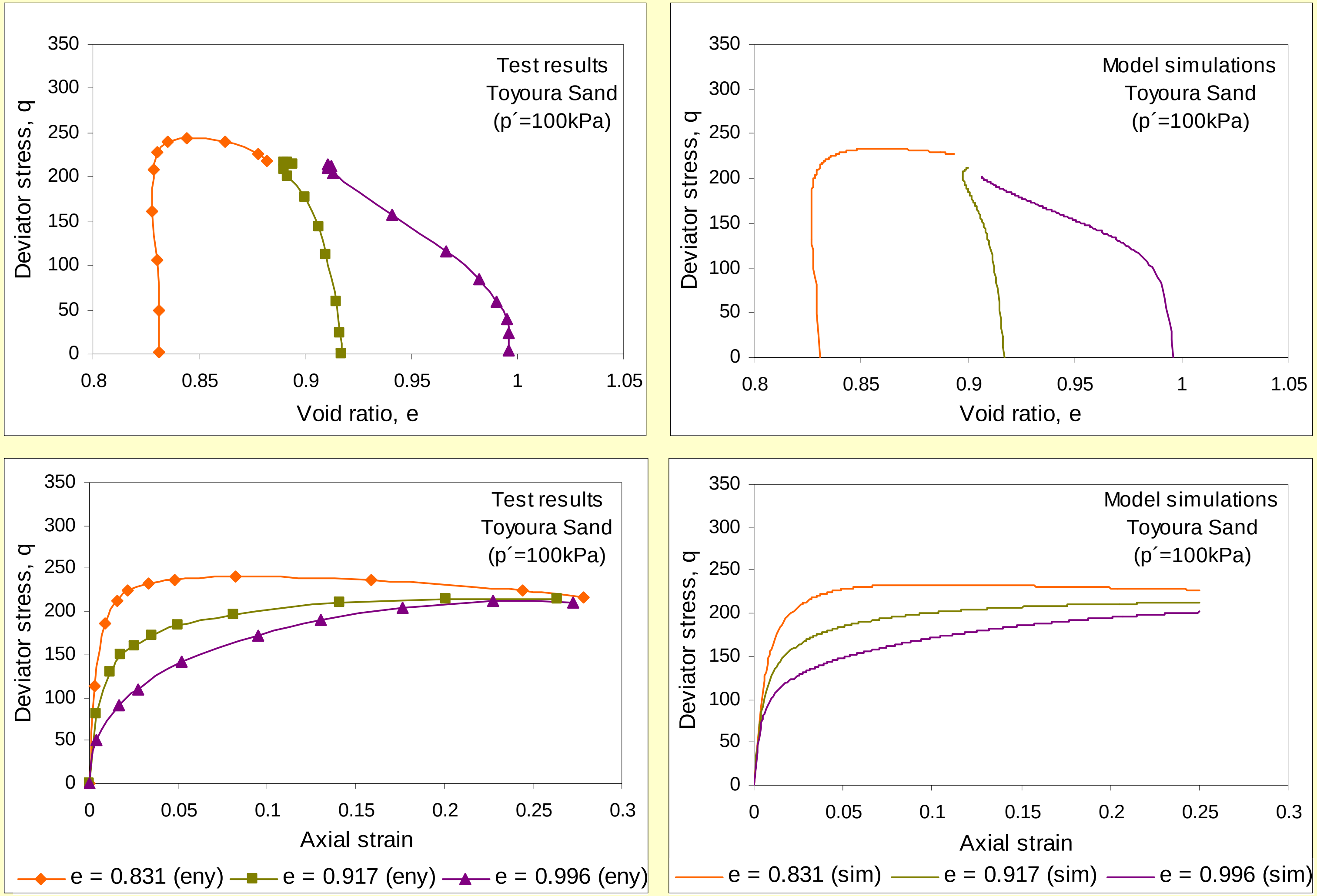
$$H_L = H_0 \cdot \sqrt{p' \cdot p'_{atm}} \cdot f\left(\eta; \boxed{\psi}\right)$$

$$f(\eta, \psi) = \underbrace{\left(1 - \frac{\eta}{\eta_f}\right)^{\mu}}_{H_f} \cdot \left[\underbrace{\left(1 - \frac{\eta}{M_g}\right)}_{H_v} + \underbrace{\boxed{\beta_2(\psi)} \cdot \exp(-\beta_0 \cdot \xi)}_{H_s} \right]$$

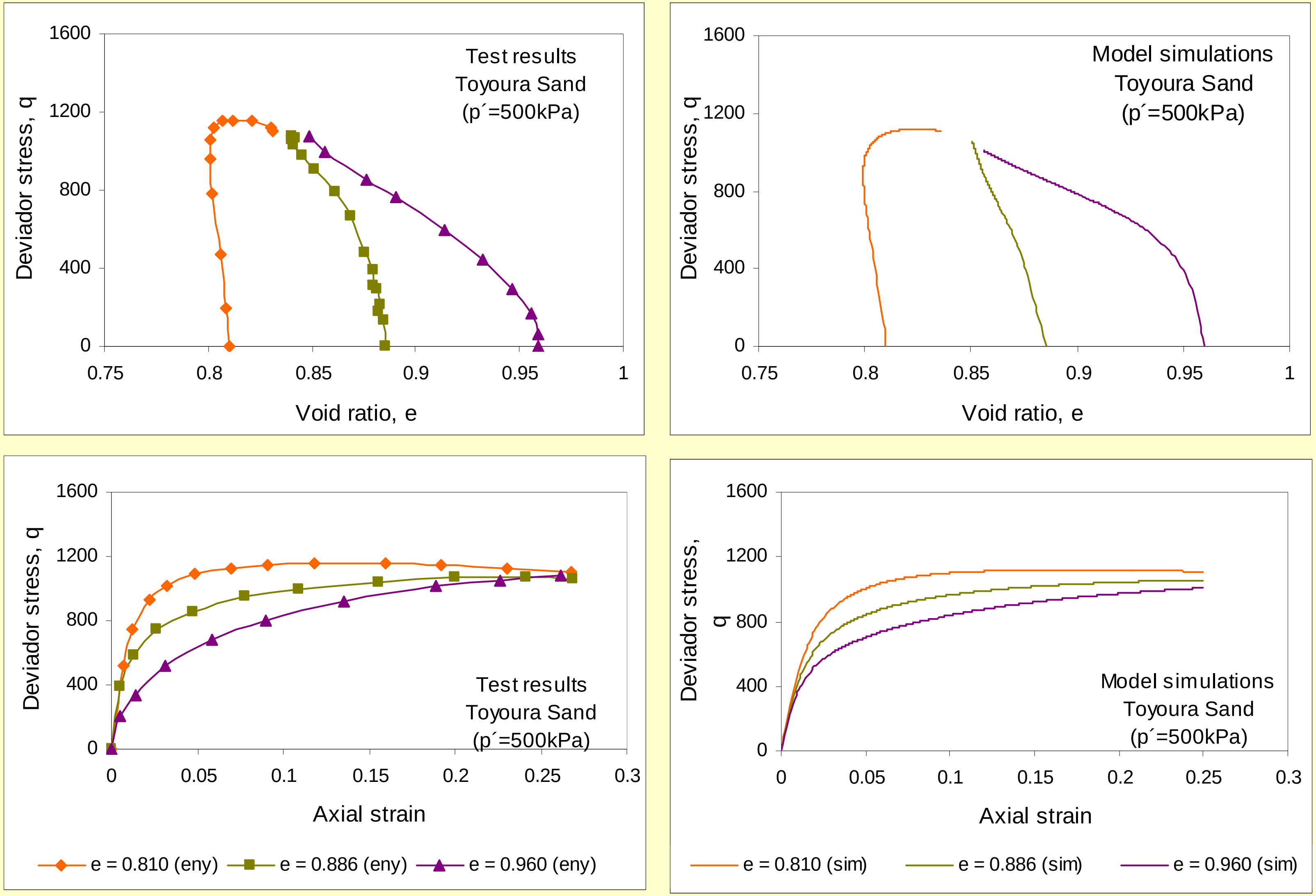
$$\eta_f = \left(1 + \frac{1}{\alpha_f}\right) \cdot \boxed{M_f}$$

$$\boxed{\beta_2(\psi) = \beta_1 \cdot \left[M_g \cdot \exp(-n\psi) - \eta \right]}$$

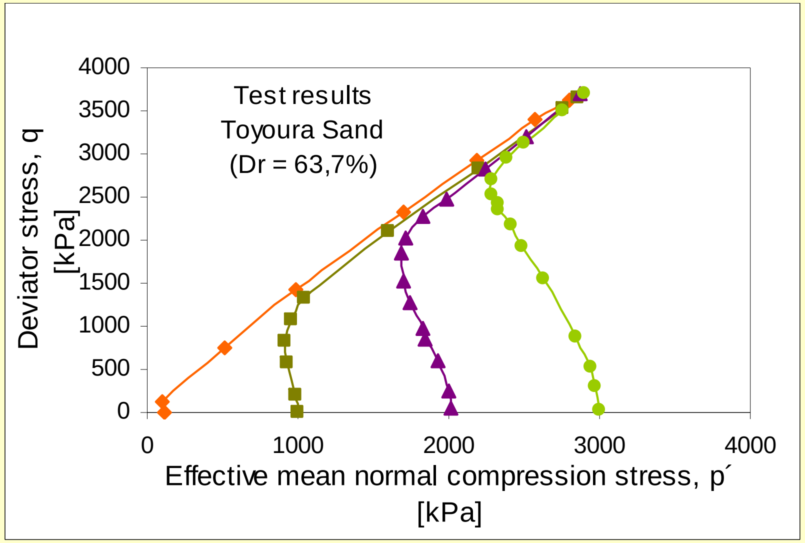
Simulations: CD Triaxial Test

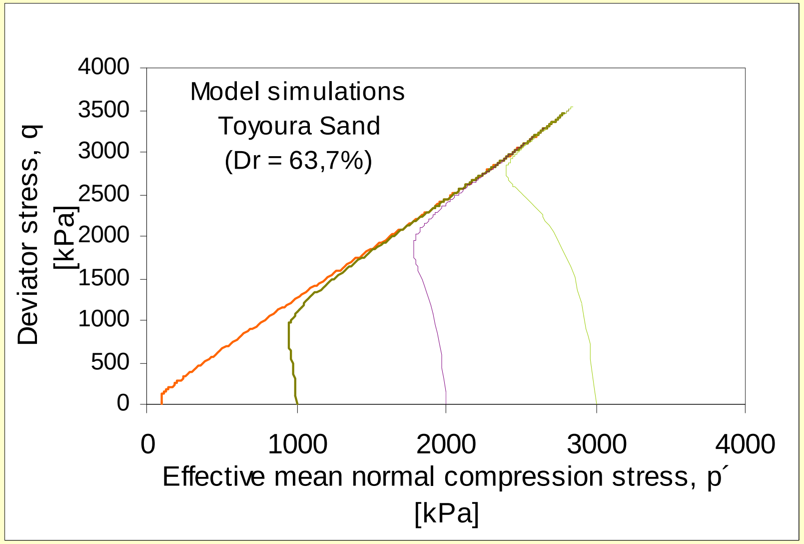


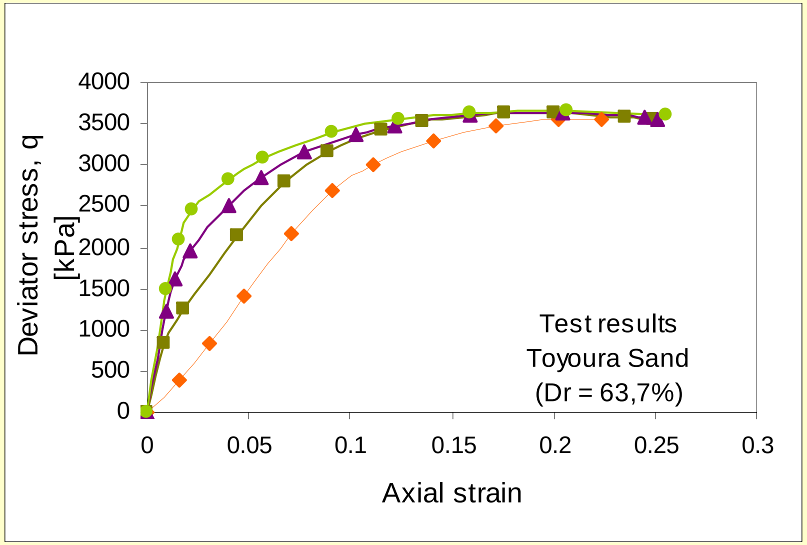
Simulations: CD Triaxial Test

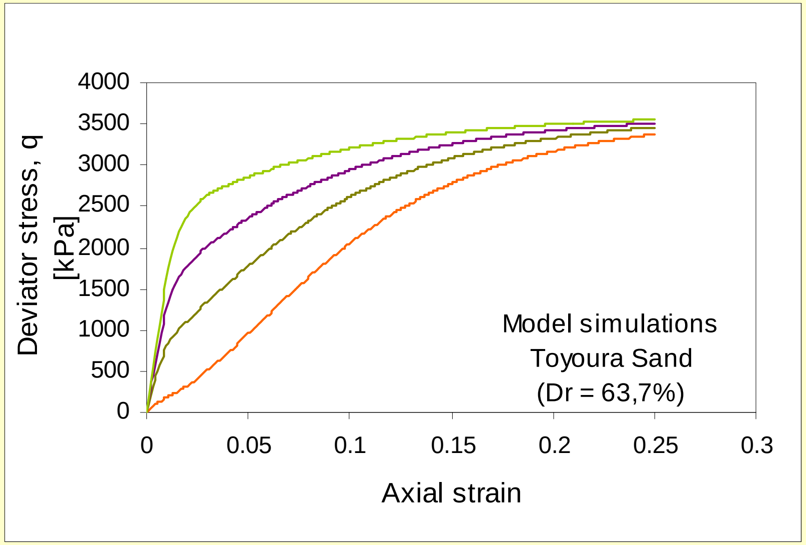


CU Triaxial Test

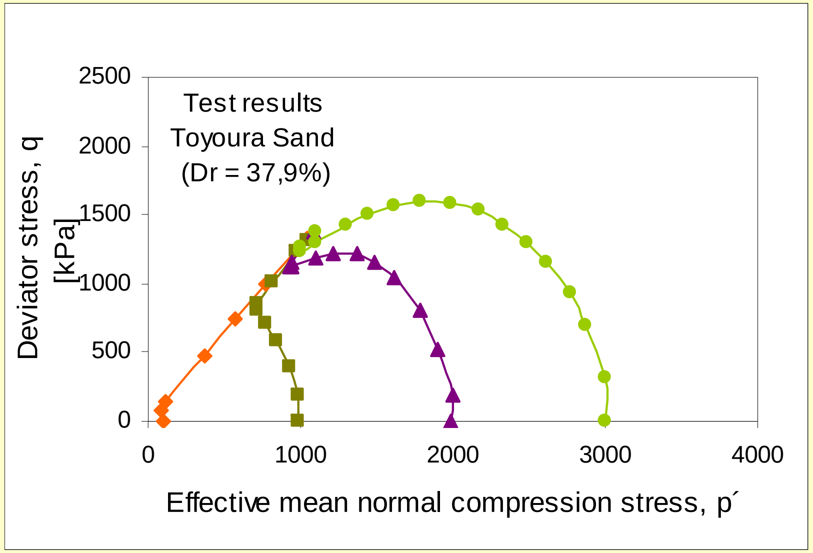


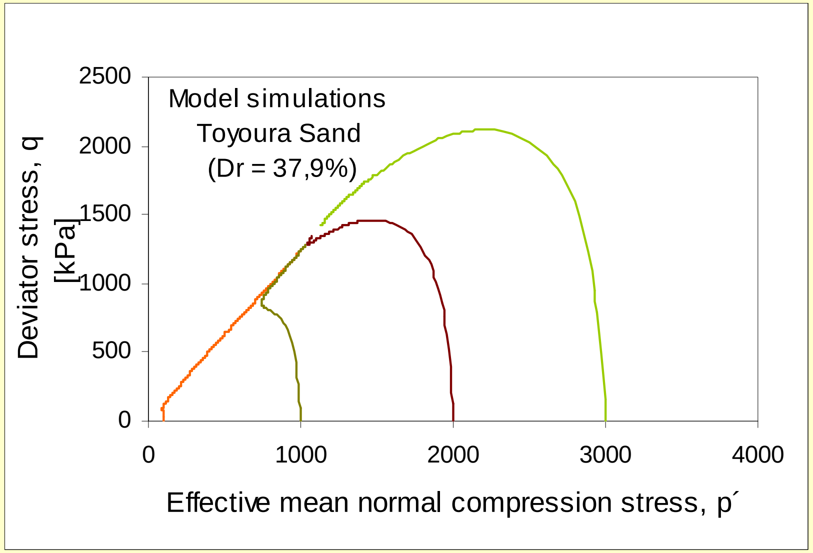


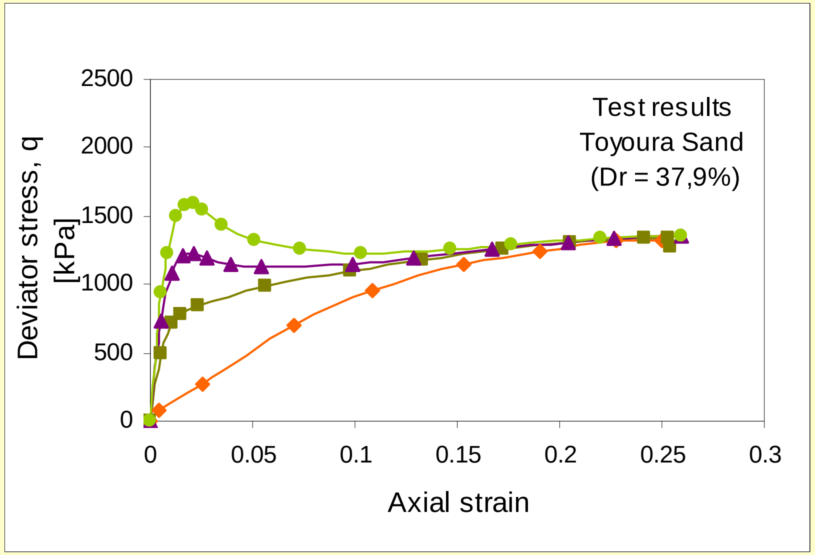


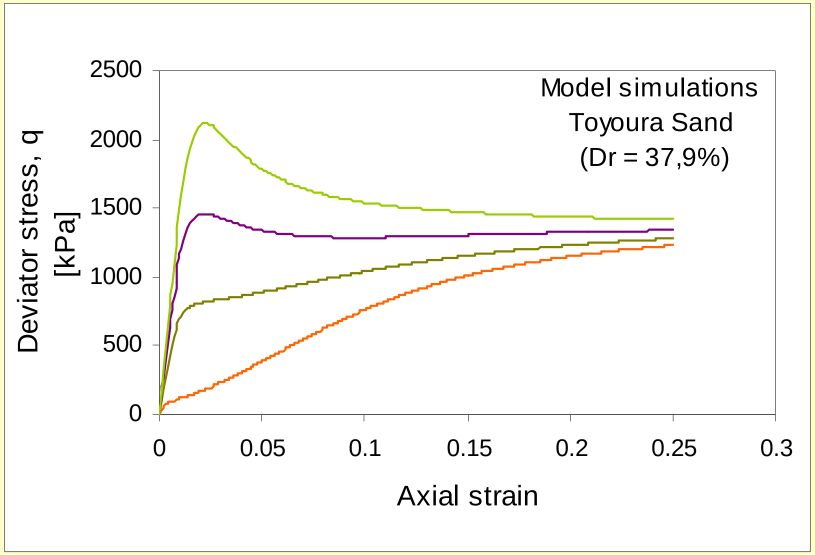


CU Triaxial Test

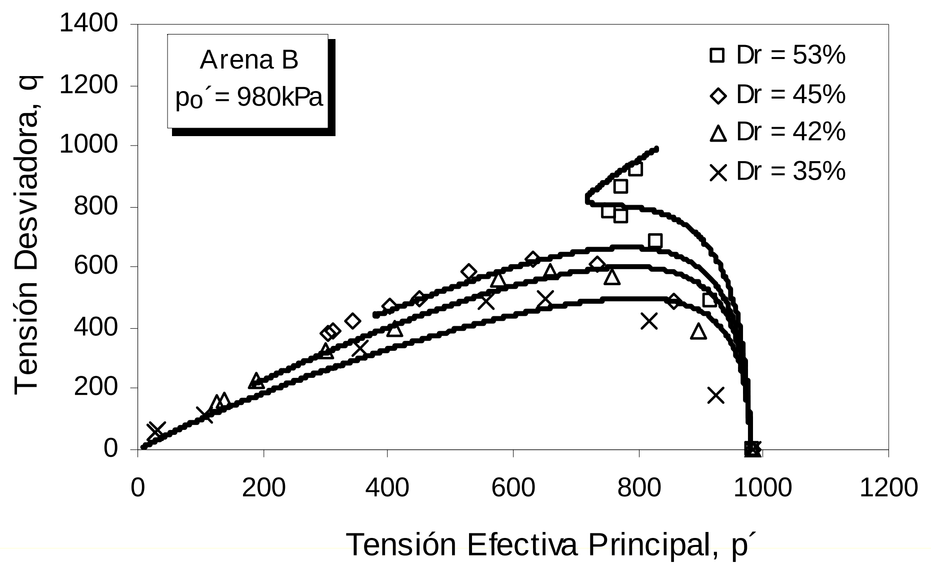


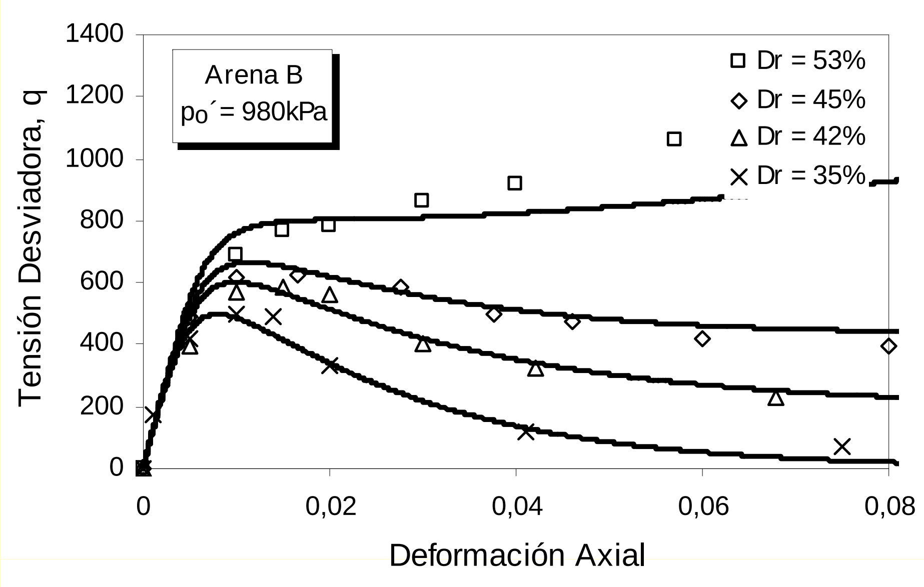




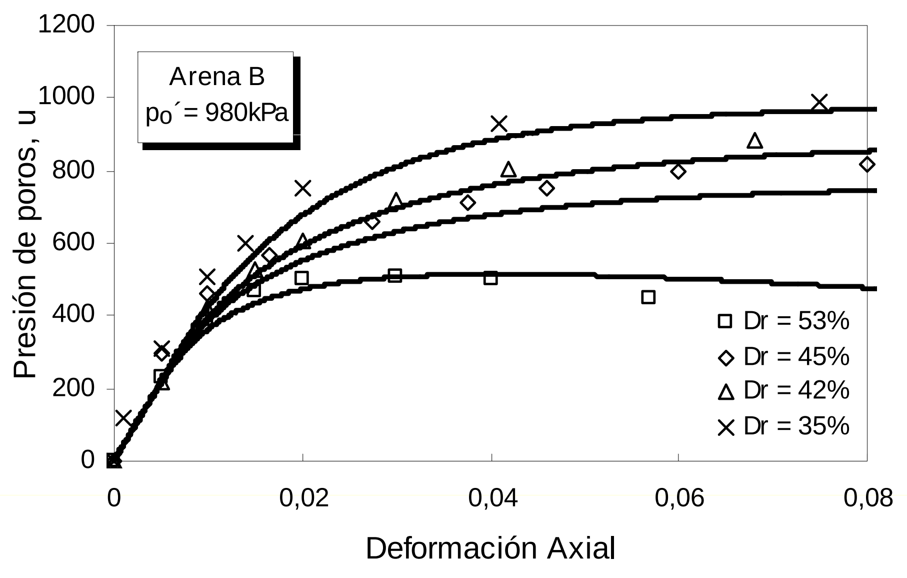


Banding sand (Castro 1969)

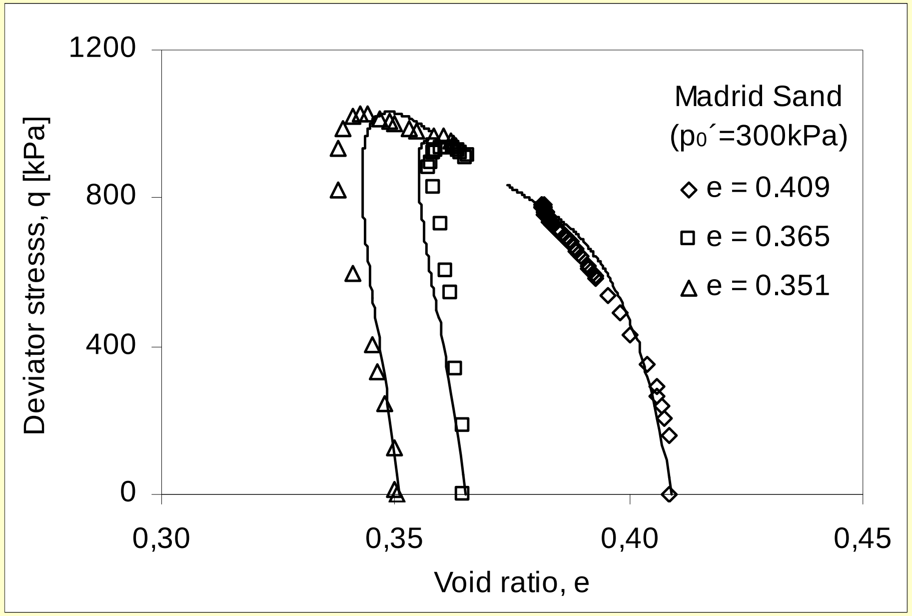




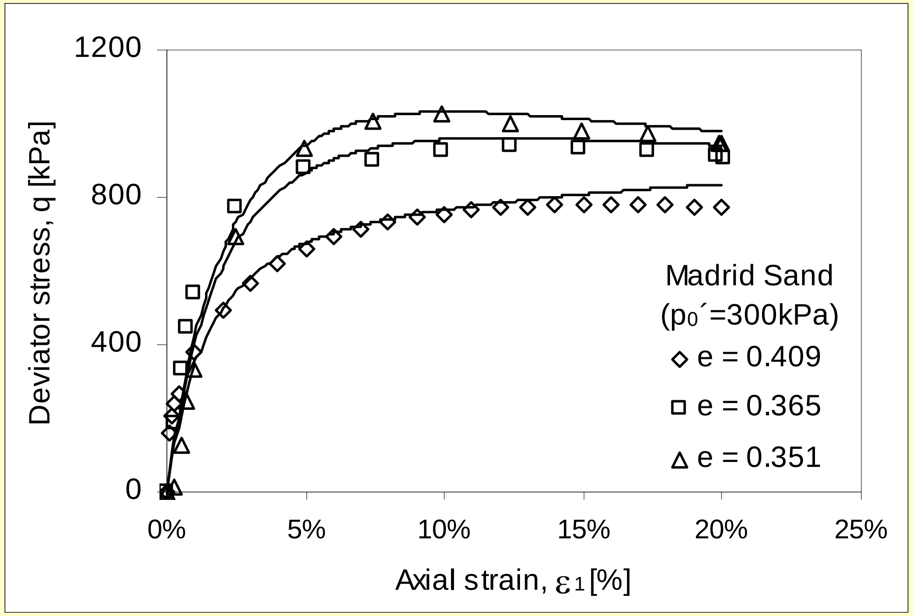
CU $p_o = 980kPa$



Madrid Sand



Deviator stresss, q [kPa]
1200
800
400
0
0,30
0,35
0,40
0,45
Void ratio, e
Madrid Sand
(p0´=300kPa)
e = 0.409
e = 0.365
e = 0.351



Deviator stress, q [kPa]
1200
800
400
0
0%
5%
10%
15%
20%
25%
Axial strain, ε1 [%]
Madrid Sand
(p0´=300kPa)
e = 0.409
e = 0.365
e = 0.351

3a3 Unsaturated

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  - Rheology
  - Dilatancy
  - A Perzyna viscoplasticity approach

Basic definitions (1/2)

- Suction $s = p_a - p_w$

- Cementation Parameter
  (Haines 1925, Fisher 1926, Gallipoli and Gens 2003)

$$f(s) = \frac{3}{4}\left\{2 - \frac{1}{2s}\left[-\frac{3T_s}{R} + \sqrt{\left(\frac{3T_s}{R}\right)^2 + \frac{8T_s}{R}s}\right]\right\}$$

Relationship between stabilizing pressure at a given suction s and at zero suction
Ts surface tension
R radius particles

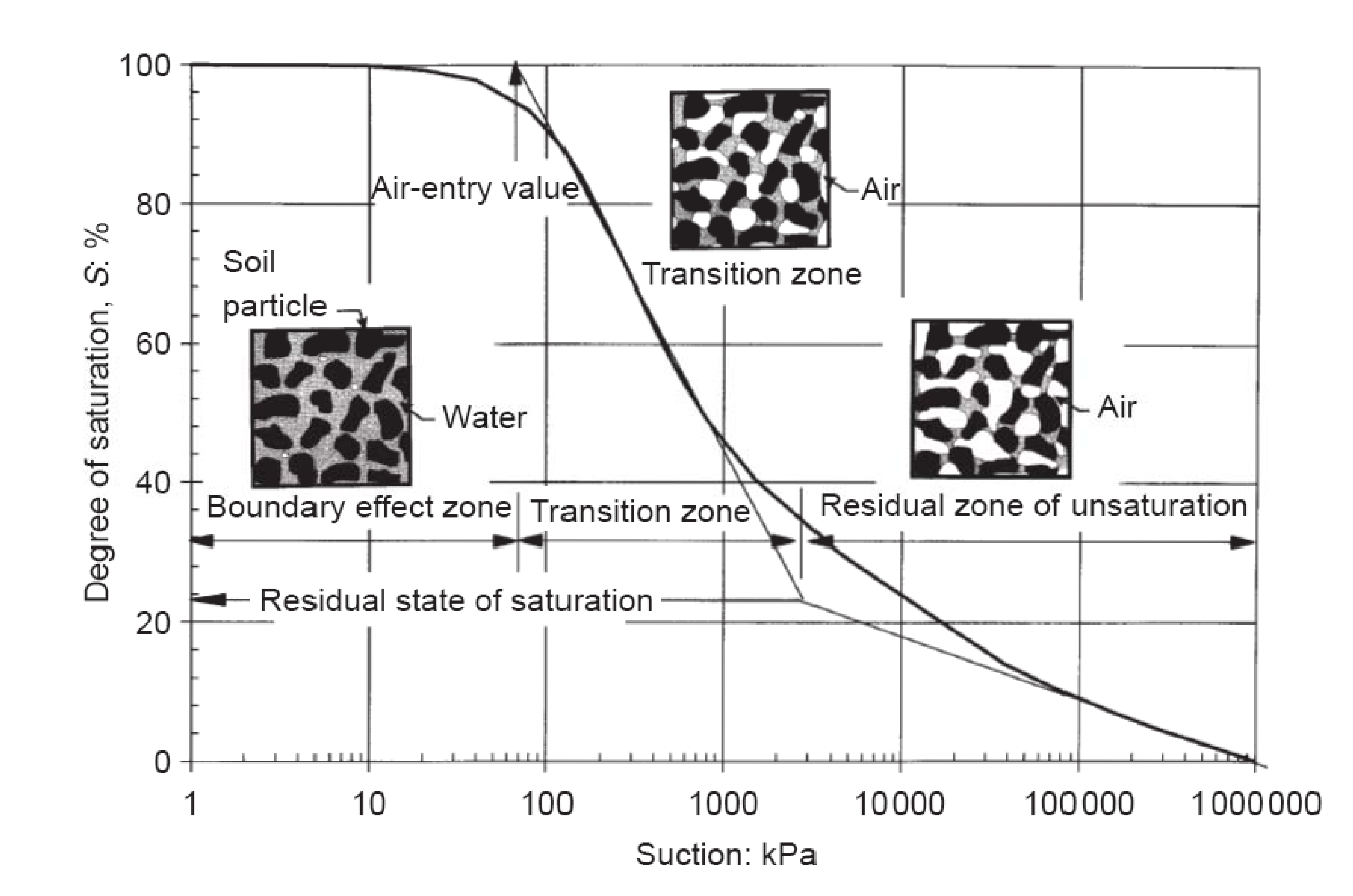
$$\xi = f(s)(1 - S_r)$$

- Effective stress (Bishop,…Schrefler)

$$\sigma' = \sigma + S_r s I - p_a I \quad (\chi = S_r)$$

- Work (Houlsby 1997)

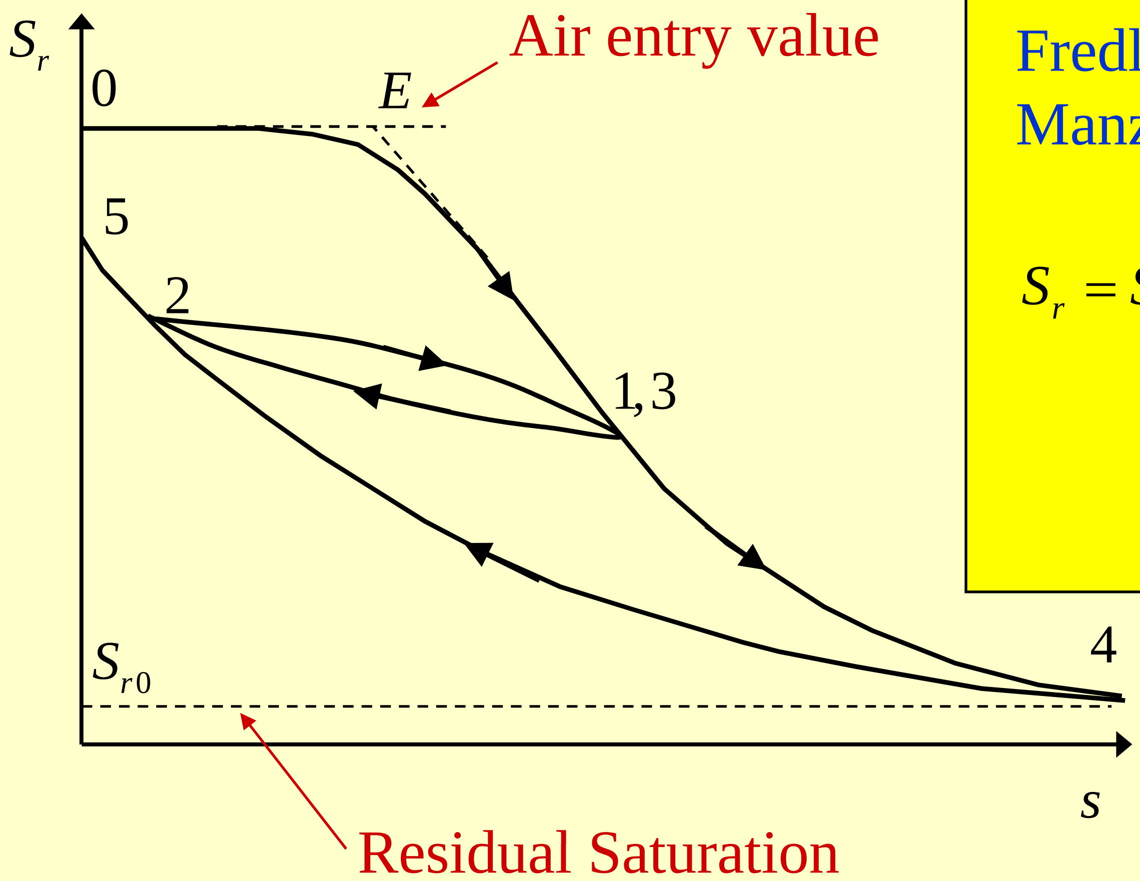
$$\delta W = (\sigma + S_r s I - p_a I)\delta\varepsilon + \boxed{s.(-n\delta S_r)}$$



From Vanapalli et al 1992

Basic definitions (2/2)

- Wetting-drying



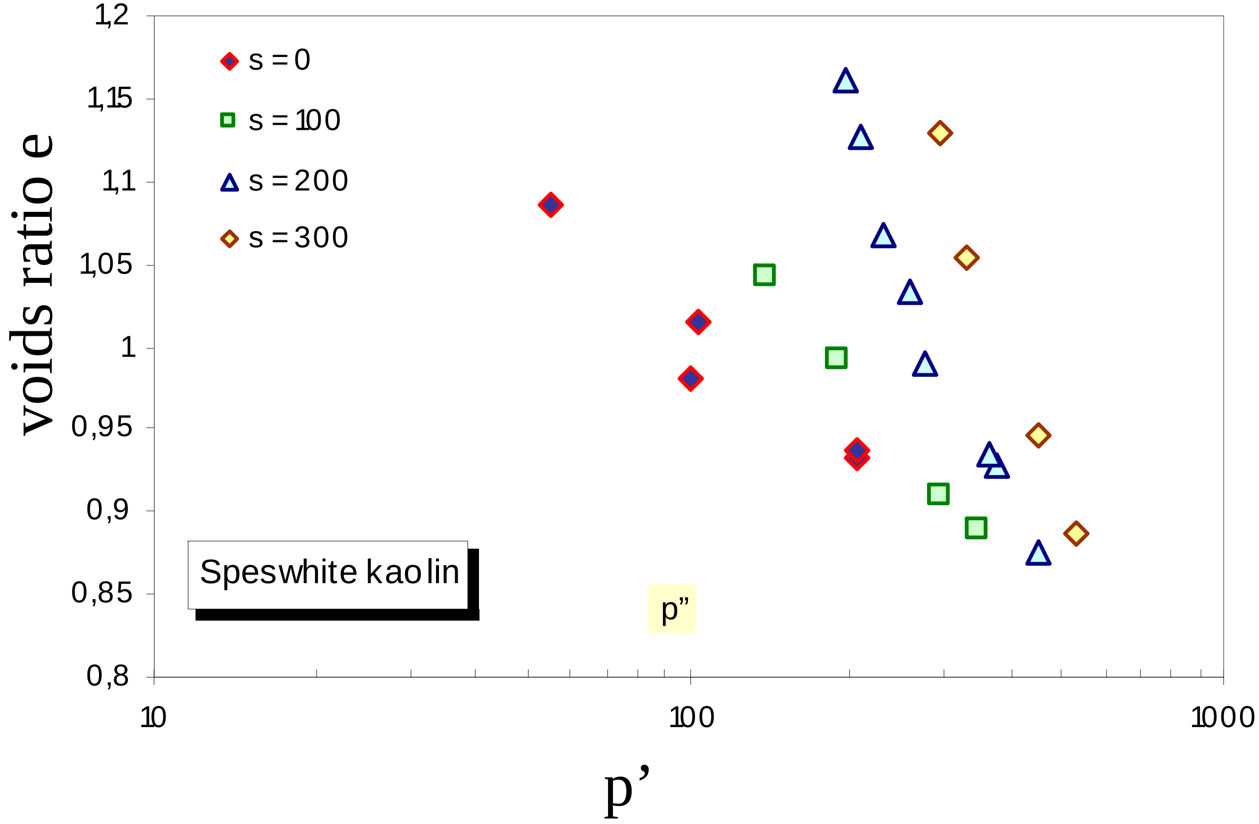
Fredlund, Romero &Vaunat Manzanal

$$S_r = S_{r0} + \frac{1 - S_{r0}}{\ln\left\{e + \left(s^* / a\right)^n\right\}^m}$$

$$s^* = e^{\Omega} s$$

Basic aspects of behaviour (1/5)

- CSL on (e,p') plane

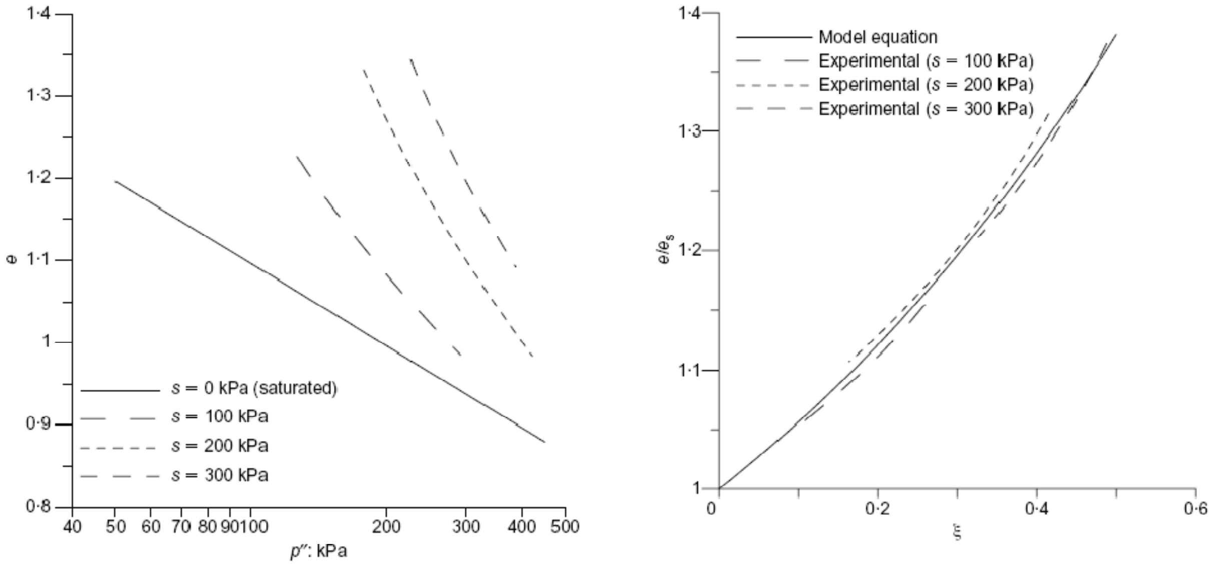


[Reanalysis of data from Sivakumar, 1993]

Basic aspects of behaviour (2/5)

- CSL on (e,p') plane: normalization Gallipoli et al 2003

Introduce $$g(\xi)=\frac{e}{e_s}=1-a\cdot\left[1-exp(b\cdot\xi)\right]$$

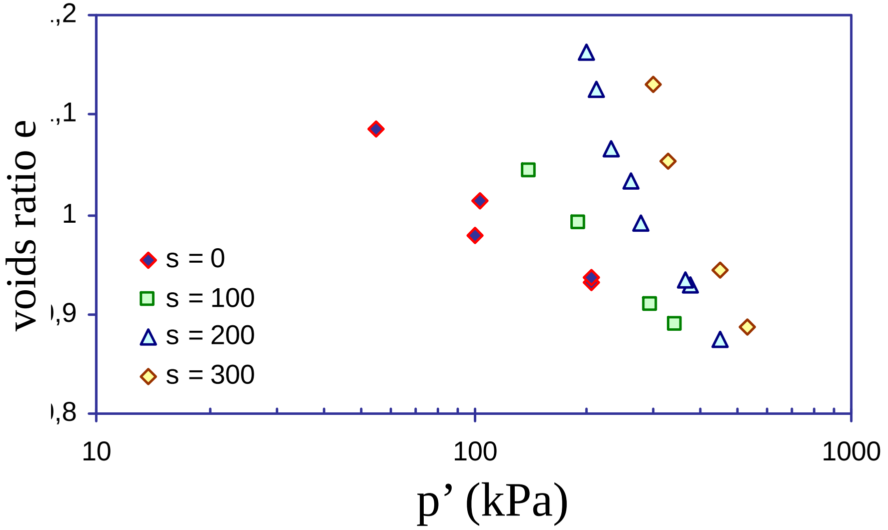


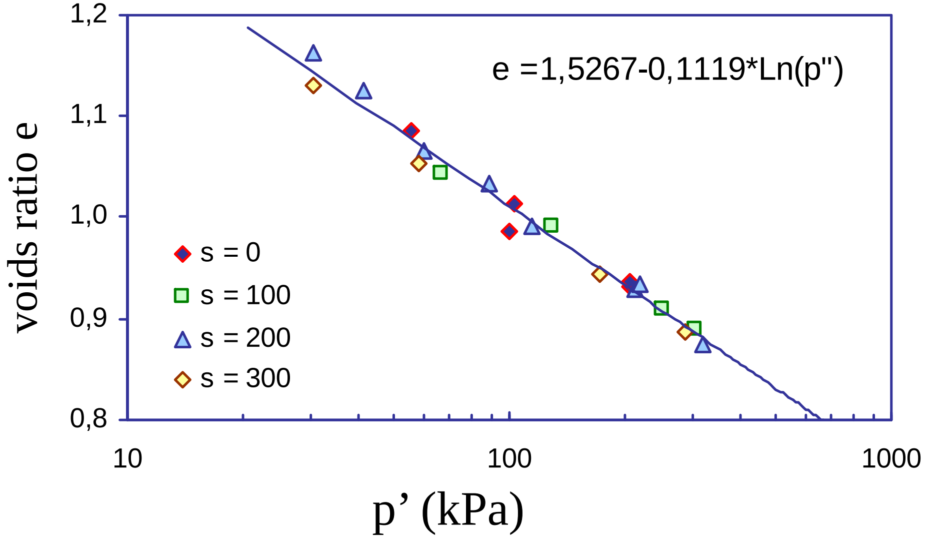
Isotropic compression lines at constant suction (left) and
normalization using the bonding factor (rigth)
Data from Sharma 1998 and predictions by Gallipoli et al (2003)

Basic aspects of behaviour (3/5)

- CSL on (e,p') plane: alternative normalization Manzanal 2008

$$\frac{p'}{p'_s} = \exp\left(g\left(\xi\right)\right) \qquad\qquad g\left(\xi\right) = a\ \exp\left(b\xi\right) - 1\}$$

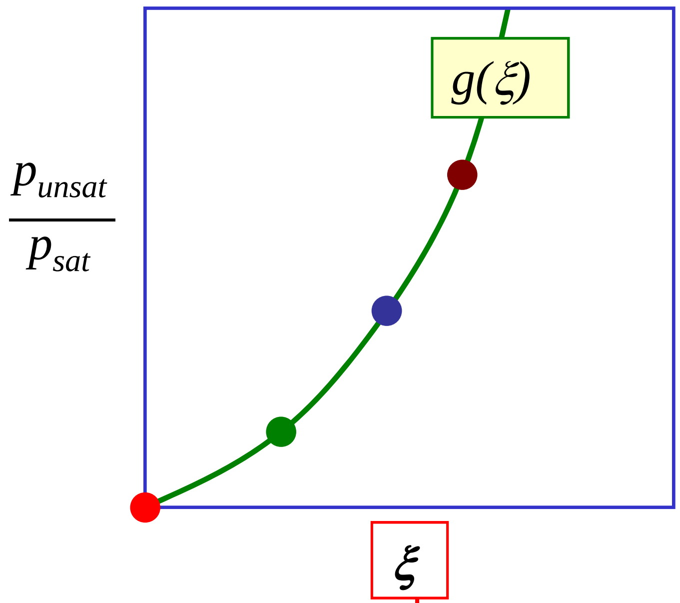


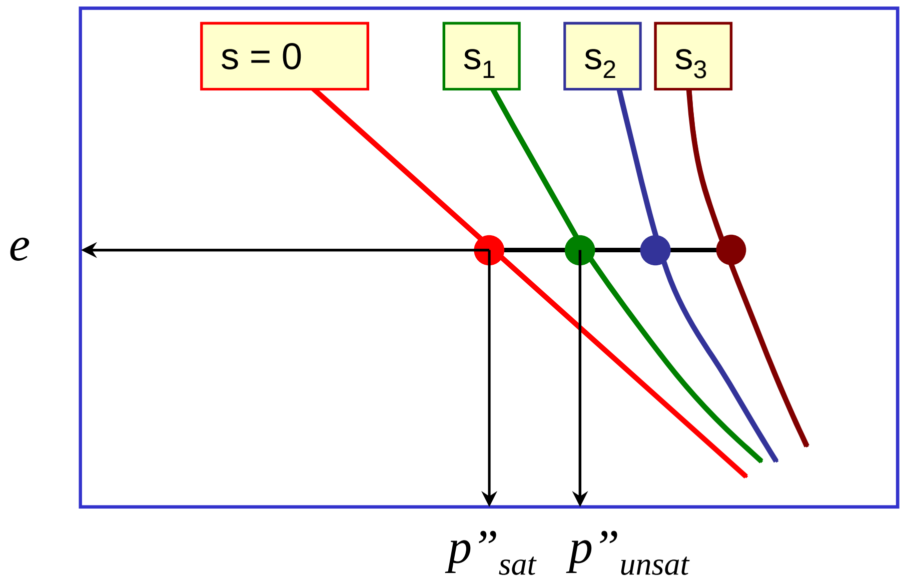


Data from Sivakumar (1993)

Basic aspects of behaviour (3b/5)

Proposed





Cementation parameter

e, p'', Sr, s_i

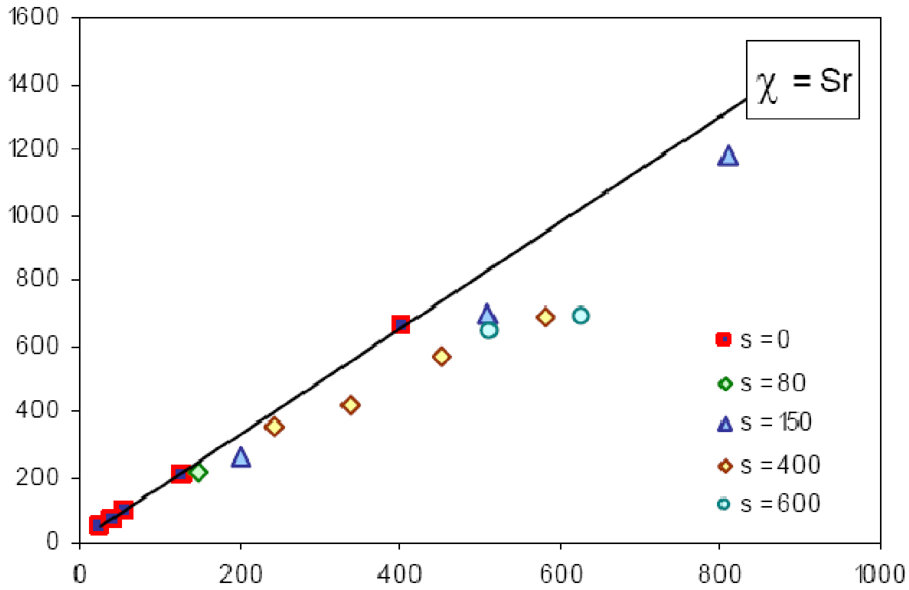
Basic aspects of behaviour (4/5)

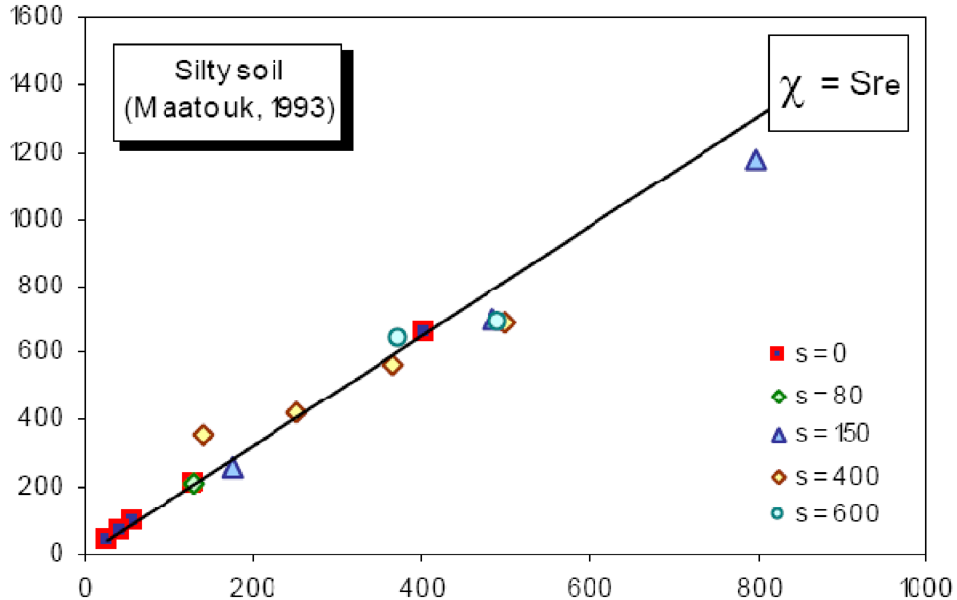
- CSL on (p’,q) plane

$$\sigma' = \sigma + S_r s I - p_a I \quad (\chi = S_r)$$

$$\sigma' = \sigma + S_r s I - p_a I \quad (\chi = S_{re})$$

$$\chi = S_{re} = \frac{S_r - S_{r0}}{1 - S_{r0}}$$

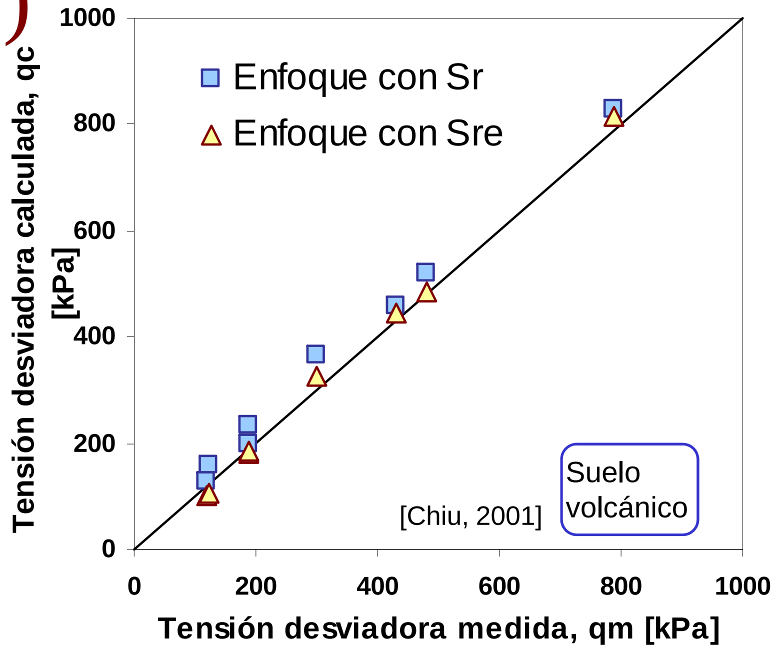


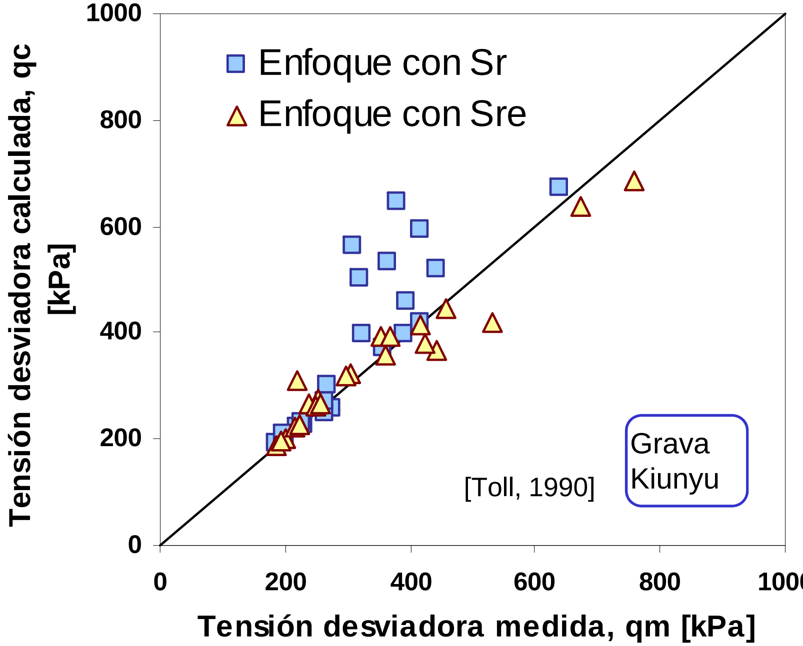


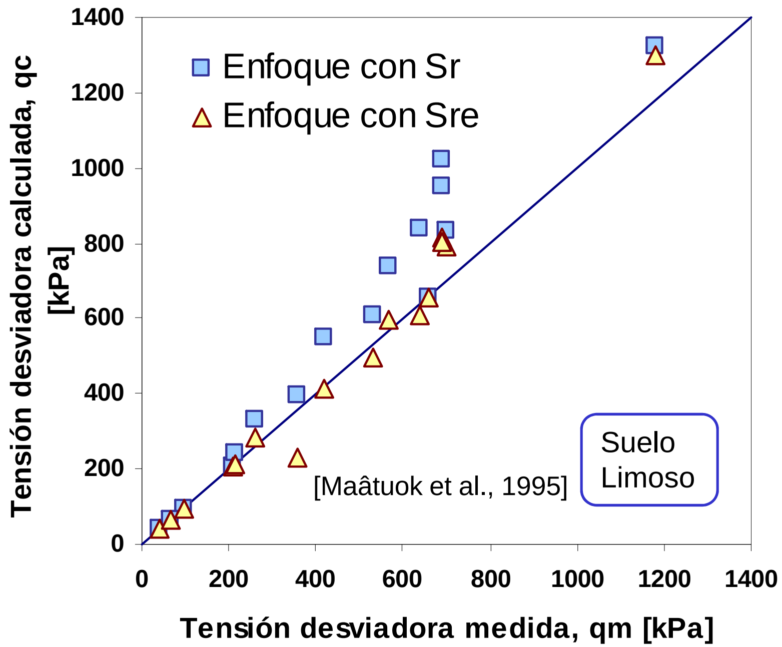
CSL on p’-q plane: without correction (left) and using the effective degree of Saturation (right

Basic aspects of behaviour (5/5)

$$q = M_{CS}\left(\bar{p} + \left(\underbrace{\boxed{\frac{S_r - S_{r0}}{1 - S_{r0}}}}_{S_{re}}\right)\cdot s\right)$$







Generalized Plasticity Model (1/2)

- Main ingredients
  - Effective stress $\sigma' = \sigma + S_r s I - p_a I \quad \left(\chi = S_r\right)$
  - Work conjugate pairs

$$\delta W = \left(\sigma + S_r s I - p_a I\right)\delta\varepsilon + s.\left(-n\delta S_r\right)$$

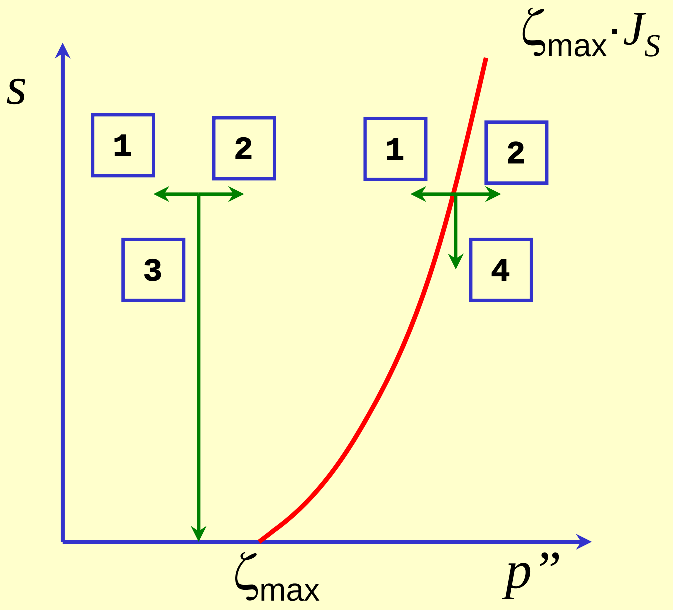
  - Wetting-drying with hysteresis
  - Increment of strain

$$d\varepsilon_{ij} = d\varepsilon_{ij}^{e} + d\varepsilon_{ij\,\sigma}^{p} + d\varepsilon_{ij\,s}^{p}$$

  - State parameter dependent

Generalized Plasticity Model (2/2)

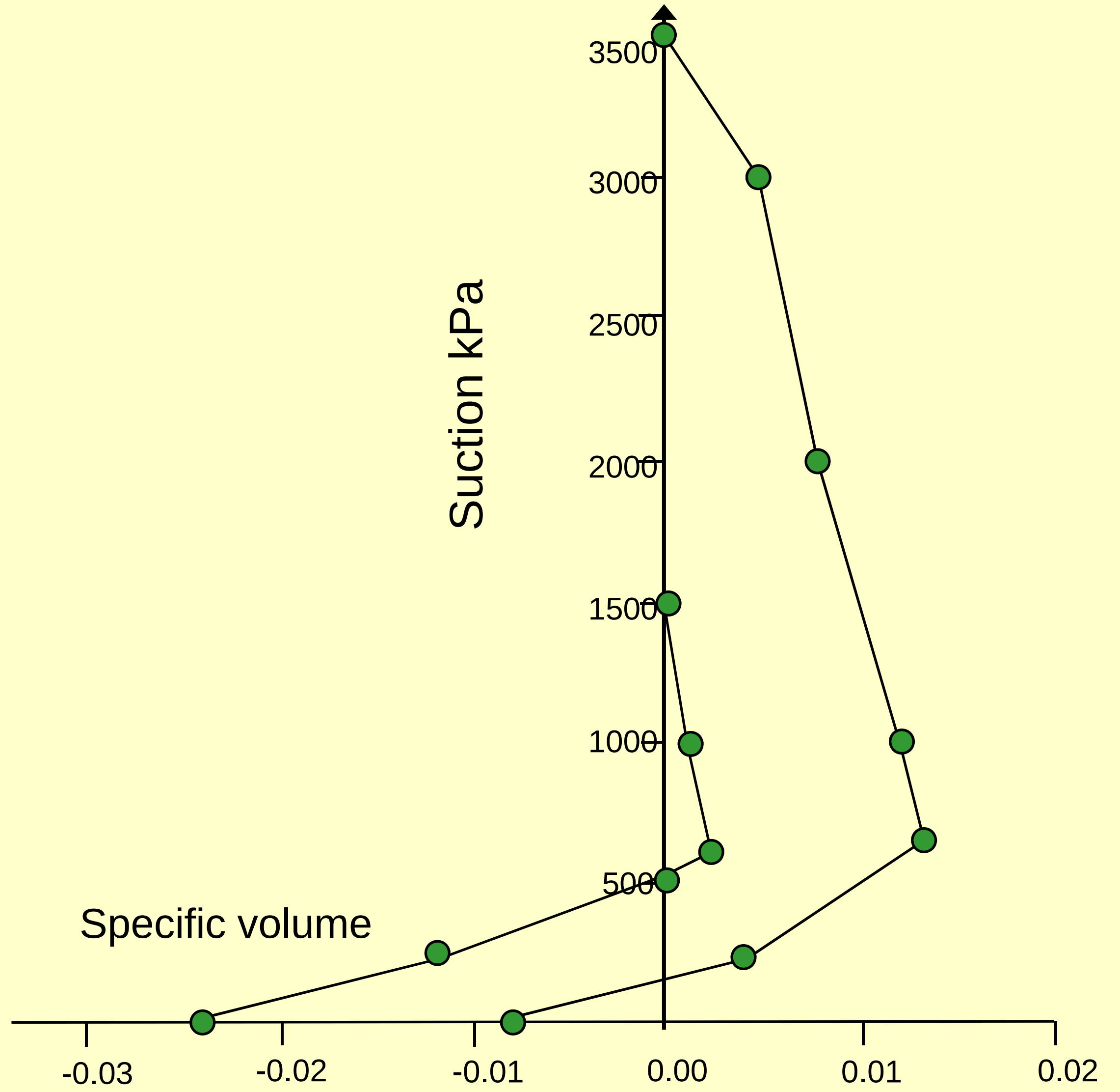
$$d\varepsilon_s^p = \frac{1}{H_b} n_g ds$$



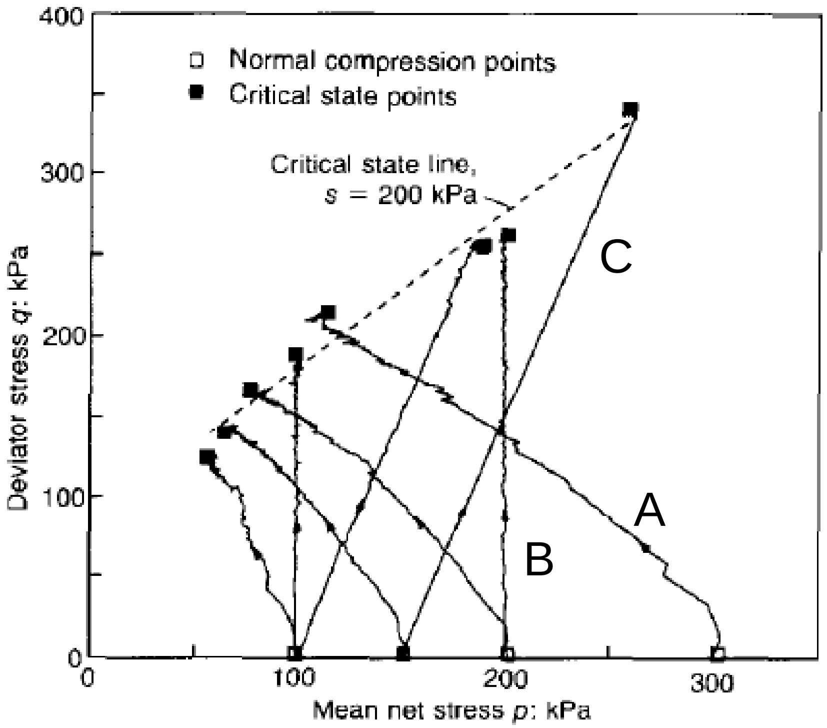
$$H_b = w(\xi) H_0 \sqrt{p' p_{atm}} H_{DM} H_v$$

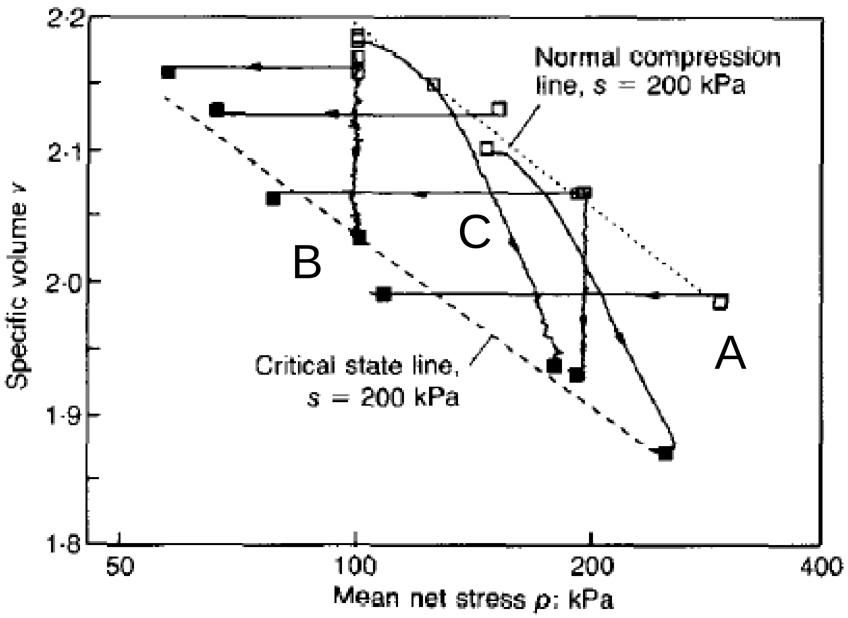
$$H_{DM} = \left(\frac{\zeta_{\max} J_s}{\zeta} \right)^{\gamma}$$

$$J_s = \exp\left(c.g\left(\xi\right)\right)$$



Suction kPa
3500
3000
2500
2000
1500
1000
500
Specific volume
-0.03
-0.02
-0.01
0.00
0.01
0.02

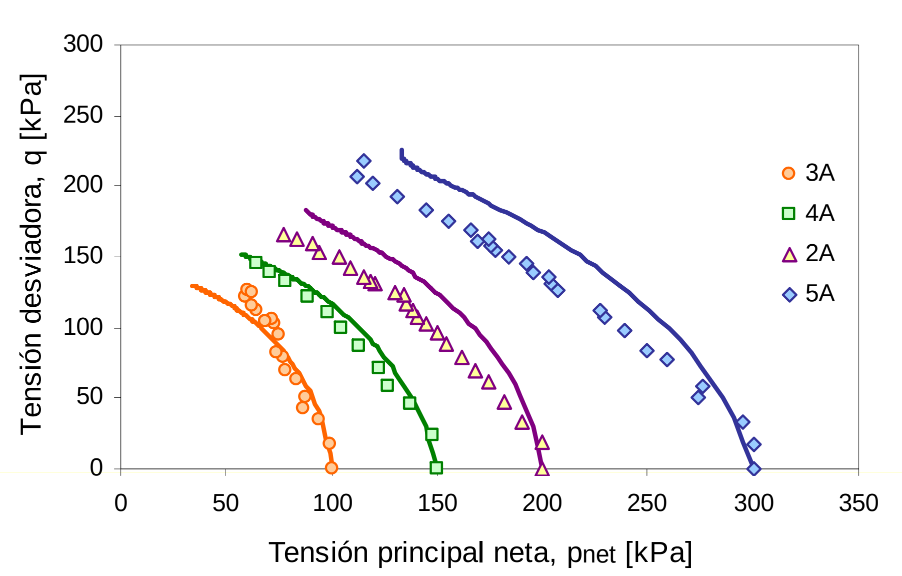




(A) Constant v , constant s tests,
pw and pa increased to keep
the volume and the suction constant

(B) Constant $\bar{p}$, constant s tests,
pw and pa increased
to keep $\bar{p}$ and the suction constant

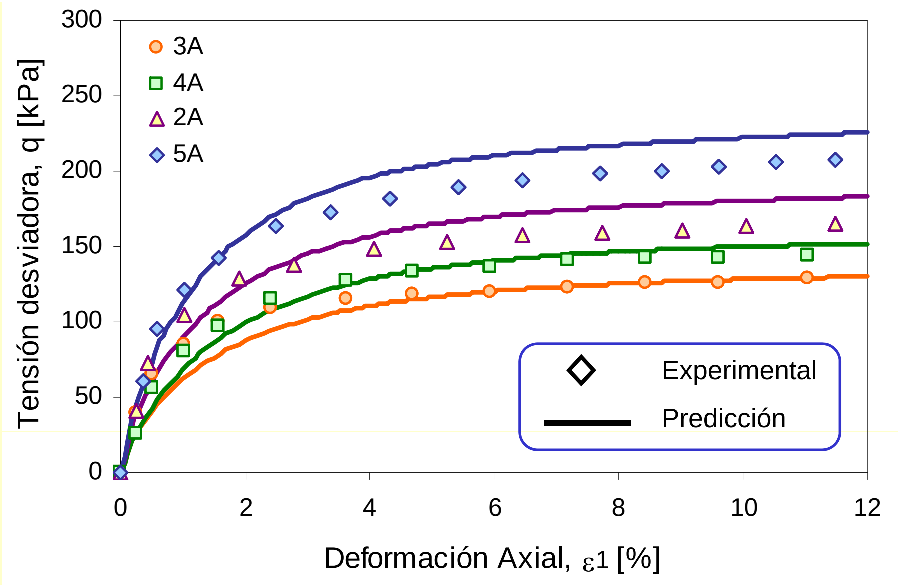
(C) Fully drained constant s test,
pw and pa kept constant

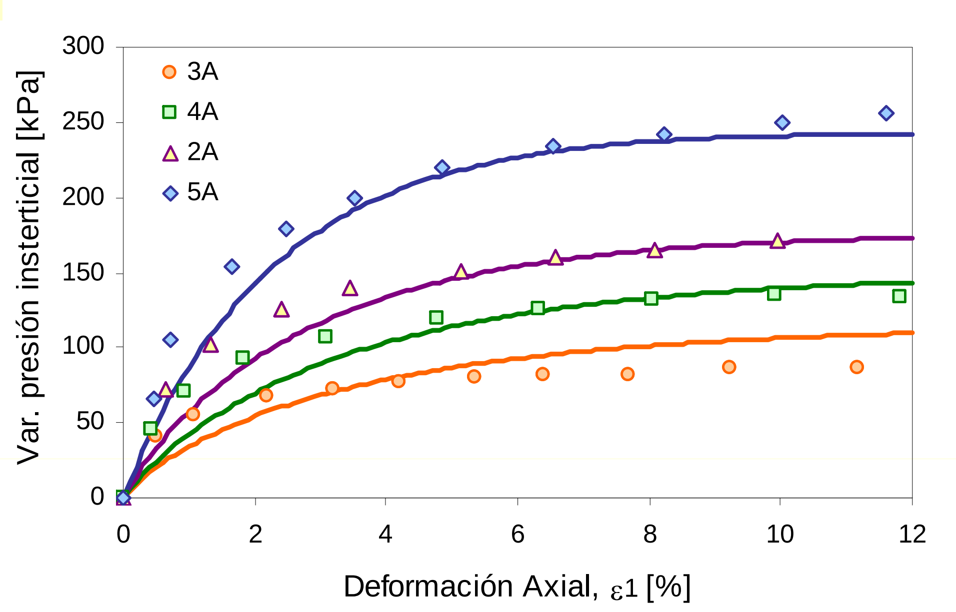


TX Constant Volume

Nº Ensayo	pneta [kPa]	s0 [kPa]	v0	Sr0 [%]
2A	200	200	2,1617	67,25
3A	100	200	2,0669	73,43
4A	150	200	2,1267	69,60
5A	300	200	1,9903	78,44

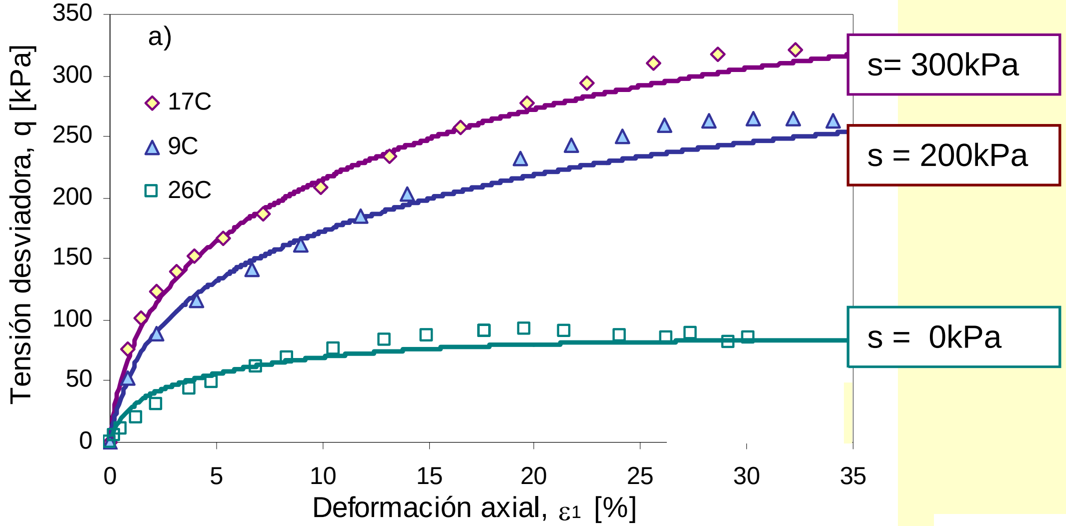
[Datos de Sivakumar, 1993]





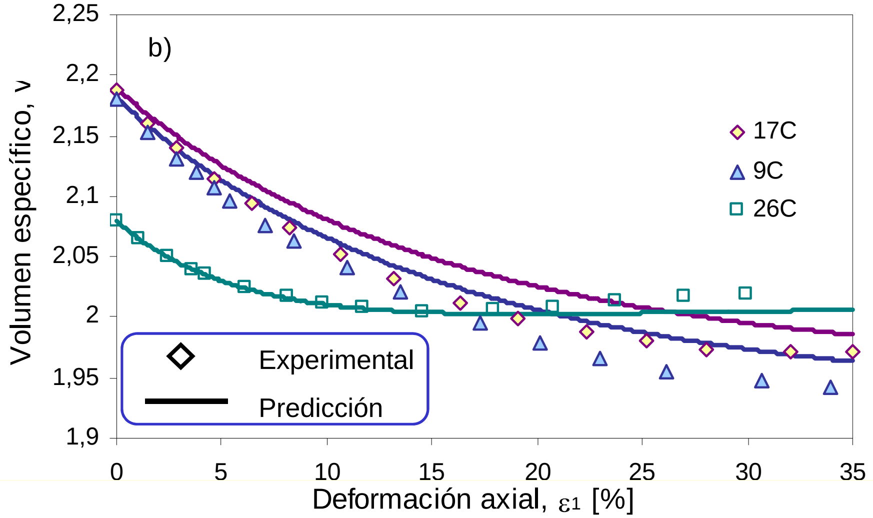
Manzanal (2008)

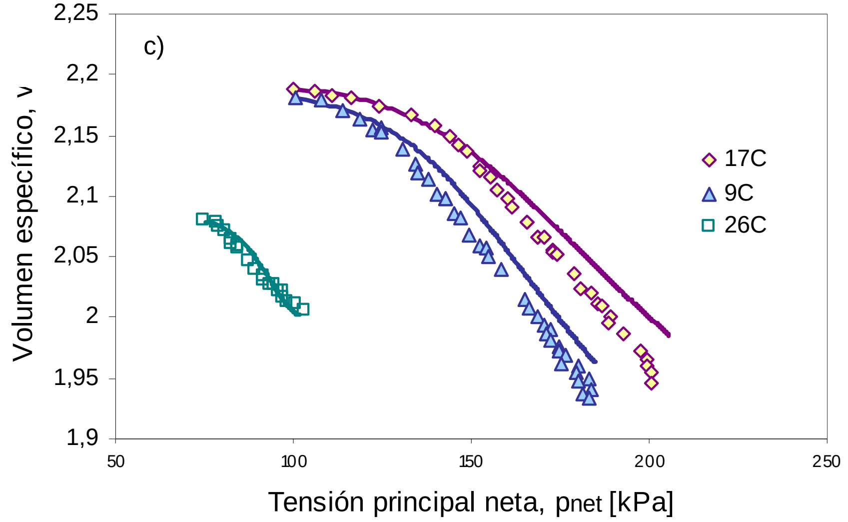
TX Drained

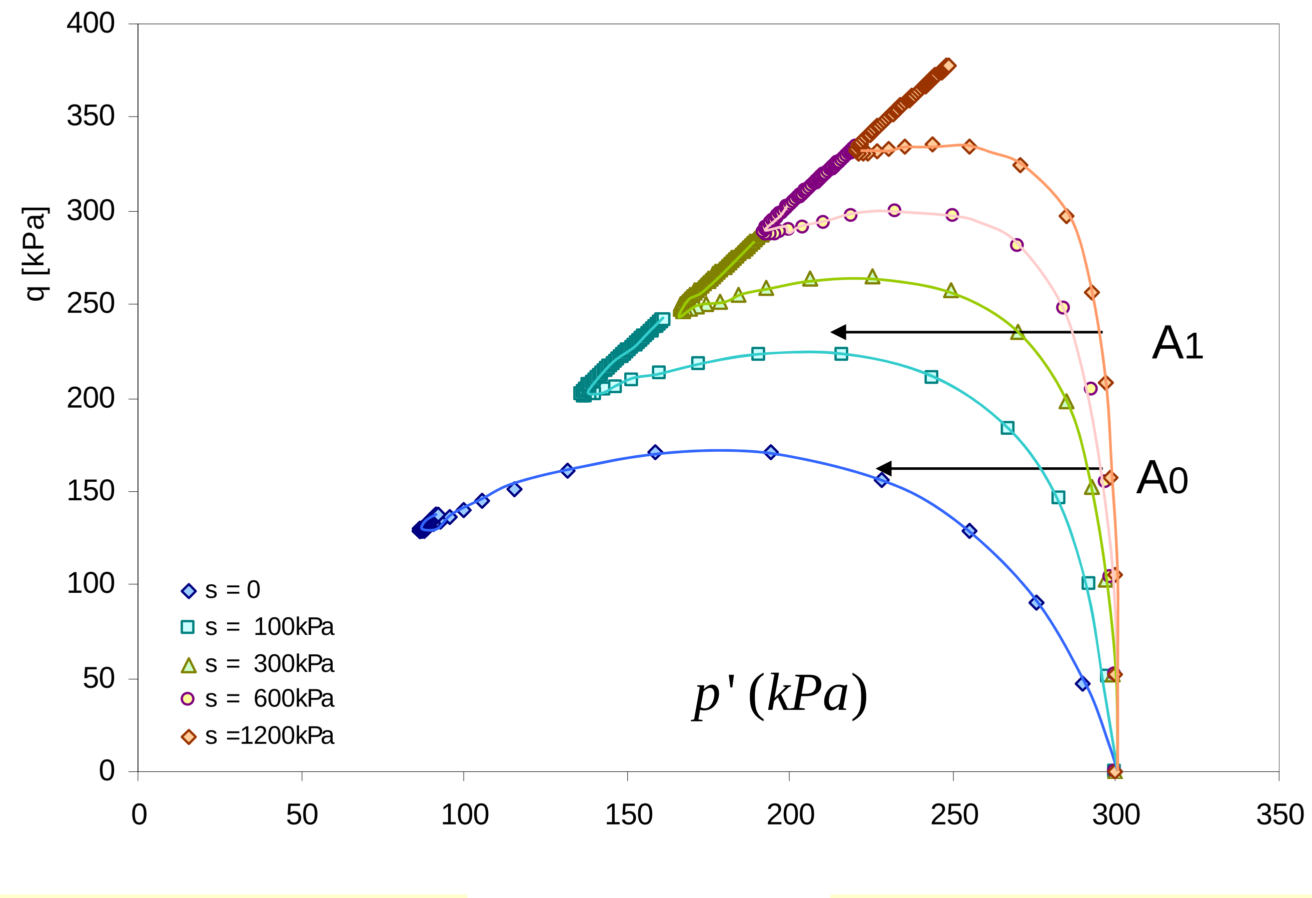


Nº Ensayo	pneta [kPa]	s0 [kPa]	v0	Sr0 [%]	p'' [kPa]
26C	75	0	2,0789	100,00	75,00
9C	100	200	2,1804	66,37	232,73
17C	100	300	2,188	61,17	283,50

[Datos de Sivakumar, 1993]







q [kPa]
400
350
300
250
200
150
100
50
0
0
50
100
150
200
250
300
350
$p'\,(kPa)$
A1
A0
s = 0
s = 100kPa
s = 300kPa
s = 600kPa
s =1200kPa

4. Rheological

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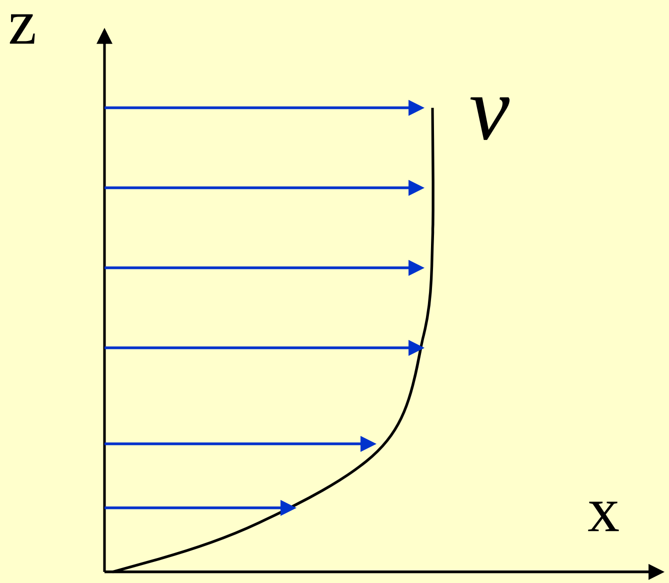
  - Rheology
  - Dilatancy
  - A Perzyna viscoplasticity approach

- R1 What is rheology?

  - In a fluid, shear stress depends on rate of shear strain

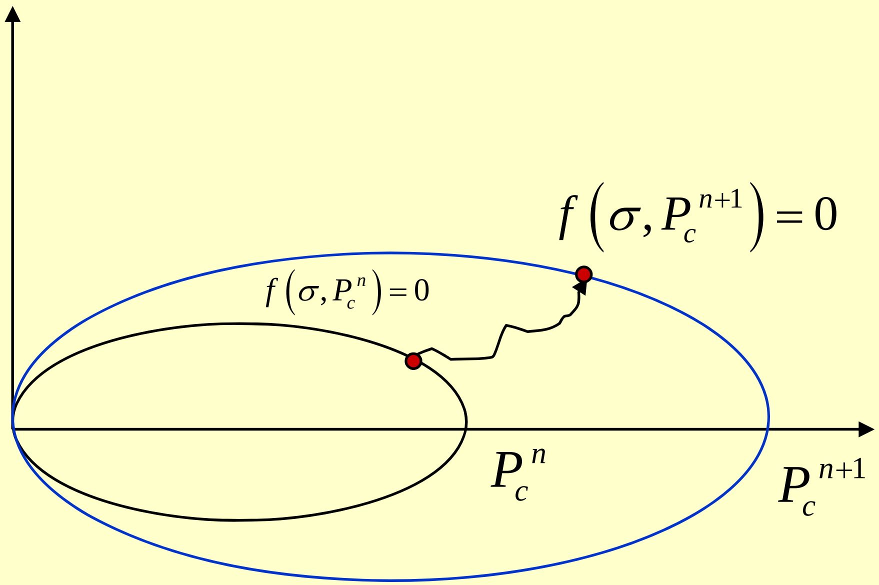


(newtonian)

$$\tau = \mu \frac{\partial v}{\partial z}$$

R1 What is rheology?

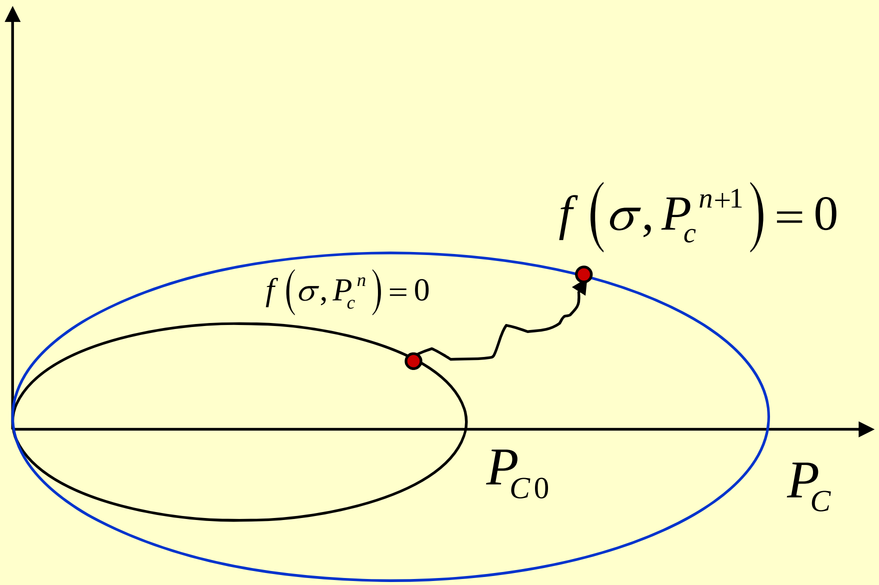
- Differences with elastoplasticity



$$\dot{\varepsilon}^p = \frac{1}{H} n_g \left(n^T . \dot{\sigma} \right) \quad \Rightarrow \quad if \;\; \sigma = ct \quad \dot{\varepsilon}^p = 0$$

R1 What is rheology?

- Differences with viscoplasticity



$$\dot{\varepsilon}^{vp}=\frac{1}{H}n_g\left(\frac{P_c-P_{c0}}{P_{c0}}\right)^n \Longrightarrow \quad if \;\; \sigma = ct \quad \dot{\varepsilon}^{vp}\neq 0$$

$$\dot{\varepsilon} \;\; = \dot{\varepsilon}^e + \dot{\varepsilon}^{vp}$$

R1 What is rheology?

- Similarities with viscoplasticity 1D

$$\dot{\varepsilon}^{vp} = \frac{1}{\hat{\mu}}\left(\frac{\tau - \tau_0}{\tau_0}\right)$$

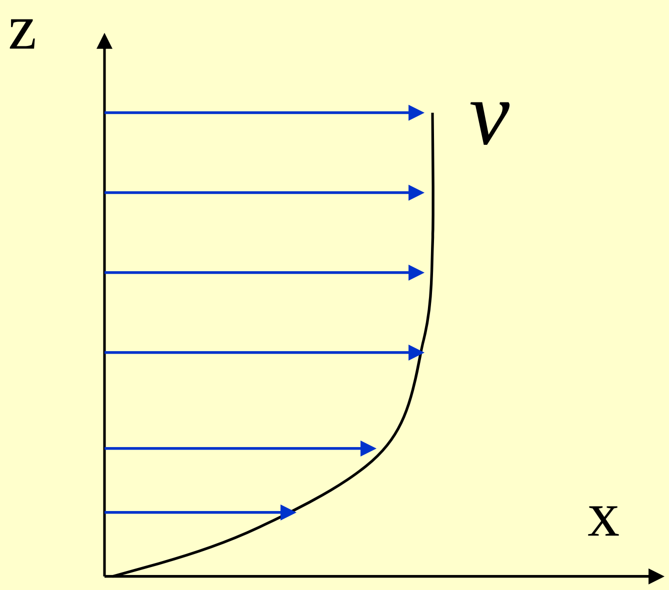
$$\frac{\tau - \tau_0}{\tau_0} = \hat{\mu}\dot{\varepsilon}^{vp}$$

$$\tau = \tau_0 + \tau_0 \hat{\mu}\dot{\varepsilon}^{vp}$$

$$\Longrightarrow \quad \tau = \tau_0 + \mu\,\dot{\varepsilon}^{vp}$$ (Bingham model)

R1 What is rheology?

- In a fluid, shear stress depends on rate of shear strain



(newtonian)

$$\tau = \mu \frac{\partial v}{\partial z}$$

R1 What is rheology? Generalization to 3D

- Rate of deformation tensor D

$$D_{ij} = \frac{1}{2}\left(\frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i}\right)$$

- Stress

$$\sigma = -pI + \Phi_0 I + \Phi_1 D + \Phi_2 D^2$$

$$\Phi_k = \Phi_k\left(I_{D1}, I_{D2}, I_{D3}\right)$$

$$I_{D1} = tr\, D$$

$$I_{D2} = \frac{1}{2} tr\, D^2$$

$$I_{3D} = \frac{1}{3} tr\, D^3$$

- Assumptions

$I_{D1} = 0$ (incompressible)

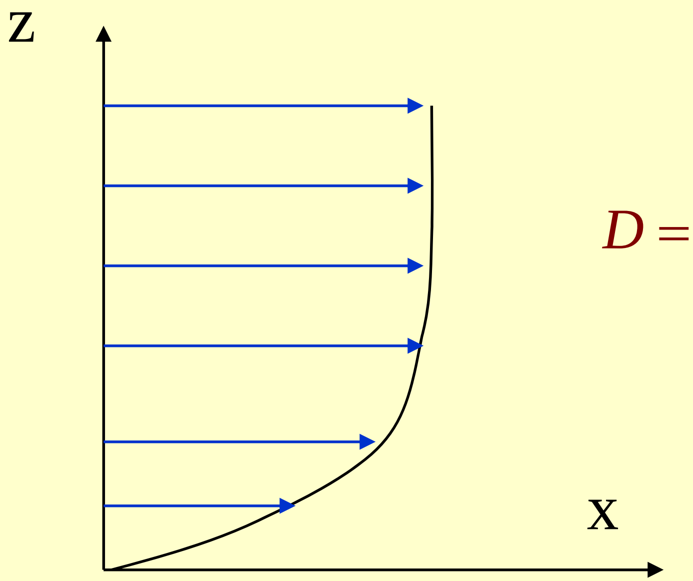
$$\Phi_k = \Phi_k\left(I_{D2}\right)$$

- Hydrostatic & deviatoric

$$p_{hyd} = -\frac{1}{3} tr(\sigma) = p - \left(\Phi_0 + \frac{1}{3}\Phi_2 trD^2\right)$$

R1 What is rheology?

- Simple shear flow $\quad v = (v, 0, 0) \quad v = v(z)$



$$D = \begin{pmatrix} 0 & 0 & \frac{1}{2}\frac{\partial v}{\partial z} \\ 0 & 0 & 0 \\ \frac{1}{2}\frac{\partial v}{\partial z} & 0 & 0 \end{pmatrix} \quad D^2 = \begin{pmatrix} \frac{1}{4}\left(\frac{\partial v}{\partial z}\right)^2 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \frac{1}{4}\left(\frac{\partial v}{\partial z}\right)^2 \end{pmatrix}$$

$$\sigma = -pI + \Phi_0 I + \Phi_1 D + \Phi_2 D^2$$

$$\sigma_{xx} = \sigma_{zz} = -p + \Phi_0 + \boxed{\frac{1}{4}\Phi_2\left(\frac{\partial v}{\partial z}\right)^2}$$

$$\sigma_{yy} = -p + \Phi_0$$

$$\sigma_{xz} = \frac{1}{2}\Phi_1 \frac{\partial v}{\partial z}$$

- R1 What is rheology?
  - Example: generalization of Newtonian fluid

$$\sigma_{xx} = \sigma_{zz} = -p + \Phi_0 + \frac{1}{4}\Phi_2\left(\frac{\partial v}{\partial z}\right)^2$$

$$\sigma_{yy} = -p + \Phi_0$$

$$\sigma_{xz} = \frac{1}{2}\Phi_1\frac{\partial v}{\partial z}$$

$$\sigma_{xx} = \sigma_{zz} = -p$$

$$\sigma_{yy} = -p$$

$$\sigma_{xz} = \mu\frac{\partial v}{\partial z}$$

$$\sigma_{xx} = \sigma_{zz} = -p + \Phi_0 + \frac{1}{4}\Phi_2\left(\frac{\partial v}{\partial z}\right)^2 = -p$$

$$\sigma_{yy} = -p + \Phi_0 = -p$$

$$\sigma_{xy} = \frac{1}{2}\Phi_1 = \mu$$

$$\Rightarrow \quad \Phi_0 = 0, \quad \Phi_1 = 2\mu, \quad \Phi_2 = 0$$

$$\sigma = -pI + \Phi_0 I + \Phi_1 D + \Phi_2 D^2 \quad \Rightarrow \quad \sigma = -pI + 2\mu D$$

Newtonian Fluid

- Experimental $\tau = \mu \dfrac{\partial v}{\partial z}$

$$\sigma_{xx} = \sigma_{zz} = -\bar{p} + \cancel{\frac{1}{4}\Phi_2 \left(\frac{\partial v}{\partial z}\right)^2}$$

$$\sigma_{yy} = -\bar{p}$$

$$\sigma_{xz} = \frac{1}{2}\Phi_1 \frac{\partial v}{\partial z} \longrightarrow \Phi_1 = 2\mu$$

- General law $\sigma = -\bar{p}\, I + 2\mu D$

Material	viscosity(Pa.s)
air	10^{-6}
water	10^{-3}
mud	10^{-2}

Bingham Fluid

- Experimental $\tau = \tau_y + \mu \dfrac{\partial v}{\partial z}$

$$\sigma_{xx} = \sigma_{zz} = -\bar{p} + \cancel{\frac{1}{4}\Phi_2 \left(\frac{\partial v}{\partial z}\right)^2}$$

$$\sigma_{yy} = -\bar{p}$$

$$\sigma_{xz} = \frac{1}{2}\Phi_1 \frac{\partial v}{\partial z}$$

$$\frac{1}{2}\Phi_1 \frac{\partial v}{\partial z} = \tau_y + \mu \frac{\partial v}{\partial z}$$

$$\Phi_1 = \frac{2\tau_y}{\partial v / \partial z} + 2\mu$$

but $I_{2D} = \dfrac{1}{2} trD^2 = \dfrac{1}{4}\left(\dfrac{\partial v}{\partial z}\right)^2$

$$\Phi_1 = \frac{\tau_y}{\sqrt{I_{2D}}} + 2\mu$$

- General law

$$\boxed{\sigma = -\bar{p}\, I + \left\{ \frac{\tau_y}{\sqrt{I_{2D}}} + 2\mu \right\} D}$$

- Bingham's fluid

	ρ	$\tau_y\left(Pa\right)$	$\mu\left(Pa.s\right)$
Jan		$100-160$	$40-60$
Johnson	$2000-2400$	$60,\ 170-150$	45
Sharp&Noble	2400		$20-60$
Pierson	2090	$130-240$	$210-810$
Rickenmann&Koch		$100-800$	$400-800$
Jeyapalan	1400	1000	50

Bagnold's rheometer

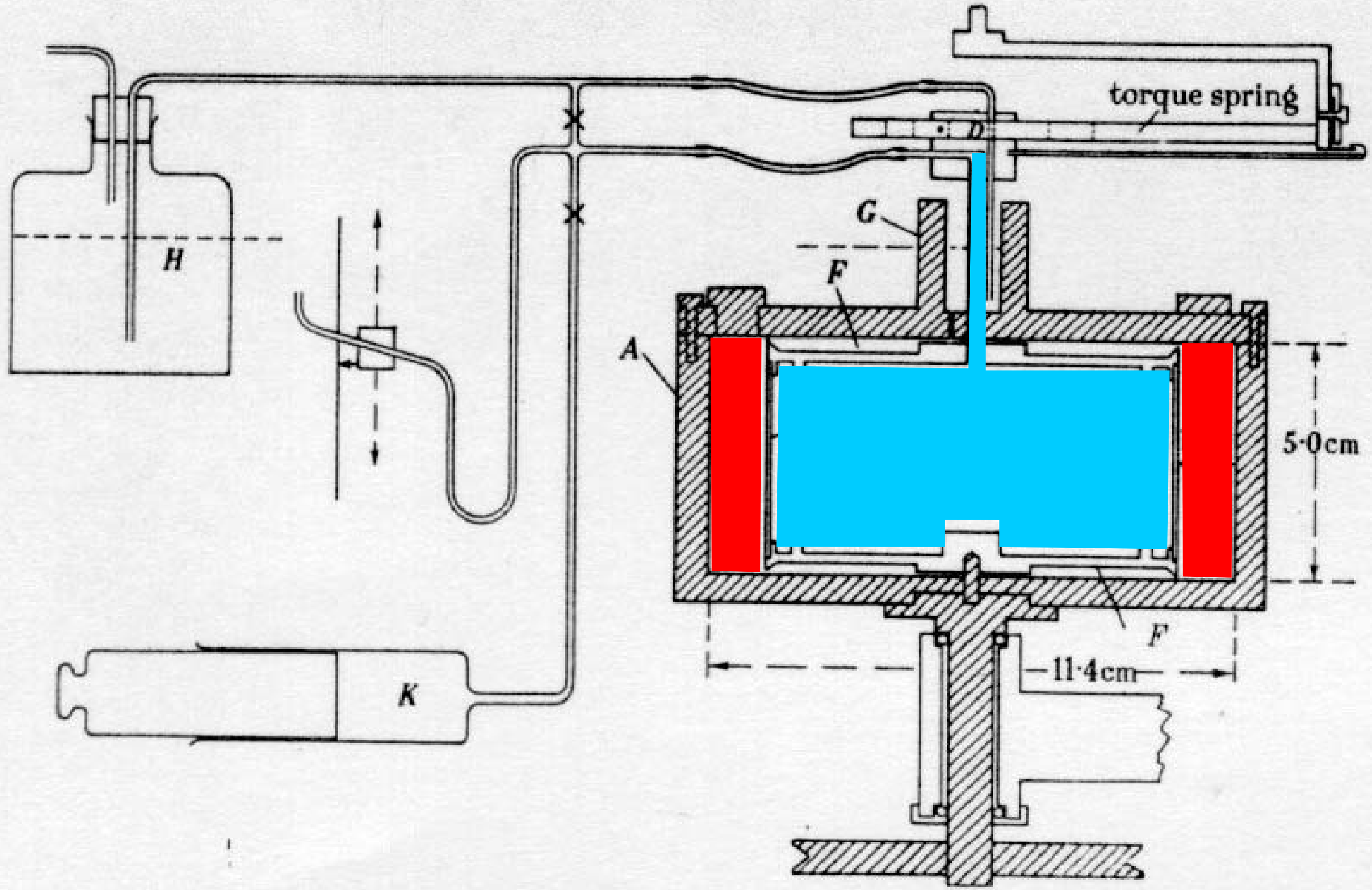
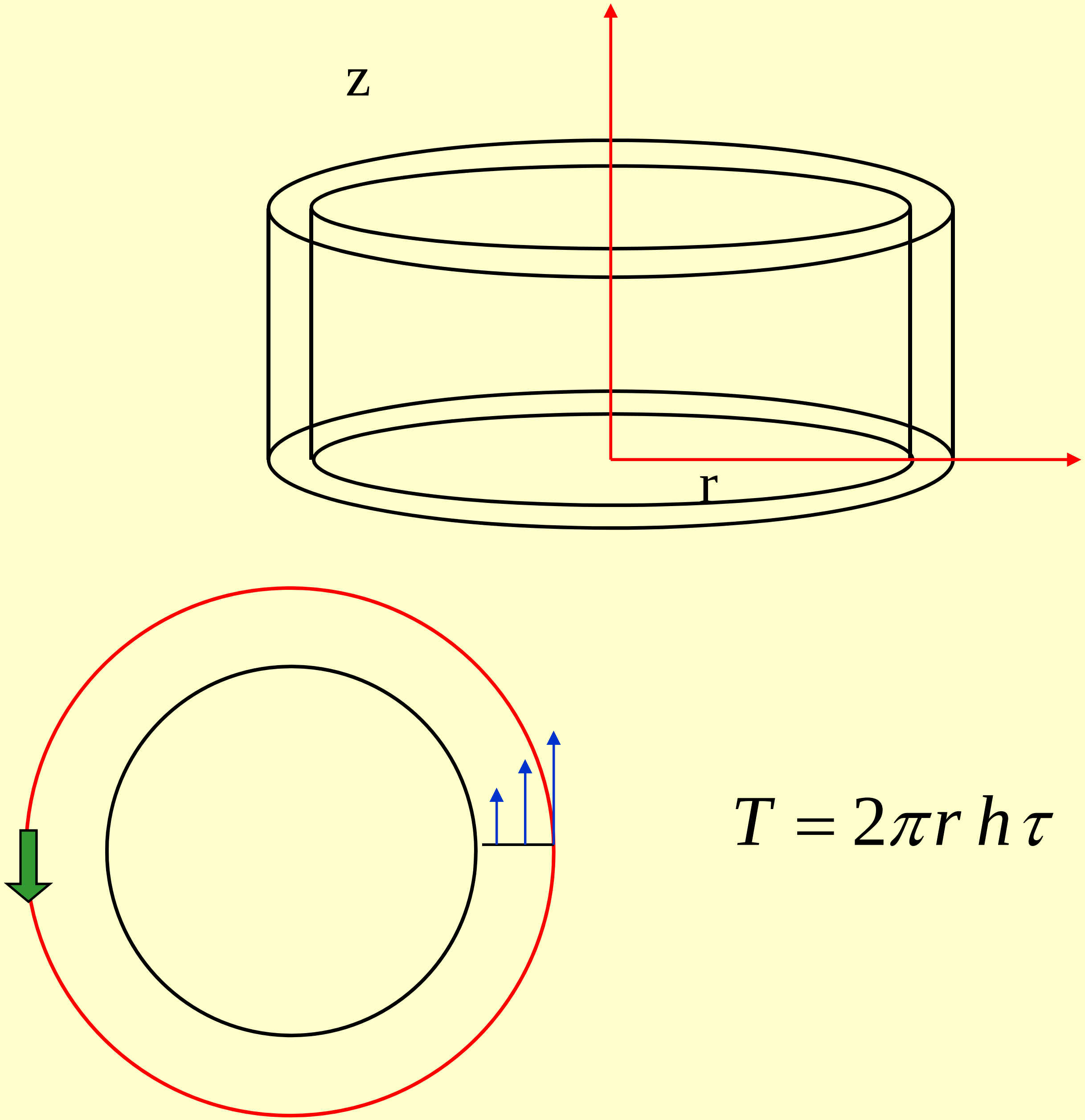


FIGURE 2. The apparatus (rotating parts shown hatched).

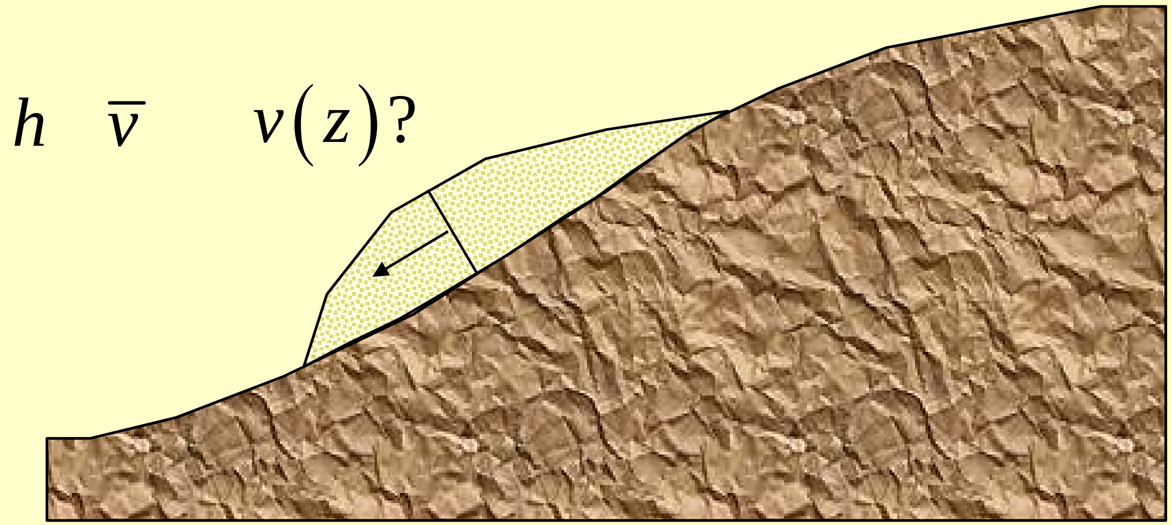
T

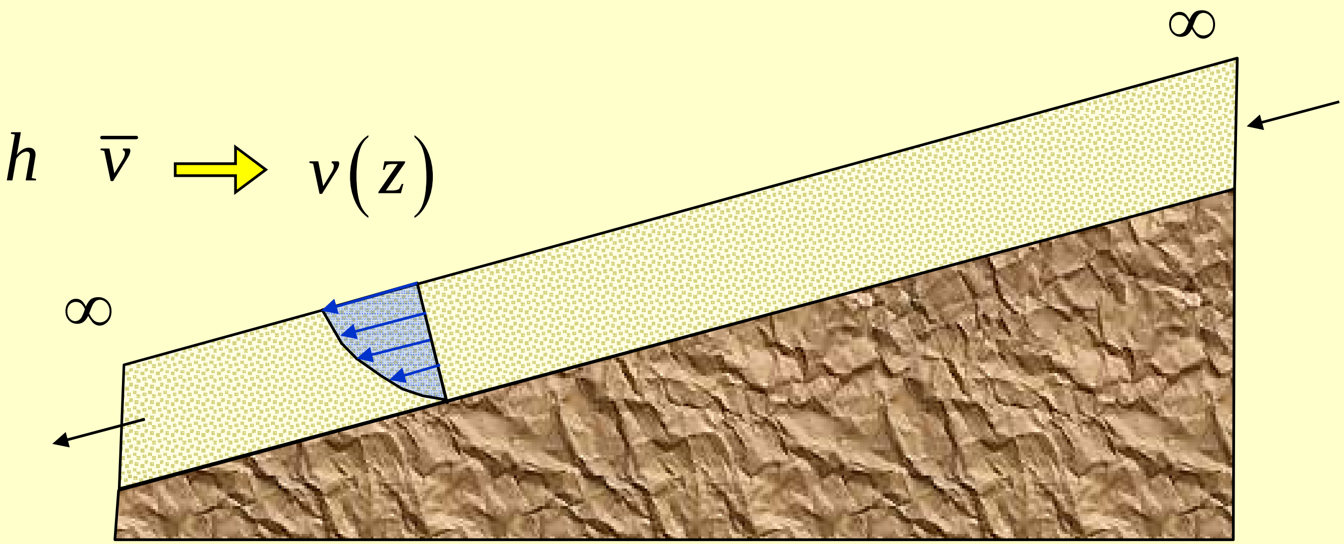
h



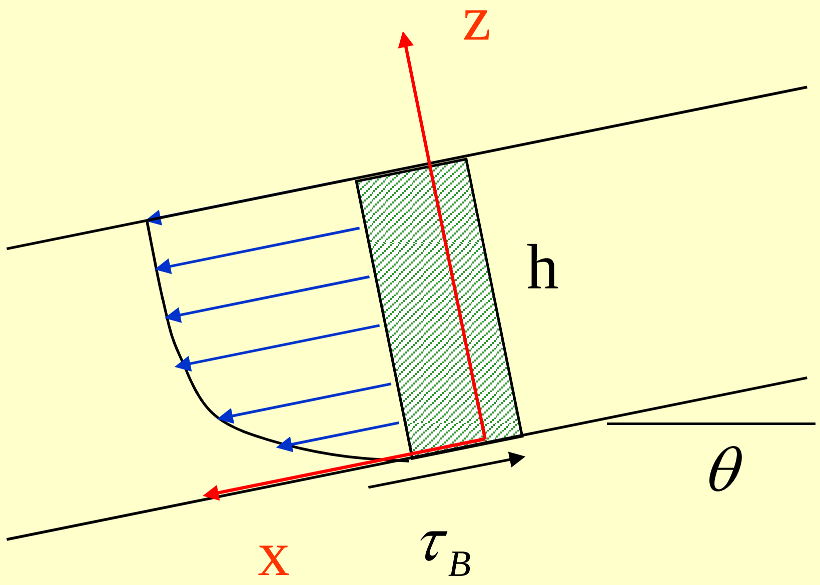
$$T = 2\pi r\,h\,\tau$$

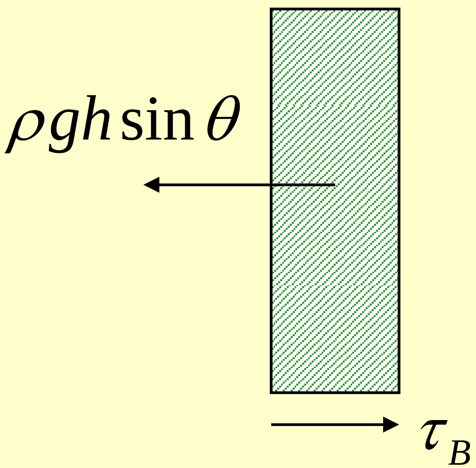
D1 Introduction. The infinite slide

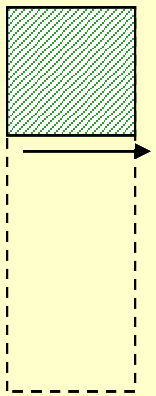




D1 Introduction. The infinite slide



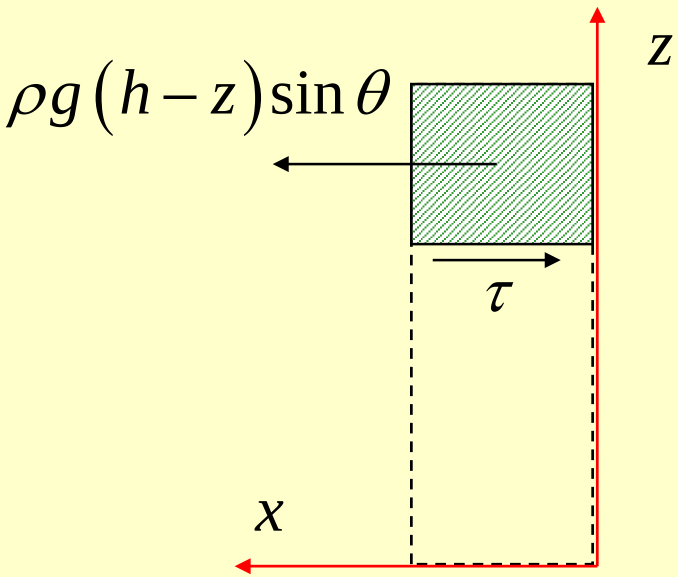




$$\tau = \rho g \left(h - z\right)\sin\theta$$

$$\underbrace{\tau = \rho g \left(h - z\right)\sin\theta = f\left(s, \frac{\partial v}{\partial z}\right)}_{\text{Integrate}}$$

D2 Some simple examples: Newtonian



$$\tau_B = \rho g h \sin\theta \qquad \xi = z/h$$

$$\mu \frac{\partial v}{\partial z} = \rho g\left(h-z\right)\sin\theta$$

$$v = \frac{\rho g \sin\theta}{\mu}\left(hz - \frac{z^2}{2}\right)$$

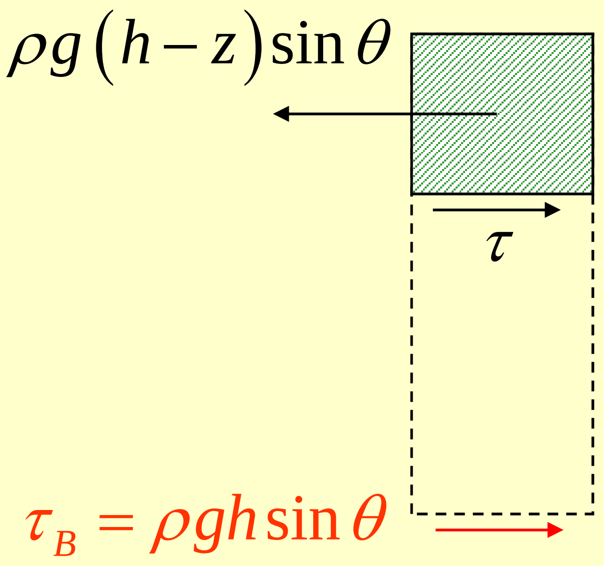
$$\boxed{v = \frac{\tau_B h}{\mu}\left(\xi - \frac{1}{2}\xi^2\right)}$$

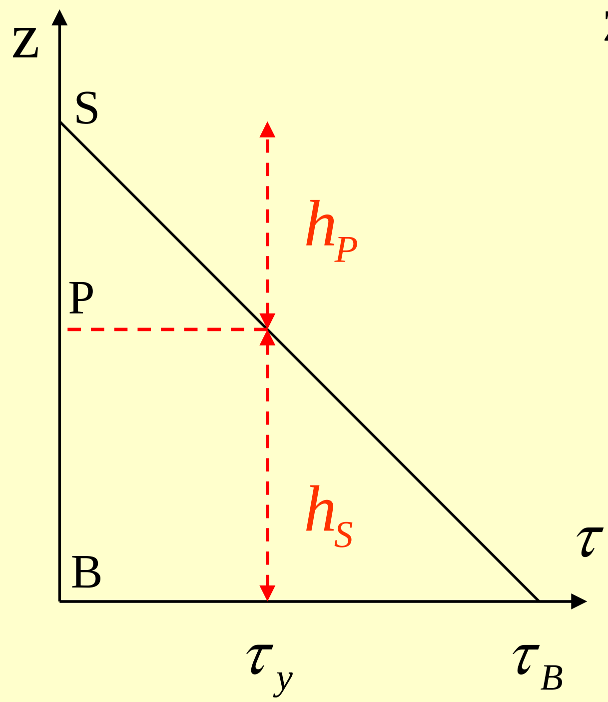
$$\boxed{\bar{v} = \frac{1}{h}\int_0^h v\left(z\right)dz = \frac{\tau_B h}{3\mu}}$$

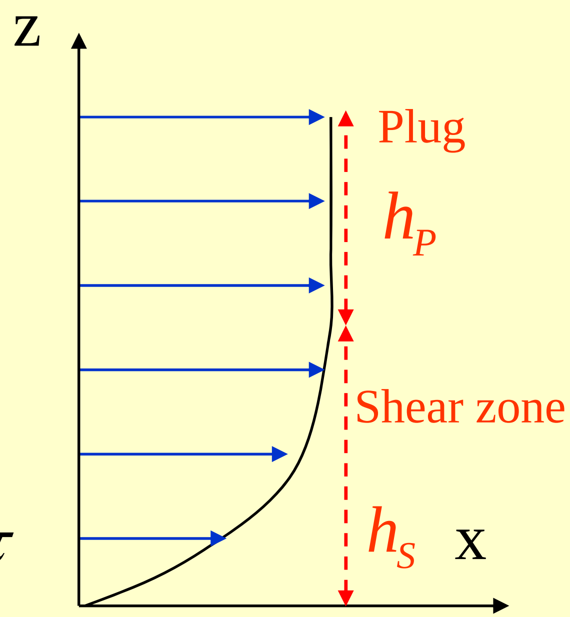
$$\boxed{\tau_B = \frac{3\mu\bar{v}}{h}}$$


Yunnan 2004

- D2 Some simple examples: <u>Bingham fluid</u> $\tau = \tau_y + \mu \dfrac{\partial v}{\partial z}$





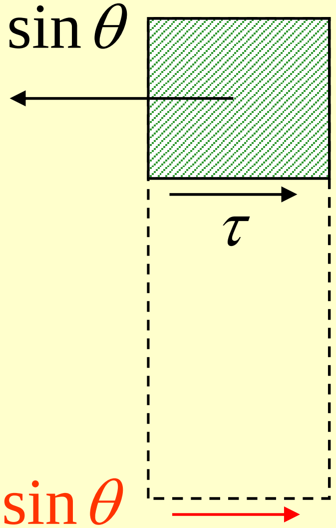


$$\rho g \sin\theta \, h_P = \tau_Y$$

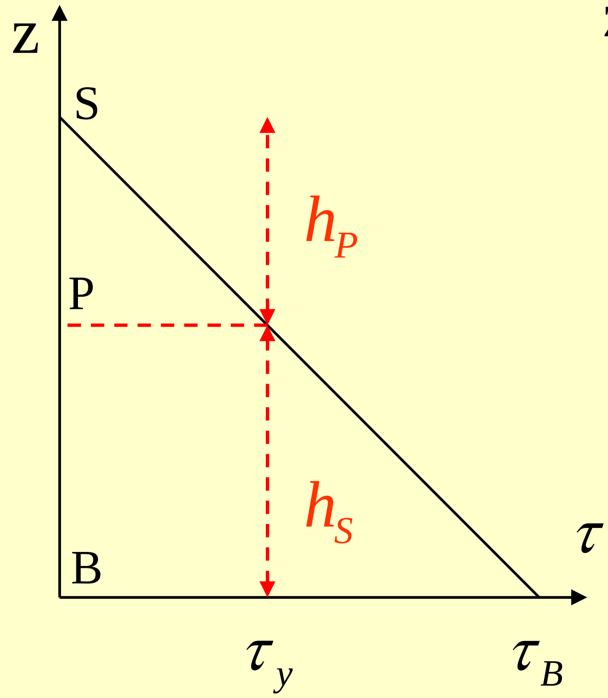
$$\boxed{h_P = \frac{\tau_Y}{\rho g \sin\theta} \qquad h_S = h - h_P}$$

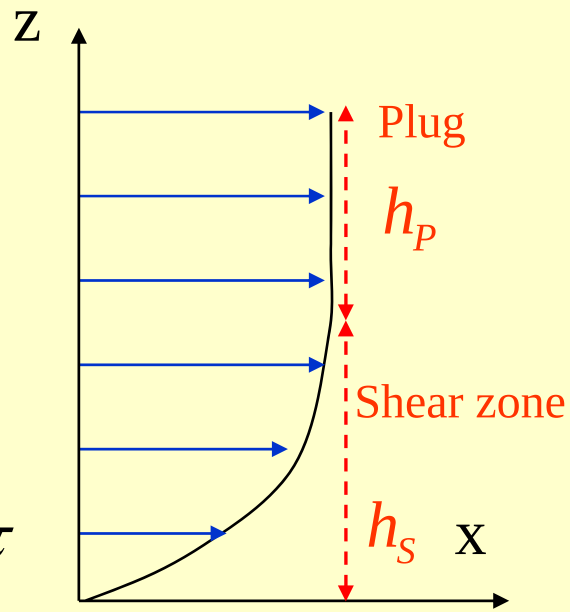
- D2 Some simple examples: <u>Bingham fluid</u> $\tau = \tau_y + \mu \dfrac{\partial v}{\partial z}$

$\rho g \left(h - z \right) \sin\theta$



$\tau_B = \rho g h \sin\theta$



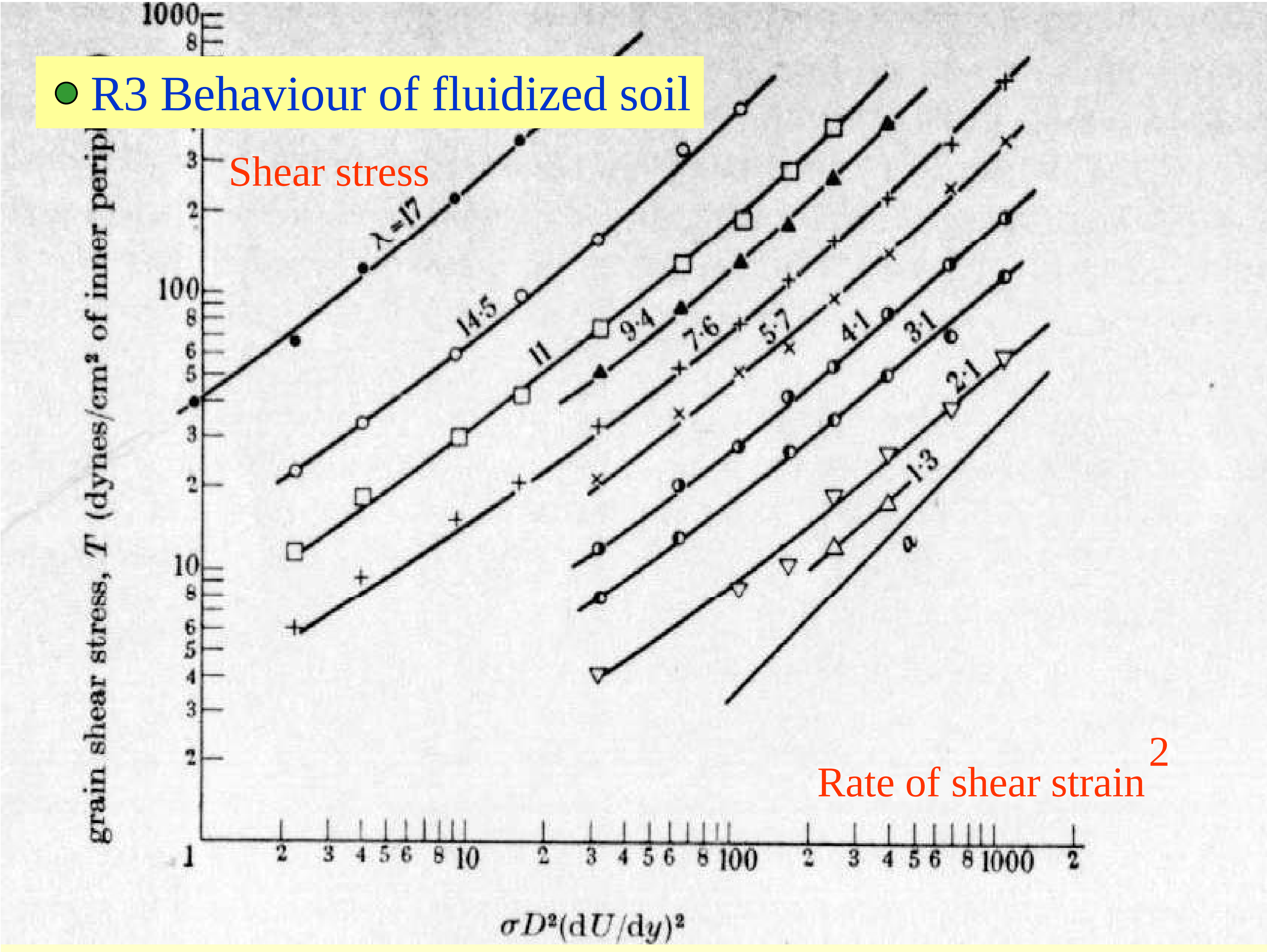


$$\rho g \sin\theta \, h_P = \tau_Y$$

$$\boxed{h_P = \frac{\tau_Y}{\rho g \sin\theta} \qquad h_S = h - h_P}$$

Contents

R3 Behaviour of fluidized soil



Bagnold's Fluid

- Experimental

$$\tau = \mu_B \sin\phi_B \left(\frac{\partial v}{\partial z}\right)^2$$

$$\sigma_v = -p - \mu_B \cos\phi_B \left(\frac{\partial v}{\partial z}\right)^2$$

$$\sigma_h = \sigma_v$$

- General law

$$\sigma_{xz} = \sigma_{zz} = -p + \frac{1}{4}\Phi_2 \left(\frac{\partial v}{\partial z}\right)^2$$

$$\sigma_{xz} = \frac{1}{2}\Phi_1 \left(\frac{\partial v}{\partial z}\right)$$

$$\sigma_{xz} = \sigma_{zz} = -p + \frac{1}{4}\Phi_2 \left(\frac{\partial v}{\partial z}\right)^2$$

$$= -p - \mu_B \cos\phi_B \left(\frac{\partial v}{\partial z}\right)^2$$

$$\sigma_{xz} = \frac{1}{2}\Phi_1 \left(\frac{\partial v}{\partial z}\right) = \mu_B \sin\phi_B \left(\frac{\partial v}{\partial z}\right)^2$$

$$\Longrightarrow$$

$$\Phi_1 = 2\mu_B \sin\phi_B \left(\frac{\partial v}{\partial z}\right)$$

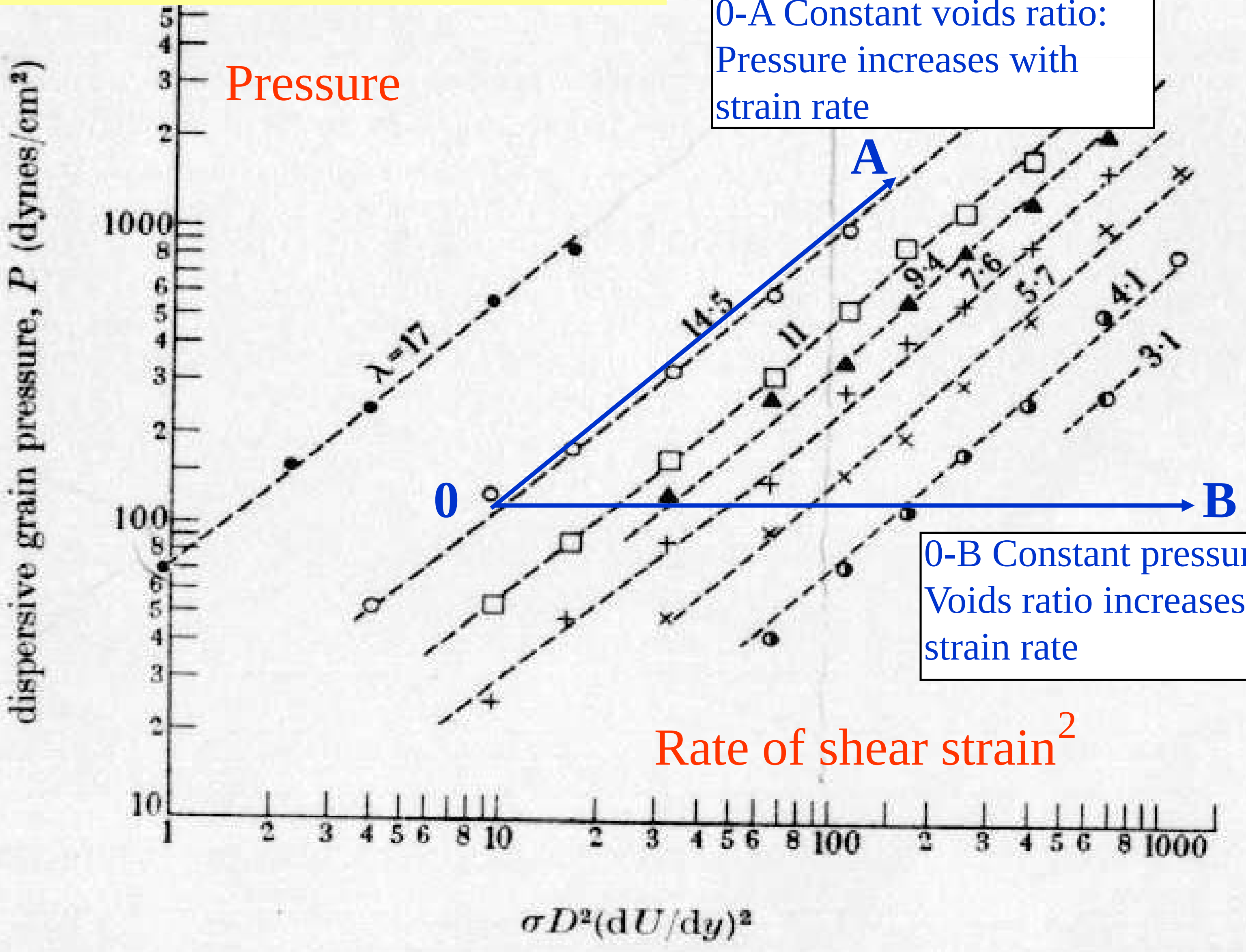
$$= 4\mu_B \sin\phi_B \sqrt{I_{2D}}$$

$$\Phi_2 = 4\mu_B \cos\phi_B$$

$$\boxed{\begin{aligned}\sigma = -pI + 4\mu_B \sin\phi_B \sqrt{I_{2D}}\,D \\ - 4\mu_B \cos\phi_B D^2\end{aligned}}$$

R3 Behaviour of fluidized soil

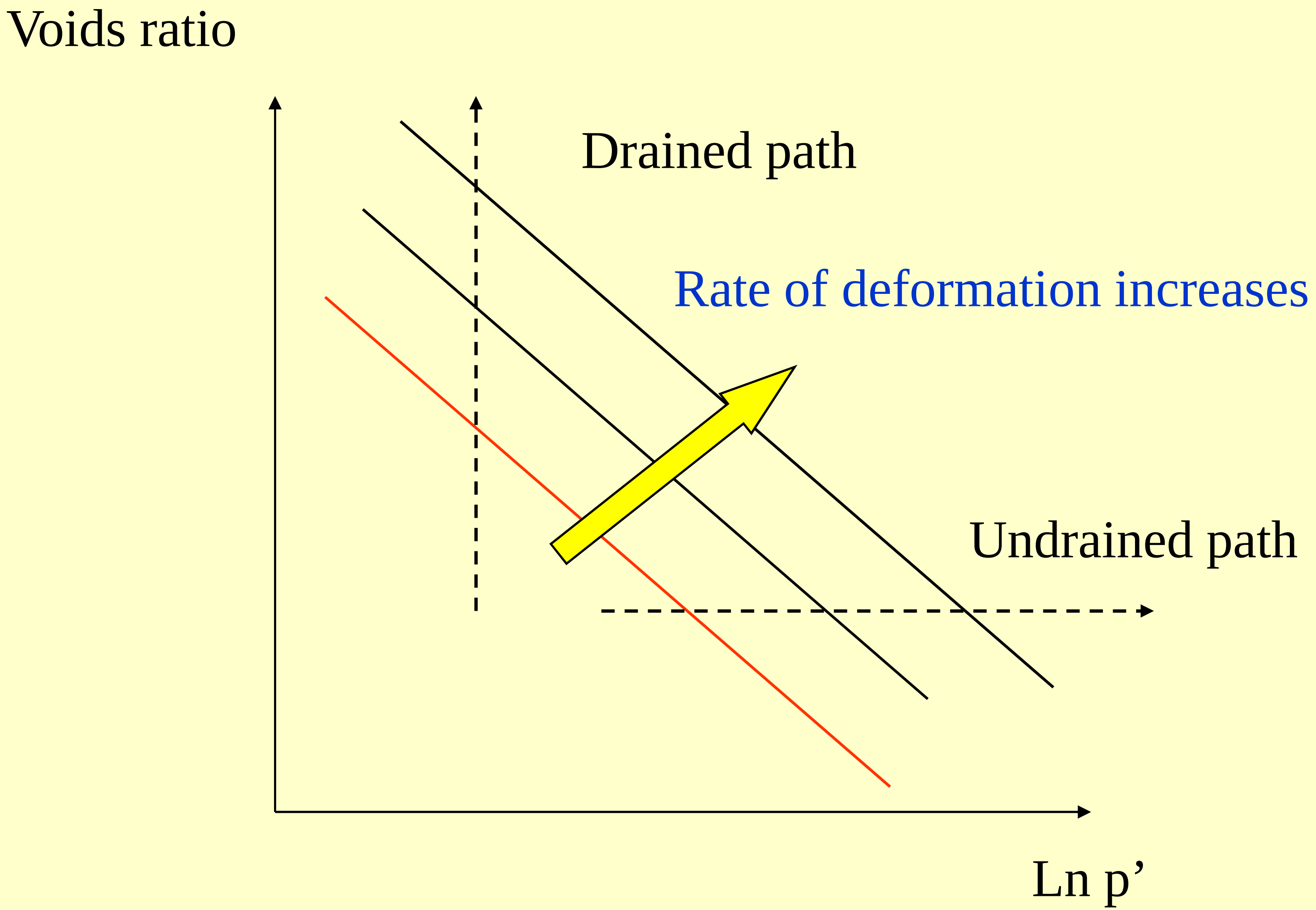
0-A Constant voids ratio: Pressure increases with strain rate



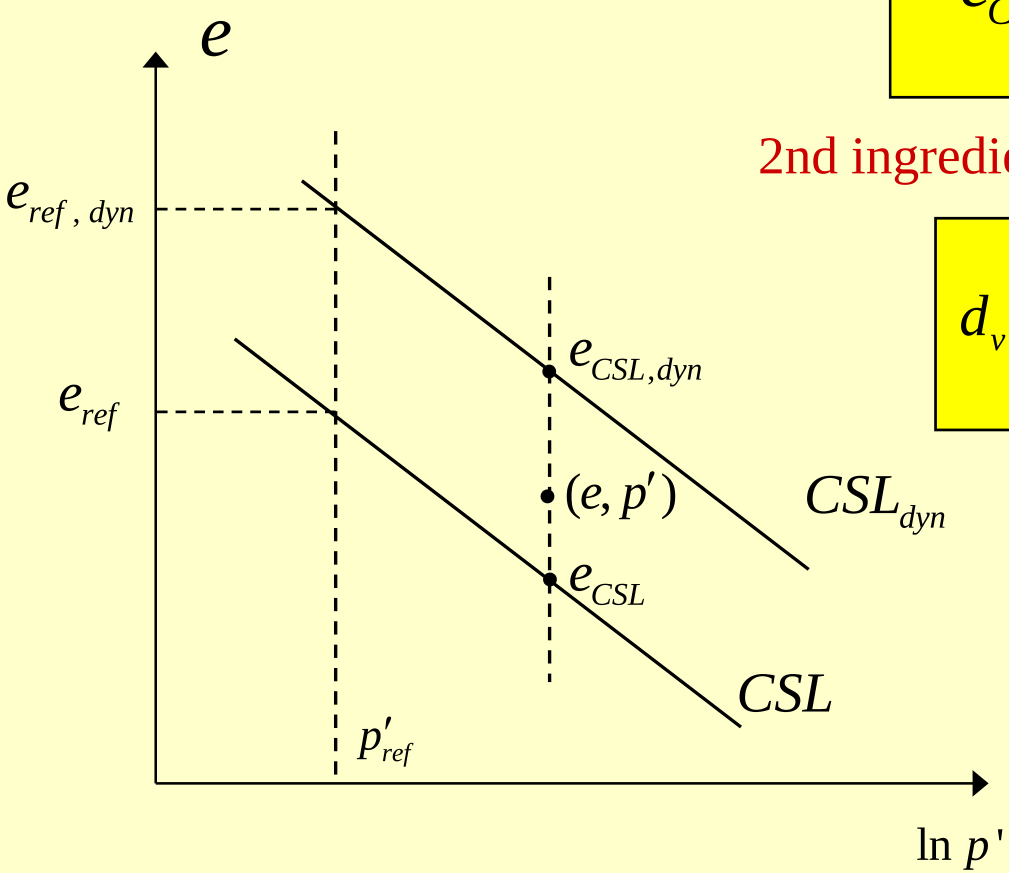
0-B Constant pressure: Voids ratio increases with strain rate

FIGURE 4

- R3 Behaviour of fluidized soil: volumetric component



R3 Behaviour of fluidized soil: volumetric component



First ingredient: dynamic CSLs

$$\boxed{e_{CSL,dyn} = e_{CSL} + \beta_1 \left(I_{2d} \right)}$$

2nd ingredient: dilatancy law at dynamic CSLs

$$\boxed{d_{v0} = -\beta_2 \frac{e_{CSL,dyn} - e}{e_{CSL}}}$$

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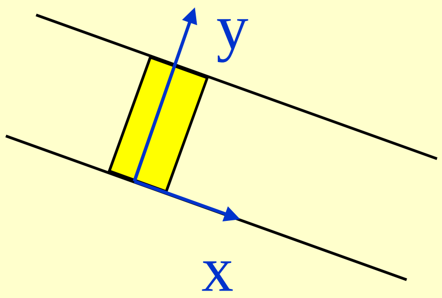
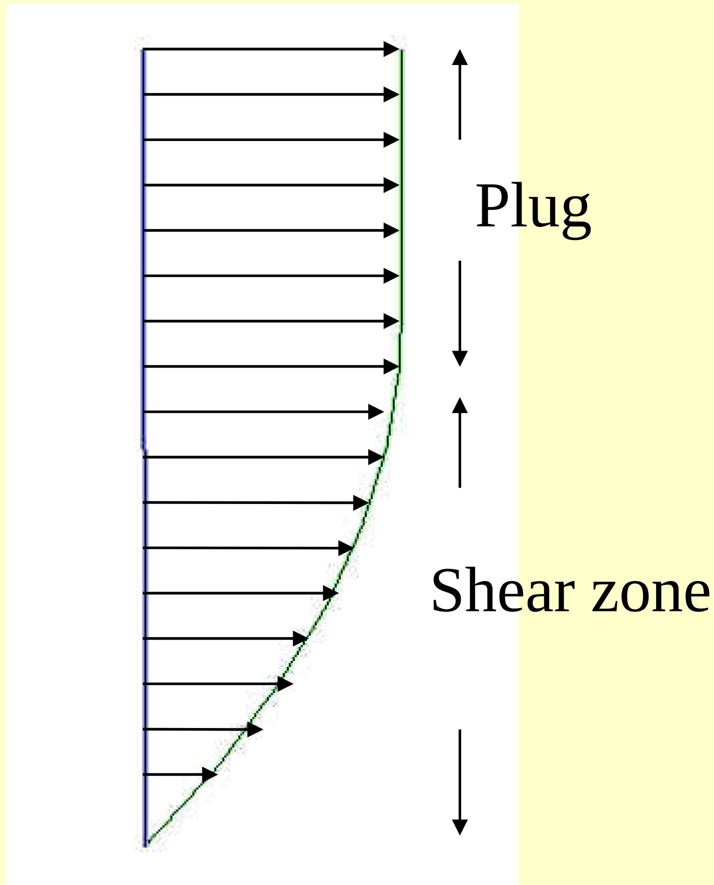
Infinite landslide: Perzyna, Von Mises Model

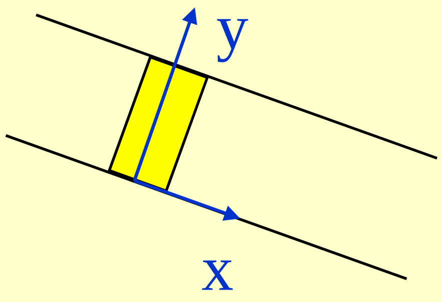


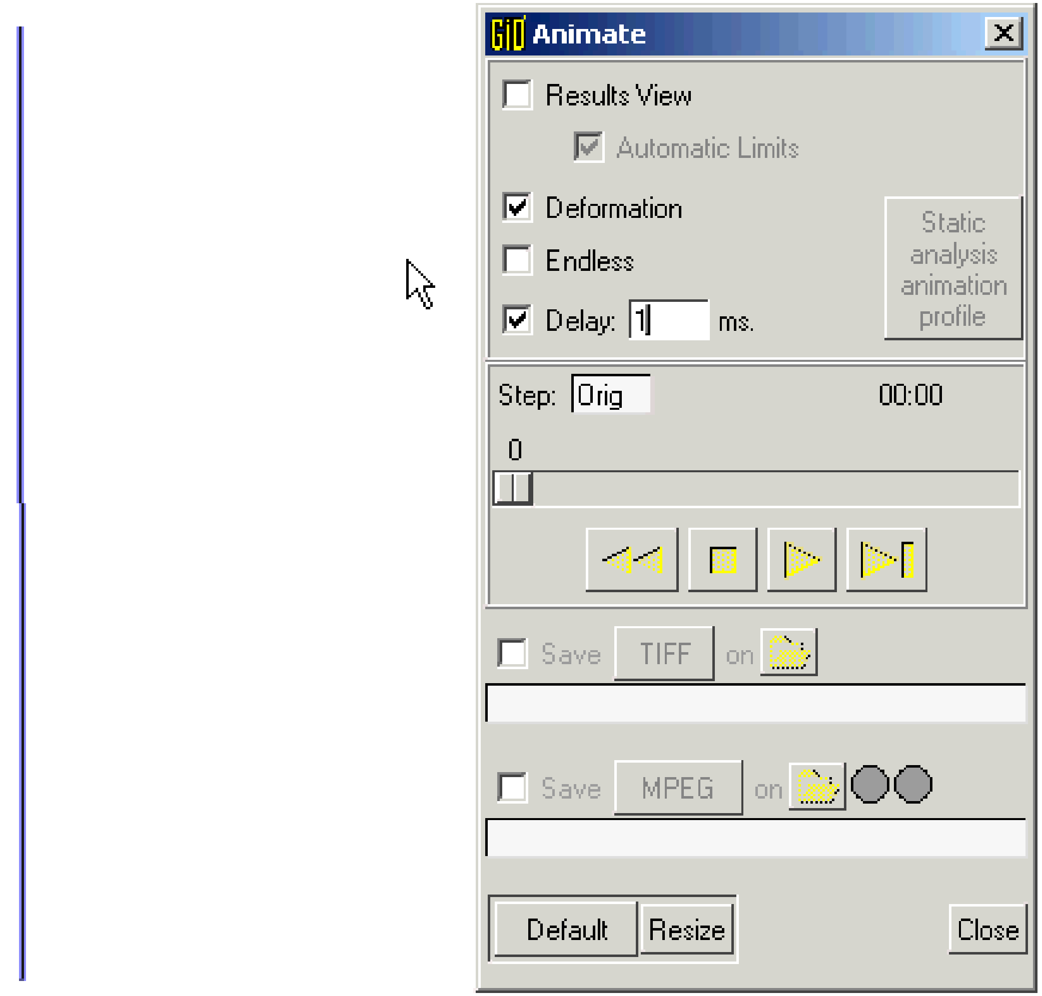
E	8.e7 Pa
Poiss	0.3
Dens	2000 Kg/m3
Yield	0.285 e5 Pa
gamma	0.1
delta	1.
Slope	1:4



Velocity Profile

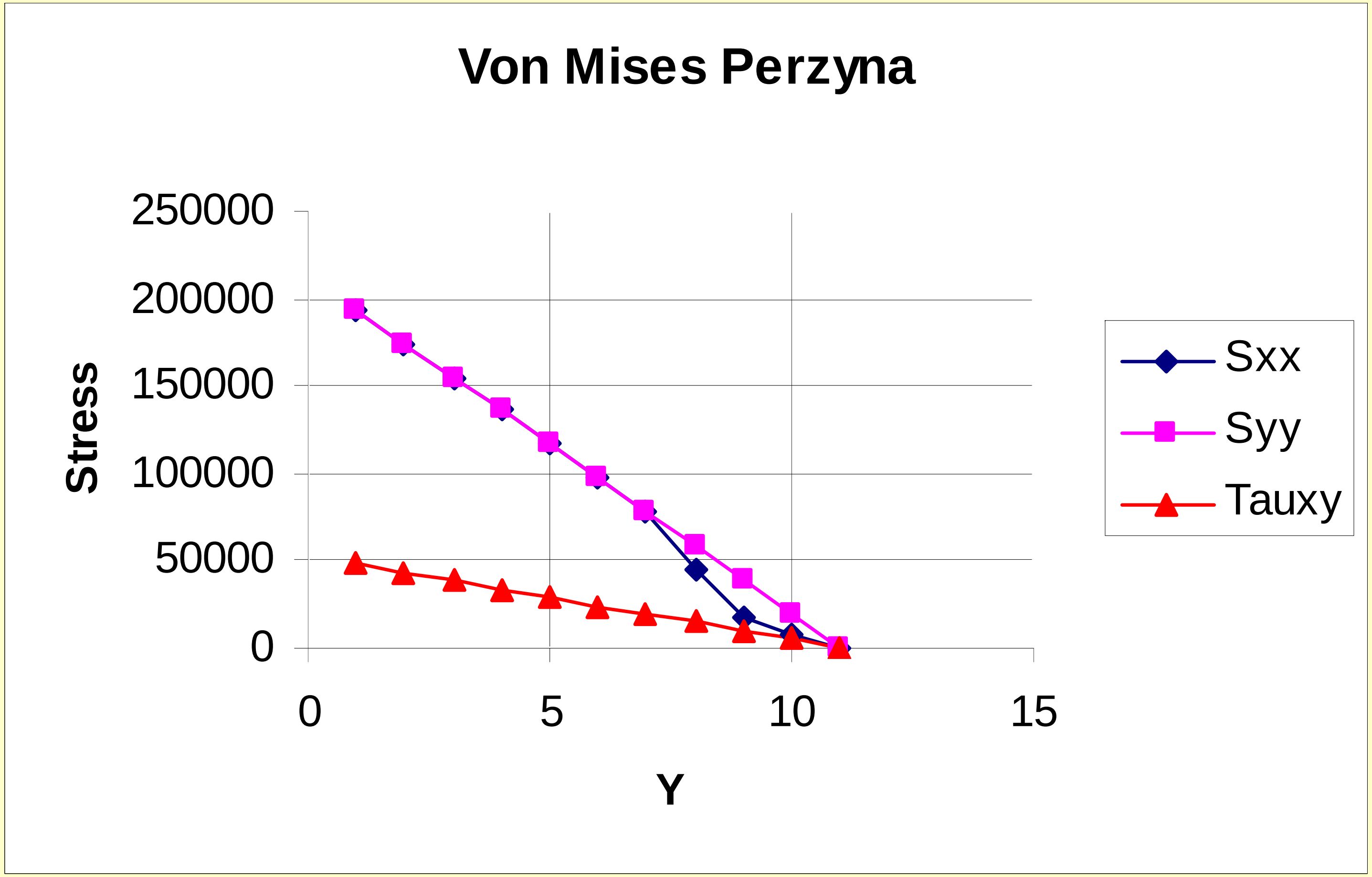
Infinite landslide: Perzyna





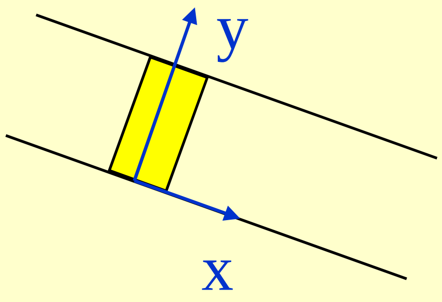
Velocity Profile

Infinite landslide: Perzyna Von Mises

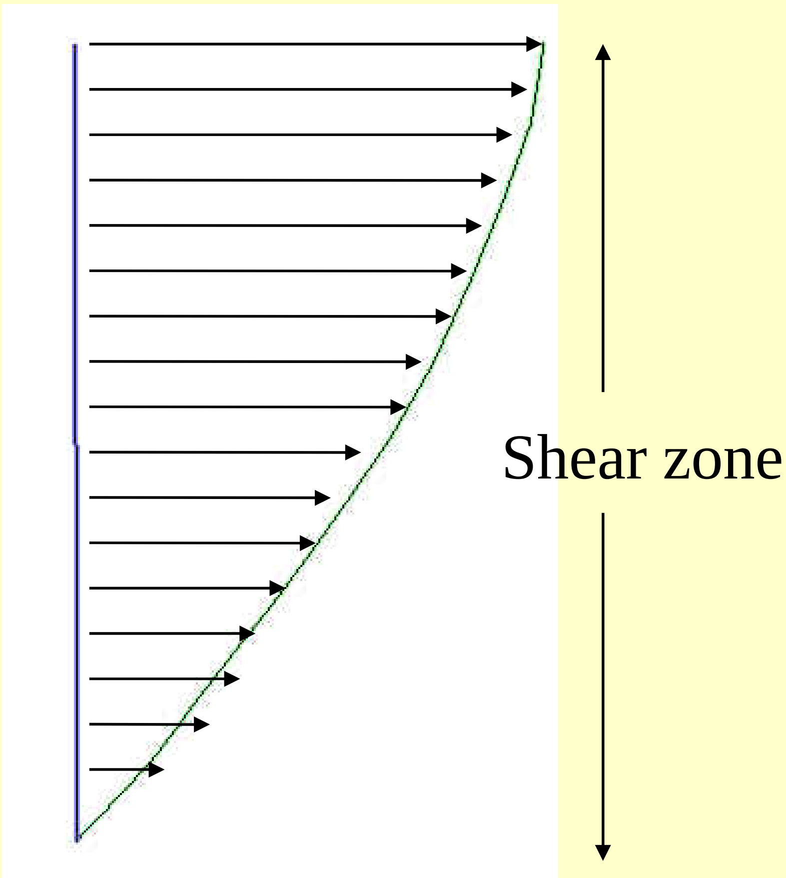


Note: Sigma x = Sigma y within shear zone!

Infinite landslide: Perzyna, Cam Clay Model

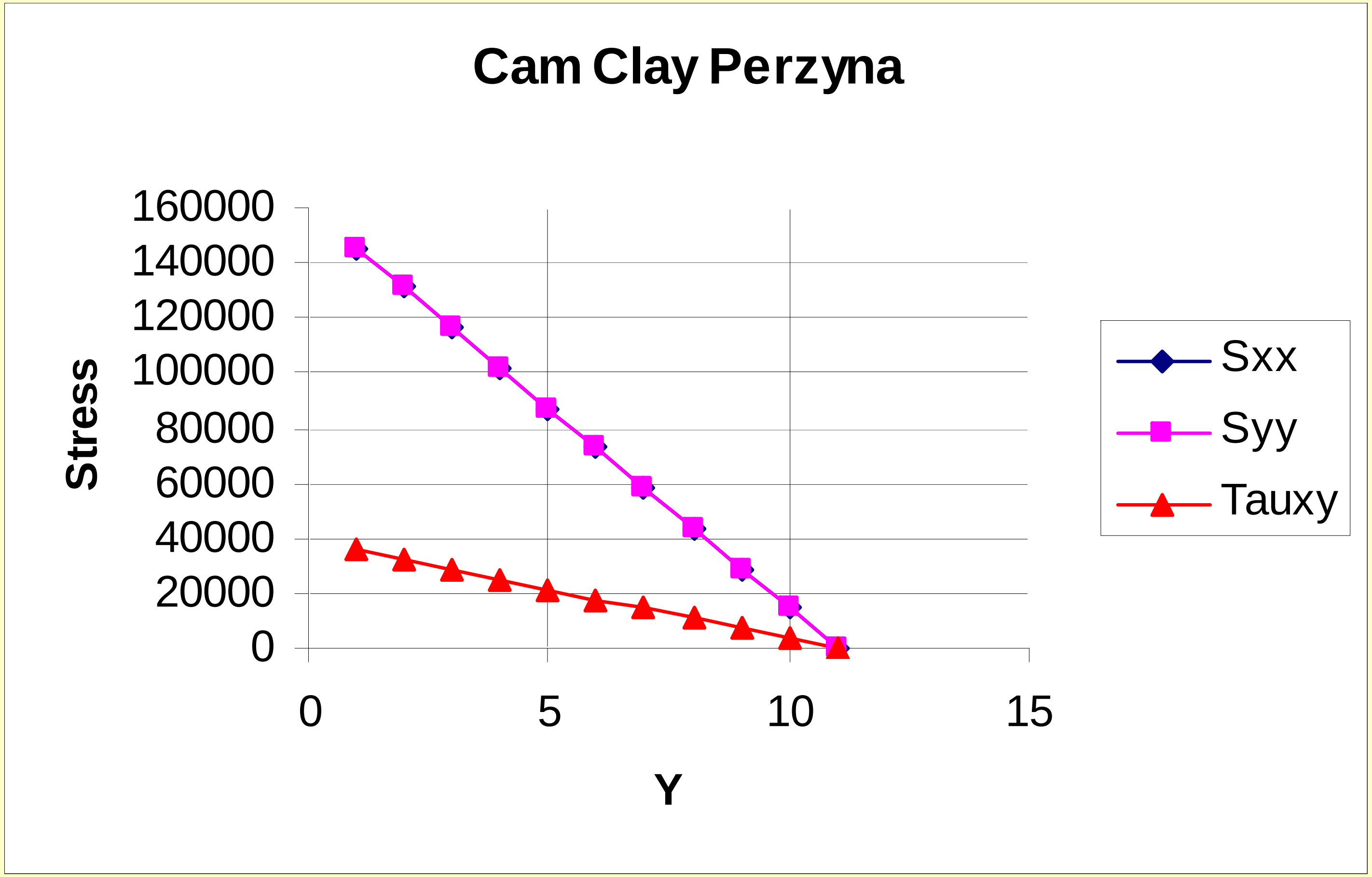


E	1.5 e7 Pa
Poiss	0.3
Dens	1500 Kg/m3
Mg	1.1
Lambda	0.51 k 0.09
Pc0	0.285 e5 Pa
gamma	0.1
delta	1.
Slope	1:4



Velocity Profile

Infinite landslide: Perzyna Cam Clay



Note: Sigma x = Sigma y within shear zone!