Hydraulic Fracturing:

Basic Concepts and Numerical Modelling

Panos Papanastasiou

Department of Civil and Environmental Engineering

University of Cyprus

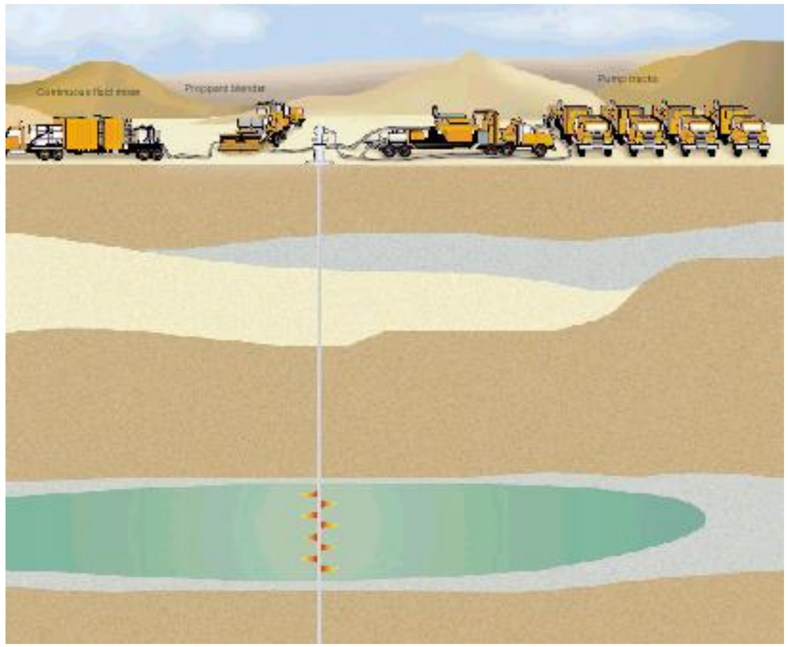
1991-2002 in Schlumberger Cambridge Research

Outline

- Applications and issues
- Basic fracturing theory: controlling parameters
  - fracture opening, propagation, modes, initiation, closure
- Perforating for fracturing
- Fracture geometry
  - deviated and horizontal wellbores
  - tortuosity and multiple fractures
- Hydraulic fracturing modeling
  - physical processes, geometrical models, height growth, net-pressure
- Fracturing weak formations

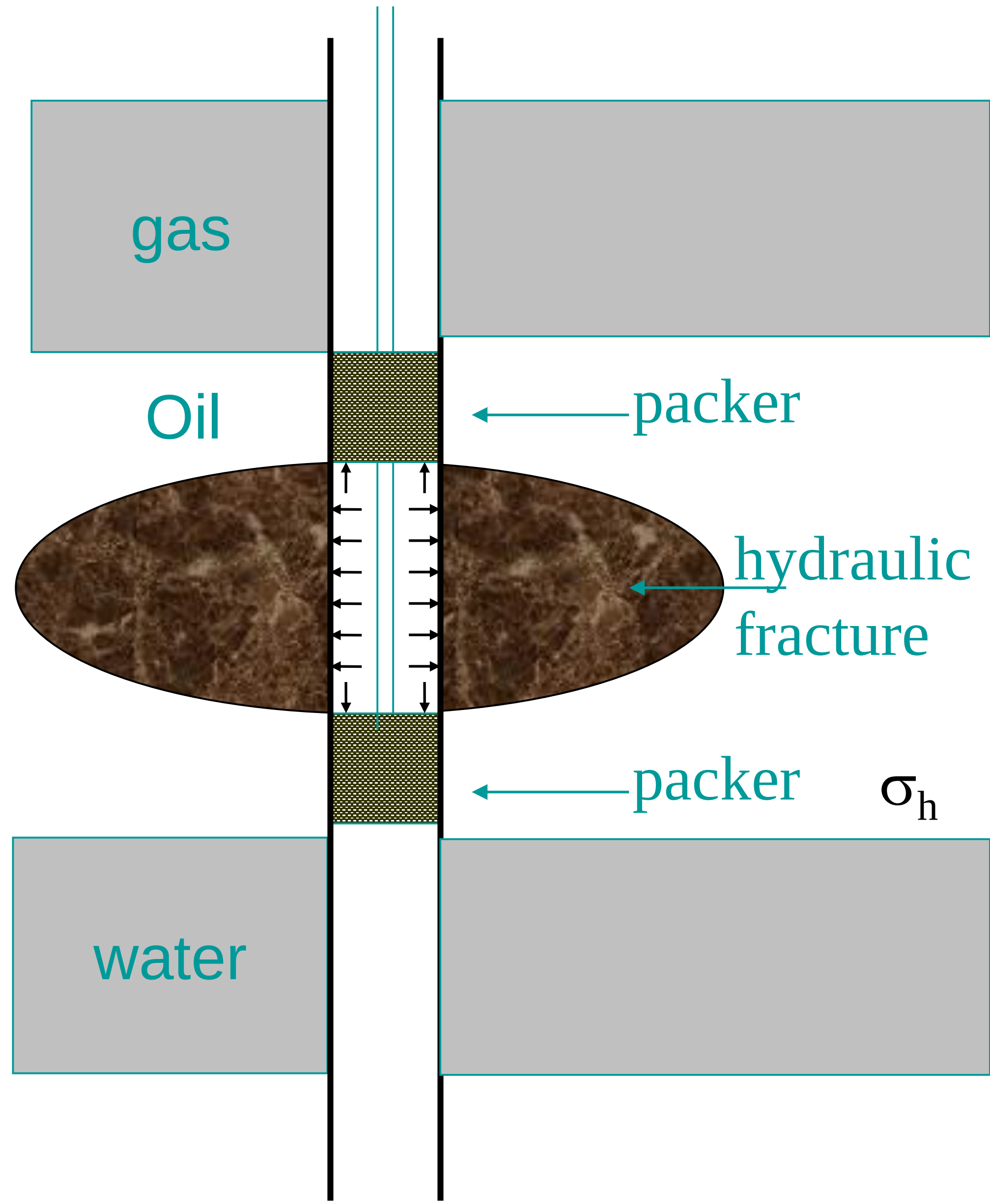
HF Applications and issues

- Petroleum engineering
  - stimulate oil and gas reservoirs, cuttings re-injection, frac-packs for sand control
  - **predict pressures and fracture dimensions and fracture containment**
  - **interval selection for HF in shale reservoirs (brittleness index)**
- Environmental engineering
  - waste disposal in shallow formations, cleaning up contaminated sites
- Geotechnical engineering
  - injection of grout, dam construction
- Enhanced Geothermal Systems
  - Maximized heat extraction



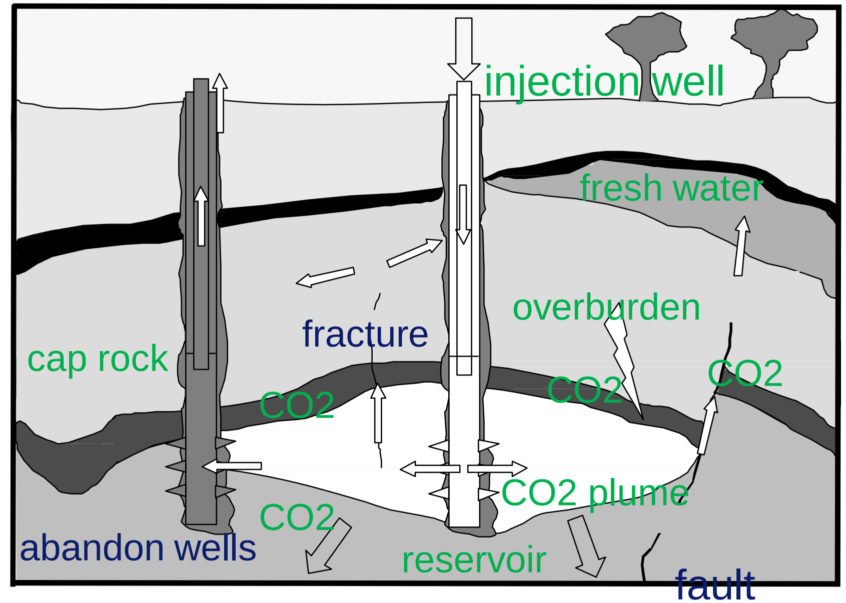
Why hydraulic fracturing in Petroleum engineering?

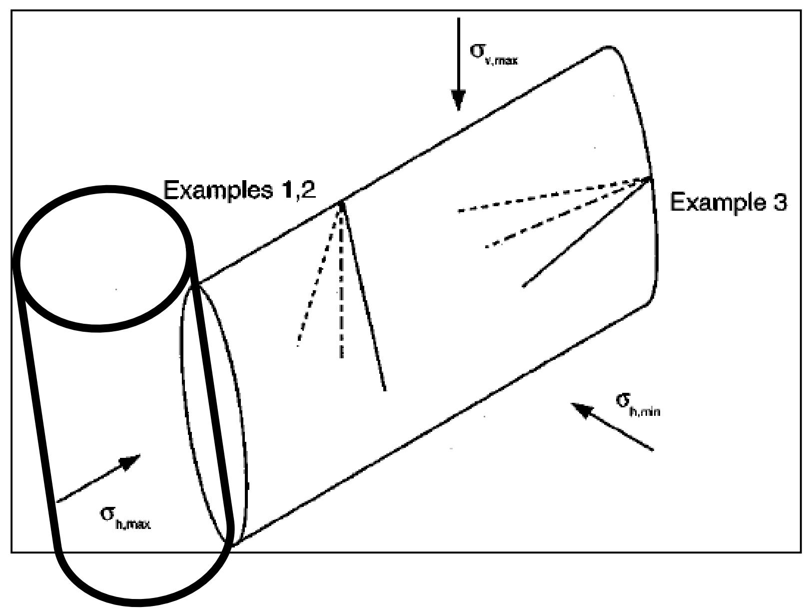
- Bypass near-wellbore formation damage
  - drilling induced, fines invasion-migration, chemical incompatibility
- Extend a conductive path deep into the formation
  - increase area exposure to flow
- Reservoir management tool
  - change flow, fewer wells, well placement, IVF, frac&pack, screen-less completion



Mechanisms of CO2 escape

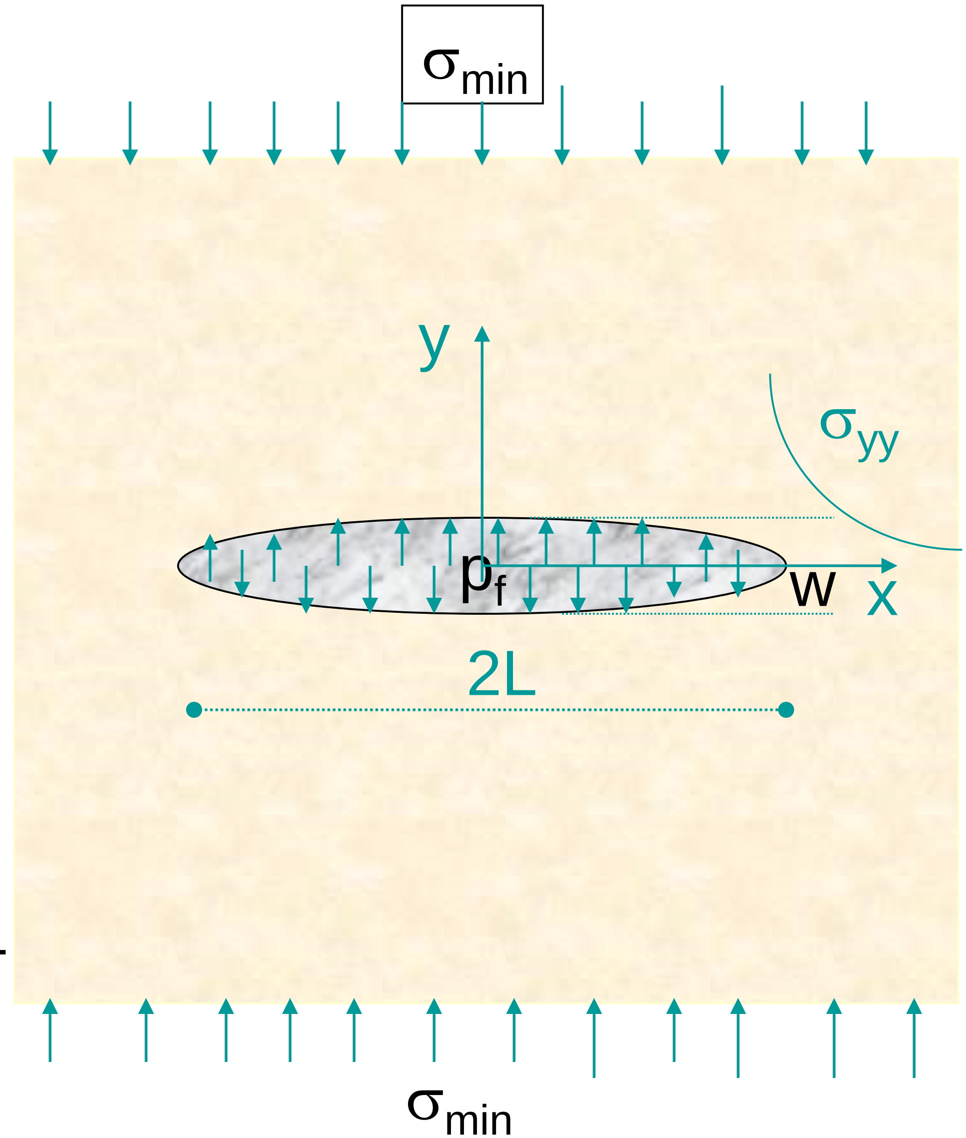
- abandon wells
  - due to bad or non-existence cement, 1000s old wells in hydrocarbon basins
- non-sealing faults
- diffusion through the cap rock
- capillary leakage
  - if pressure exceeds capillary pressures in the cap
- **induced hydraulic fractures**
  - CO2 pressure exceeds the closure stress + ….
  - but if propagate horizontally may solve wellbore injectivity problem (Andre et al, 2016) and storage capacity



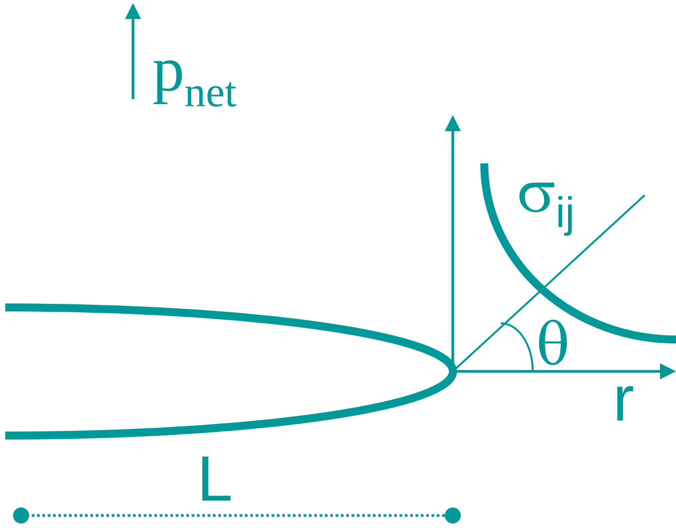


Fracture Opening

- Fracture opens if the net pressure
  - $p_{net} = p_f - \sigma_{min} > 0$
- Fracture opening
  - $w(x) = 4\, p_{net}\, (L^2 - x^2)^{1/2} / E'$
  - $E' = E/(1-\nu^2)$ is the plane strain modulus
- maximum width for x=0
  - for constant height: $W = 4\, p_{net}\, L/E'$
  - for radial fracture: $W = 8\, p_{net}\, R/(\pi E')$
- singular stress at the crack tip, for x=L
  - $\sigma_{yy} = p_{net}\, [x/(x^2 - L^2)^{1/2} - 1]$

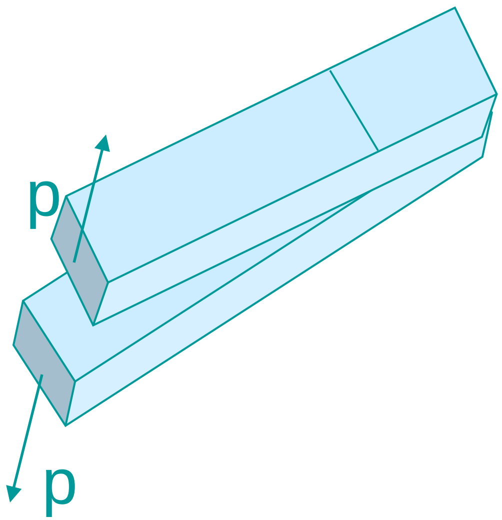


Fracture Propagation

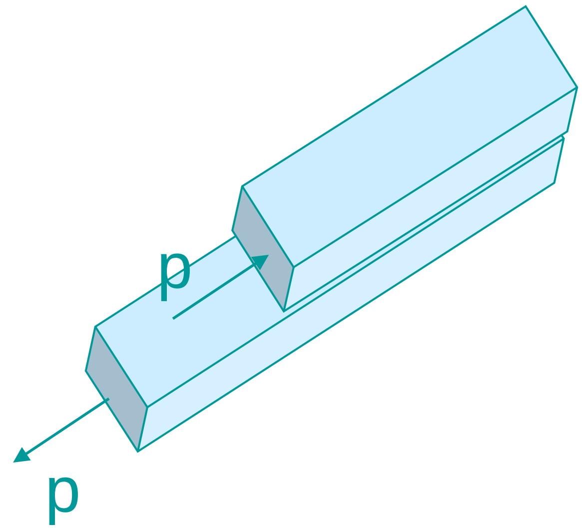


- The stresses ahead of the crack tip are singular characterized by the stress intensity factor K_I
  - $\sigma_{ij} = [K_I / (2\pi\ r)^{1/2}]\ f(\theta) + \ldots$
  - example: an elliptical crack, $K_I = p_{net}\ L^{1/2}$
- A crack will propagate if
  - $K_I = K_{IC}$
  - K_{IC} is a material parameter called fracture toughness. Typical values for rocks are 0.1 - 2 MPa $m^{1/2}$

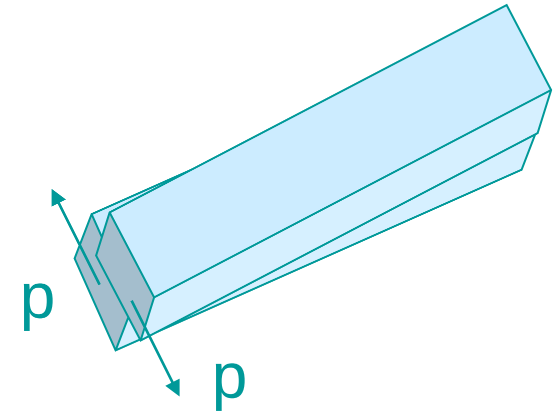
Fracture Modes



I. opening mode



II. sliding mode



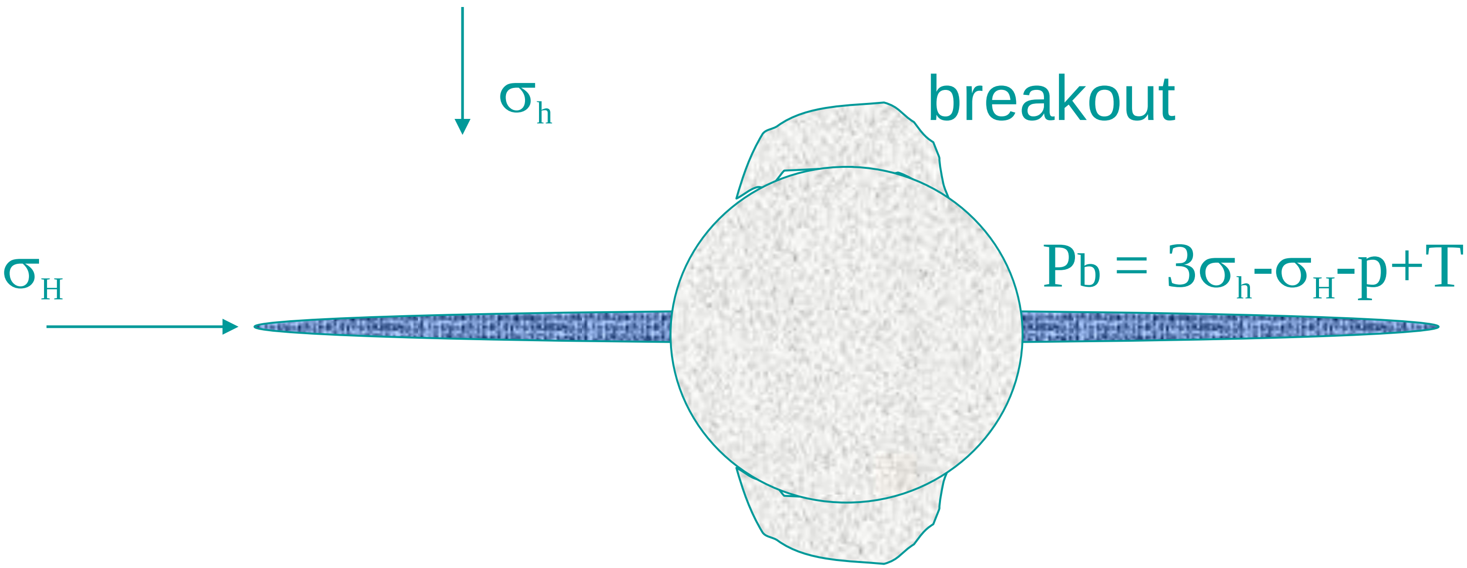
III. tearing mode

tensile fractures
hydraulic fractures
drilling induced

faults
shear fractures and
turning of fractures
near wellbore

splitting of the
crack front,
multiple fractures

Fracture Initiation in Open Holes



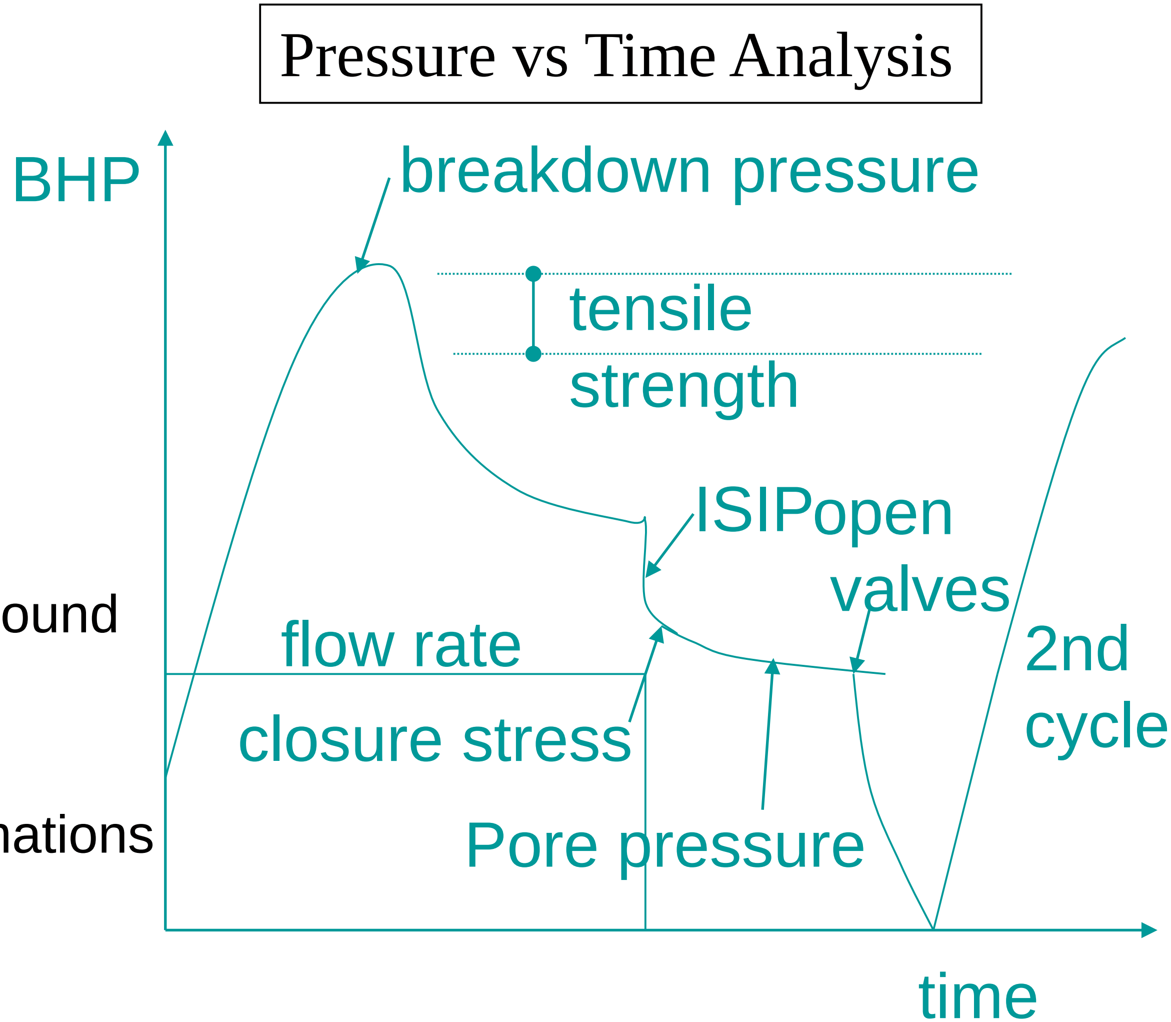
p is the formation pressure

T is the tensile strength

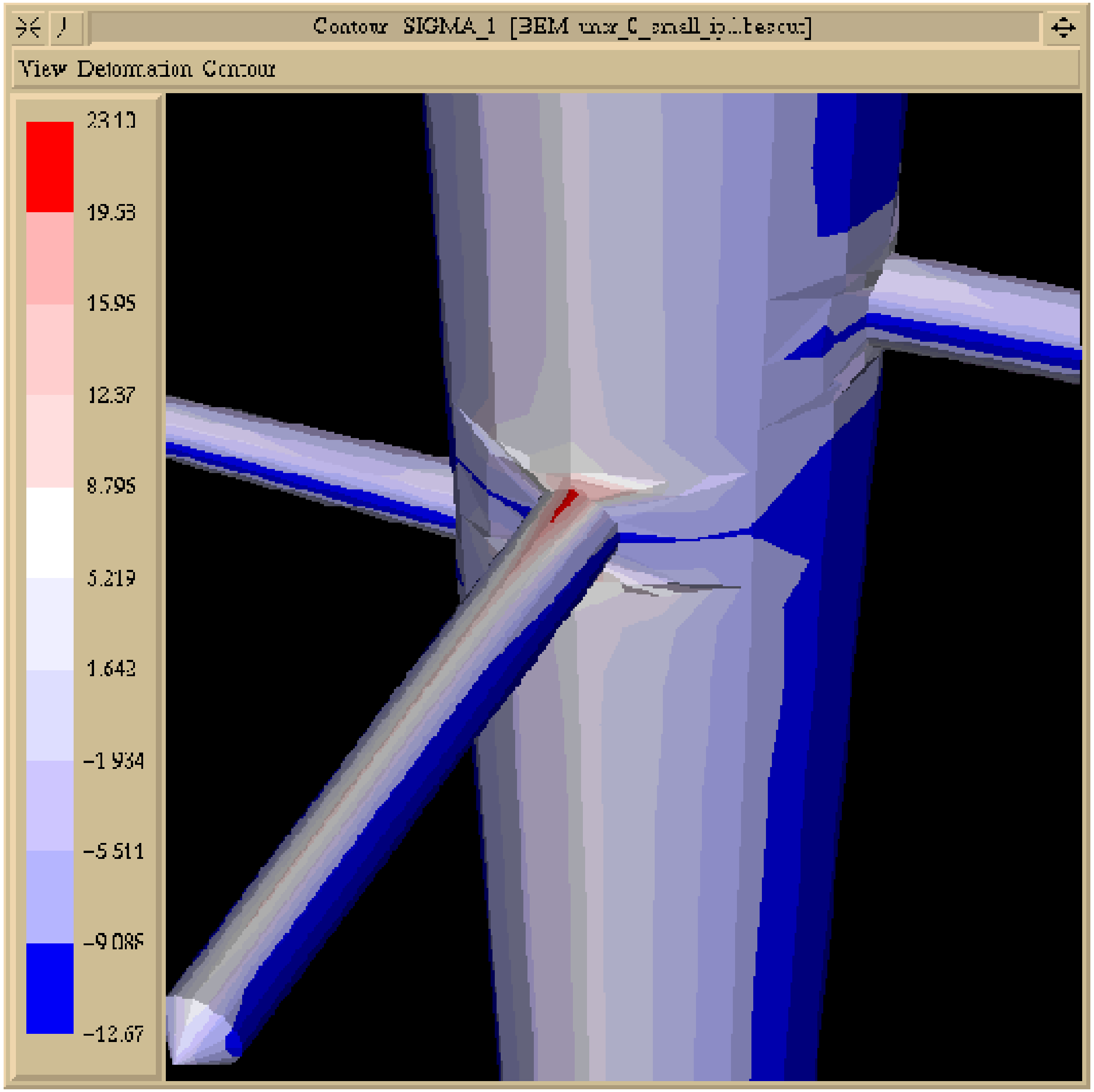
- Fracture initiation at lower pressures
  - large contrast between insitu stresses
  - high pore pressure, e.g. eject at low rates prior pressurization
  - preexisting flaws and natural fractures

Fracture Initiation and Closure

- Mini-frac calibration test
- Breakdown pressure
  - $Pb = 3\sigma_h - \sigma_H - p + T$
  - p is the formation pressure
  - T is the tensile strength
  - no fluid penetration, upper bound
- Closure stress
  - ISIP in low permeability formations

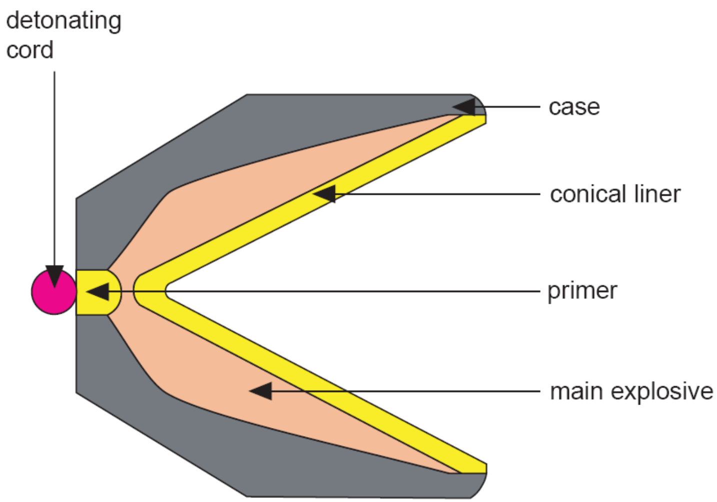


Perforated Cased Holes

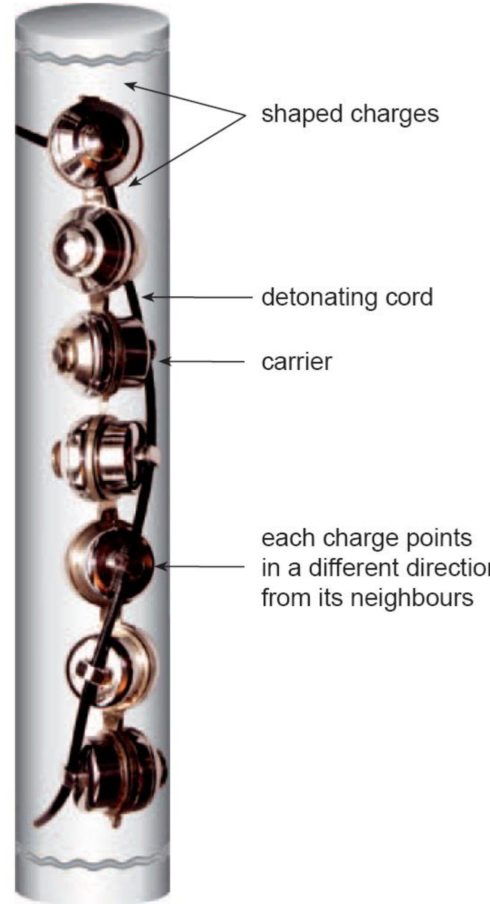


Perforations

– The perforation guns contain many shaped charges in different directions (phased and non-phased perforations). In each shaped charge there is a cone of explosive. When detonated, this sends out a high-pressure unidirectional jet which punches through the casing, the cement, and 1-2 feet into the formation

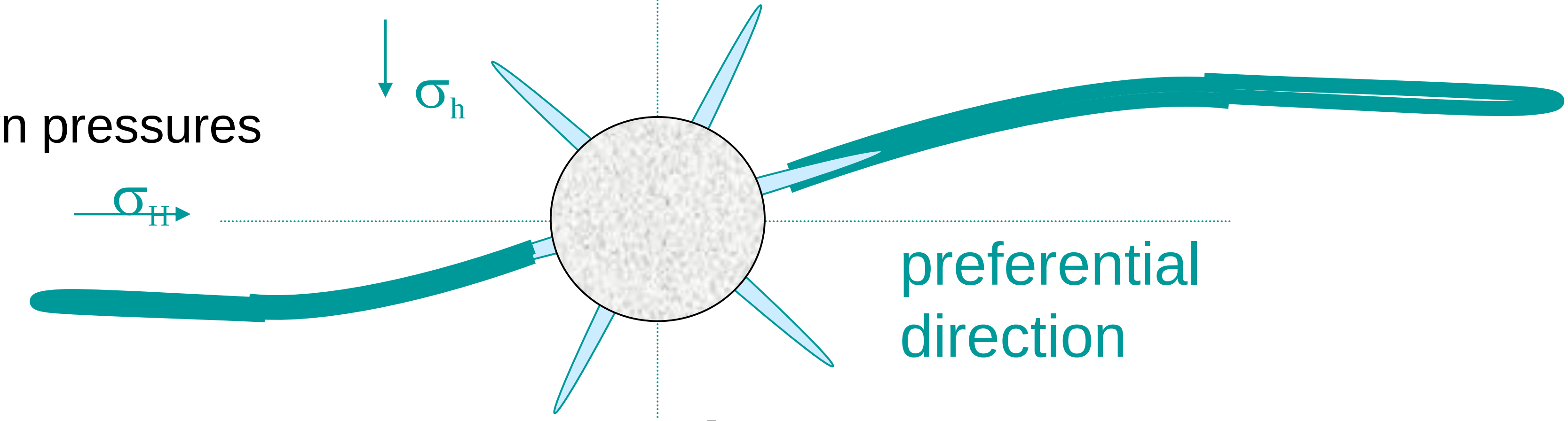


Shaped charge

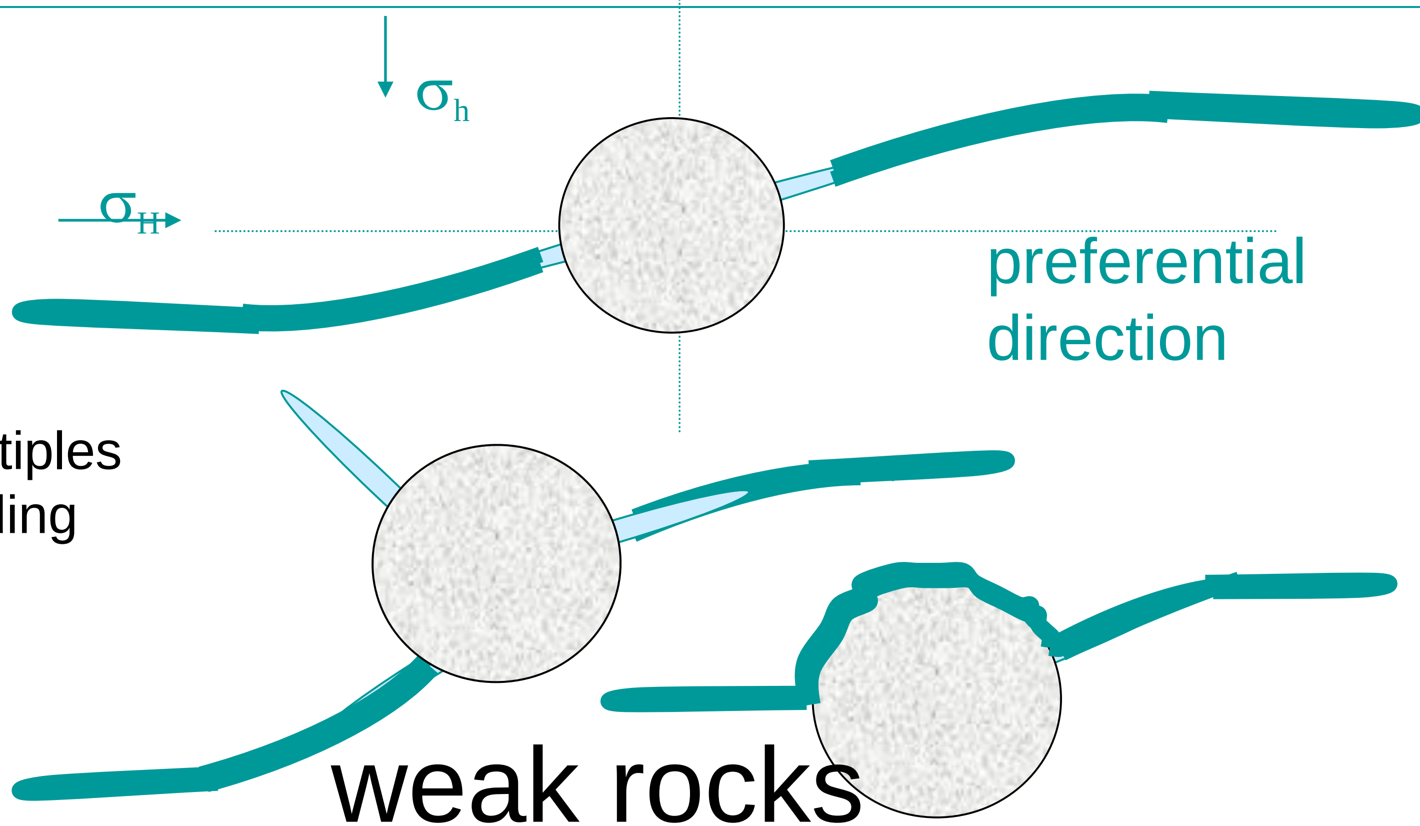


Perforating gun

1. low breakdown pressures



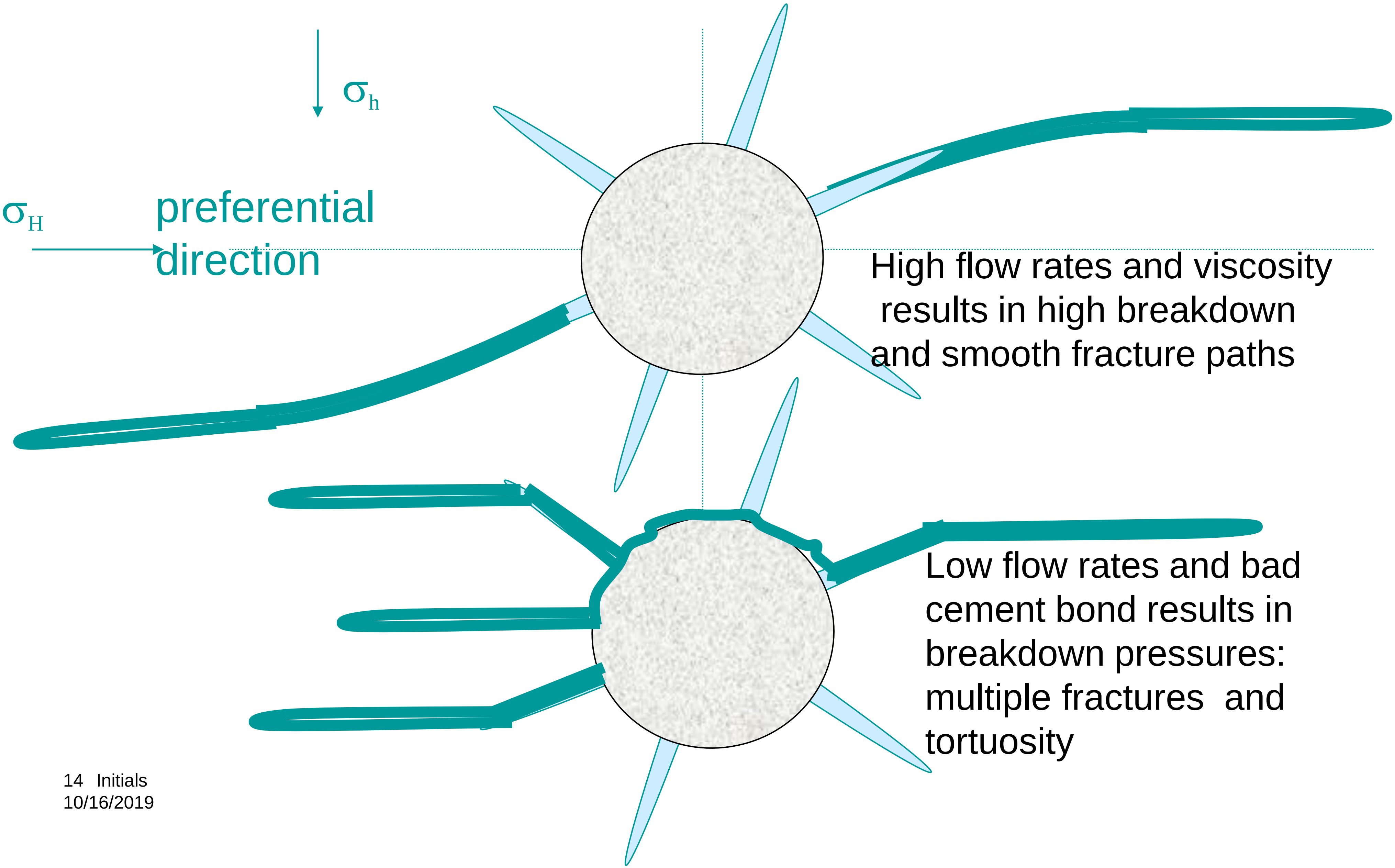
strong rocks



1. reduce multiples
2. risk of sanding

weak rocks

Near Wellbore Fracture Geometry

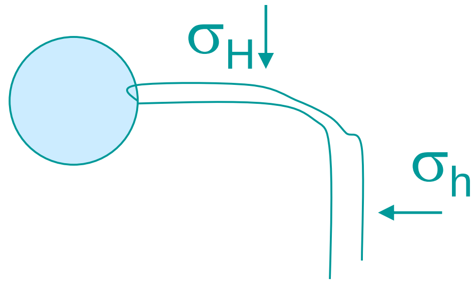


High flow rates and viscosity results in high breakdown and smooth fracture paths

Low flow rates and bad cement bond results in breakdown pressures: multiple fractures and tortuosity

Fracture Tortuosity

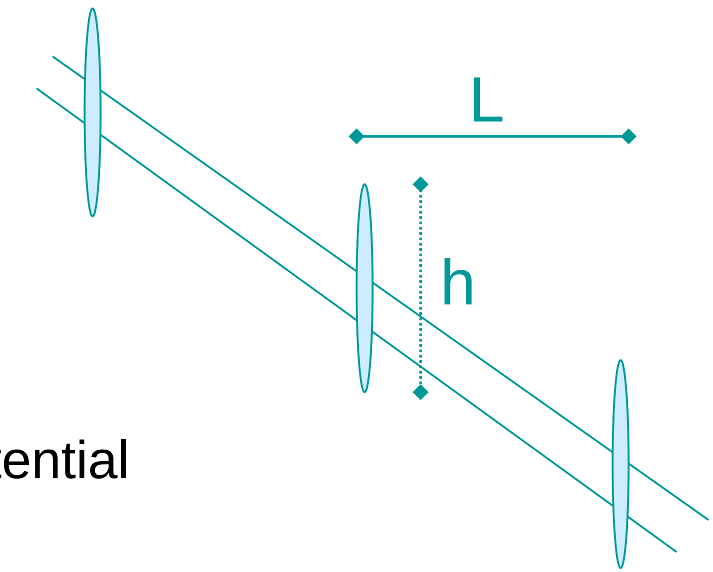
- Gradual or sharp fracture re-orientation to the preferred plane results in width restriction near the well
- Tortuosity occurs
  - in high differential stress fields
  - in deviated wells
  - in long perforated intervals and in phased perforations
  - in reservoirs with natural fractures



- Problems
  - near-wellbore friction resulting in pressure drop
  - premature screen-out due to proppant bridging

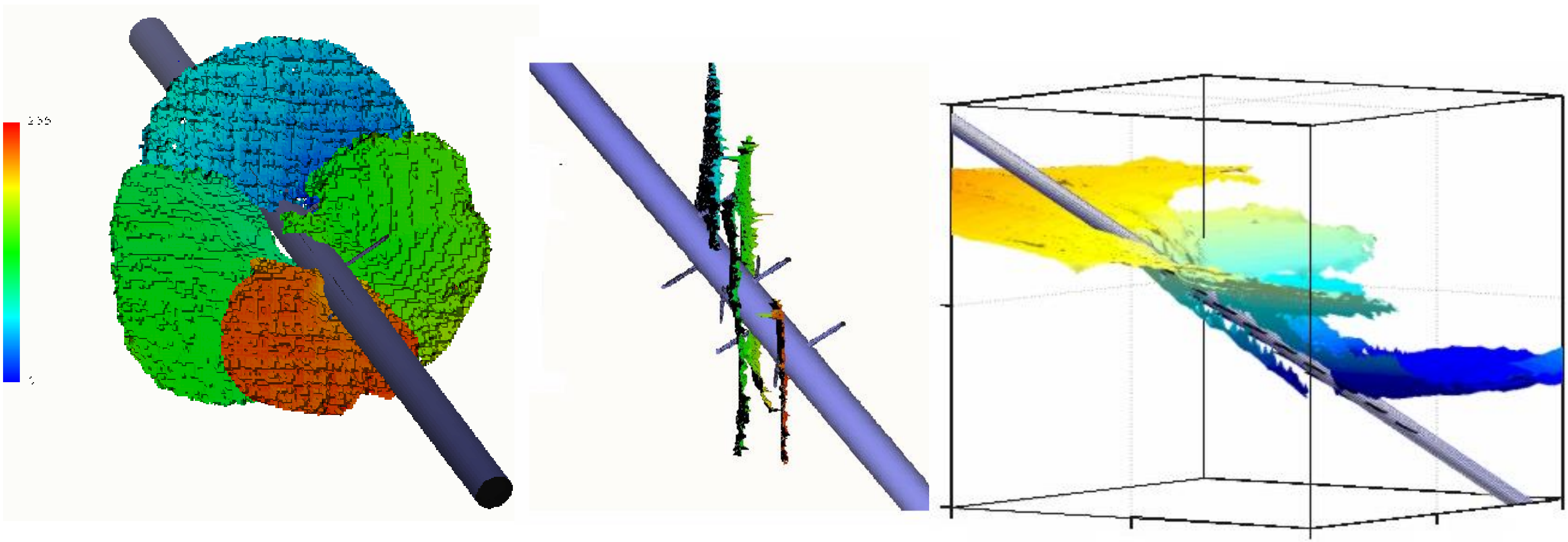
Multiple Fractures

- Propagation of multiple fractures away from the wellbore area
- Multiples occur
  - in multiple or long perforated intervals with phased perforations
  - in deviated wells where the separation between fractures is large compared to the fracture height
  - in reservoirs with natural fractures



- Problems
  - increase treating net-pressure
  - reduced fracture widths: increase screenout potential
  - increased leakoff: lower efficiency
  - Reduced fracture length

Deviated and Horizontal Wells

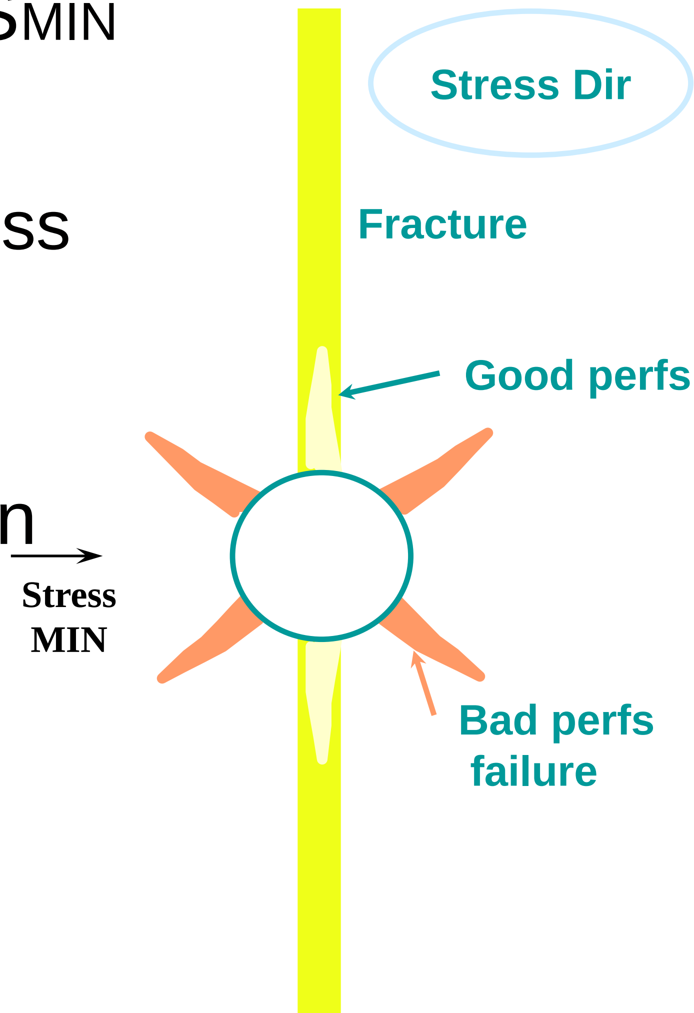


Experiments in Delft Fracturing Consortium (1997)

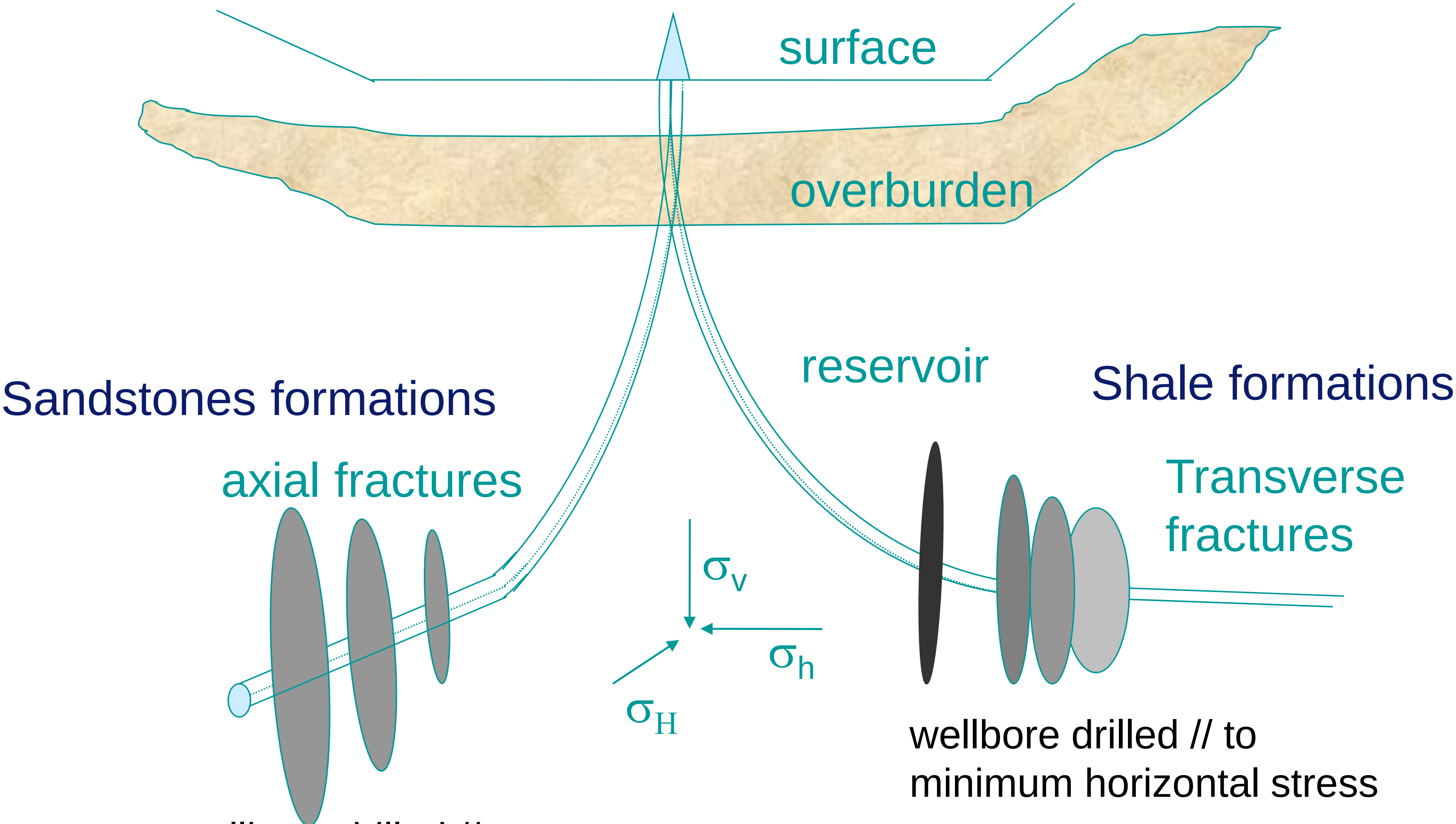
Hydraulic Fracture Optimization

- Oriented Perforations normal to S_{MIN}
  – hard rock
  – soft rock : Frac & Pack, Screenless Completion

- Creating single, bi-wing fracture in PFP
  – Minimizing near-wellbore tortuosity
  – Minimizing frac pressures
  – Eliminating multiple, competing fractures
  – Minimizing risk of premature screen-outs



Horizontal Wellbores



wellbore drilled // to
minimum horizontal stress

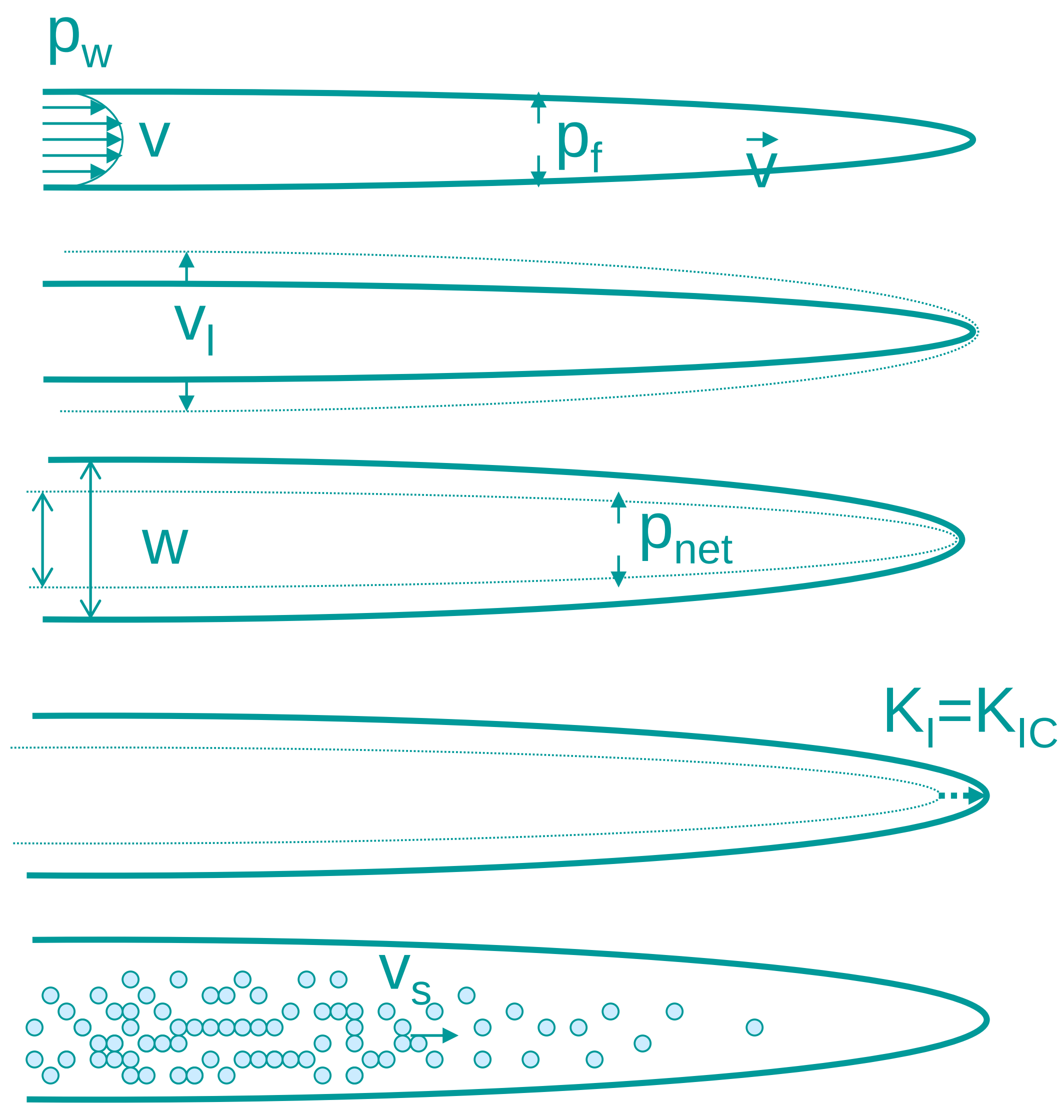
wellbore drilled // to
maximum horizontal stress

Modelling Hydraulic Fracture Propagation

- Optimize the treatment (pumping schedule, proppant stages)
  - increase well production
  - reduce cost
- Control where the fracture is growing
  - avoid fracturing near layers with different content: oil, gas, water
  - create long fractures in some layers
- Predict the response during treatment
- Post-evaluation of the treatment

Physical Processes in Hydraulic Fracturing

- Viscous fluid flow in the fracture
- Fluid leakoff in the formation
- Rock deformation
- Fracture propagation
- Proppant transport

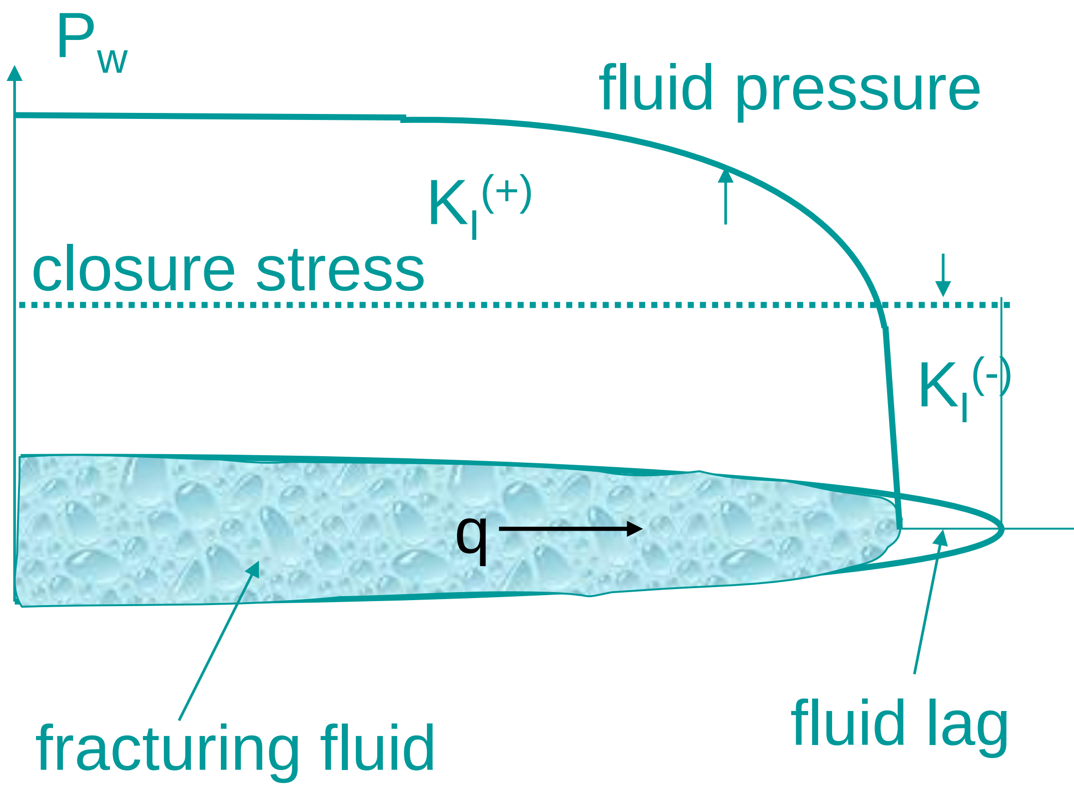


Pressure Loading on Fracture Surfaces

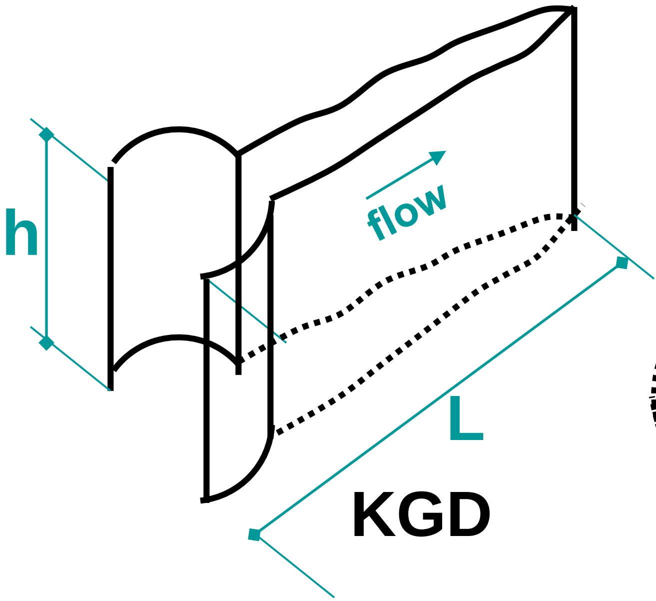
- Pressure drop: $dp/dx=12\ \mu\ q/w^3$
- Net pressure $p_{net}=p_f - \sigma_h$ gives $K_I^{(+)}>0$
- Closure stress over fluid-lag gives $K_I^{(-)}$ <0
- Fracture propagates when

$$K_I^{(+)} + K_I^{(-)} = K_{IC}$$

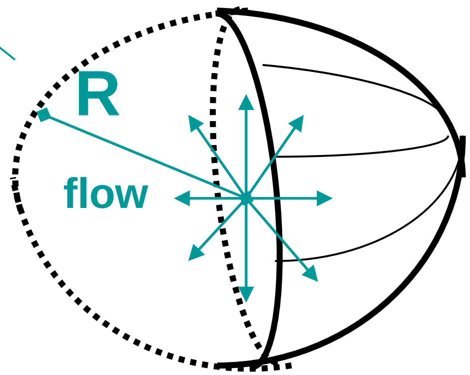
- Fracture toughness K_{IC} is small but plasticity may increase it to large values of an apparent fracture toughness



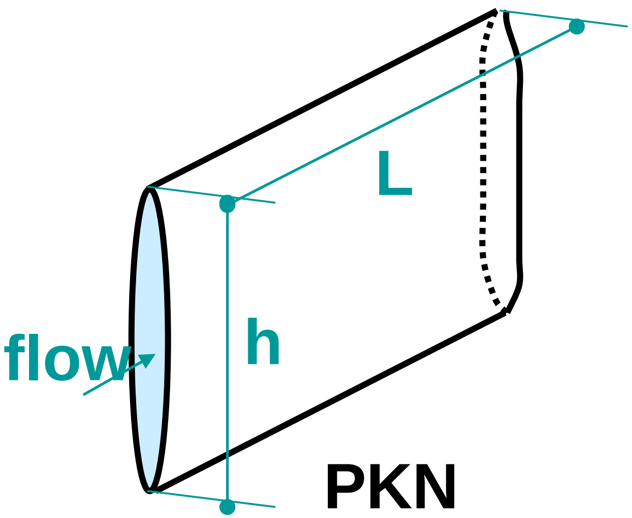
HF Geometrical Models



KGD

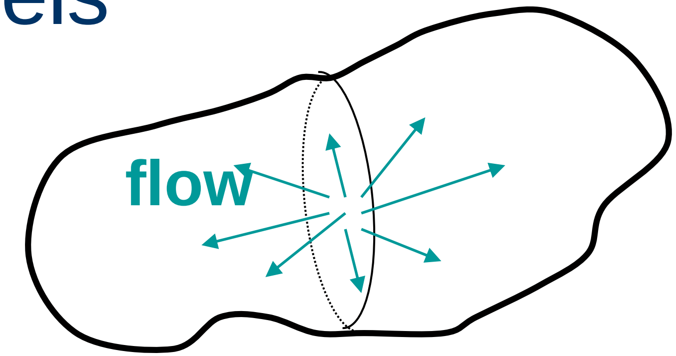


Radial

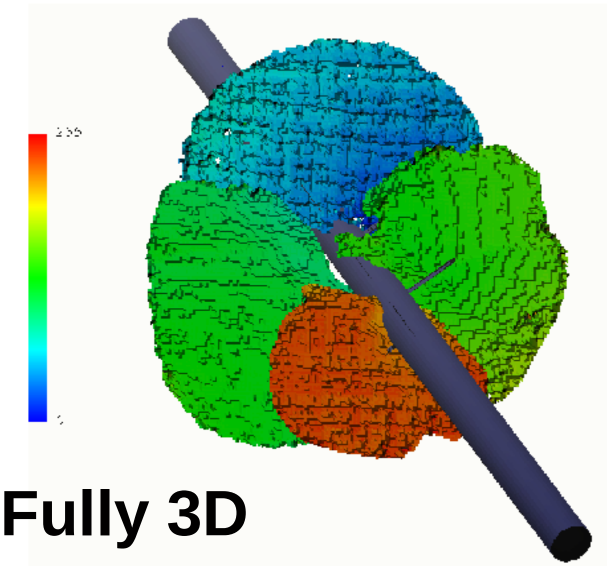


PKN

1D MODELS

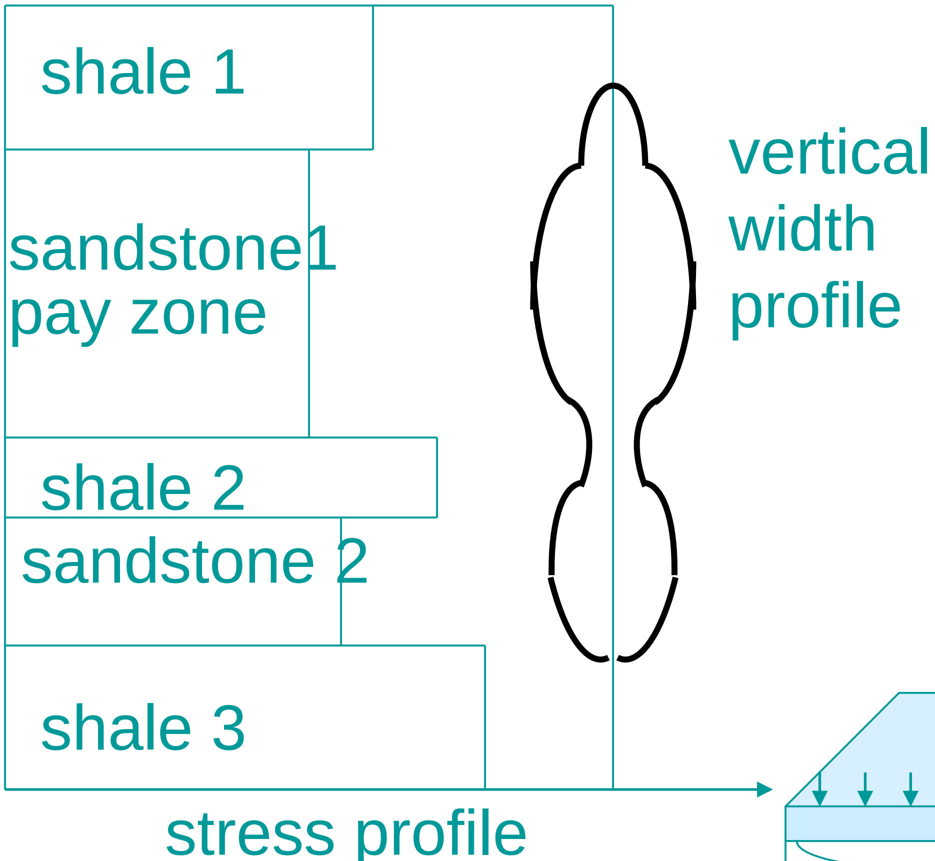


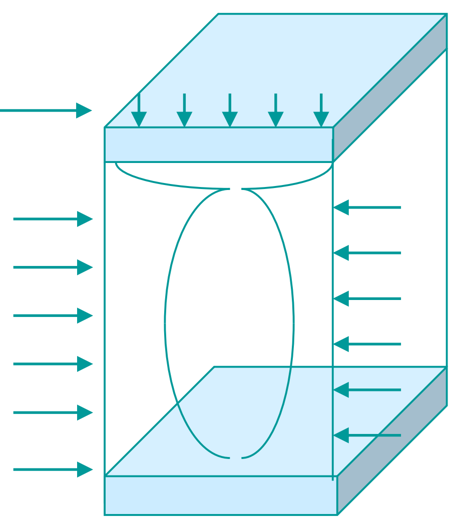
2D or Planar 3D



Fully 3D

Fracture Profiles in Layered Formations

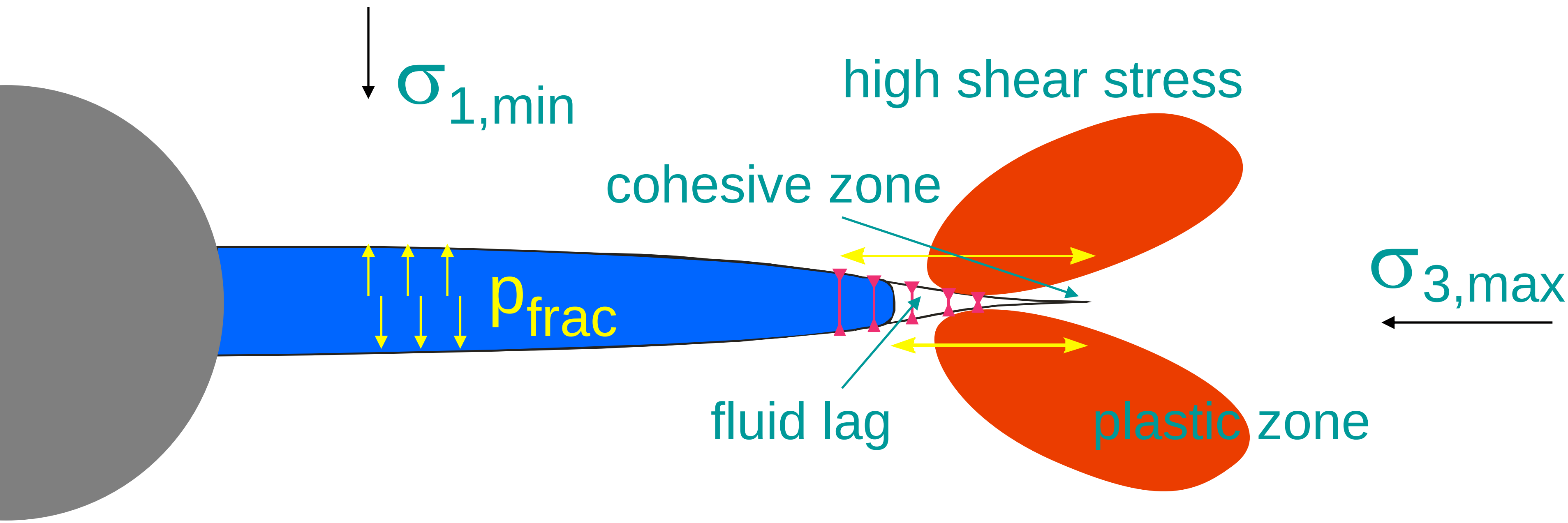




T-shape fracture

- Fracture may not penetrate deep to the optimum length
- Fracture may connect several pay zones separated by shale layers
- Fracture may grow in non-productive layers
- Problems with proppant placement
- Indirect Vertical Fracturing (IVF) for sand control

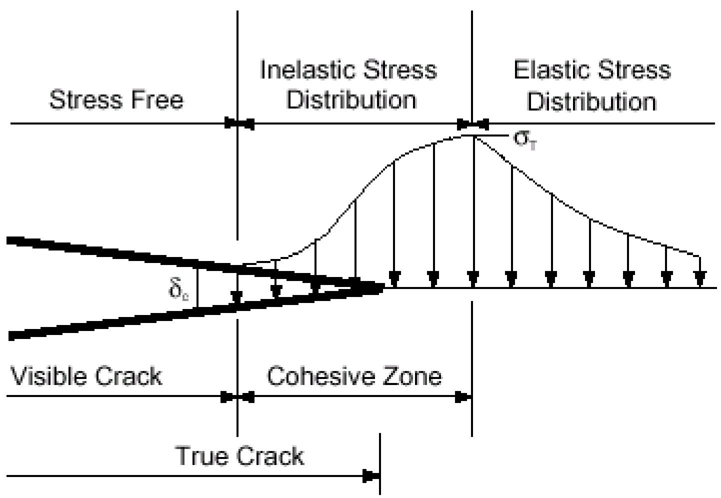
Fracture tip



- High net-pressures ($p_{net}=p_{frac}-\sigma_{min}$)
  - flow behaviour near the tip: **fluid-lag**, rock dilation
  - **high apparent fracture toughness**: due to scale effect, confining pressure, heterogeneities and plasticity
  - underestimation of the **closure stress** (**σ_{min}**)

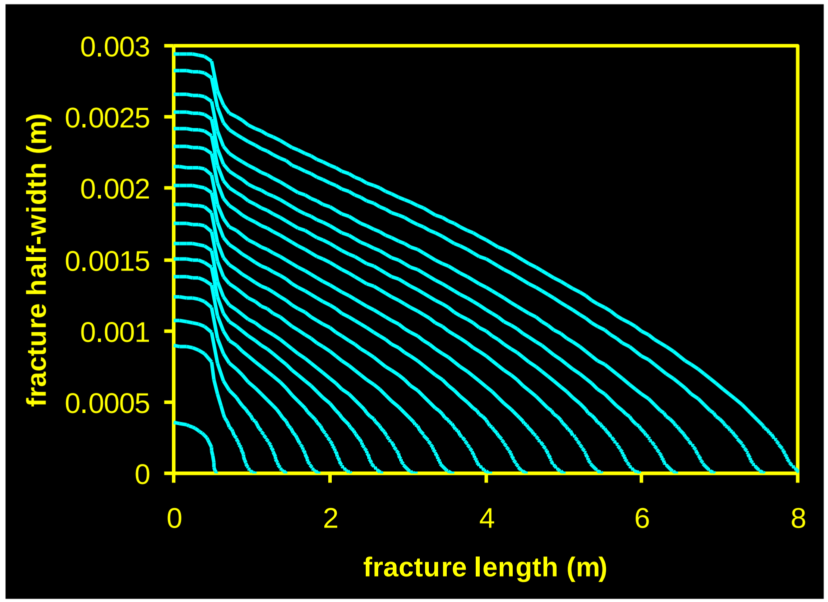
Elasto-plastic HF model

- Fluid-flow in the fracture
  - Newtonian viscous fluid, lubrication theory: $dp/dx=12\ \mu\ q/w^3$
- Rock deformation
  - Mohr-Coulomb flow theory of plasticity
- Fracture propagation
  - Cohesive model

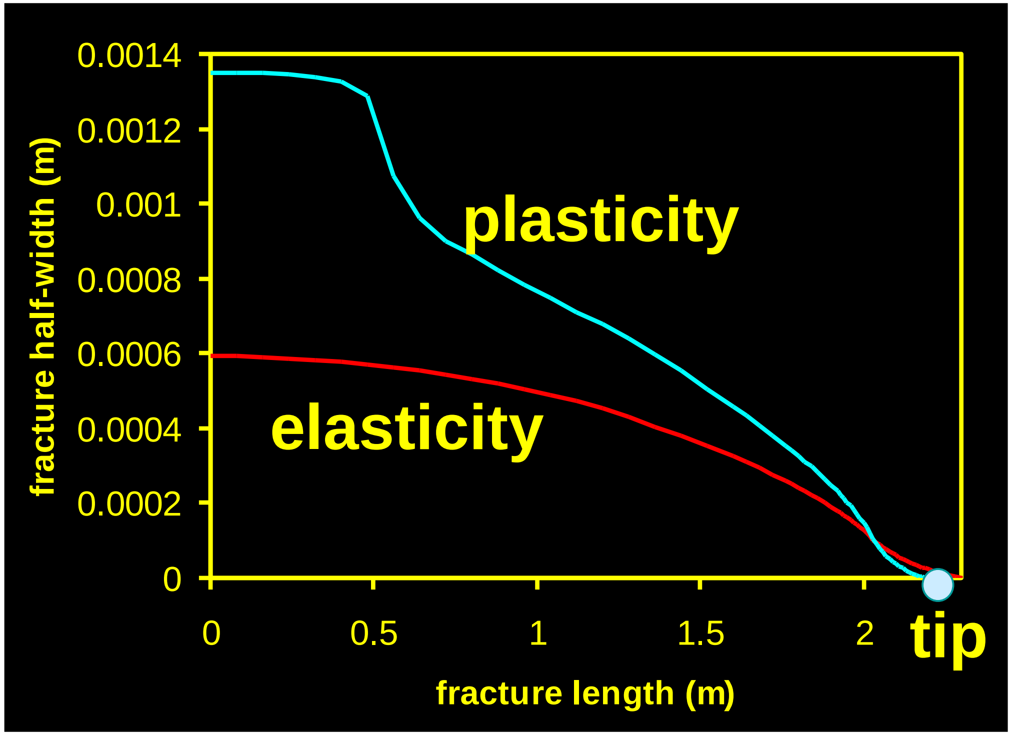


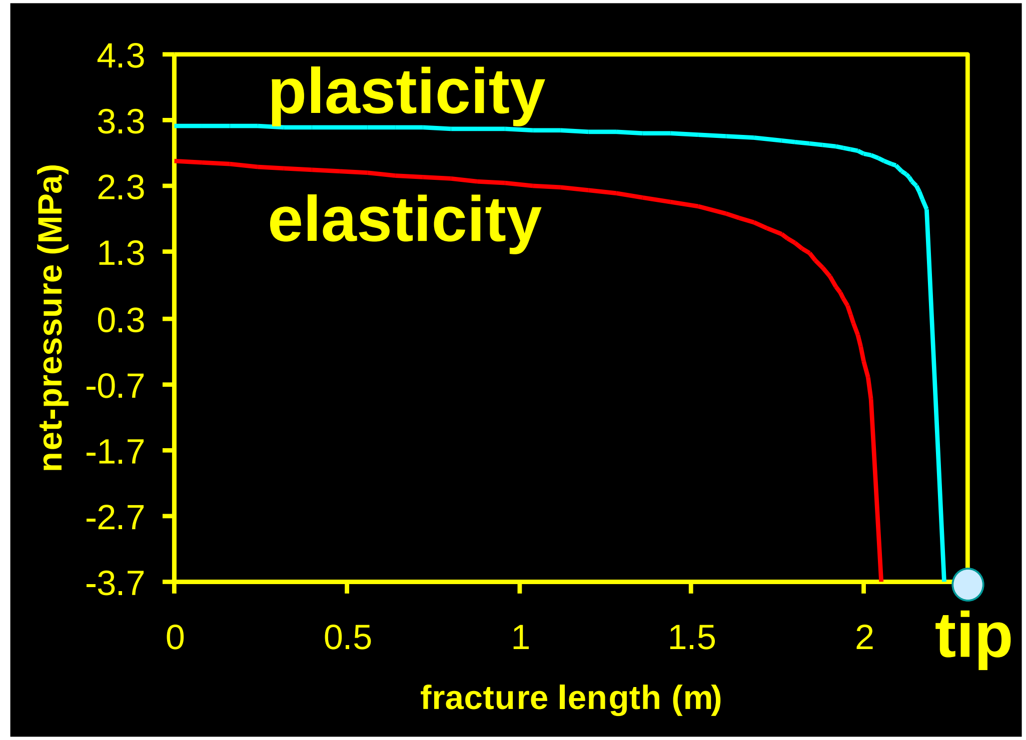
- Finite element analysis
  - fully coupled solution, special continuation algorithm
  - meshing/remeshing

Papanastasiou (Comp. Mech. J,1999)



Fracture propagation



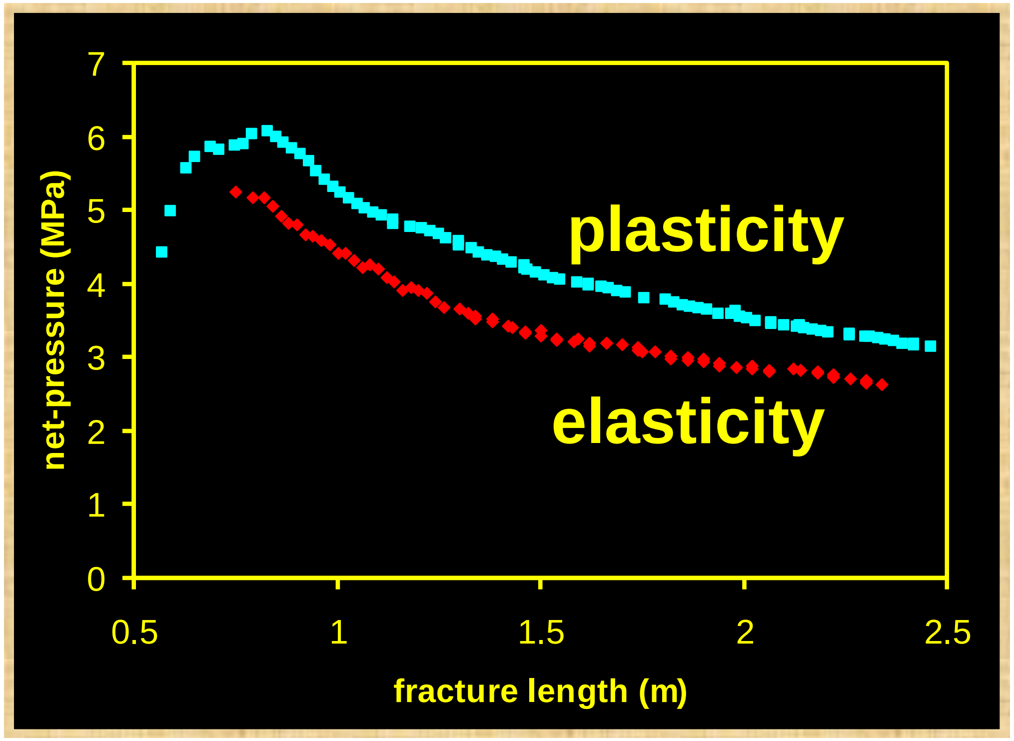


Plastic fractures are wider and shorter than the elastic fractures

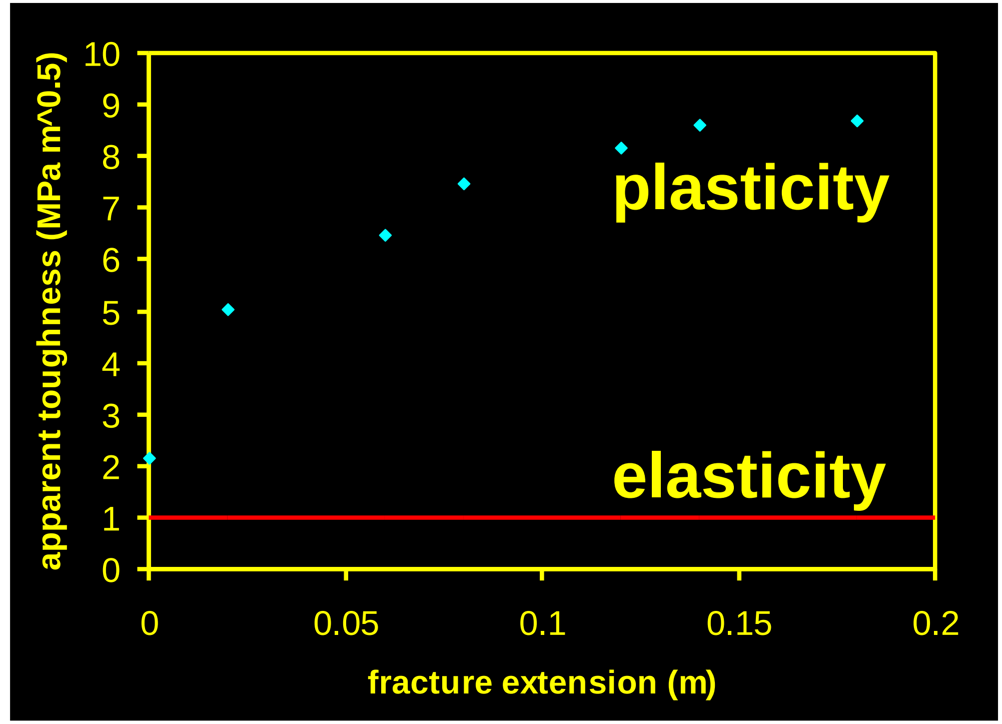
Fluid-lag is smaller in the plastic fracture

Papanastasiou (I J Frac,1997)

Apparent fracture toughness



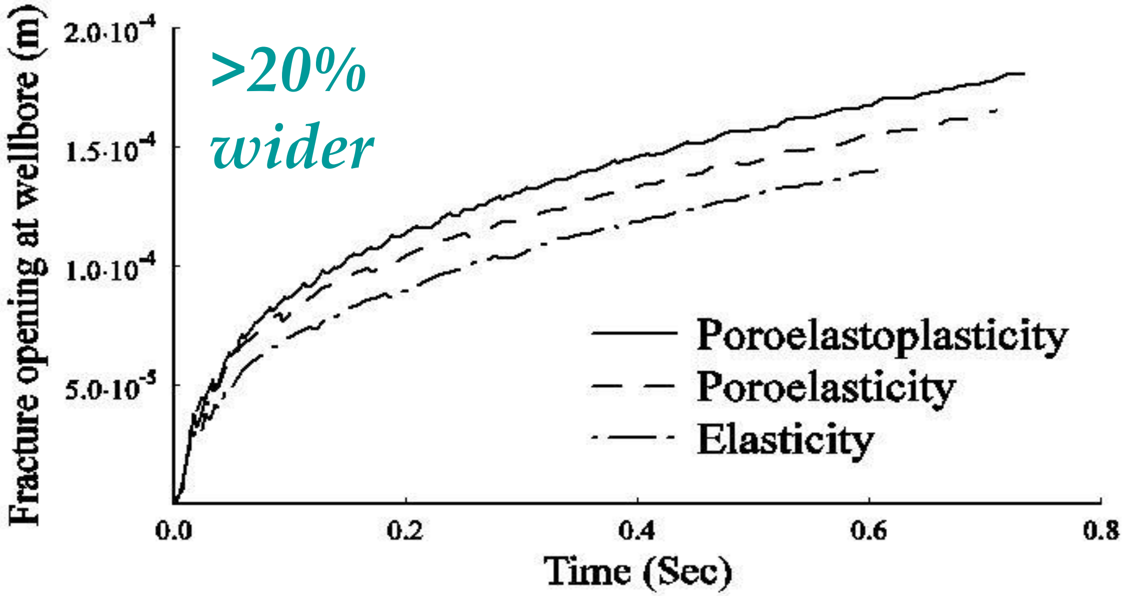
Propagating pressures are higher in plastic fractures



Effective fracture toughness, determined from J-integral, is higher for the plastic fracture

Papanastasiou (I J Frac,1999)

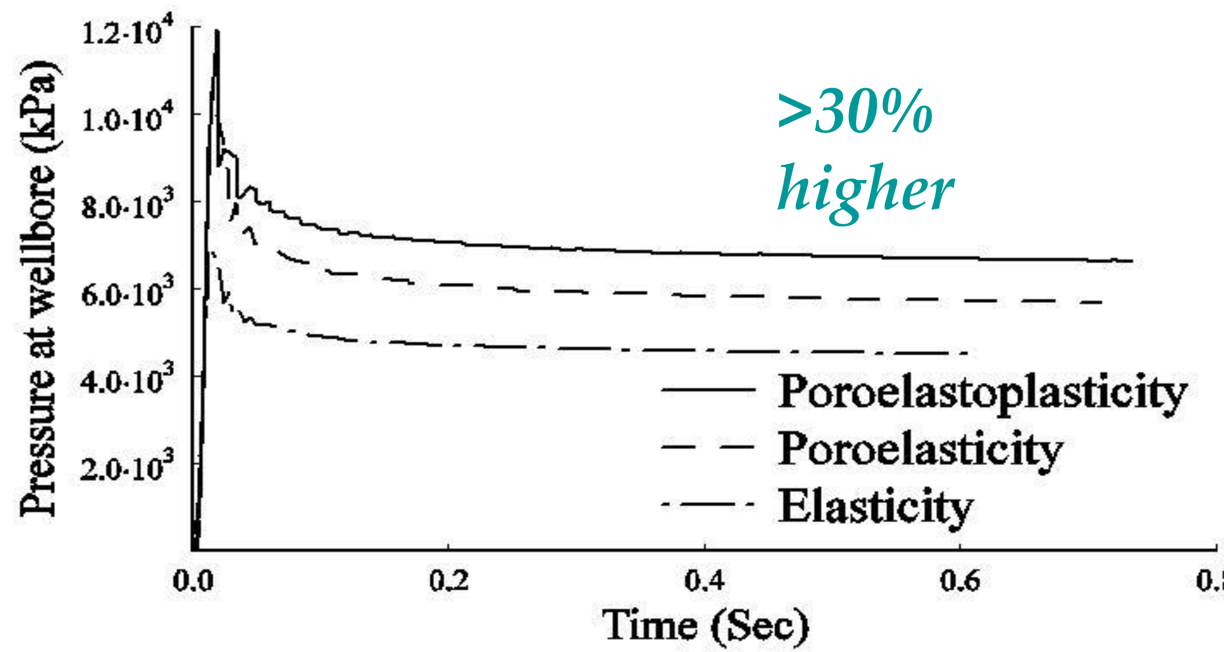
Theories together



Fracture opening at wellbore

Sarris and Papanastasiou(2010,2011,2014)

Pressure at wellbore



Scaling of plastic zones

Plastic Zones: function of { *Deviator stress*, *In-situ stress* } → $l_p = \sim \frac{\sigma_1 - \sigma_3}{\sigma_1 + \sigma_2 + \sigma_3 - 3P}$

Characteristic length of the problem

$$l_p = \frac{1}{3\pi}\left(\frac{K}{\sigma_t}\right)^2$$

LEFM Case (Small scale yielding)

$$l_p \sim \frac{E' \mu v}{\sigma_c}$$

H.F analytic solutions

{ *Higher: a) E′*
b) μ
c) u
d) K
Lower: a) σ_c } → *high stress concentration at the tip is expected* → *Larger Plastic Zone Development*

Papanastasiou (I J Frac,1999)

Effective fracture toughness (EFT)

Table 3. Variation of EFT with stress field and rock strength.

plane strain modulus	$E' = 31.25\ GPa$		
friction and dilation angles	$\phi = \psi = 30^o$		
rock fracture toughness	$K_{IC} = 2MPa\sqrt{m}$		
pumping parameters	$\mu v = 10^{-8} \sim 10^{-7} MPa\ m$		
	rock strength $\frac{\sigma_c}{\sigma_T}$		
stress field $\frac{\sigma_3}{\sigma_1}$	$\frac{60}{6}$	$\frac{20}{6}$	$\frac{20}{2}$
$\frac{30}{30} = 1.0$	2.0	2.0	2.0
$\frac{45}{30} = 1.5$	2.0	4.60	7.31
$\frac{60}{30} = 2.0$	2.0	7.03	15.48

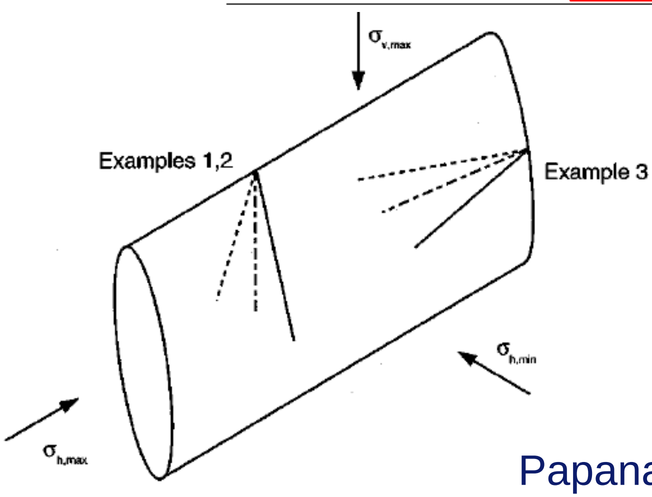


Table 4. Variation of EFT with stress field and pumping parameters.

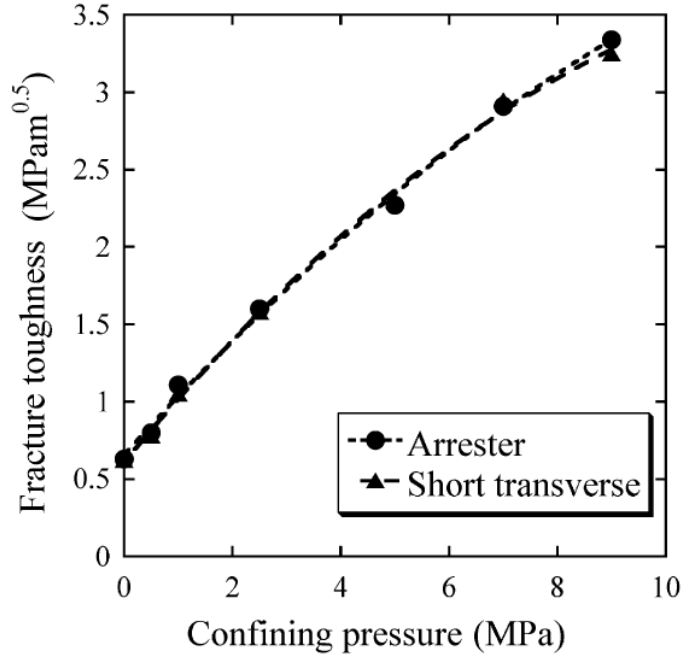
plane strain modulus	$E' = 31.25\ GPa$		
friction and dilation angles	$\phi = \psi = 30^o$		
rock fracture toughness	$K_{IC} = 1MPa\sqrt{m}$		
rock strength	$\frac{\sigma_c}{\sigma_T} = \frac{20}{2}$		
	pumping parameters $\mu\ v\ \ MPa\ m$		
stress field $\frac{\sigma_3}{\sigma_1}$	10^{-8}	10^{-7}	10^{-6}
$\frac{30}{30} = 1.0$	1.0	1.0	1.0
$\frac{45}{30} = 1.5$	1.0	1.88	5.66
$\frac{60}{30} = 2.0$	3.87	5.07	13.04

Papanastasiou (I J Frac,1999)

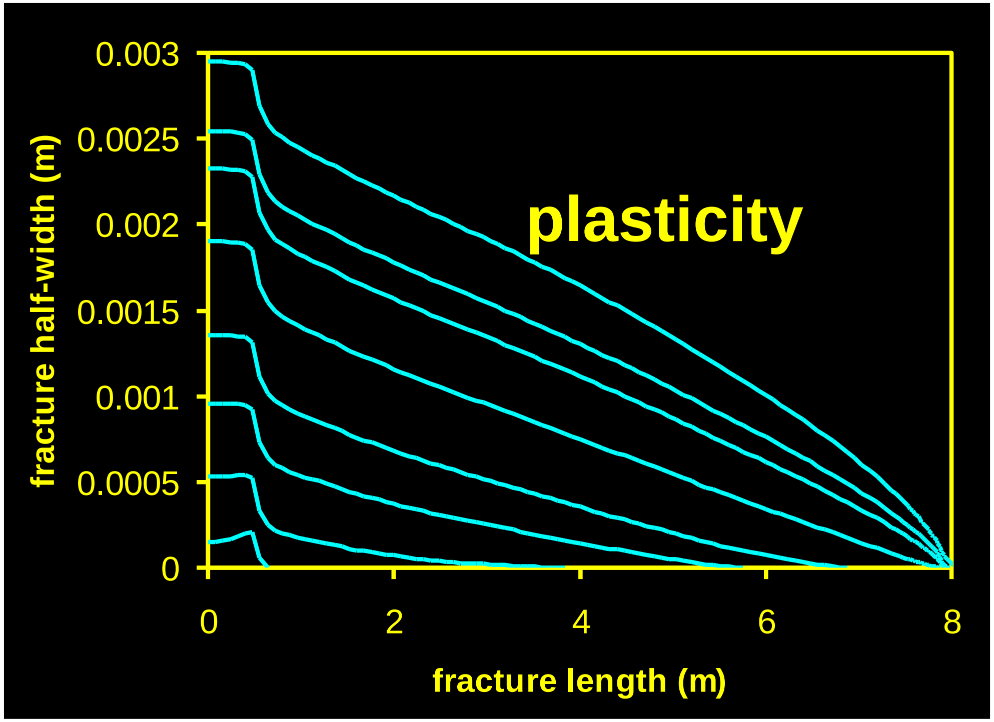
Experimental results on fracture toughness

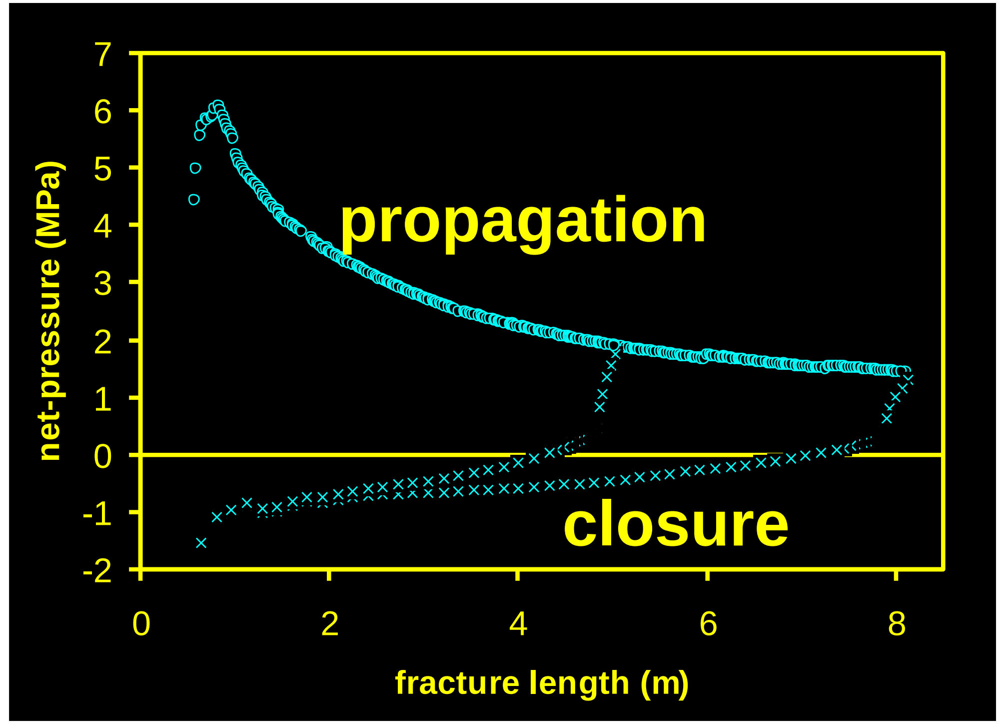
(Funatsu et al. Int. J. Rock Mech. & Min. Sci , 2004)

- Significant Increase of the rock fracture toughness with confining pressure
  - the fracture toughness of Kimachi sandstone increased by approximately 470% at 9 MPa confinement over its value at atmospheric pressure
  - similar variation of fracture toughness is caused by the combined effects of temperature and confining pressure.



Fracture closure

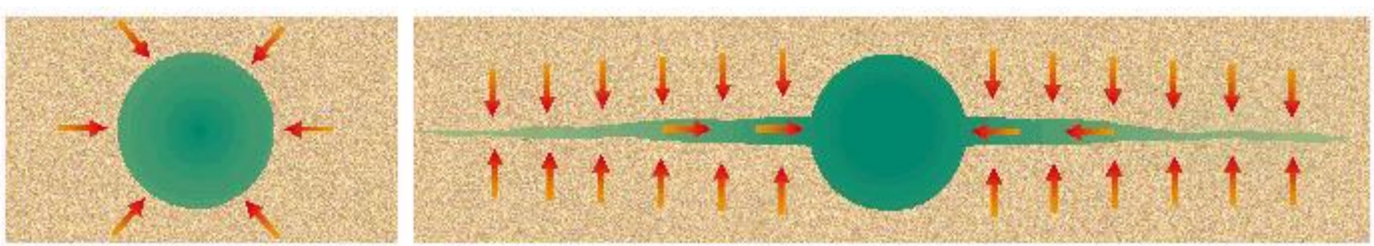




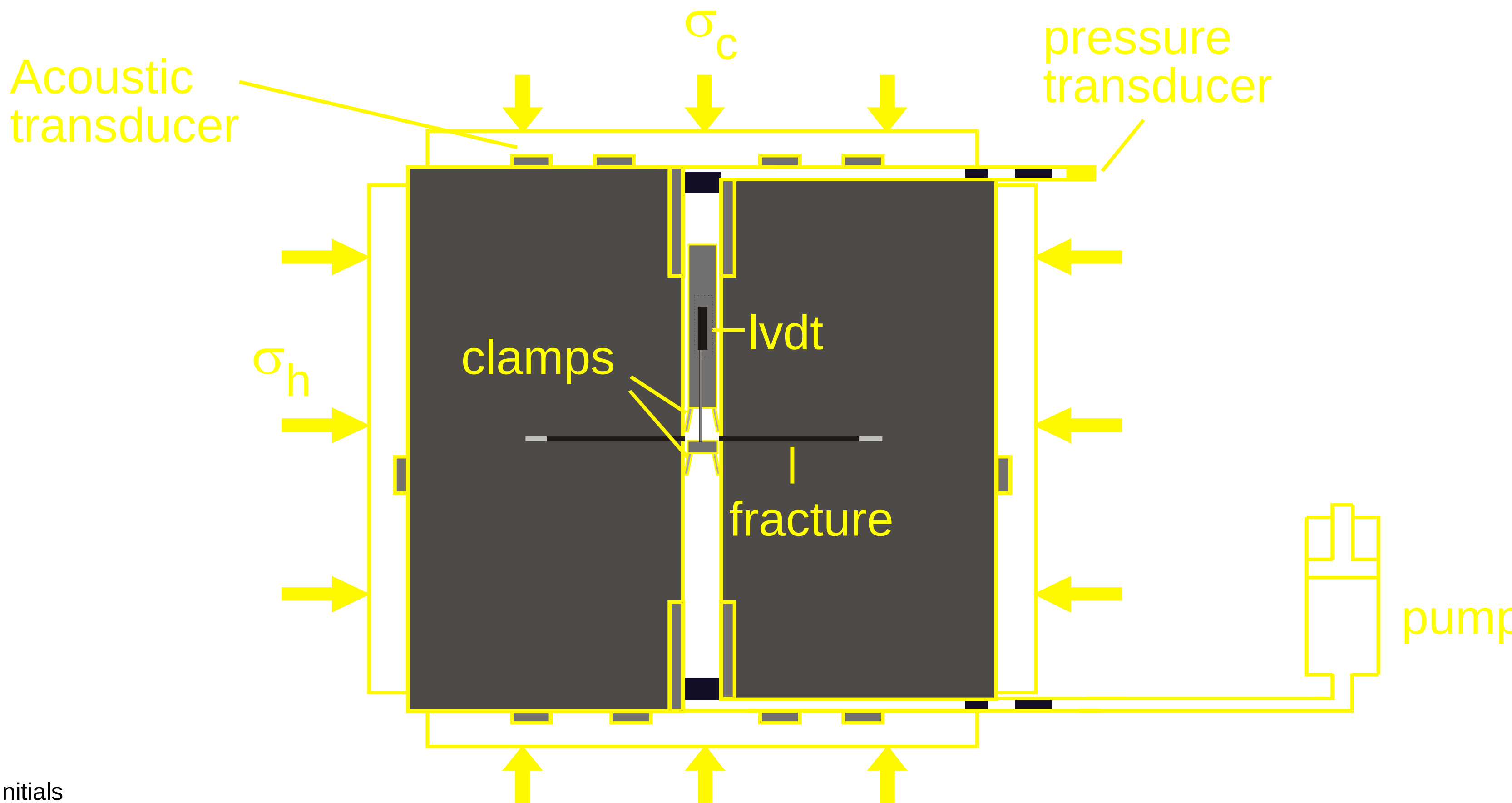
Plastic fracture closes first near the tip

Fracture is open at zero net-pressure and closes at negative values

Papanastasiou (I J Frac,2000)

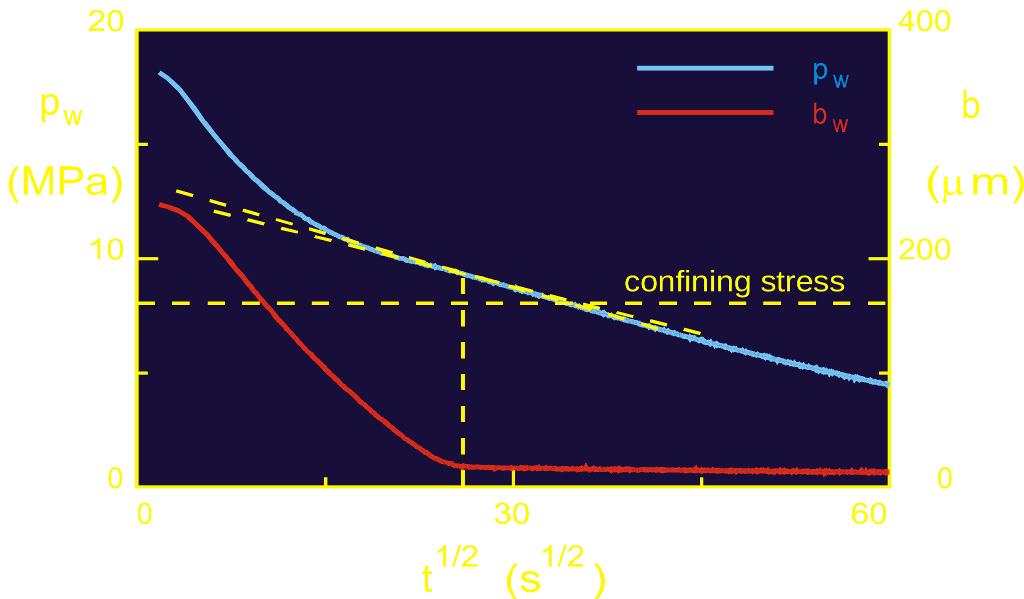


Experimental Set-up (DelFrac)

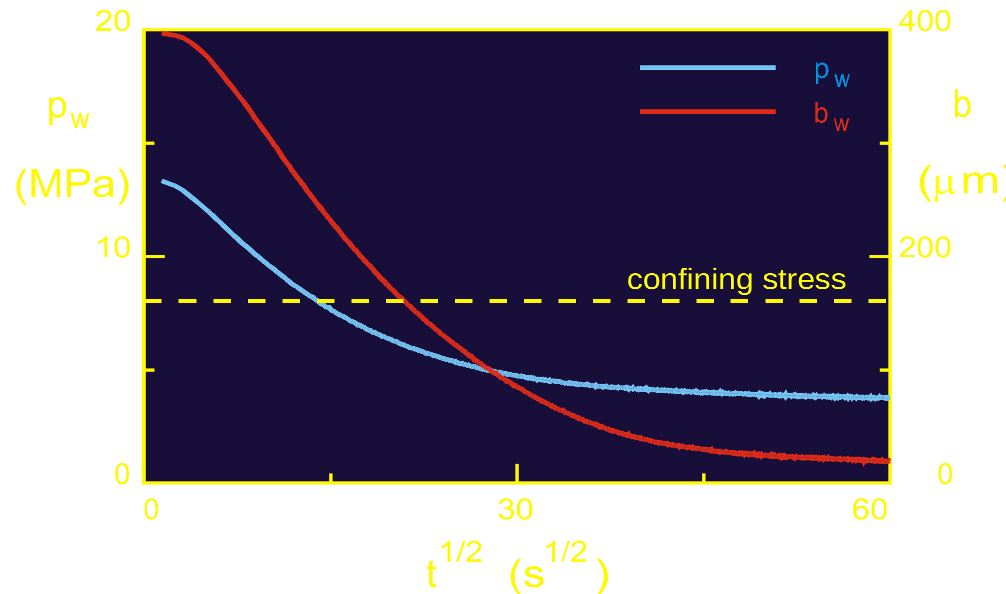


Closure in plaster

Strong Plaster:



Weak Plaster



van Dam D.B., et al *J. SPE Prod. & Fac*, (2002)

Dislocation model

- Fracture loading

$$\sigma = p + \sigma_1 \quad \text{(net-pressure)}$$

$$K_{el} = \sigma (\pi a)^{1/2}$$

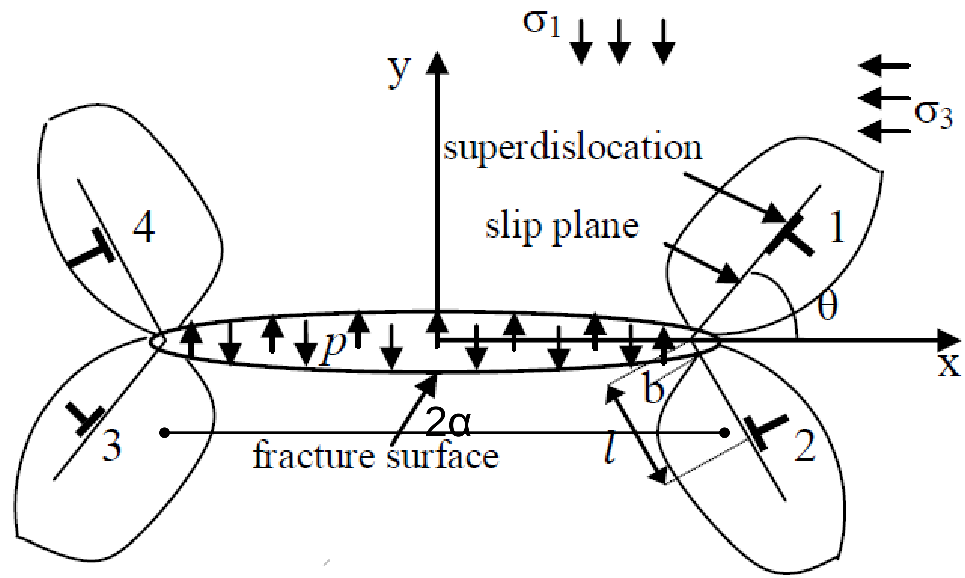
- Position and strength of super-dislocations $z = a + \ell e^{i\theta}$
  - stress intensity factor at the crack tip

$$\sigma\sqrt{\pi\alpha} - \frac{1}{8(\pi\alpha)^{1/2}} \frac{E}{(1-\nu^2)} b\, f = K_{IC}$$

  - stresses satisfy Mohr-Coulomb yield criterion at dislocations

$$\tau + \sigma_n \tan\phi = c$$

  - total crack-opening-displacement is maximized θ



Papanastasiou and Atkinson IJ Frac (2000)

For small scale yielding $l \ll \alpha$ and $K_{IC} = 0$ (Papanastasiou and Atkinson (2000))

$$\frac{l^{1/2}}{K_{el}/c} = \left(\frac{2}{\pi}\right)^{1/2} \frac{f_1(\theta)+\tan\varphi f_2(\theta)}{f_0}$$

$$\frac{b}{(1-\nu^2)K_{el}^2/(Ec)} = \frac{4}{3}\frac{f_1(\theta)/f_3(\theta)+\tan\varphi f_2(\theta)/f_3(\theta)}{f_0}$$

$$\frac{\delta}{(1-\nu^2)K_{el}^2/(Ec)} = \frac{8}{3}\frac{f_1(\theta)/f_4(\theta)+\tan\varphi f_2(\theta)/f_4(\theta)}{f_0}$$

$$\frac{F}{(1-\nu^2)K_{el}^2/(Ec)} = \frac{8}{3}\frac{1}{f_3(\theta)}\left\{\left[\frac{f_1(\theta)+\tan\varphi f_2(\theta)}{f_0}\right]\left[f_0 + \frac{(\sigma_3-\sigma_1)}{2c}\sin(2\theta)\right] - \tan\varphi\, f_2(\theta)\right\}$$

$$f_0\left(\frac{(\sigma_3-\sigma_1)}{2c}, \frac{p}{c}, \varphi, \theta\right) = \left[1 - \frac{(\sigma_3-\sigma_1)}{2c}\sin(2\theta)\right] + \tan\varphi\left[\frac{p}{c} + \frac{(\sigma_3-\sigma_1)}{2c}(1-\cos(2\theta))\right]$$

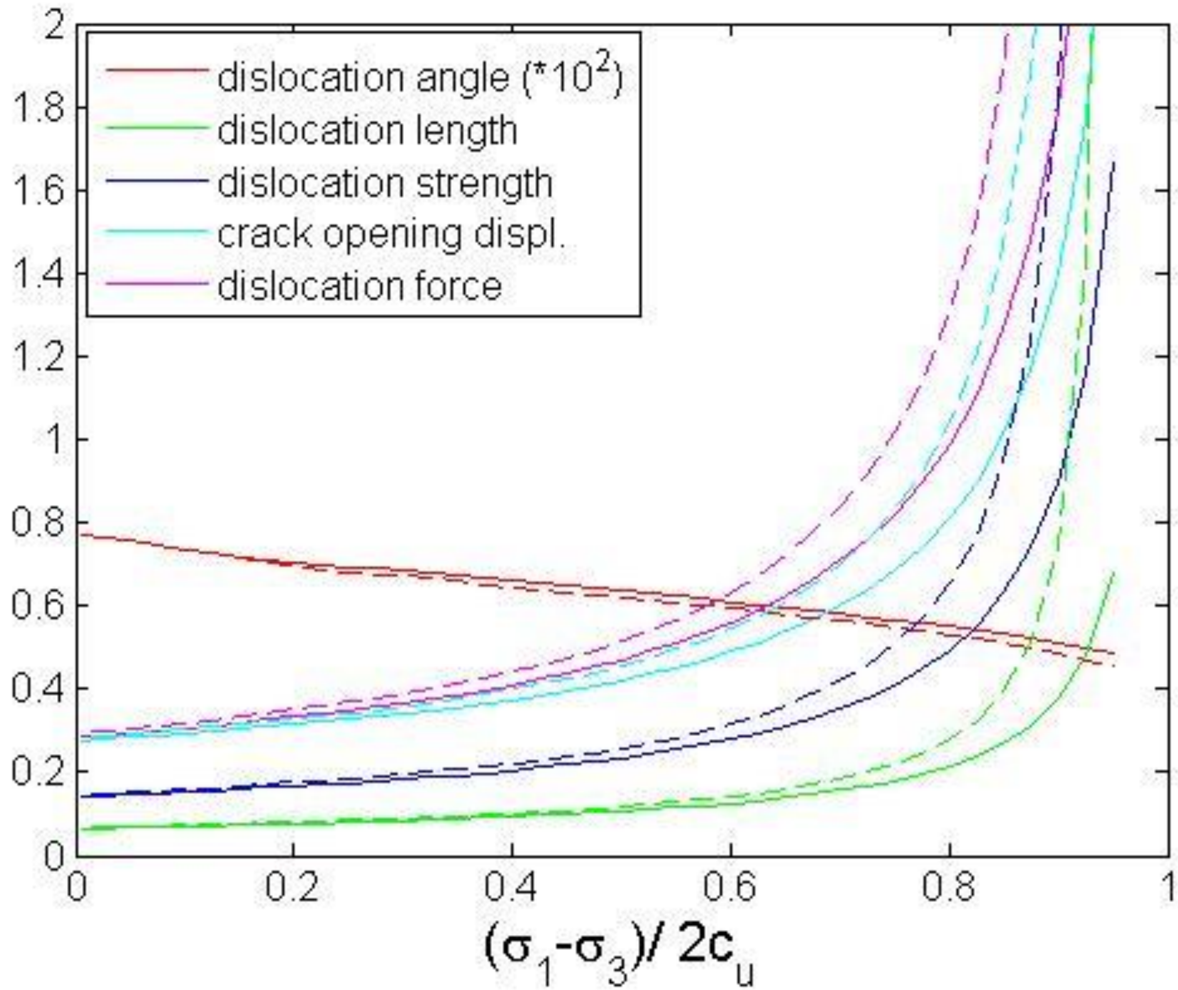
$$f_1(\theta) = \frac{2\sin^2\theta\cos^2\left(\frac{\theta}{2}\right)-1}{12\sin\theta\cos\left(\frac{\theta}{2}\right)}$$

$$f_2(\theta) = \frac{\cos\theta/\sin\theta + \sin(2\theta) - 1.5\sin\theta\cos^2\left(\frac{\theta}{2}\right)}{12\sin\theta\cos\left(\frac{\theta}{2}\right)} + \frac{\cos^3\left(\frac{\theta}{2}\right)}{2}$$

$$f_3(\theta) = \sin\theta\cos\left(\frac{\theta}{2}\right)$$

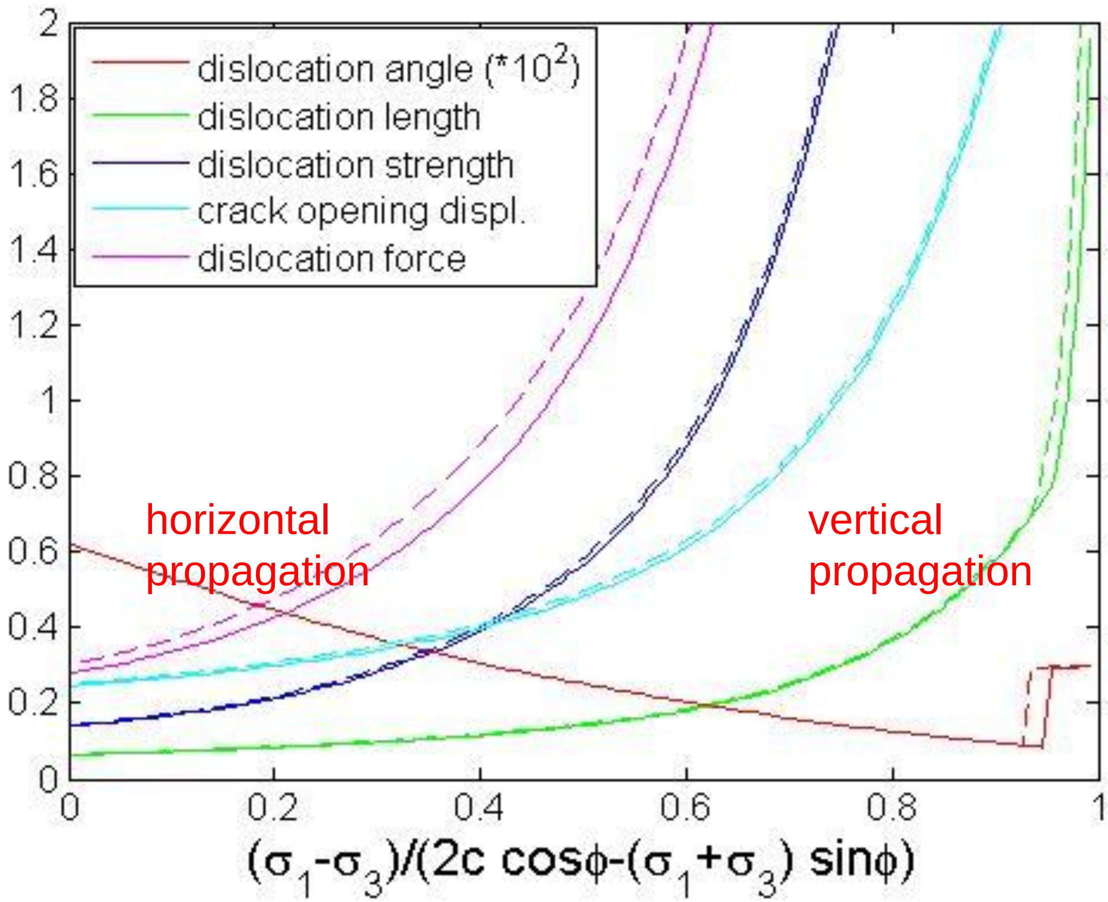
$$f_4(\theta) = \cos\left(\frac{\theta}{2}\right)$$

Frictionless or undrained analysis



Dimensionless quantities vs ductility number for an undrained material for small scale yielding (solid lines) and large scale yielding (dashed lines).

Cohesive-frictional material



The values between 0 and 1 correspond to fracture propagation of increasing ductility from brittle to small and large scale yielding and finally to 1 for a fracture that requires infinite energy release per unit advance

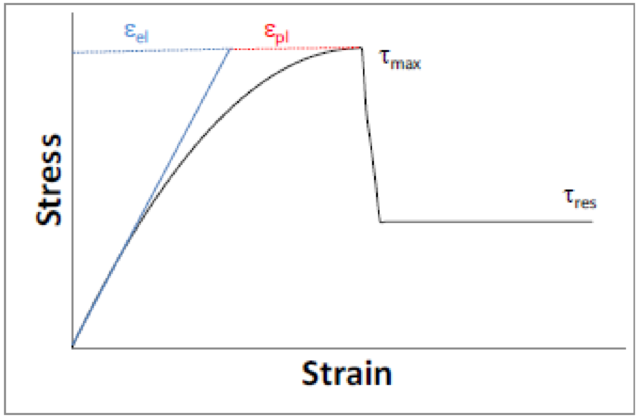
Dimensionless quantities vs ductility number for a cohesive-frictional material for small scale yielding (solid lines) and large scale yielding (dashed lines).

Papanastasiou et al (IJNAMG, 2016)

Brittleness parameters

- Brittleness parameters reported in Holt et al Int. J. Petr. Sc. Eng. (2015)

Definition of brittleness parameter	Comment	Reference
$B_1 = \frac{\varepsilon_{el}}{\varepsilon_{tot}}$	Elastic vs. total strain prior to failure	Hucka and Das, 1974
$B_2 = \frac{W_{el}}{W_{tot}}$	Elastic vs. total strain energy prior to failure	Hucka and Das, 1974
$B_3 = \frac{C_0 - T_0}{C_0 + T_0}$	Difference between unconfined compressive (C_0) and tensile (T_0) strengths, normalized by their sum	Hucka and Das, 1974
$B_4 = \sin\varphi$	φ is friction angle, measured from the failure envelope at zero normal stress	Hucka and Das, 1974
$B_5 = \frac{\tau_{\max} - \tau_{res}}{\tau_{\max}}$	Stress drop from peak (τ_{max}) to residual strength (τ_{res}) normalized by the peak stress	Bishop, 1967
$B_6 = \left\| \frac{\varepsilon_f^p - \varepsilon_c^p}{\varepsilon_c^p} \right\|$	ε_f^p and ε_c^p represent plastic strain at failure and at some specific strain level beyond	Hajiabdolmajid and Kaiser, 2003
$B_7 = OCR^b$	OCR is the overconsolidation ratio, i.e. the ratio between maximum historic and present in situ effective vertical stress	Ingram and Urai. 1999
$B_8 = \frac{1}{2}\left(\frac{E_{dyn}[Mpsi](0.8-\phi)-1}{8-1} + \frac{\nu_{dyn}-0.4}{0.15-0.4}\right)\cdot 100$	E_{dyn} and ν_{dyn} are dynamic Young's modulus and Poisson's ratio, respectively	Rickman et al., 2008
$B_9 = \frac{c_{qtz}}{c_{qtz} + c_{cl} + c_{carb}}$	c_{qtz}, c_{cl} and c_{carb} are weight fractions of quartz, clay and carbonate, respectively	Jarvie et al., 2007



- New parameter

$$B = 1 - t = 1 - \frac{(\sigma_1 - \sigma_3)}{2c\cos\varphi - (\sigma_1 + \sigma_3)\sin\varphi}$$

Propagation direction: Effective toughness vs closure stress gradient

- Net pressure

$$P_{net} = P_f - \sigma_{h\min}$$

- Linear elastic fracture mechanics

$$K_I = P_{net}\sqrt{\pi a} = (P_f - \sigma_{h\min})\sqrt{\pi a} = K_{IC}$$

$$P_f = \sigma_{h\min} + K_{IC} / \sqrt{\pi a}$$

- Local stress gradient for $\sigma_{h\min}$ is 15.8 kPa/m (0.7 psi/ft)
- As fracture propagates vertically upward the resistance decreases by 15.8 *KPa/m* but may increase proportionally to

$$K_{eff} \sim 10 MPa\sqrt{m}$$

- As fracture propagates horizontally the closure stress does not change and the resistance due to EFT is smaller

Conclusions

- Plasticity plays a shielding mechanism around the tip resulting in a significant increase of the apparent fracture toughness.
- Plasticity results in higher pressures and larger width everywhere in the fracture even though dilatancy tends to close the fracture near the tip.
- Closure pressure in plastic rock may be significantly lower than the far field stress on the fracture plane.
- Stress redistribution after closure of the fracture is important in weak rocks for sand avoidance. The shear stress near the wellbore is significantly lower than expected from elastic behaviour.

- A new definition of a brittleness index for interval selection for HF in shale reservoirs

$$B = 1 - t = 1 - \frac{(\sigma_1 - \sigma_3)}{2c\cos\varphi - (\sigma_1 + \sigma_3)\sin\varphi}$$

  - a combination of material strength parameters and in-situ stresses.
  - it varies between 0 and 1 with the value 1 to correspond to brittle propagation and 0 to a fracture that requires infinite energy release per unit advance

- CO2 related applications
  - Less resistance for a fracture to propagate horizontally than vertically. This decreases the risk of CO2 escape and increases the wellbore injectivity and reservoir storage capacity.
  - if the corrosive CO2 damages both cohesion and fracture toughness of rock proportionally then higher energy will be needed to propagate a mode I fracture