GRAINSIZE DYNAMICS

MIXING, SEGREGATION, CRUSHING AND THEIR HETERARCHY

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and Itai Einav

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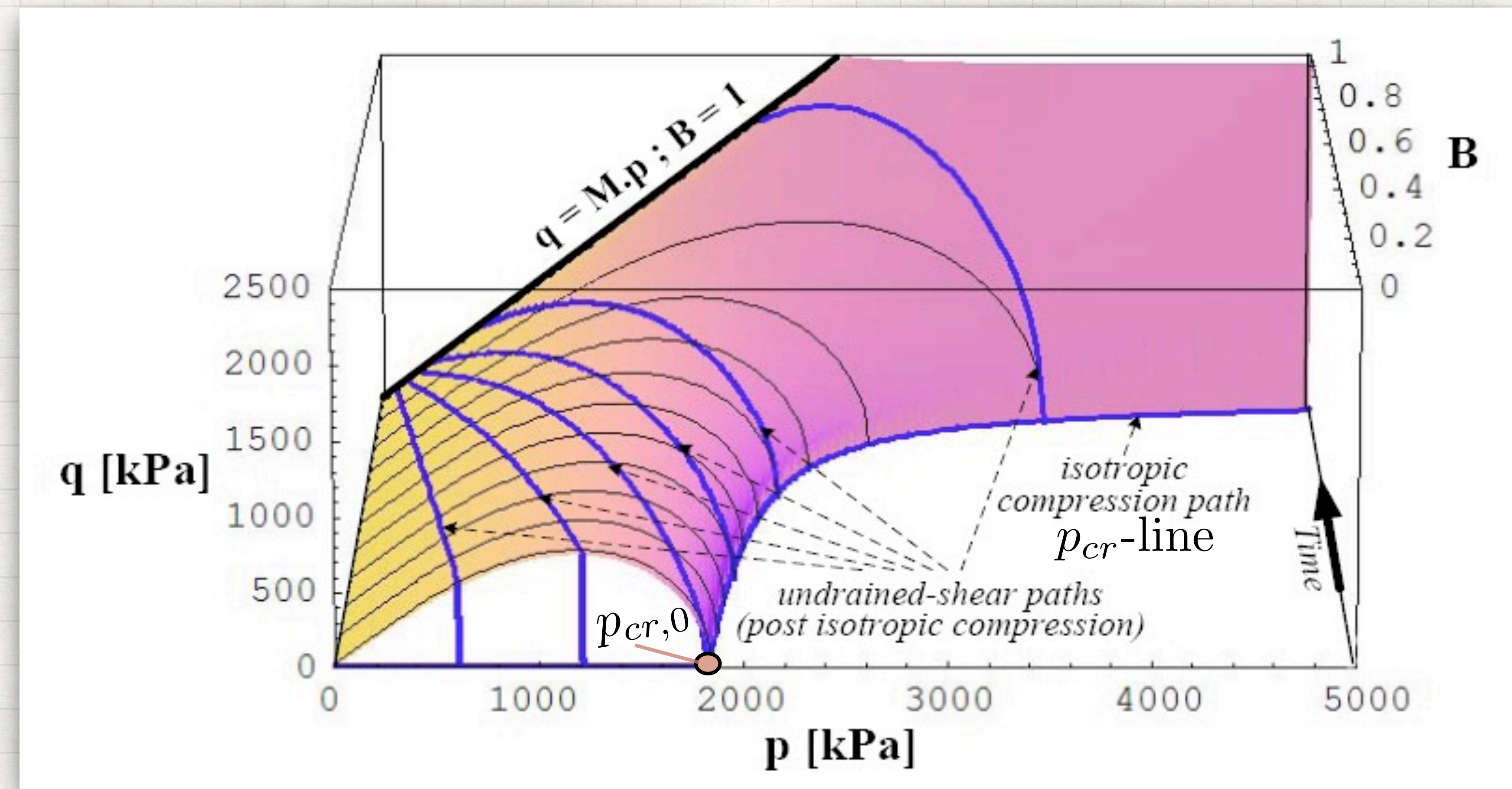
THE UNIVERSITY OF SYDNEY
AUSTRALIA

SCIENCE ATTACK

Climbing Masada, Israel (2009)

BACKGROUND

Vardoulakis first to support Breakage mechanics by a young researcher (in 2006, before publication)!



Einav (2007)

BACKGROUND

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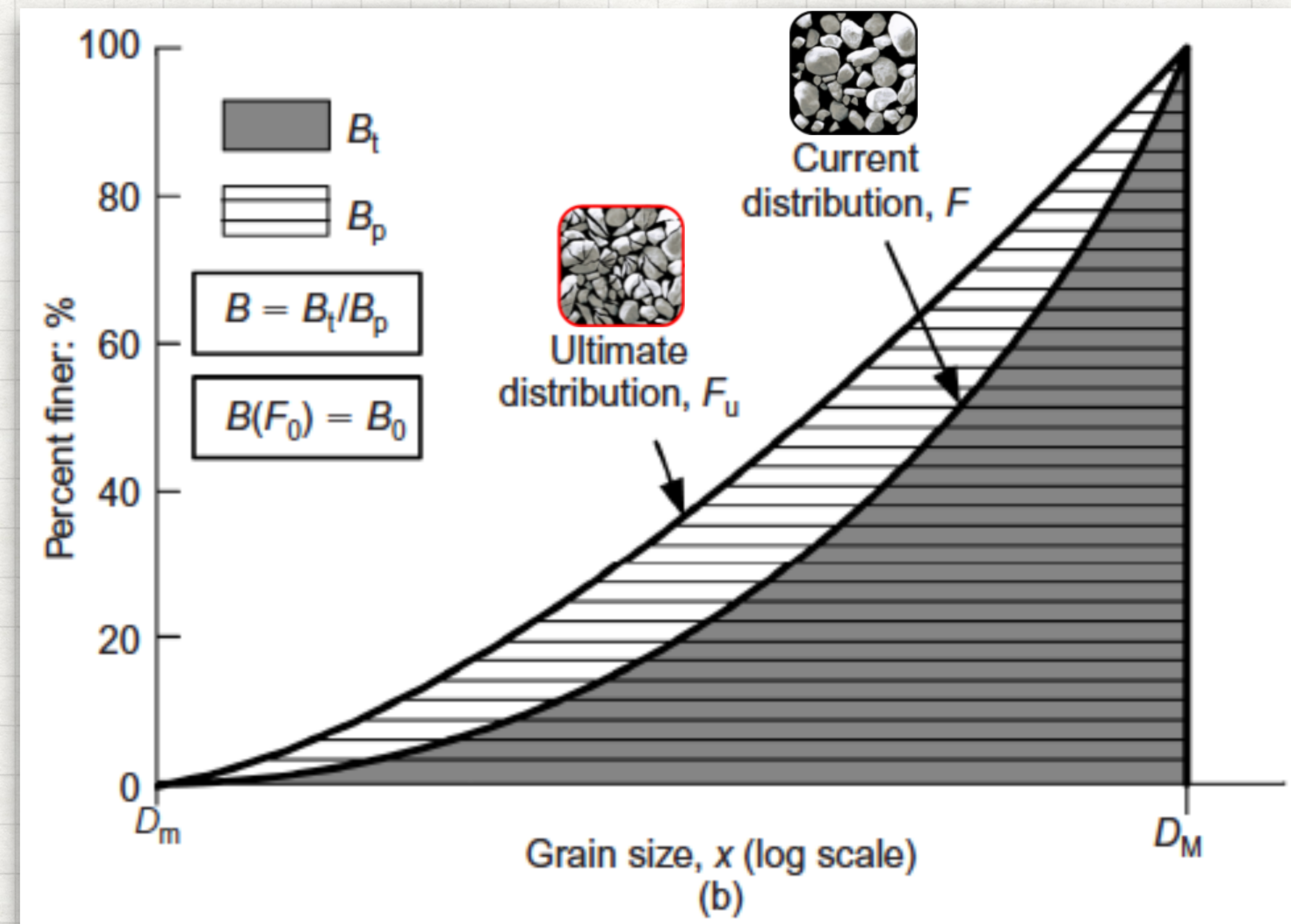
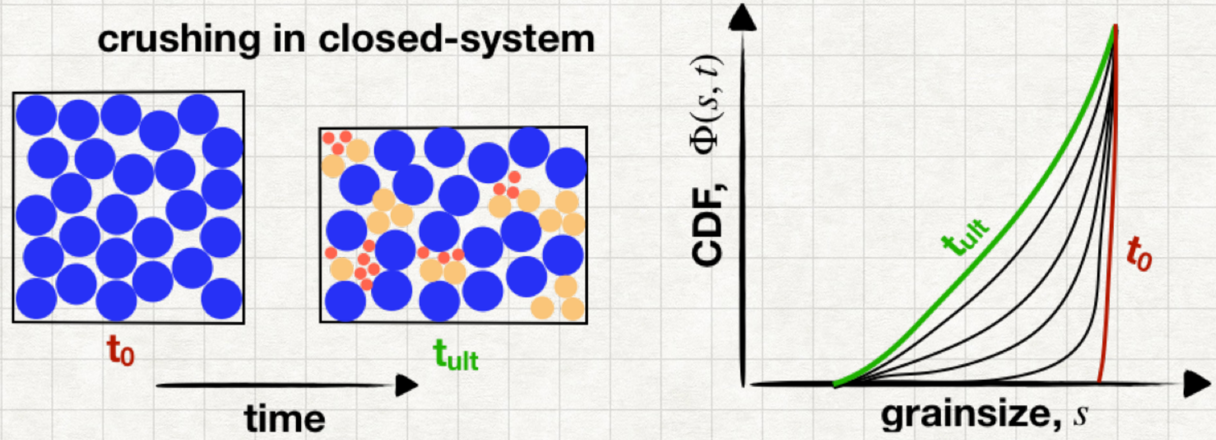


Einav (2007)

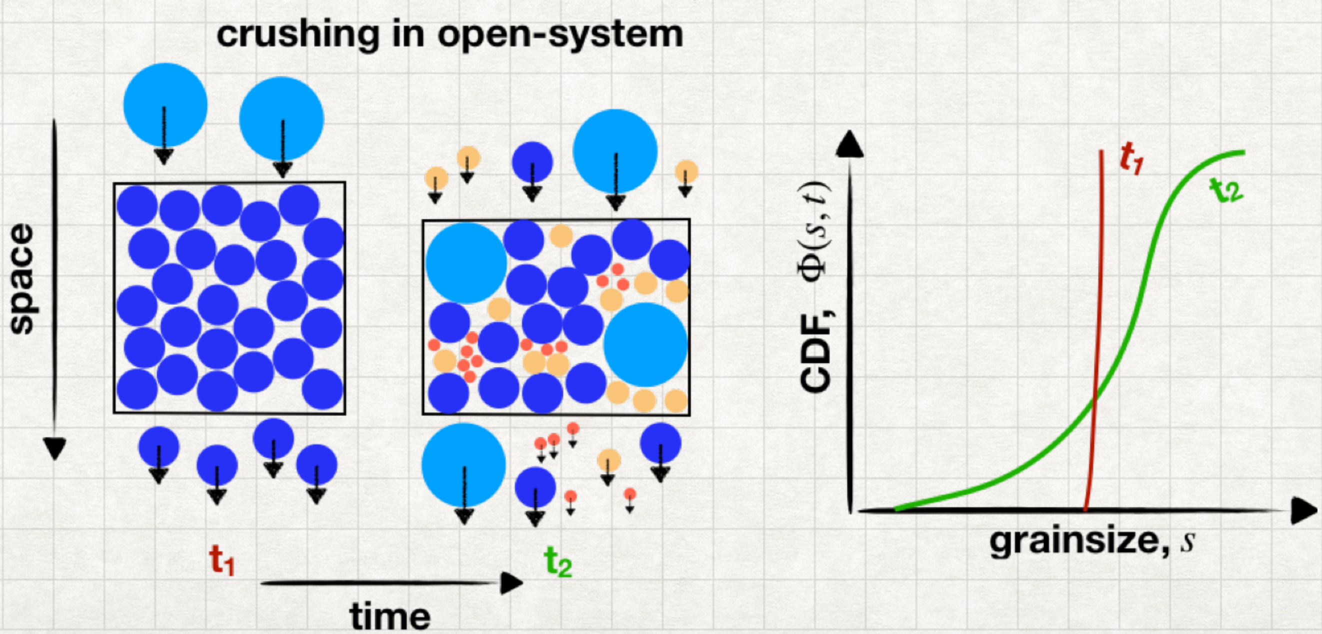
VARDOULAKIS'S CHALLENGE (ATHENS, JUNE 2007)

Resolve mathematically the oldest industrial problem in human history – the simultaneous crushing, mixing and segregation of flour in stonemills.

CLOSE VS OPEN SYSTEMS



Where Breakage Mechanics work...



Where Breakage Mechanics does not work...

CONTENT

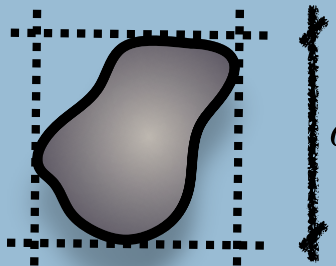
- WHY GRAIN SIZE
- ‘GRAINSIZE DYNAMICS’
- ‘HETERARCHY’
- GRAINSIZE DYNAMICS BY SEGREGATION
- GRAINSIZE DYNAMICS BY MIXING
- GRAINSIZE DYNAMICS BY CRUSHING
- COMBINED GRAINSIZE MECHANISMS
- DISCUSSION

WHY
GRAIN SIZE

WAIT A SECOND – SO WHAT IS 'GRAIN SIZE'?

Three of the more common measures:

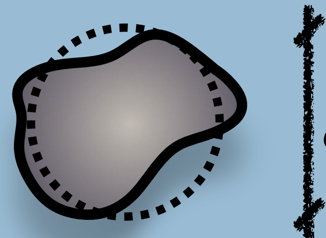
1st measure



d_{sieve}

$$d \equiv d_{sieve}$$

2rd measure

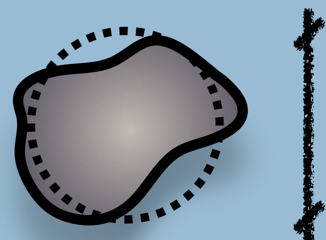


d_{Aeq}

$$A_p = \frac{\pi}{4} d_{Aeq}^2$$

$$d \equiv d_{Aeq} = \left(\frac{4A_p}{\pi} \right)^{1/2}$$

3rd measure



d_{eq}

$$V_p = \frac{\pi}{6} d_{Veq}^3$$

$$d \equiv d_{Veq} = \left(\frac{6}{\pi} V_p \right)^{1/3}$$

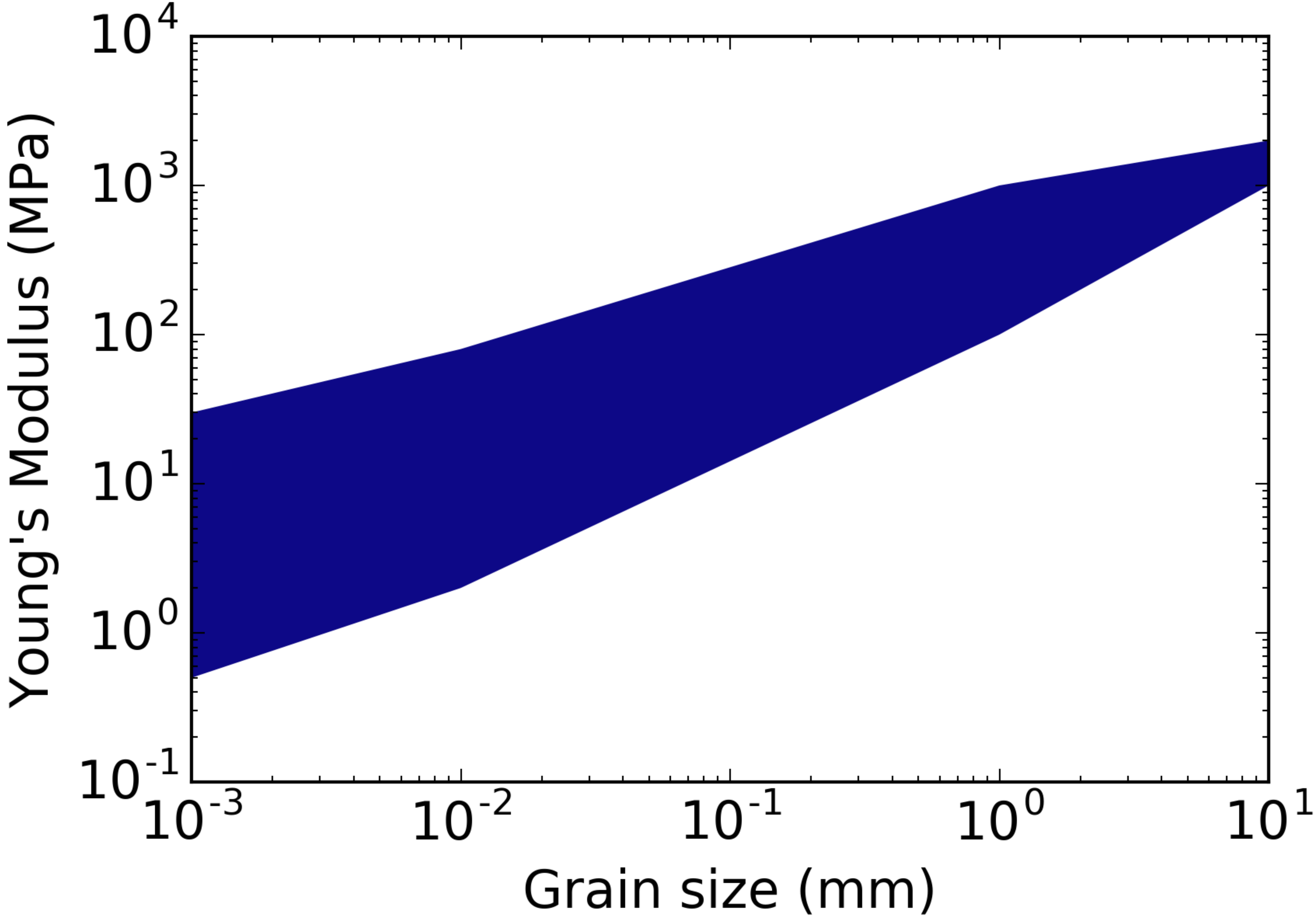
Disclaimer

The following graphs contain **nominal** values, and are not to be used for design purposes. The data is collated from the following sources:

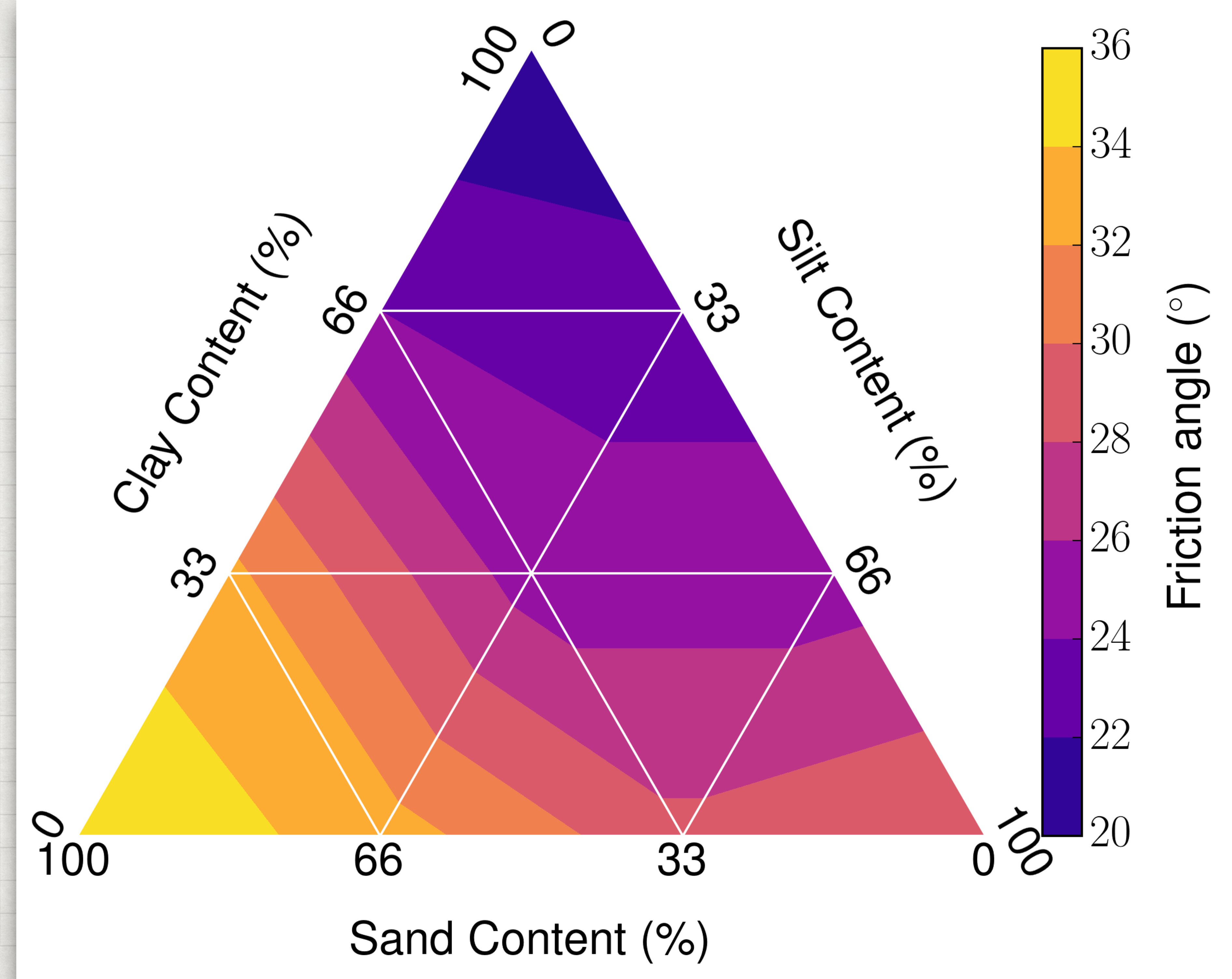
Obrzud R. & Truty, A. The Hardening Soil Model — A Practical Guidebook. *Zace Services*, 2010.

Engineering and Design Settlement Analysis, USACE *Engineering Manual 1110-1-1904*, 1990.

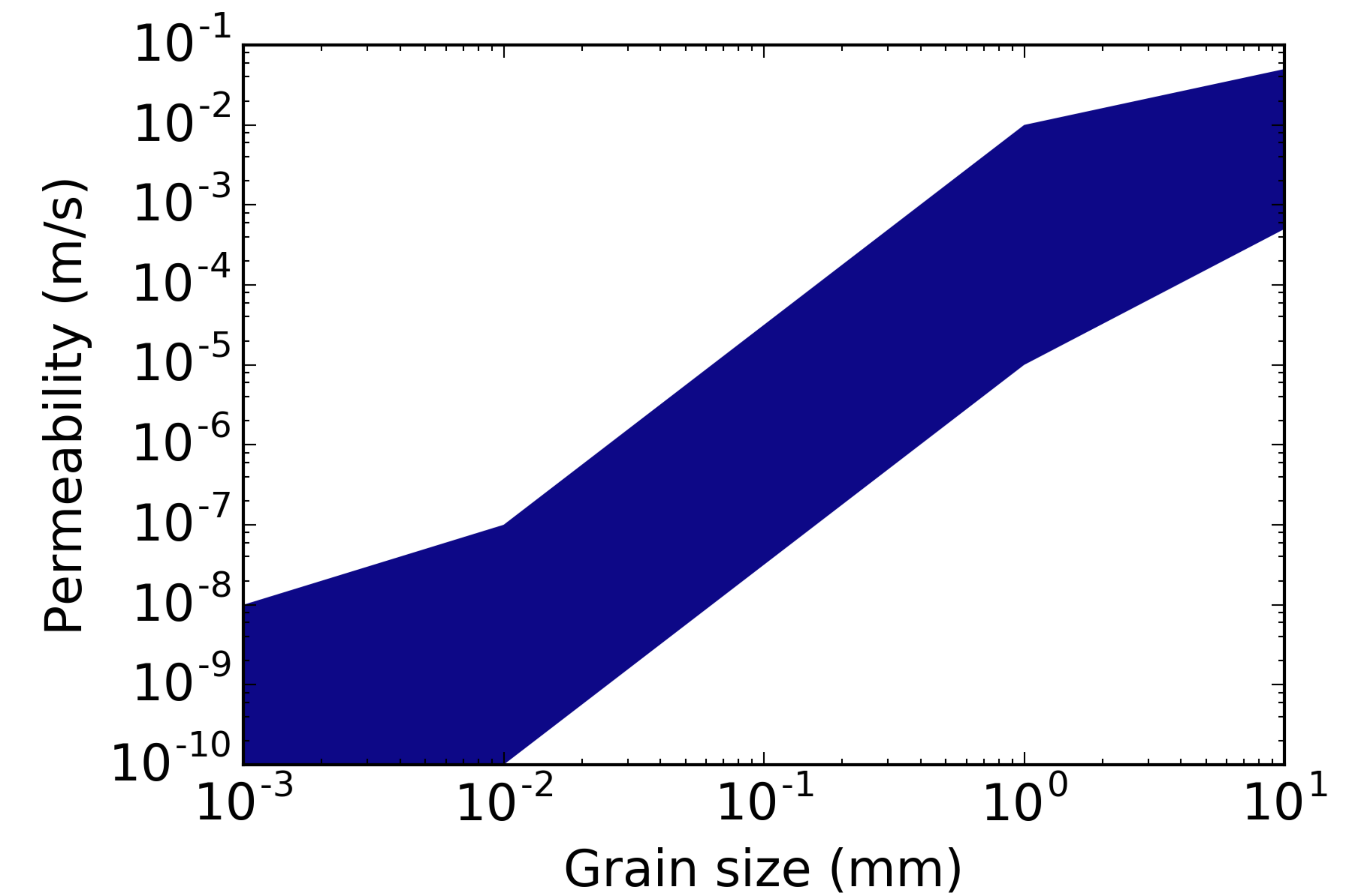
Yang, H., Rahardjo, H., Leong, E. C., & Fredlund, D. G. (2004). Factors affecting drying and wetting soil-water characteristic curves of sandy soils. *Canadian Geotechnical Journal*, 41(5), 908-920.



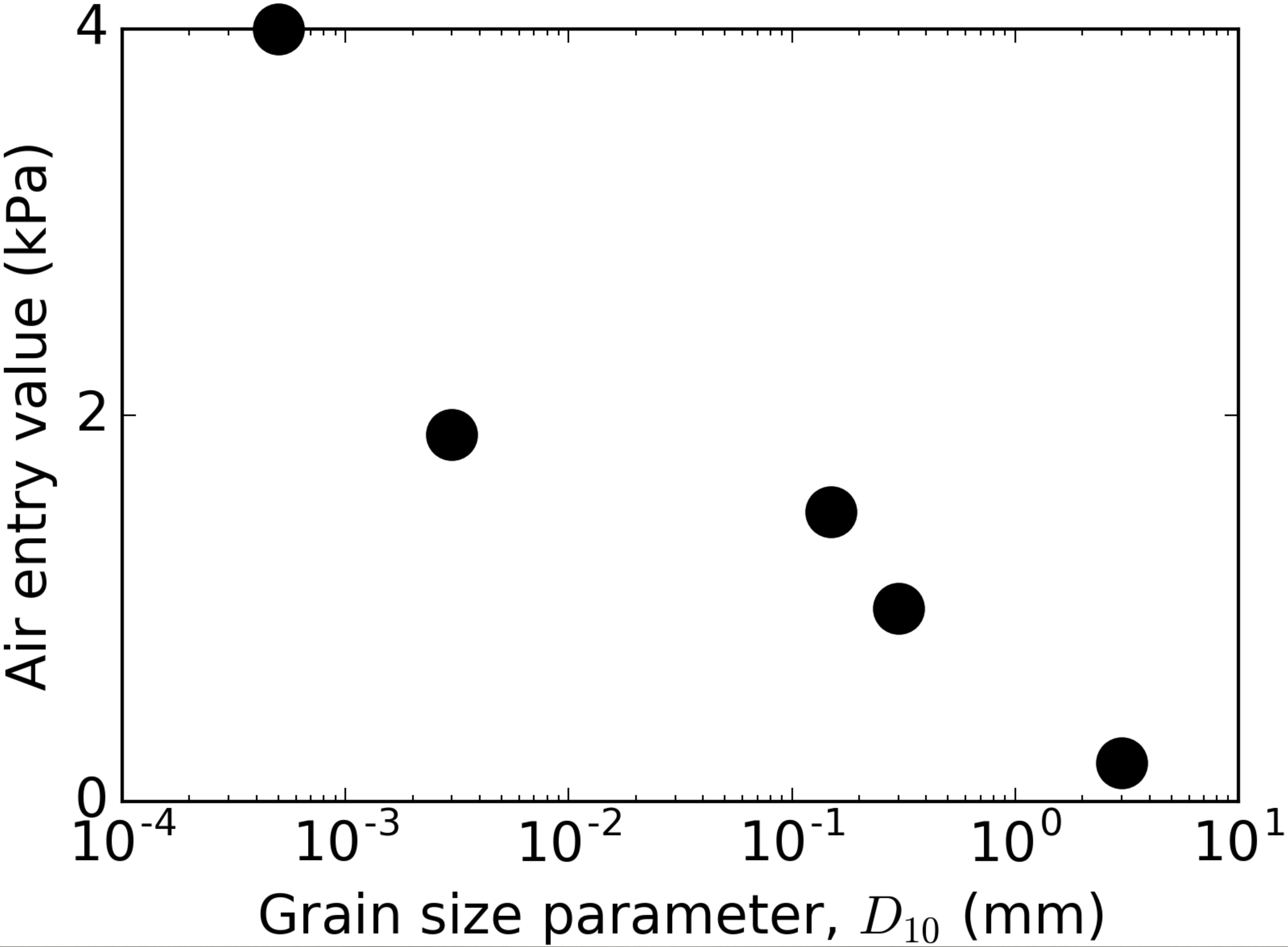
Young's Modulus (MPa)
10^4
10^3
10^2
10^1
10^0
10^{-1}
10^{-3}
10^{-2}
10^{-1}
10^0
10^1
Grain size (mm)



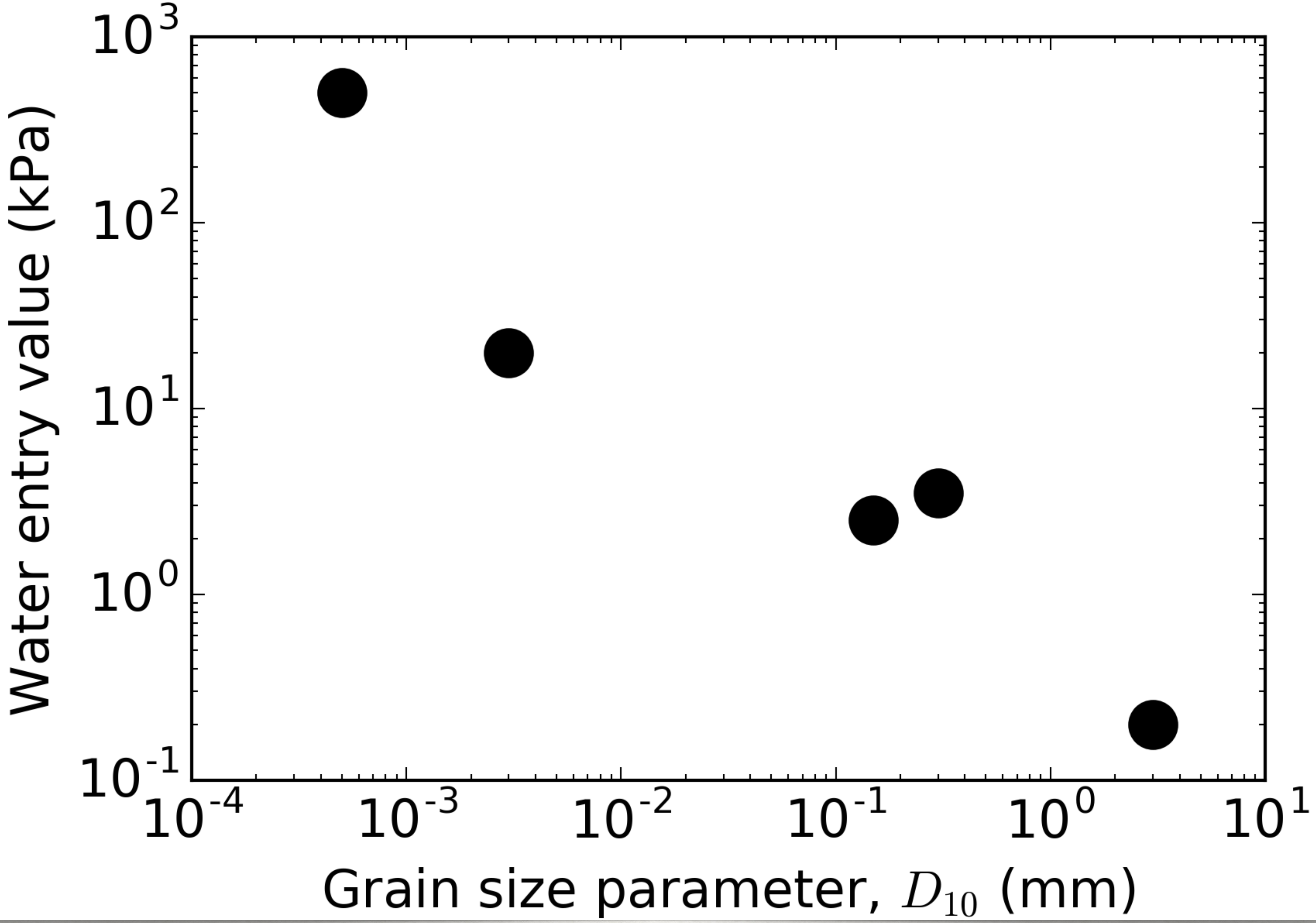
100
0
66
33
Clay Content (%)
33
66
Silt Content (%)
0
100
66
33
0
100
Sand Content (%)
36
34
32
30
28
26
24
22
20
Friction angle (°)



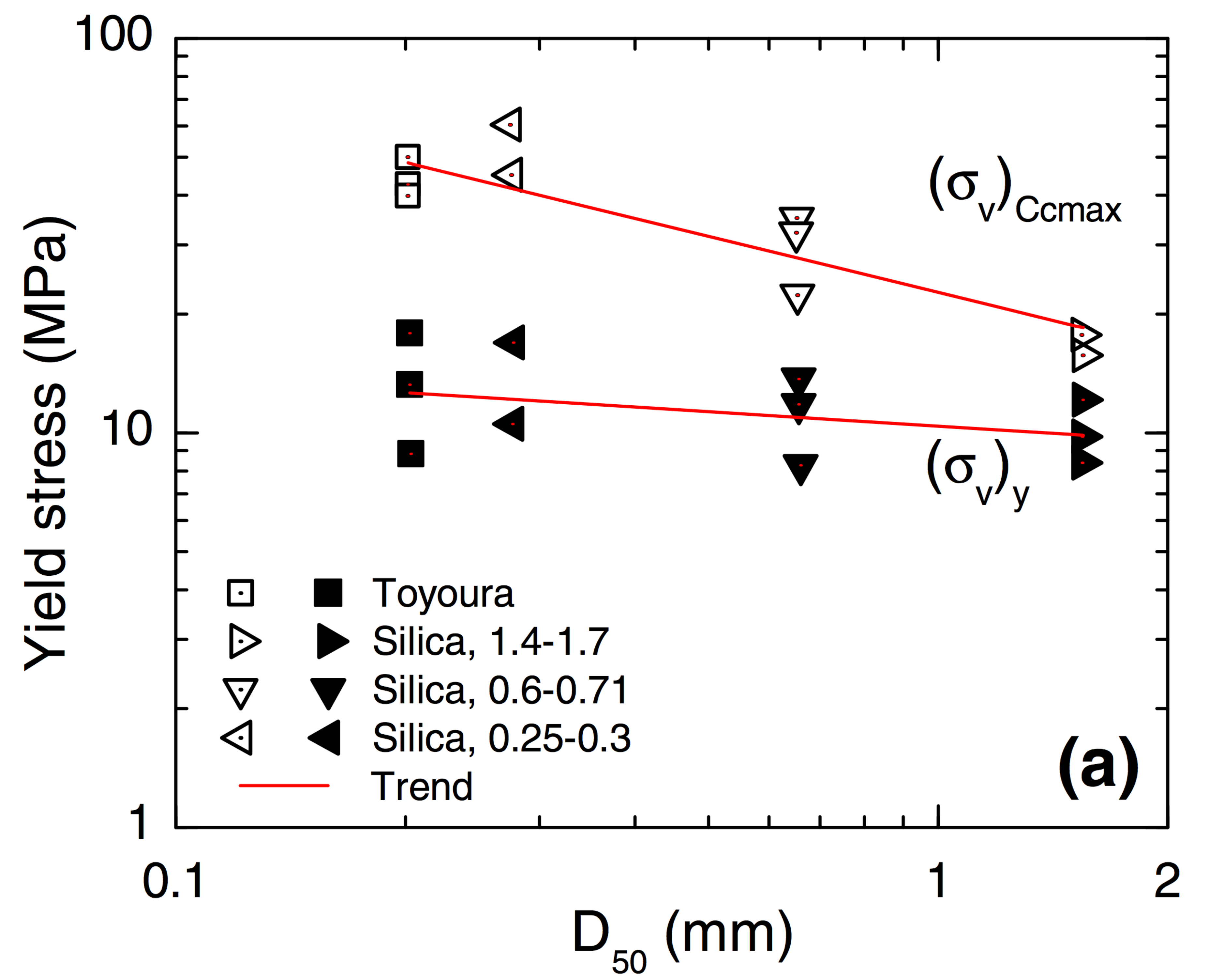
Permeability (m/s)
10-1
10-2
10-3
10-4
10-5
10-6
10-7
10-8
10-9
10-10
10-3
10-2
10-1
100
101
Grain size (mm)



Air entry value (kPa)
4
2
0
10-4
10-3
10-2
10-1
100
101
Grain size parameter, D_{10} (mm)



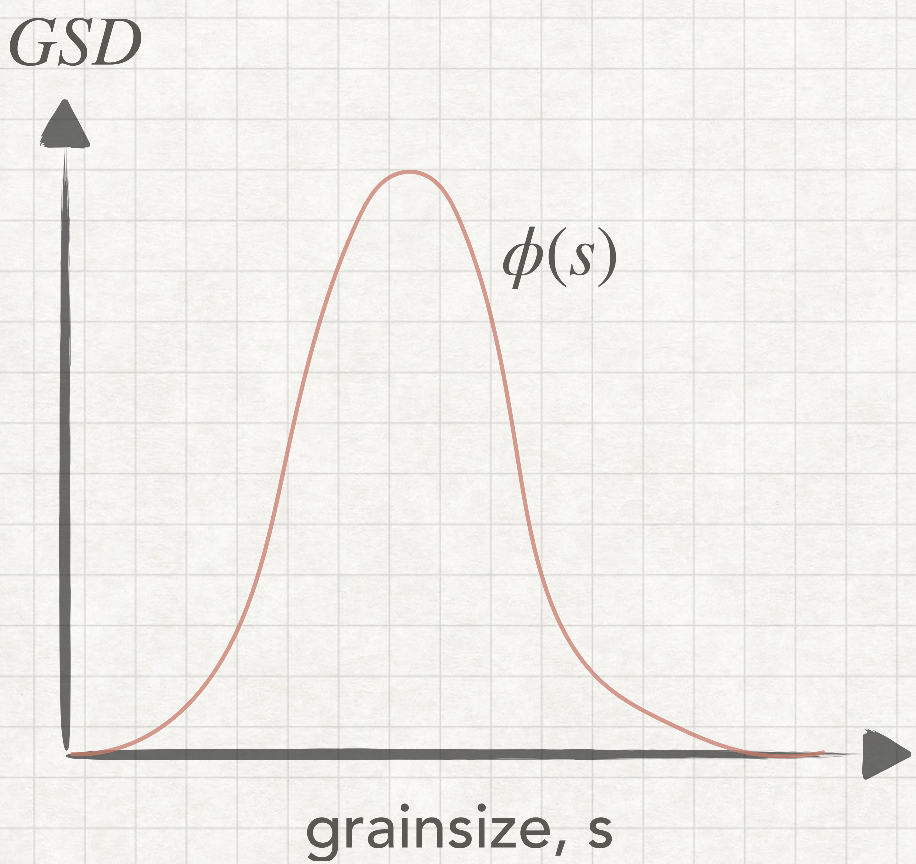
Water entry value (kPa)
10^3
10^2
10^1
10^0
10^-1
10^-4
10^-3
10^-2
10^-1
10^0
10^1
Grain size parameter, D_{10} (mm)

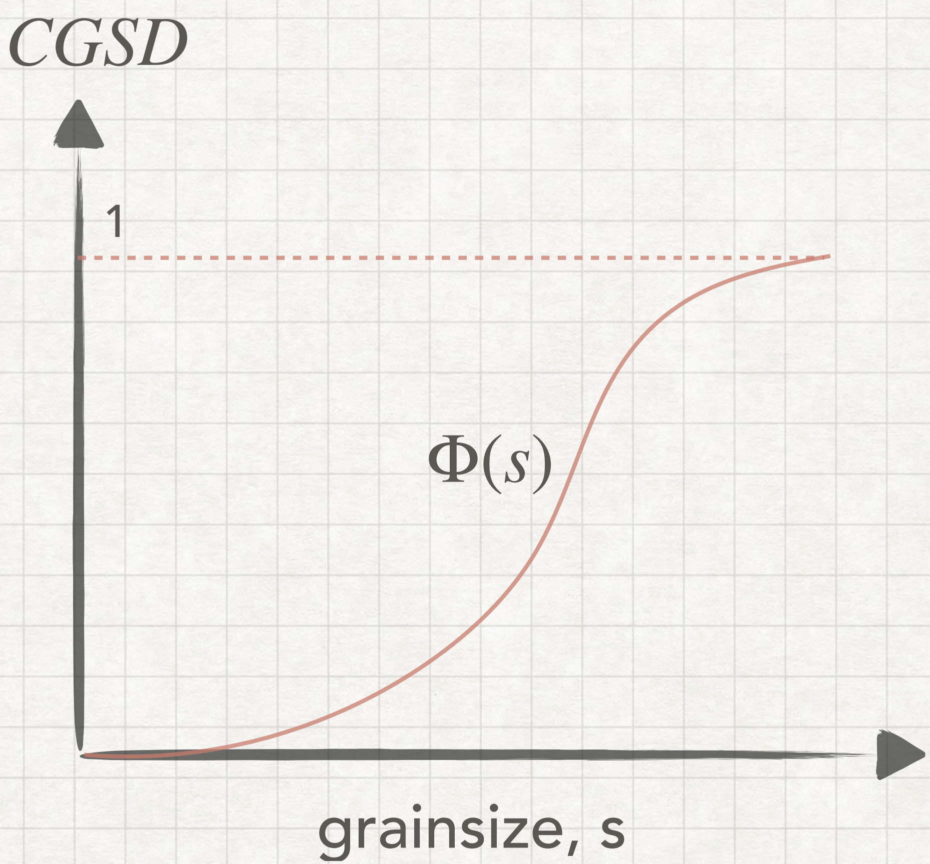


100
10
1
0.1
1
2
Yield stress (MPa)
D50 (mm)
(σv)Ccmax
(σv)y
Toyoura
Silica, 1.4-1.7
Silica, 0.6-0.71
Silica, 0.25-0.3
Trend
(a)

GRAINSIZE DYNAMICS

GRAINSIZE DISTRIBUTION



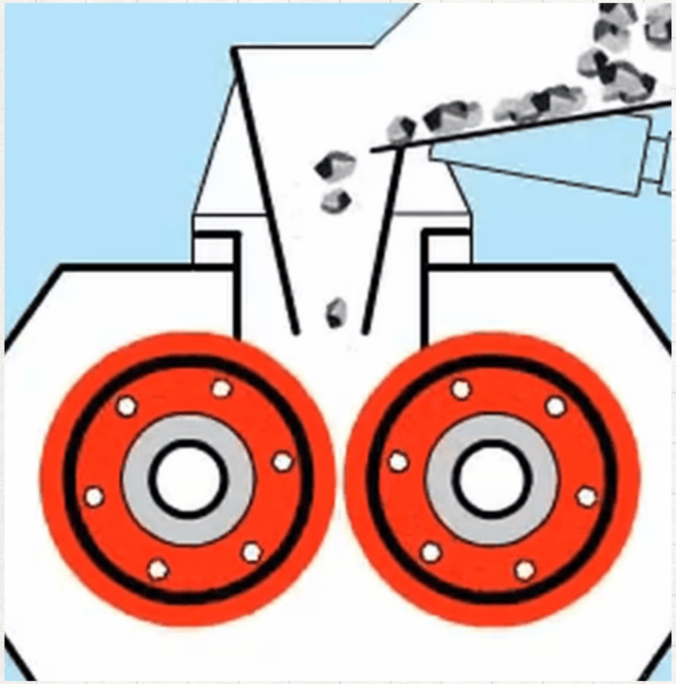


$$\int_0^\infty \phi(s)\ ds = 1$$

$$\Phi(s) = \int_0^s \phi(x)\ dx$$

MOTIVATION

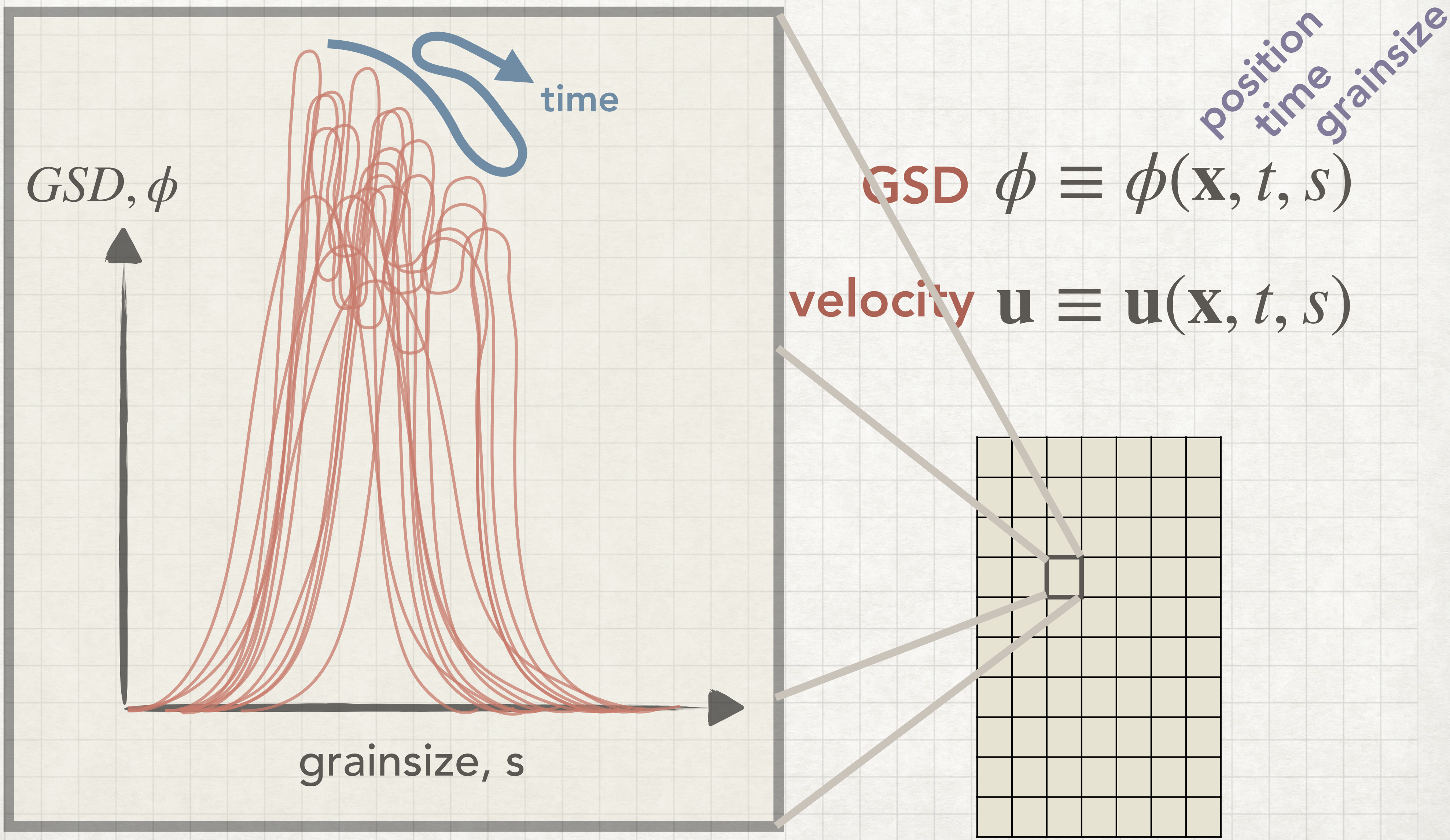
- ROLL MILL



GSD varies in time and space

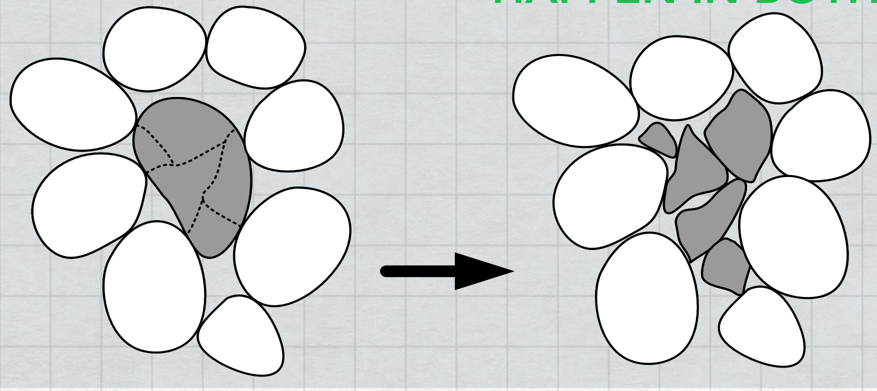
MOTIVATION

- CONTINUUM MECHANICS IN 5D

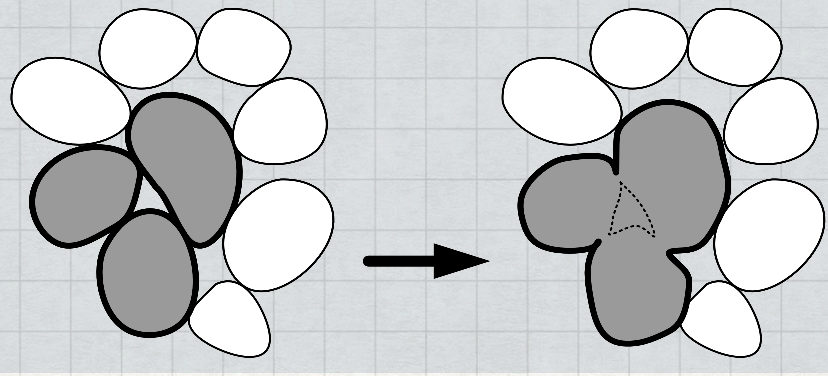


GRAINSIZE DYNAMICS – MECHANISMS

HAPPEN IN BOTH CLOSE & OPEN SYSTEM

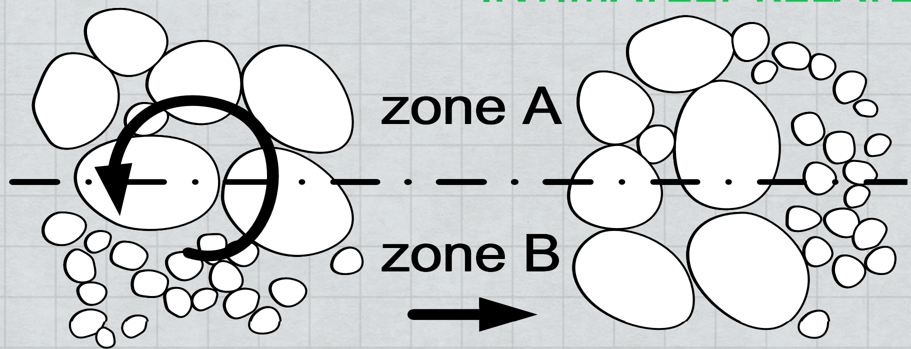


crushing

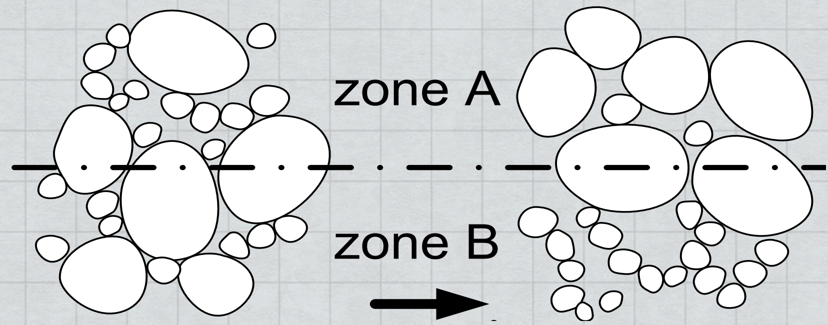


agglomeration

INTIMATELY RELATED TO OPEN SYSTEMS



mixing



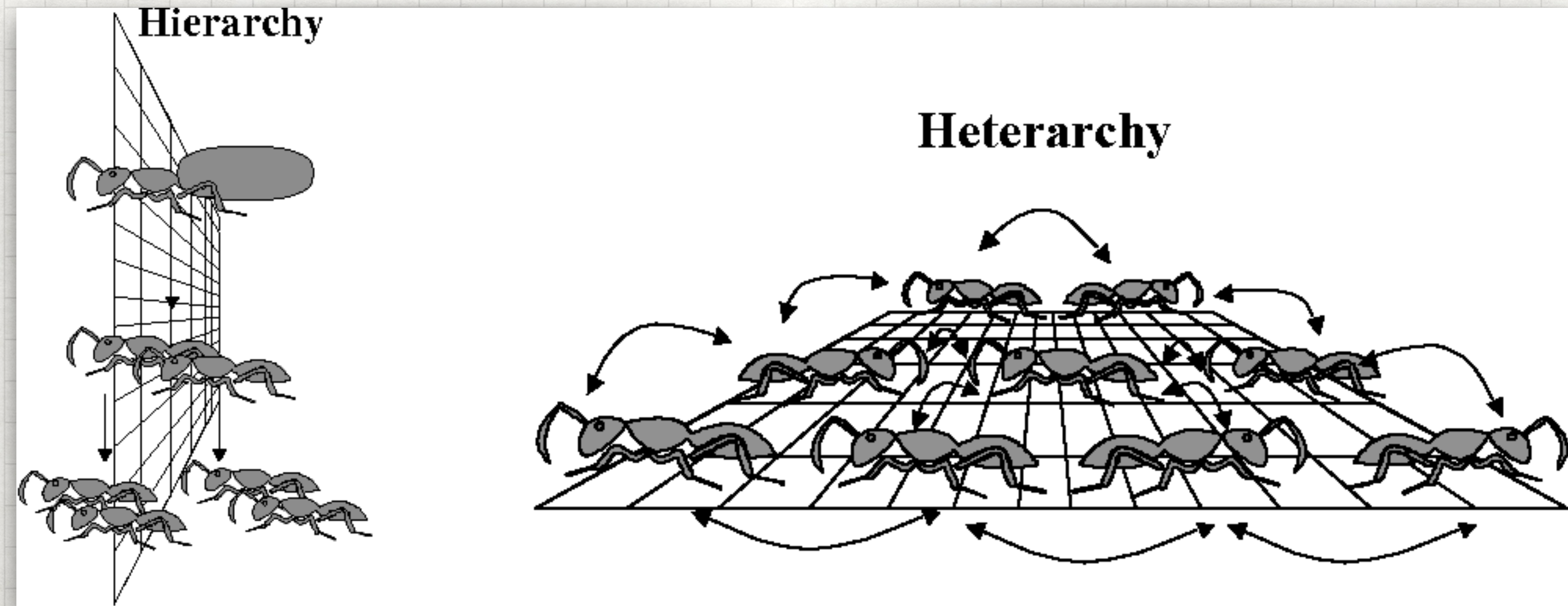
segregation

HETERARCHY

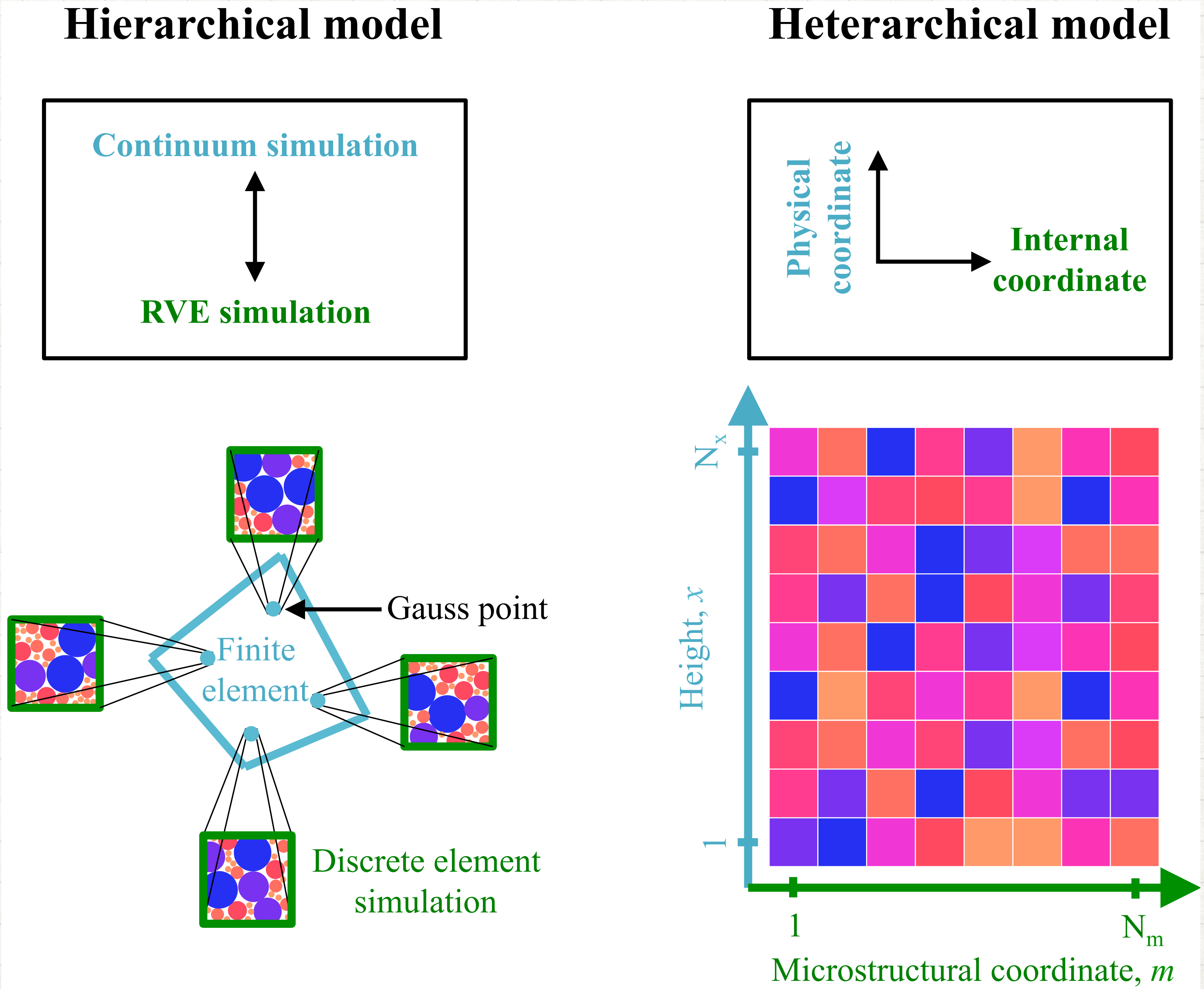
HETERARCHY

- WIKI DEFINITION

A **HETERARCHY** IS A SYSTEM OF ORGANIZATION WHERE THE ELEMENTS OF THE ORGANIZATION ARE UNRANKED (NON-HIERARCHICAL)...



MULTI-SCALE MODELS



Marks-Einav, GM, 2017

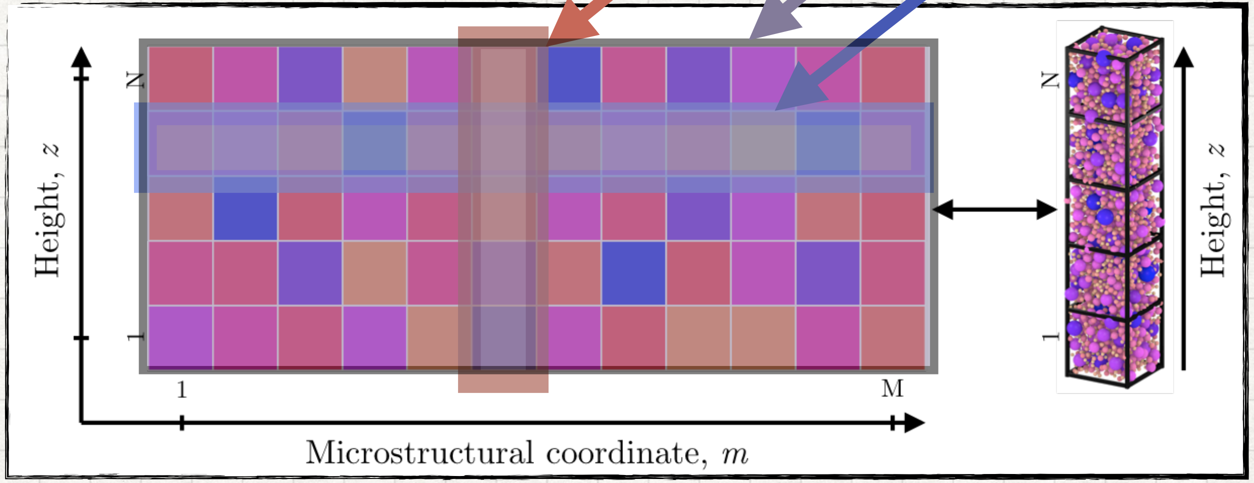
MOTIVATION FOR HETERARCHY

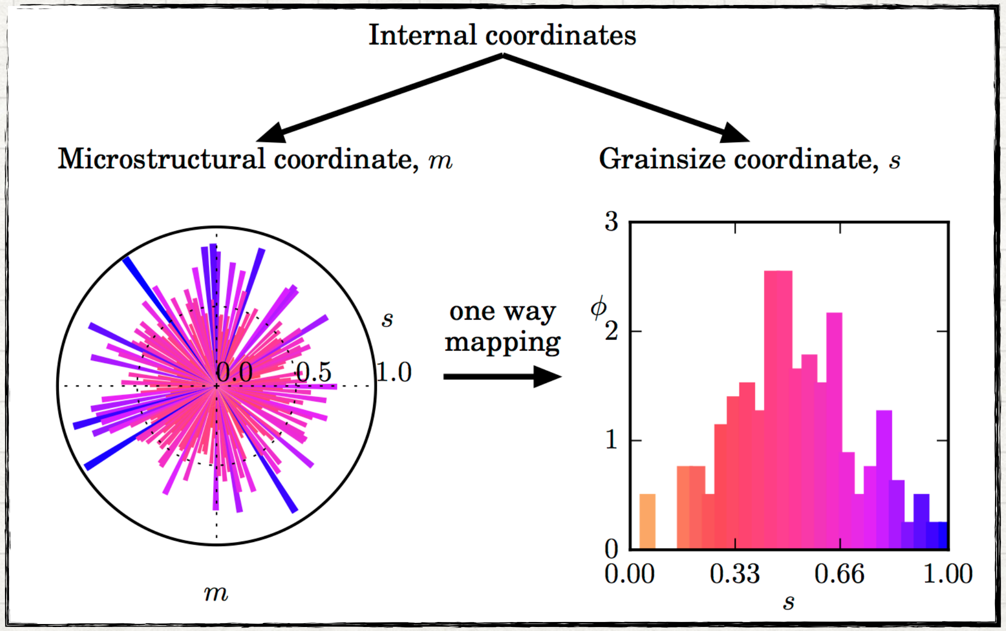
- INTERNAL COORDINATES

intimately related to open systems

General Heterarchy

develops even in close systems





SEGREGATION

SEGREGATION OF A SKIER

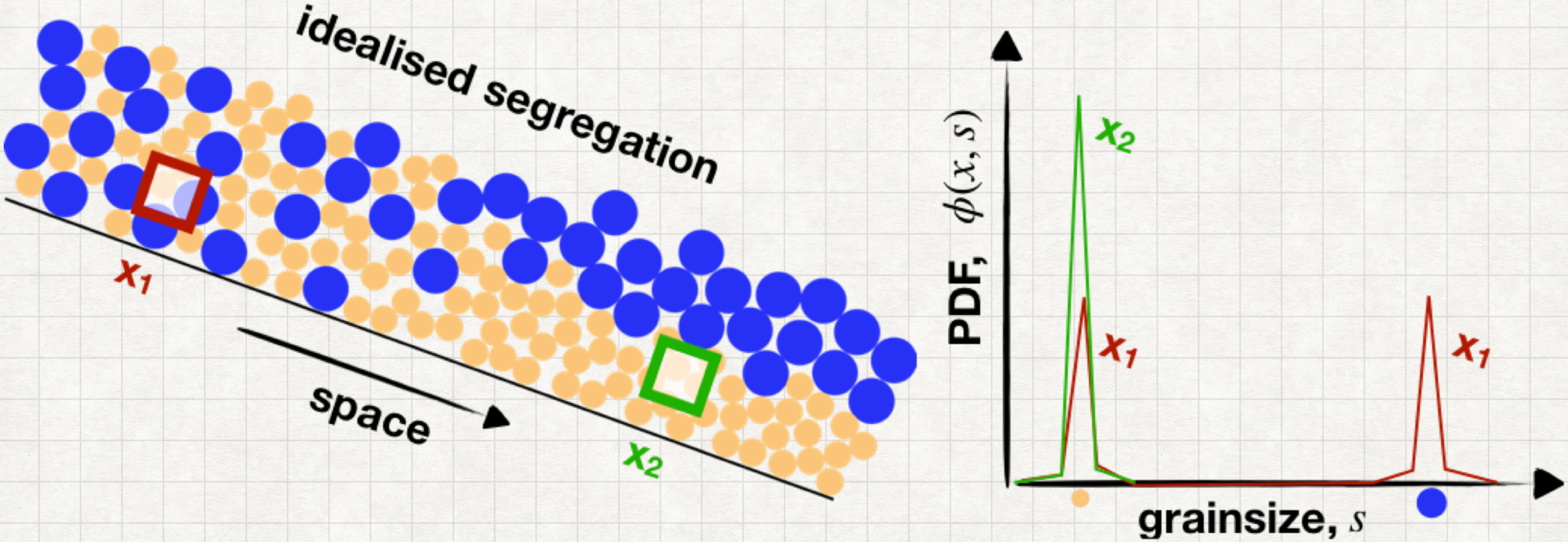
- AVALANCHE (WHISTLER, CANADA 01/11/2017)

SEGREGATION OF A SKIER

- ABS TWINBAG

SEGREGATION OF GRAINS

- IN ONE DIMENSION



SEGREGATION OF GRAINS

- HANDED BY VARDOULAKIS TO SOLVE HIS CHALLENGE

PROCEEDINGS OF THE ROYAL SOCIETY A

Proc. R. Soc. A (2005) **461**, 1447–1473
doi:10.1098/rspa.2004.1420
Published online 26 April 2005

A theory for particle size segregation in shallow granular free-surface flows

By J. M. N. T. Gray and A. R. Thornton

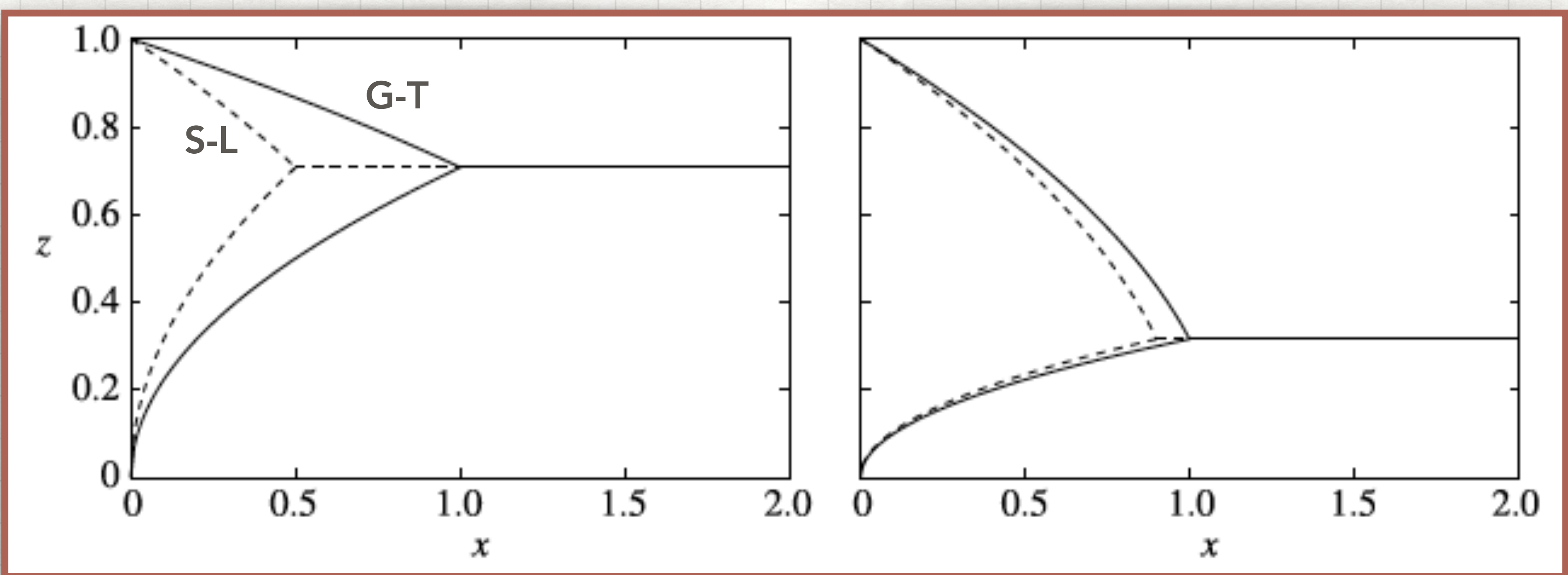
OR

J. Fluid Mech. (1988), *vol.* 189, *pp.* 311–335 311
Printed in Great Britain

Particle size segregation in inclined chute flow of dry cohesionless granular solids

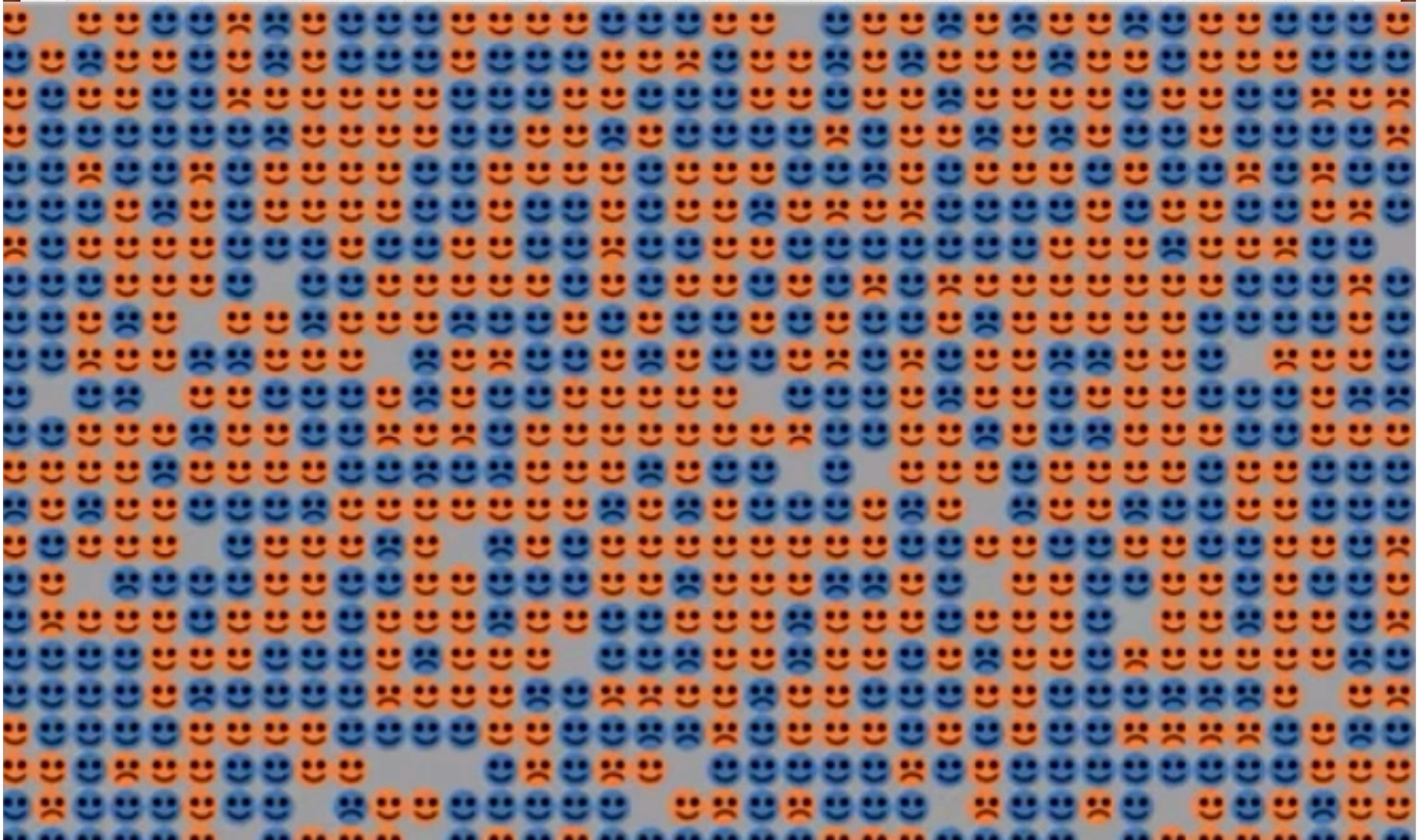
By S. B. Savage and C. K. K. Lun

?



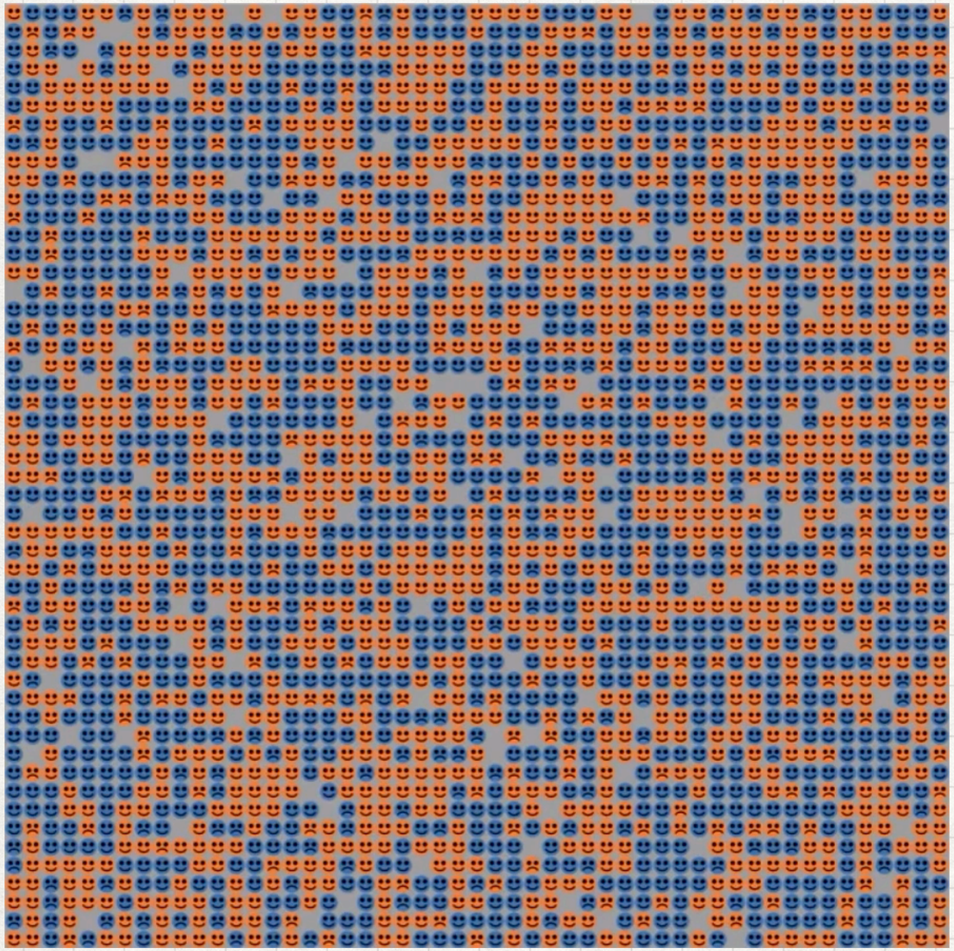
HUMAN SEGREGATION

- SCHELLING MODEL/AUTOMATA (1971)

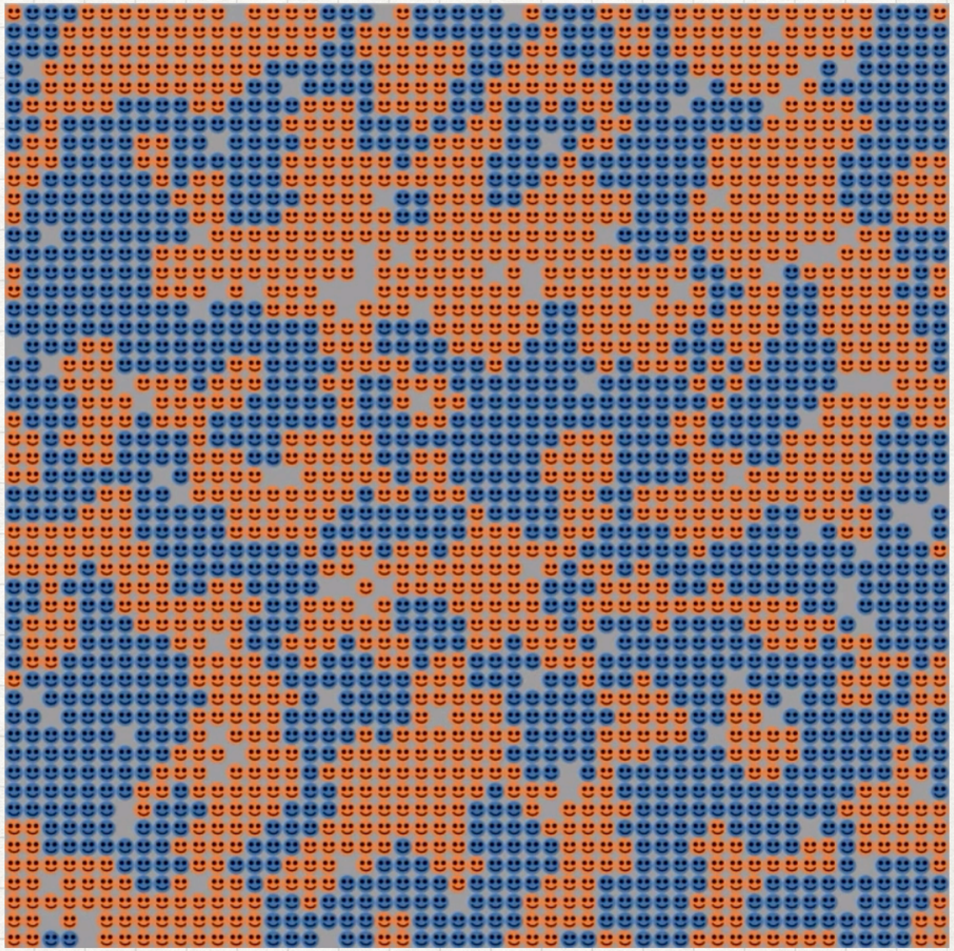


HUMAN SEGREGATION

- SCHELLING MODEL/AUTOMATA (1971)



At the start

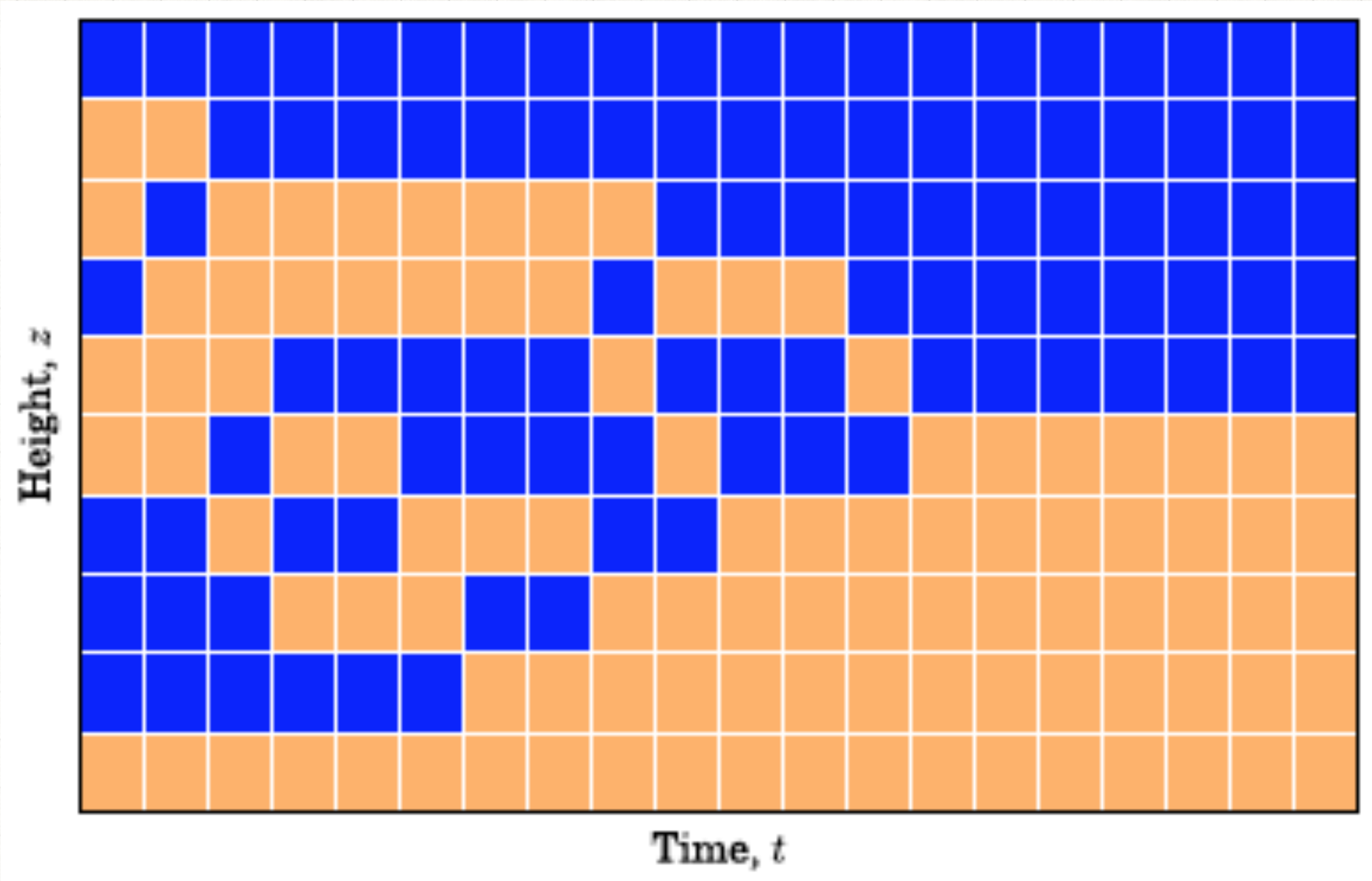


At the end

GRAIN SEGREGATION

- AUTOMATA IN 1D (STOCHASTIC LATTICE MODEL)

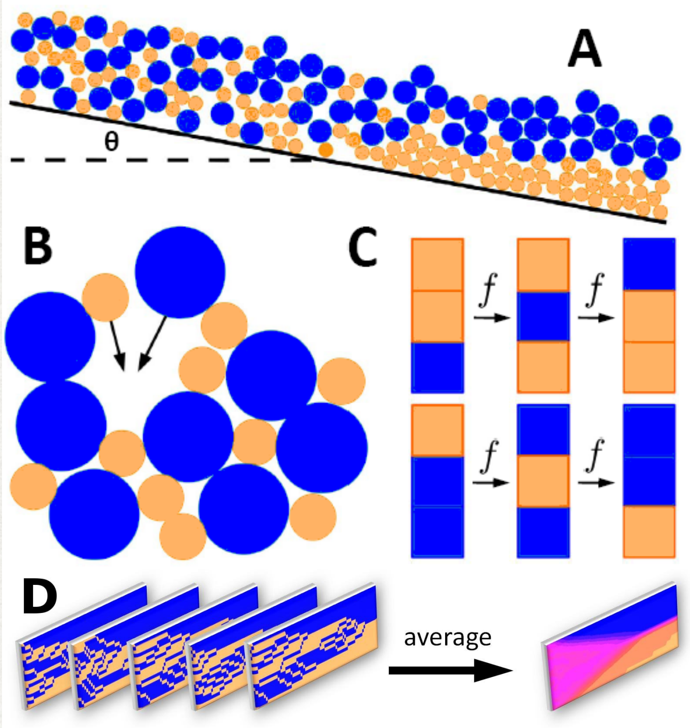
$$x(s, t + \Delta t) = x(s, t) + \hat{u}(x, s, t)\Delta t$$

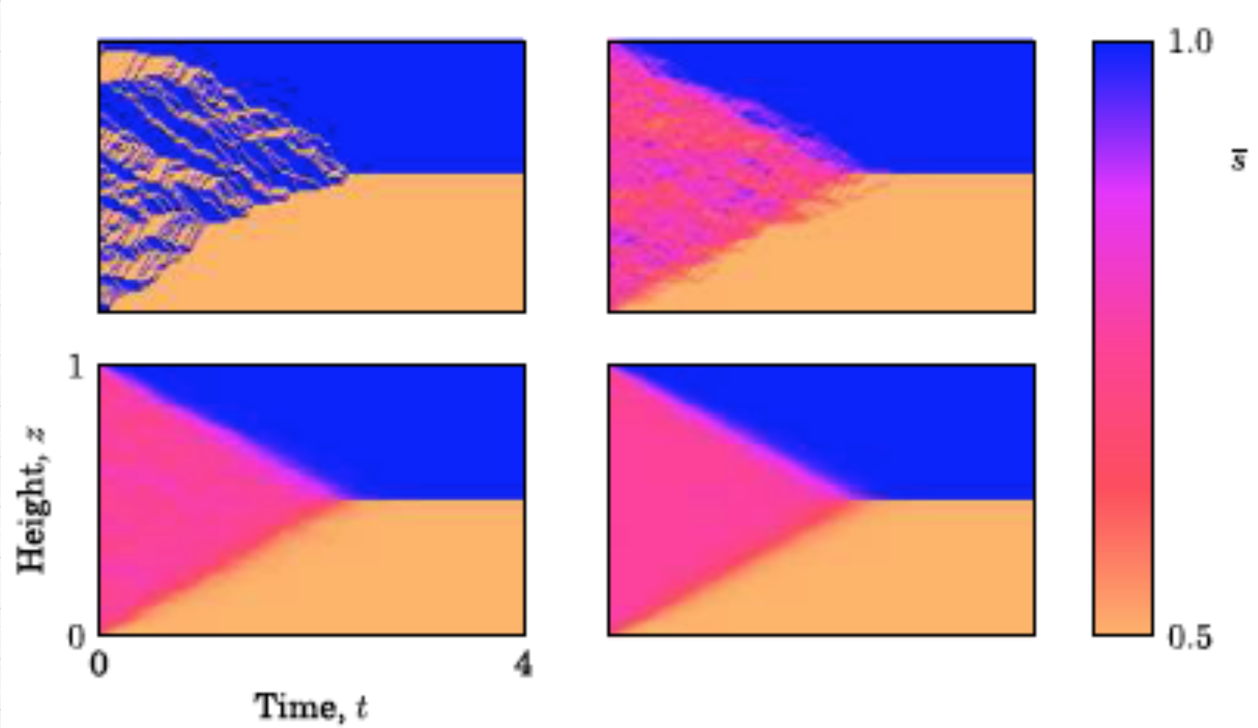


1) move cells with size-dependent frequency $f = \hat{u}(s)/\Delta z$, for now taken constant

GRAIN SEGREGATION

- AUTOMATA IN 1D (STOCHASTIC LATTICE MODEL)





$\hat{u} \propto f$ for now taken constant

2) average many simulations

GRAIN SEGREGATION

- AUTOMATA IN 1D (STOCHASTIC LATTICE MODEL)

Mass balance of stochastic grainsize fluxes (eg, bi-mixtures)

$$\left[\Phi_s(x,t+\Delta t)-\Phi_s(x,t)\right]\Delta x=\Big[\hat{u}_s(x+\tfrac{1}{2}\Delta x)\Phi_s(x+\Delta x,t)\big(1-\Phi_s(x,t)\big)$$

$$-\hat{u}_s(x-\tfrac{1}{2}\Delta x)\Phi_s(x,t)\big(1-\Phi_s(x-\Delta x,t)\big)\Big]\Delta t$$

Taylor expansion

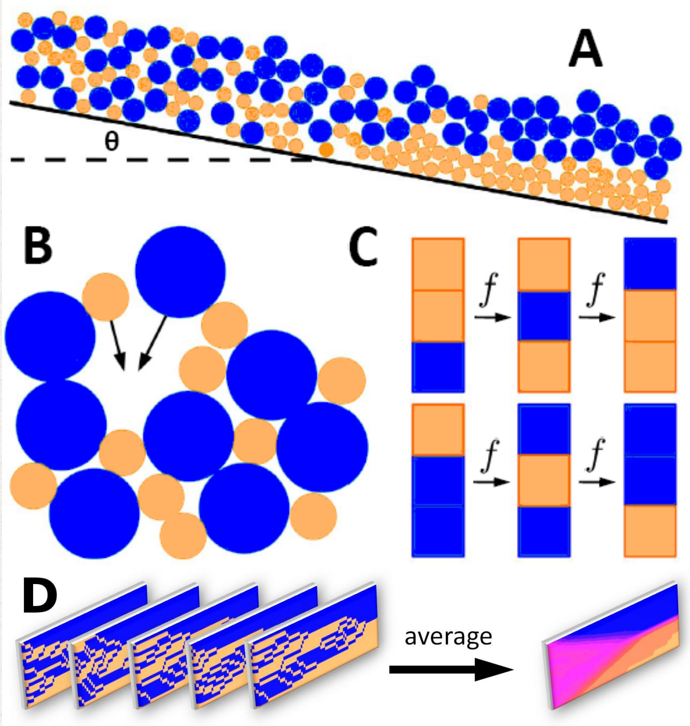
mass fraction of small particles

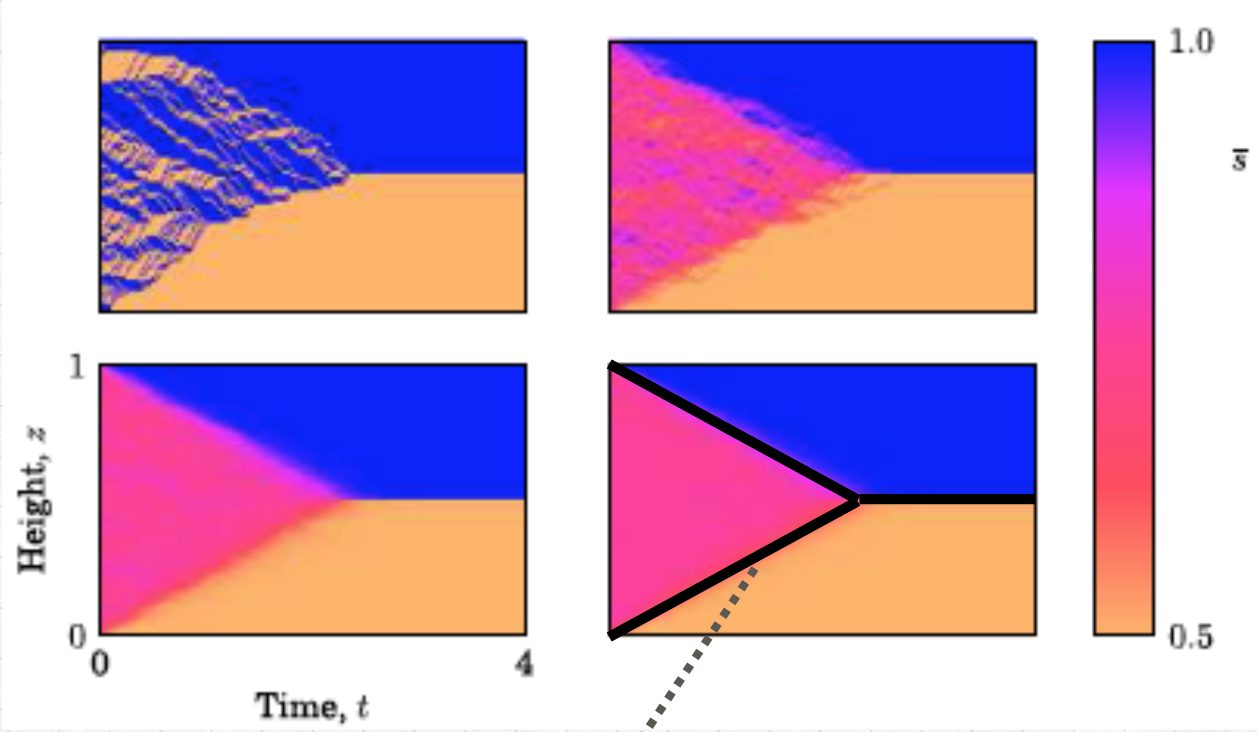
$$\partial_t\Phi_s=\partial_s\left[\hat{u}_s\Phi_s(1-\Phi_s)\right]$$

Marks-Einav, GM, 2011

GRAIN SEGREGATION

- AUTOMATA IN 1D (STOCHASTIC LATTICE MODEL)





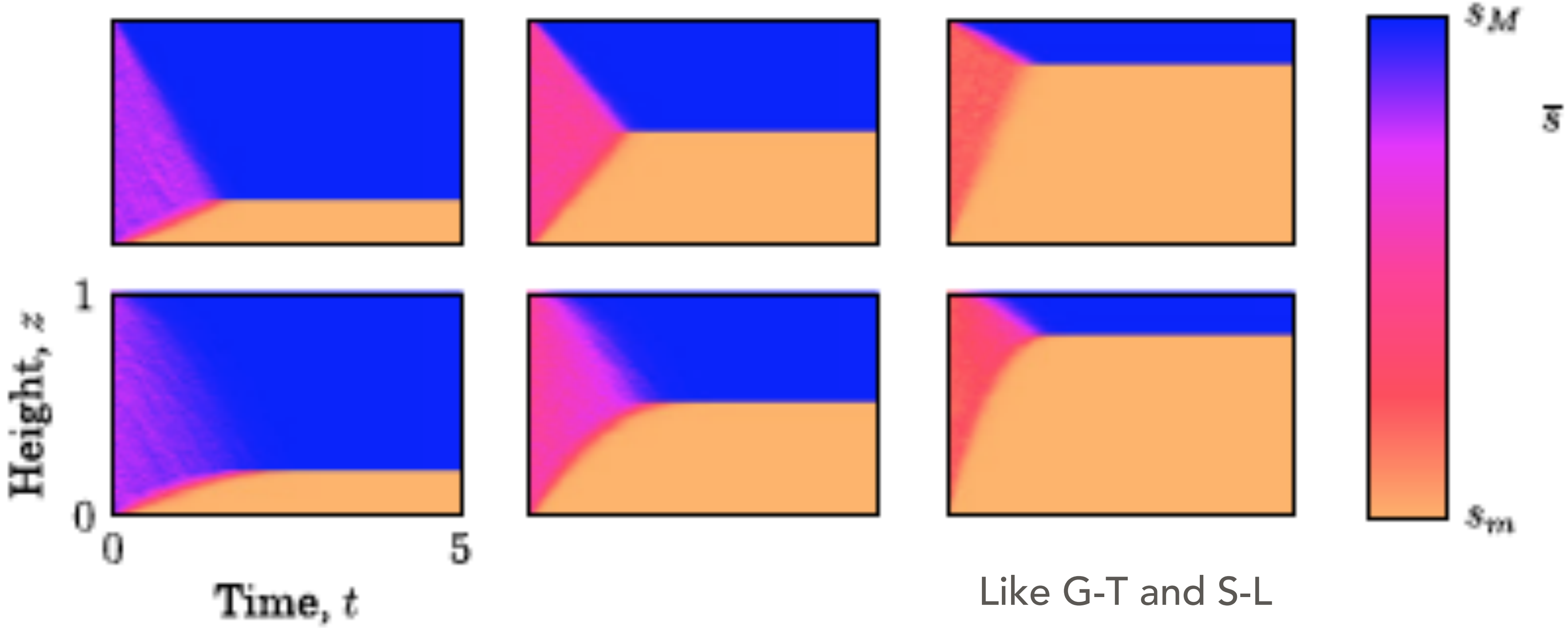
$$\partial_t \Phi_s = \partial_s \left[\hat{u}_s \Phi_s (1 - \Phi_s) \right]$$

2) average many simulations

$\hat{u}$ for now taken constant

GRAIN SEGREGATION

- AUTOMATA IN 1D (STOCHASTIC LATTICE MODEL)



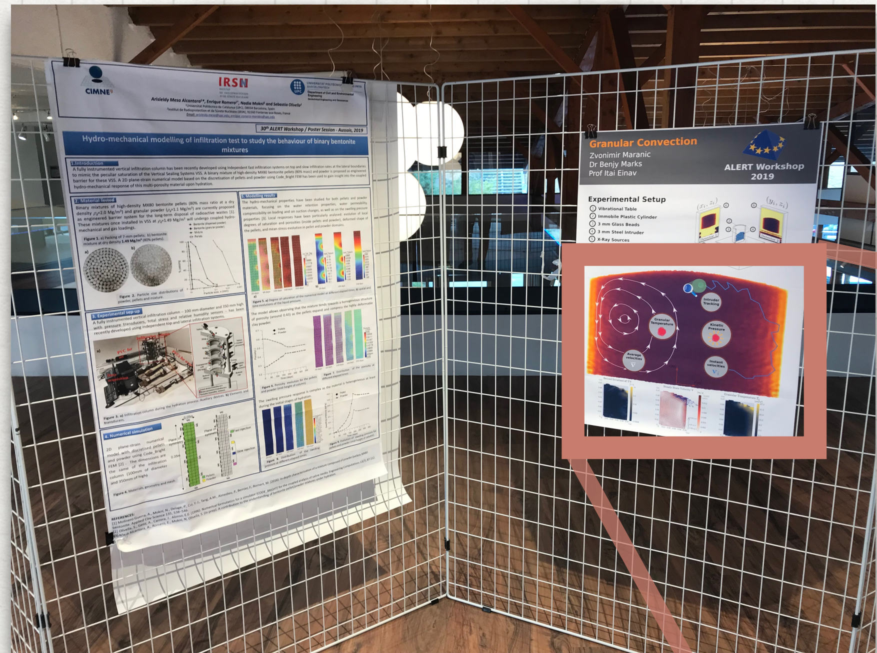
Like G-T and S-L

$$\partial_t \Phi_s = \partial_s \left[\hat{u}_s \Phi_s (1 - \Phi_s) \right]$$

$$\hat{u}_s \propto |\dot{\gamma}(z)| (s/\bar{s} - 1)$$

GRAIN SEGREGATION

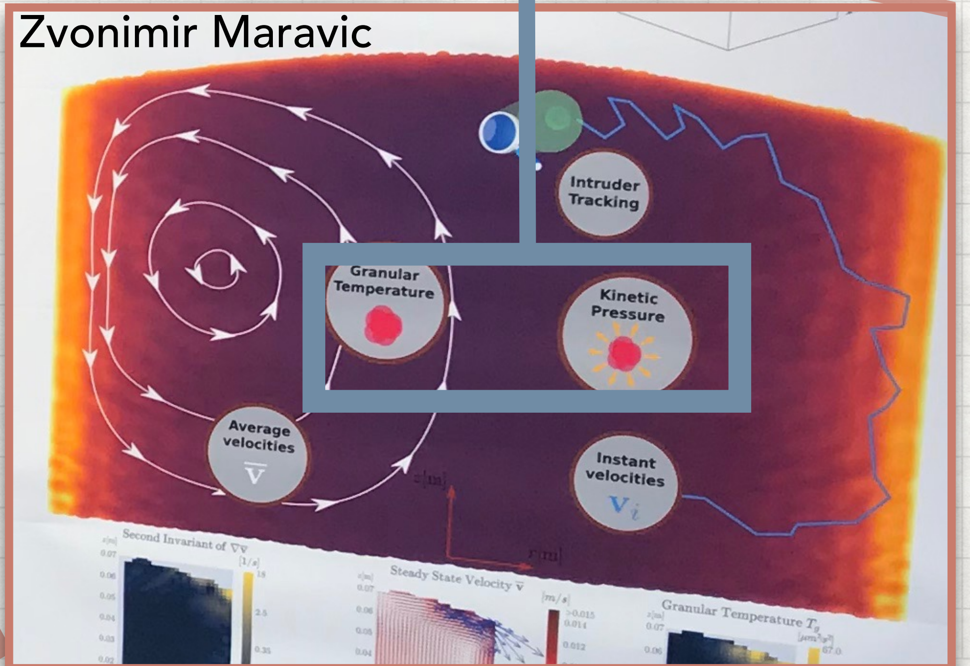
- POSTERSIZE DYNAMICS



Ebrahim Alaei

$$\hat{u}_s \propto \nabla p_k \propto \nabla (T_g^2)$$

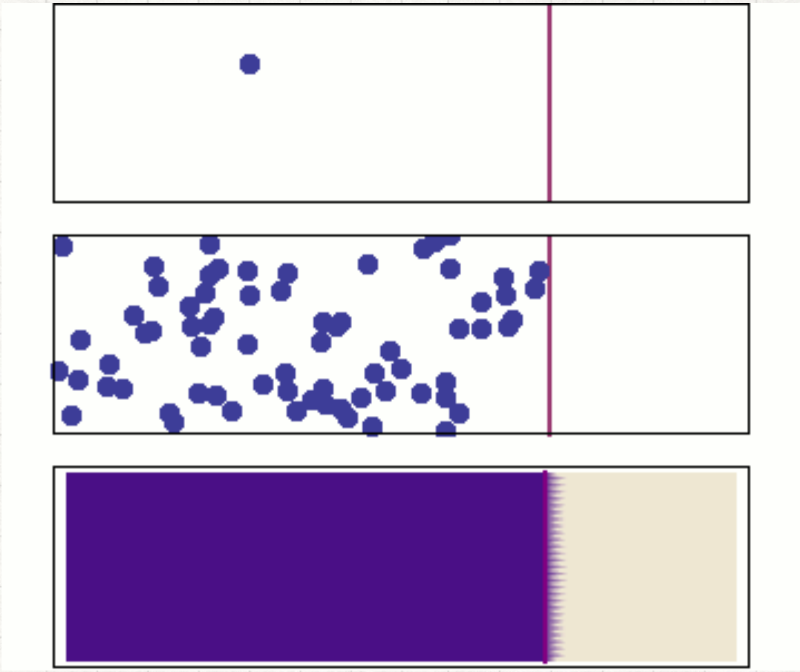
Zvonimir Maravic



MIXING

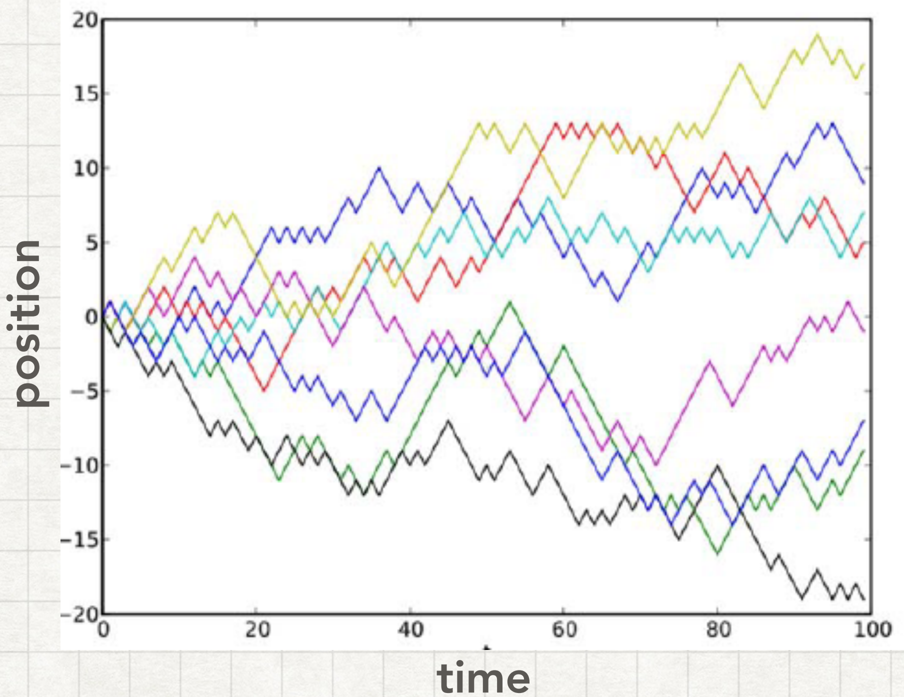
MIXING

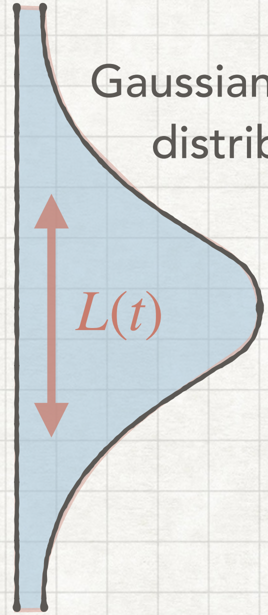
- DIFFUSION IN FLUIDS



MIXING

- STOCHASTIC BROWNIAN MOTION (BROWN, 1827)





1D	β=2
2D	β=4
3D	β=6

Averaged position $\langle \mathbf{x_p(t) - x_p(0)} \rangle \approx \mathbf{0}$

Averaged squared position $L^2(t) = \langle \left(\mathbf{x_p(t) - x_p(0)} \right)^2 \rangle = \beta D t$

Diffusivity in m^2/s

MIXING

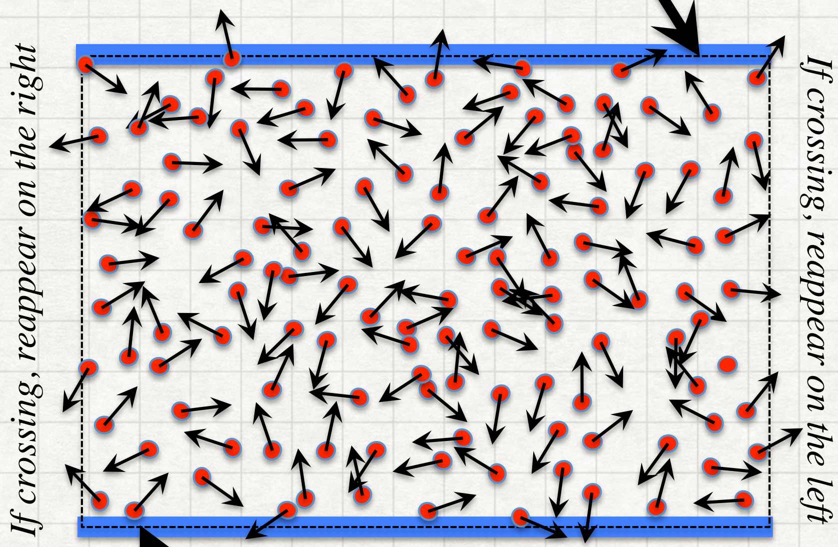
- ANALOGOUS CONTINUUM FICK LAW

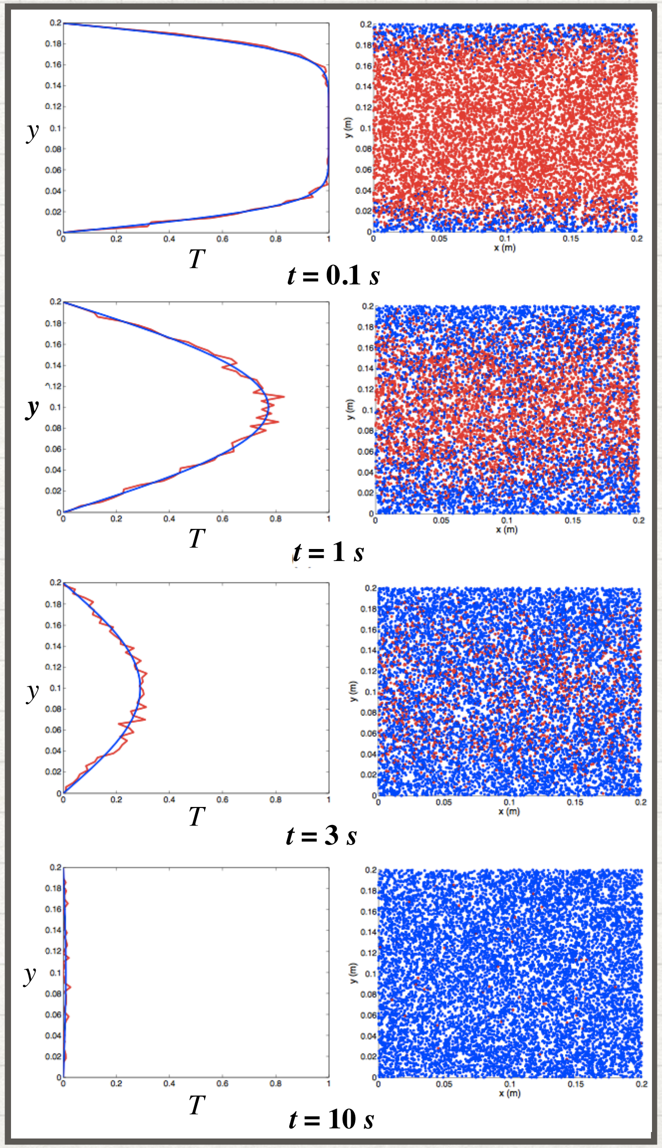
$$\partial_t c(x) = D\,\nabla^2 c(x)$$

* For derivation from Brownian motion see a paper by one called Einstein (1905)

MIXING

- FICK LAW VS STOCHASTIC SIMULATION (CHECK AT HOME)

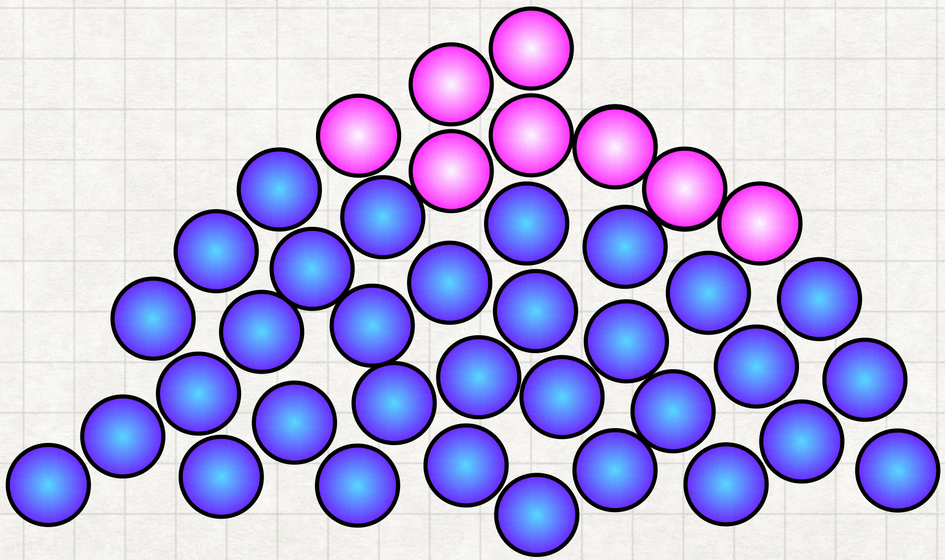




$\beta = 4 \rightarrow v_p = 2\sqrt{\frac{D}{t}}$

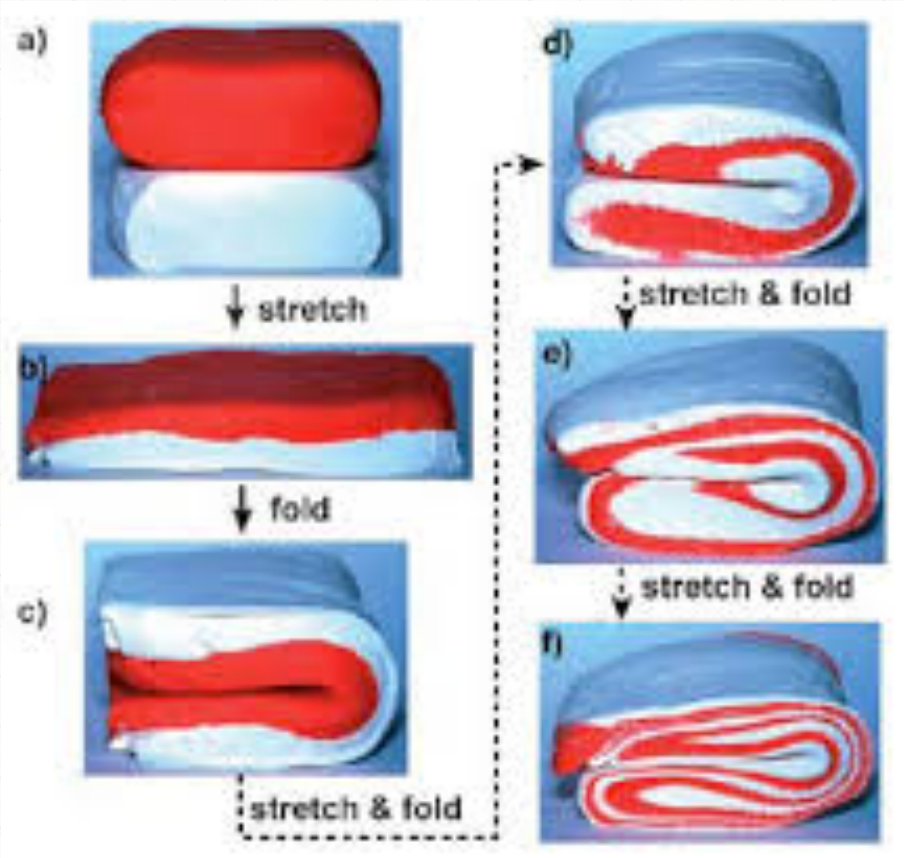
Griffani et al., Phys Fluids, 2013

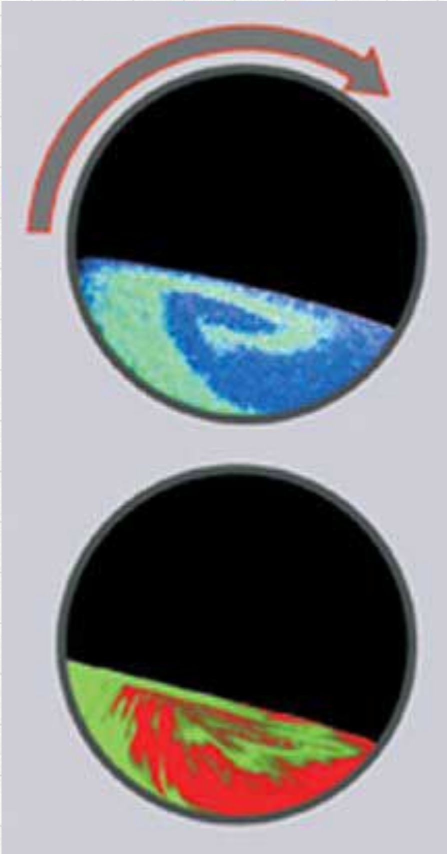
Wait, wait, and wait, but unlike atoms, why not mixing?



MIXING OF GRAINS

- CHAOTIC ADVECTION

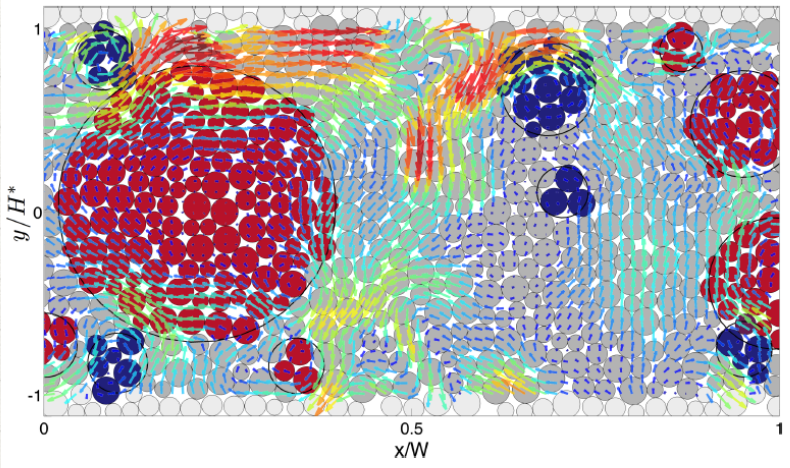


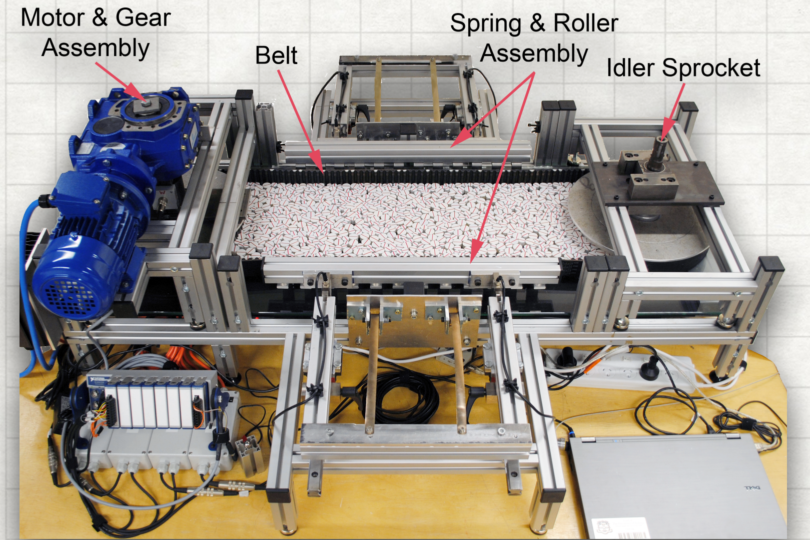


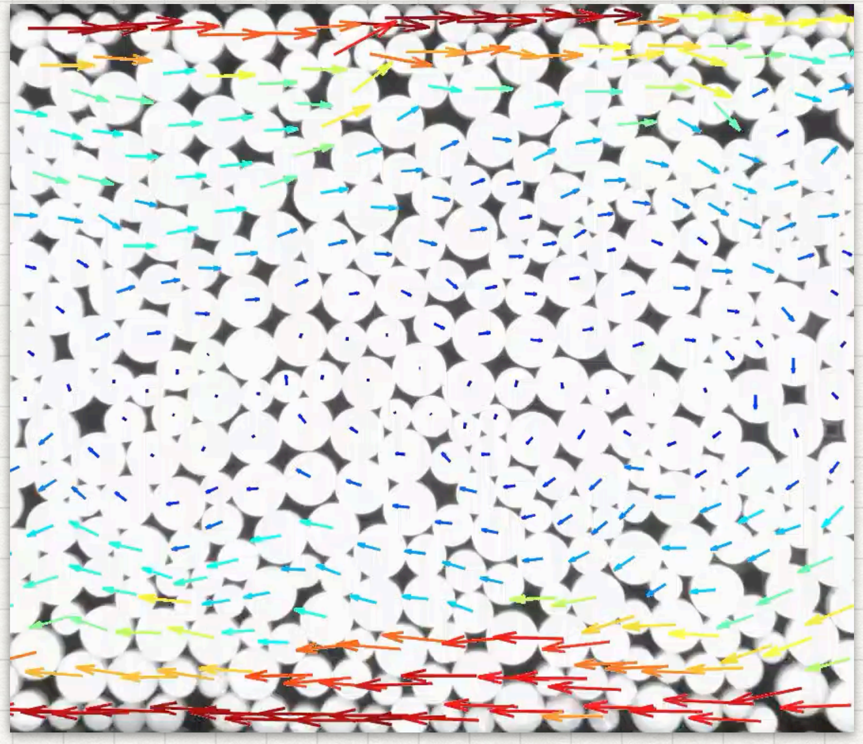
Explainable by momentum and mass conservation equations for bulk motion...

MIXING OF GRAINS

- SHEAR INDUCED DIFFUSION







Inertial number $I = \dot{\gamma} d \sqrt{\frac{\rho}{p}}$

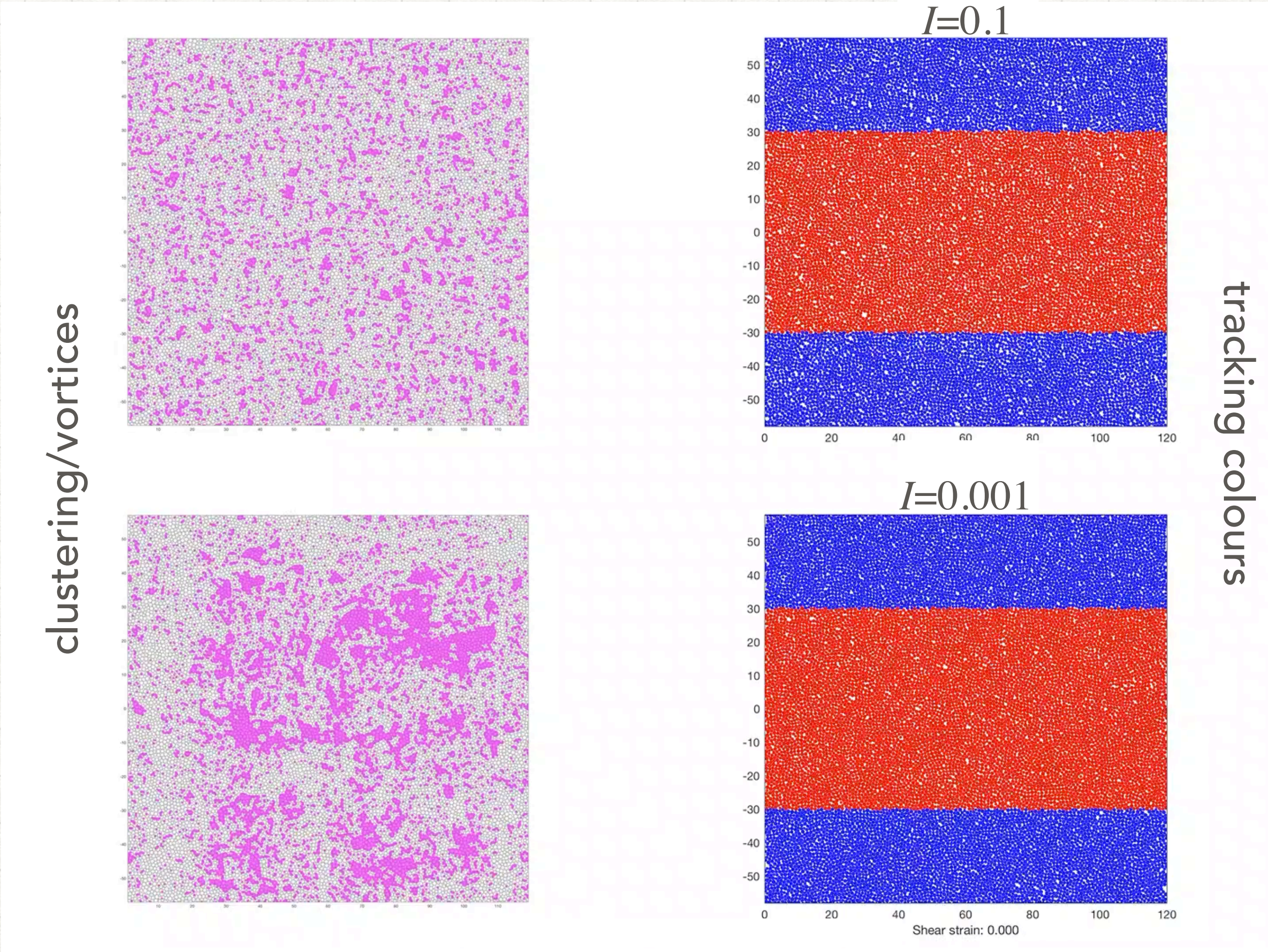
Vortex size $l_{vortex} \propto \frac{d}{\sqrt{I}}$

Stadium Shear Device

Miller-Rognon-Einav, PRL 2013

MIXING OF GRAINS

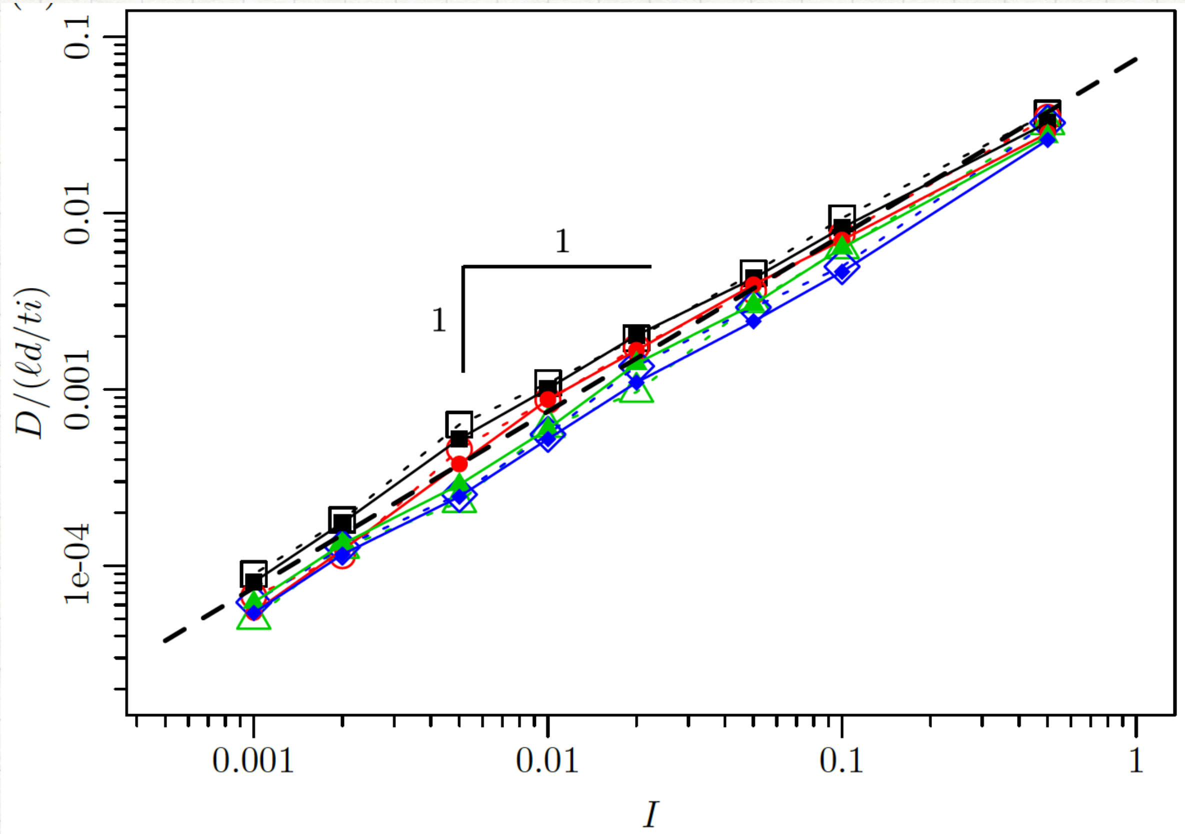
- SHEAR INDUCED DIFFUSION



Discrete Element Method simulations

MIXING OF GRAINS

- SHEAR INDUCED DIFFUSION



$$D \propto [l_{vortex} d]\dot{\gamma} \propto \frac{\dot{\gamma} d^2}{\sqrt{I}}$$

Kharel-Rognon, PRL 2017

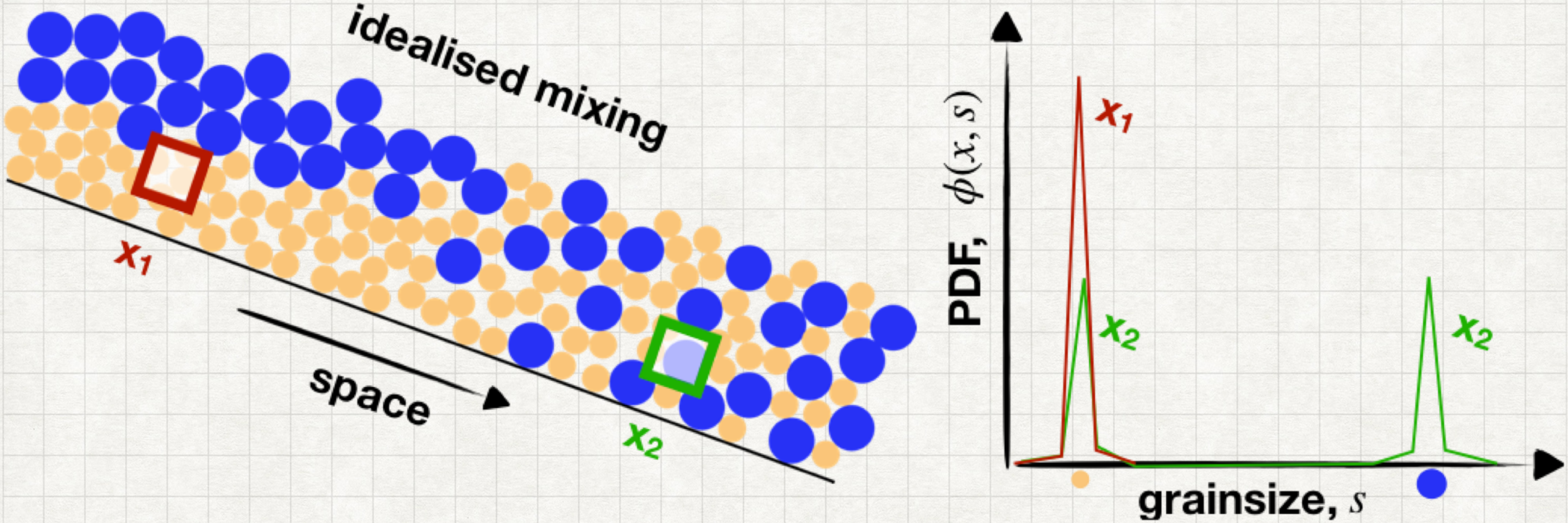
MIXING OF GRAINS

- GRAINSIZE DYNAMICS

$$\partial_t \left[\phi(\mathbf{x}, \mathbf{s}, \mathbf{t})\right] = \nabla^2 \left[D\phi\right]$$

MIXING OF GRAINS

- IN ONE DIMENSION

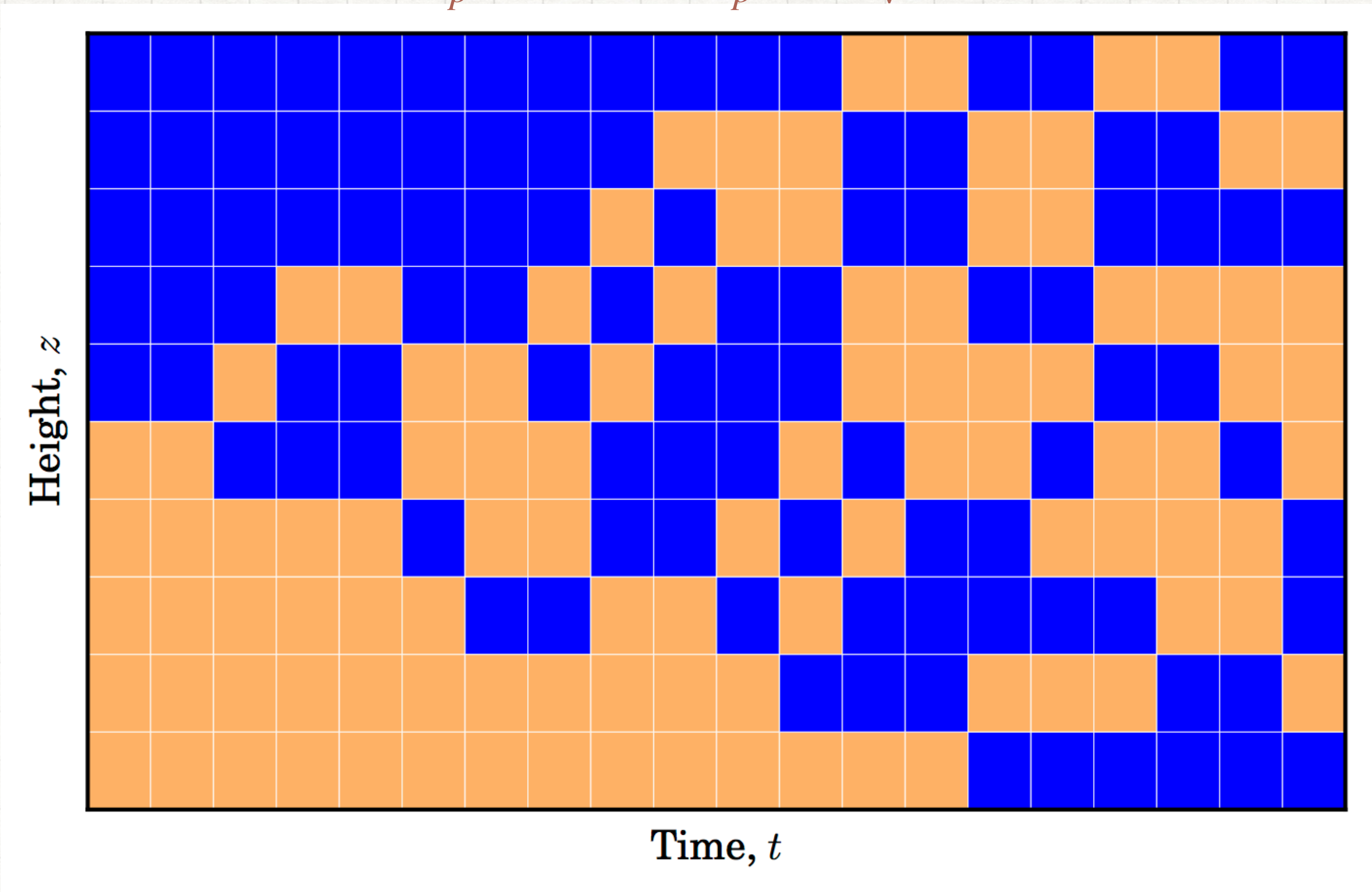


MIXING OF GRAINS

- STOCHASTIC LATTICE MODEL

$\beta = 2$

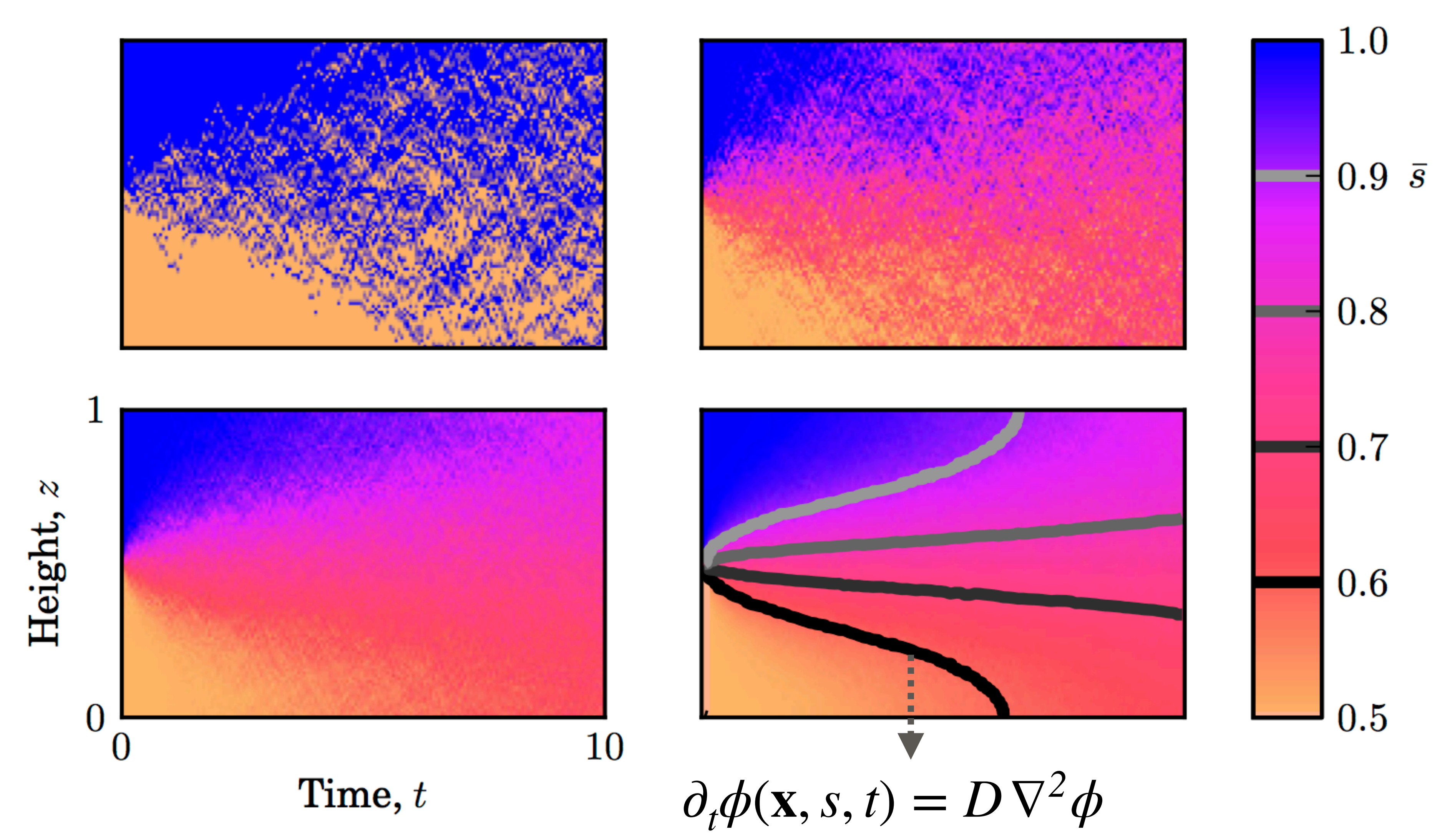
$$x_p(t + \Delta t) = x_p(t) \pm \sqrt{2D\Delta t}$$



1) Swap cells at a (stochastic) frequency determined by 1D diffusivity *D*

MIXING OF GRAINS

- STOCHASTIC LATTICE MODEL

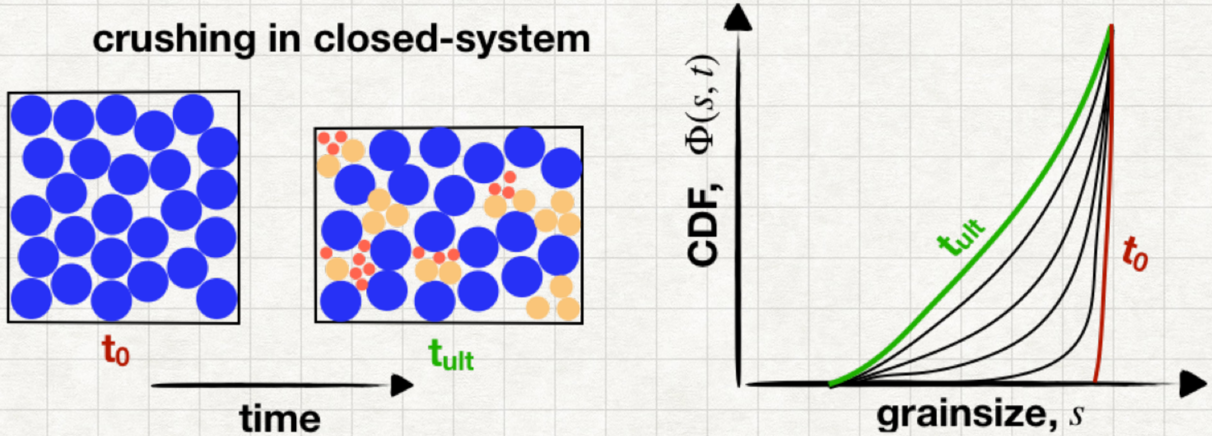


2) Average many simulations

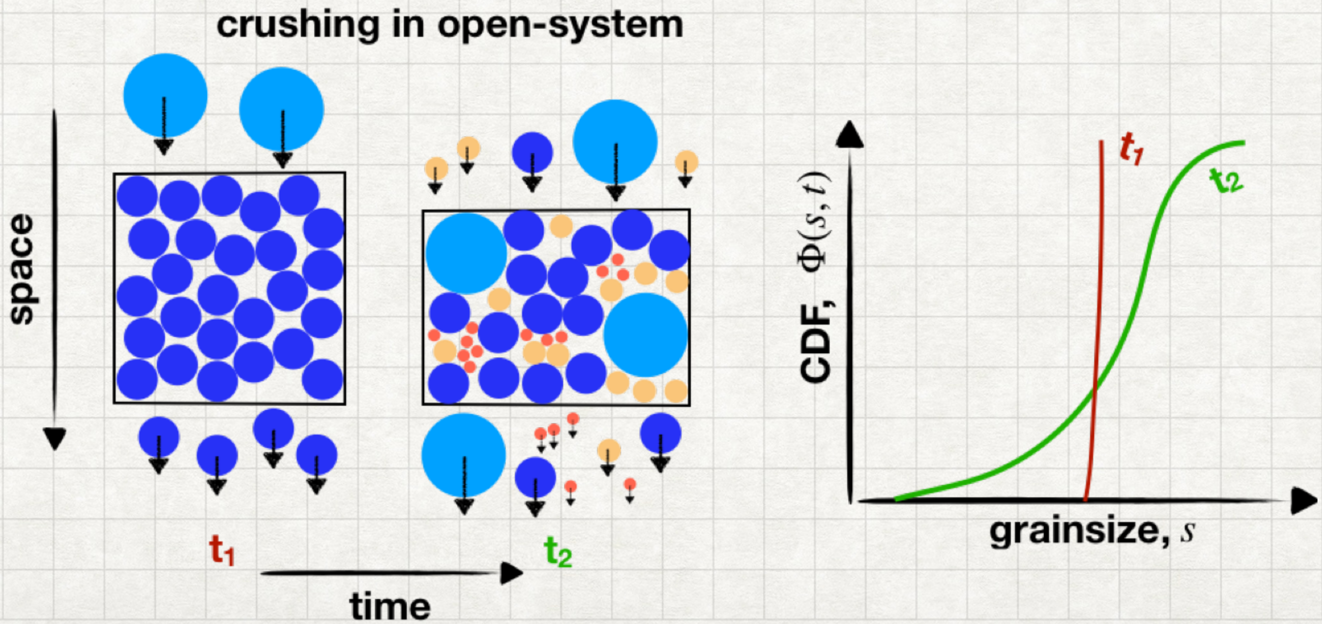
CRUSHING

GRAIN CRUSHING

- OPEN VS CLOSED SYSTEMS



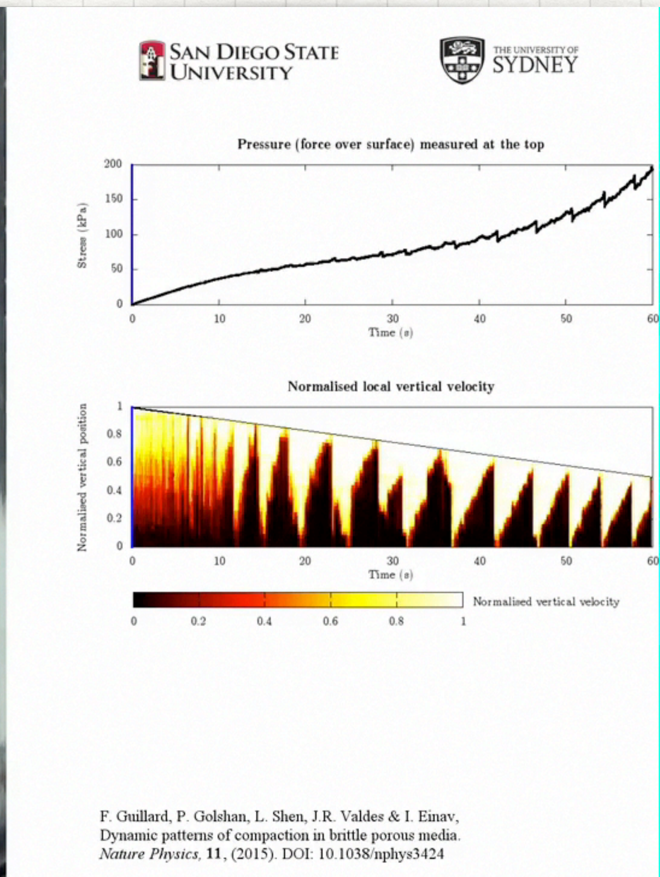
Where Breakage Mechanics work...

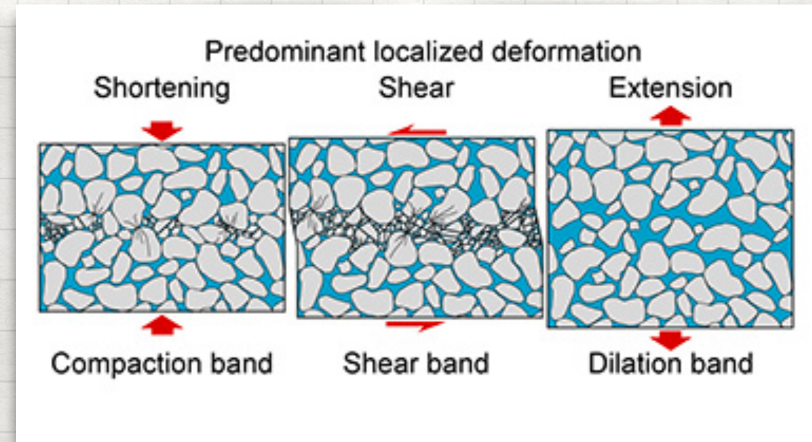


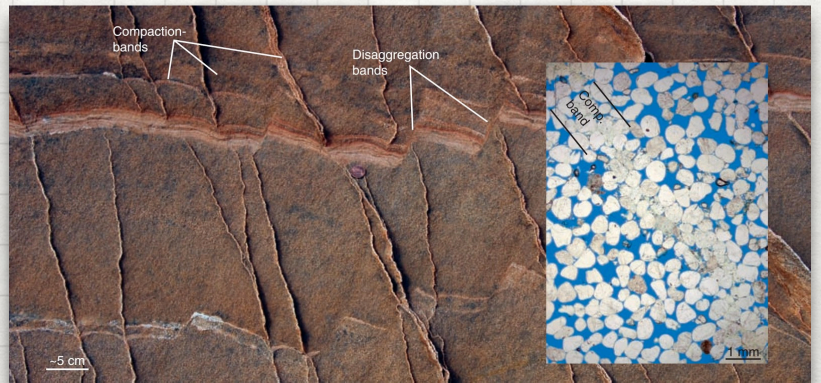
Where Breakage Mechanics does not work...

GRAIN CRUSHING

- CLOSED-SYSTEM TREATMENT SUFFICIENT







GRAIN CRUSHING

- OPEN-SYSTEM APPROACH REQUIRED

CRUSHING OF A SINGLE PARTICLE

- WEIBULL'S WEAKEST LINK THEORY (FOR CHAINS)

critical failure force = ?

$F_{cr} = ?$

critical failure force = ?

$F_{cr} = ?$

Survival probability of a chain with N links

$$P_s(F_{cr}, N) = \exp\left[-N\left(\frac{F_{cr}}{F_{\text{ref}}}\right)^w\right]$$

for a given survival probability

$$F_{cr} \propto N^{1/w}$$

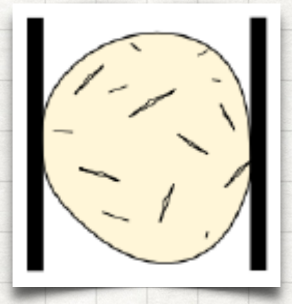
Weibull (1931)

CRUSHING OF A SINGLE PARTICLE

- WEIBULL'S WEAKEST LINK THEORY (FOR PARTICLES)

critical crushing force = ?

$$F_{cr} = ?$$



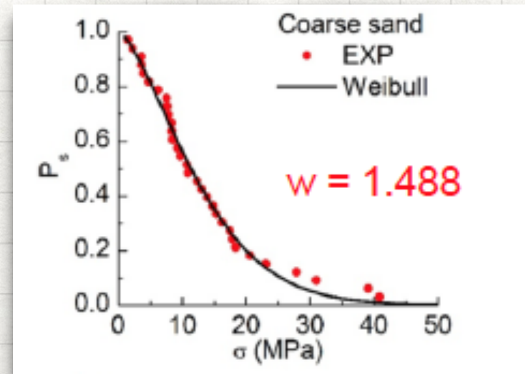
critical crushing force = ?

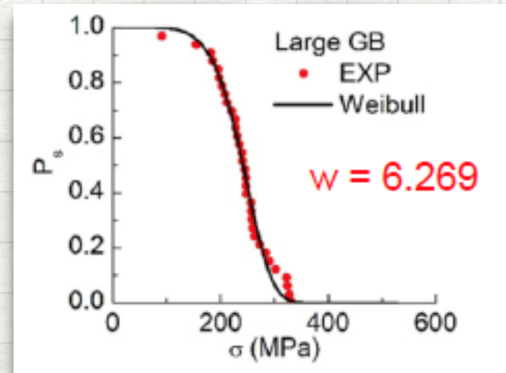
$$F_{cr} = ?$$

Survival probability of a sphere with volume V_p

$$P_s(\sigma_{pc}, V_p) = \exp\left[-\frac{V_p}{V_{\text{ref}}}\left(\frac{\sigma_{pc}}{\sigma_{\text{ref}}}\right)^w\right]$$

Sand particles (quite heterogeneous)





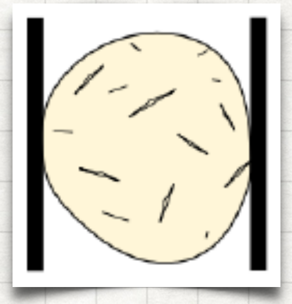
Glass Bead (quite homogeneous)

CRUSHING OF A SINGLE PARTICLE

- WEIBULL'S WEAKEST LINK THEORY (FOR PARTICLES)

critical crushing force = ?

$F_{cr} = ?$



critical crushing force = ?

$F_{cr} = ?$

Survival probability of a sphere with volume V_p

$$P_s(\sigma_{pc}, V_p) = \exp\left[-\frac{V_p}{V_{\text{ref}}}\left(\frac{\sigma_{pc}}{\sigma_{\text{ref}}}\right)^w\right]$$

for a given survival probability

$$\sigma_{pc} \propto V_p^{-1/w} = d_p^{-3/w}$$

Mogi (1962); McDowell-Bolton (1996)

$$F_{cr} \approx \sigma_{pc} d_p^2 \propto d_p^{2-\frac{3}{w}}$$

CRUSHING OF A SINGLE PARTICLE

- SIZE DEPENDENCE

$F_{cr} = ?$  $F_{cr} = ?$

$F_{cr} \propto d_p^{3/2}$	Centre crack $(w = 6)$	$\sigma_{pc} \propto d_p^{-1/2}$
$F_{cr} \propto d_p^{2-\frac{3}{w}}$	Weibull scaling	$\sigma_{pc} \propto d_p^{-3/w}$
$F_{cr} \propto d_p^{1/2}$	$(w = 2)$ Contact crack	$\sigma_{pc} \propto d_p^{-3/2}$

Zhang-Buscarnera-Einav, Géotechnique 2016

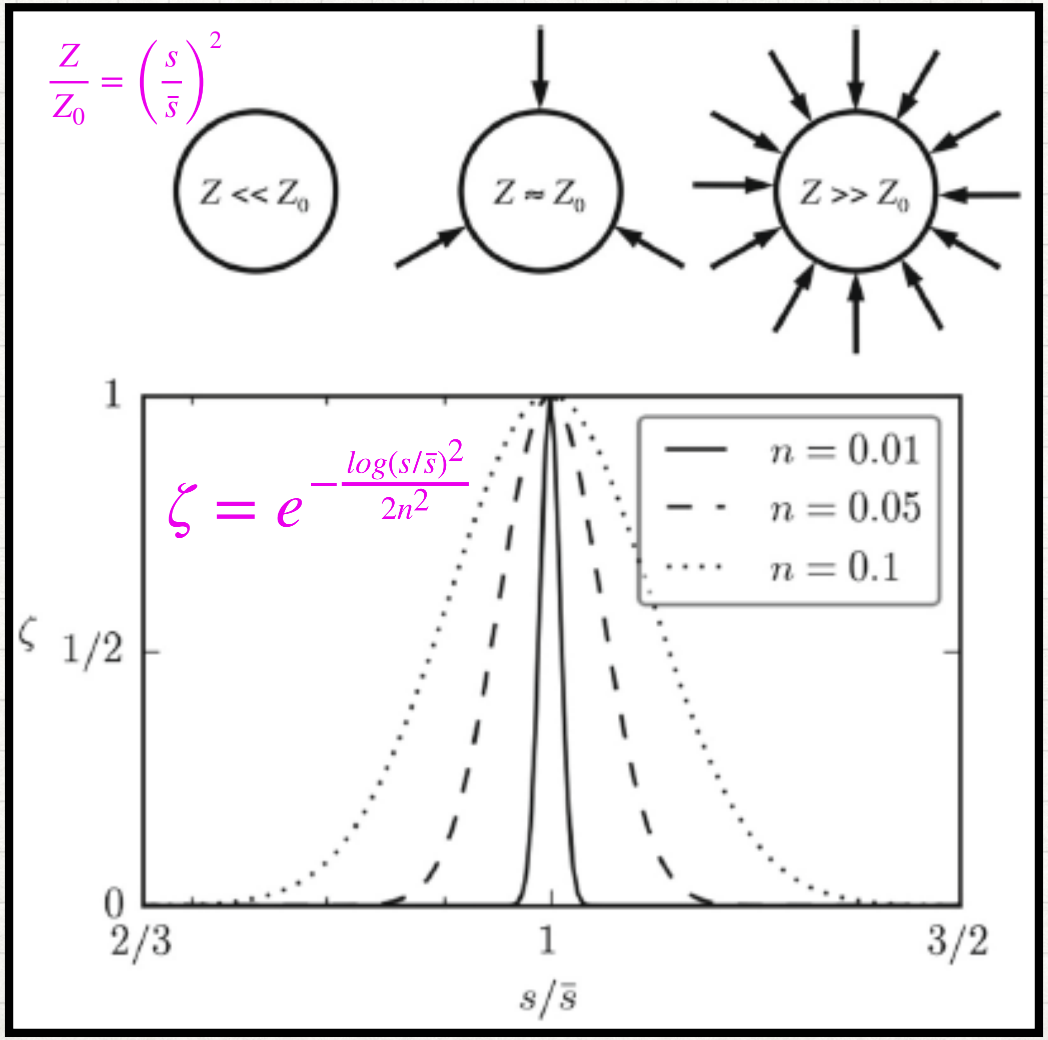
CRUSHING OF A SINGLE PARTICLE

- CUSHIONING EFFECT

CRUSHING OF A SINGLE PARTICLE

- CUSHIONING FACTOR ζ

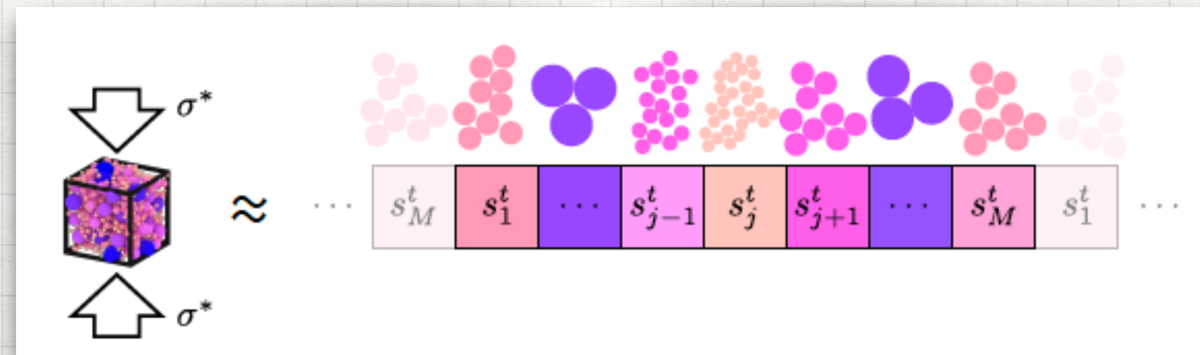
max tensile stress → $\sigma_t(s, \bar{s}) \propto \zeta(s, \bar{s})\sigma$ ← bulk stress



Marks-Einav, GM, 2017

CRUSHING OF GRANULAR ASSEMBLY

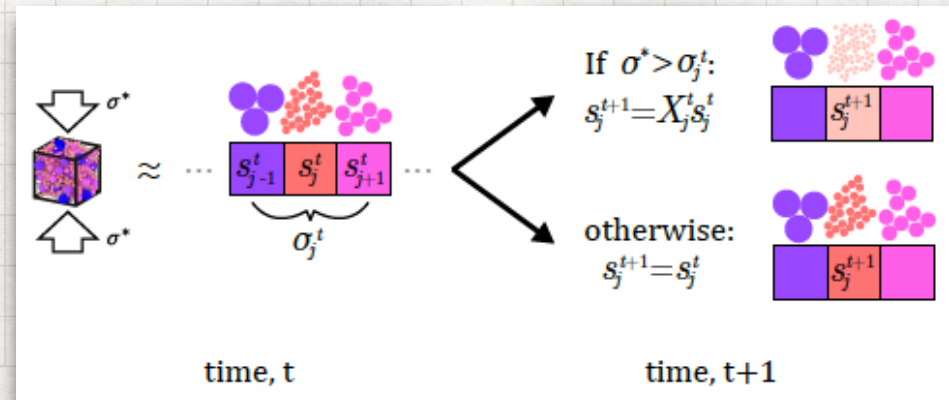
- STOCHASTIC LATTICE MODEL



Crushing criterion
$$s_j^{t+1} = \begin{cases} X_j^t s_j^t & \text{if } \sigma^* \geq \sigma_j^t\left(s_{j-1}, s_j, s_{j+1}\right) = \left(\frac{s_j}{s_{\max}}\right)^{-3/w} \exp\left(\frac{\log\left(s_j^t/\bar{s}_j^t\right)^2}{2n^2}\right) \\ s_j^t & \text{otherwise,} \end{cases}$$

size dependence   cushioning

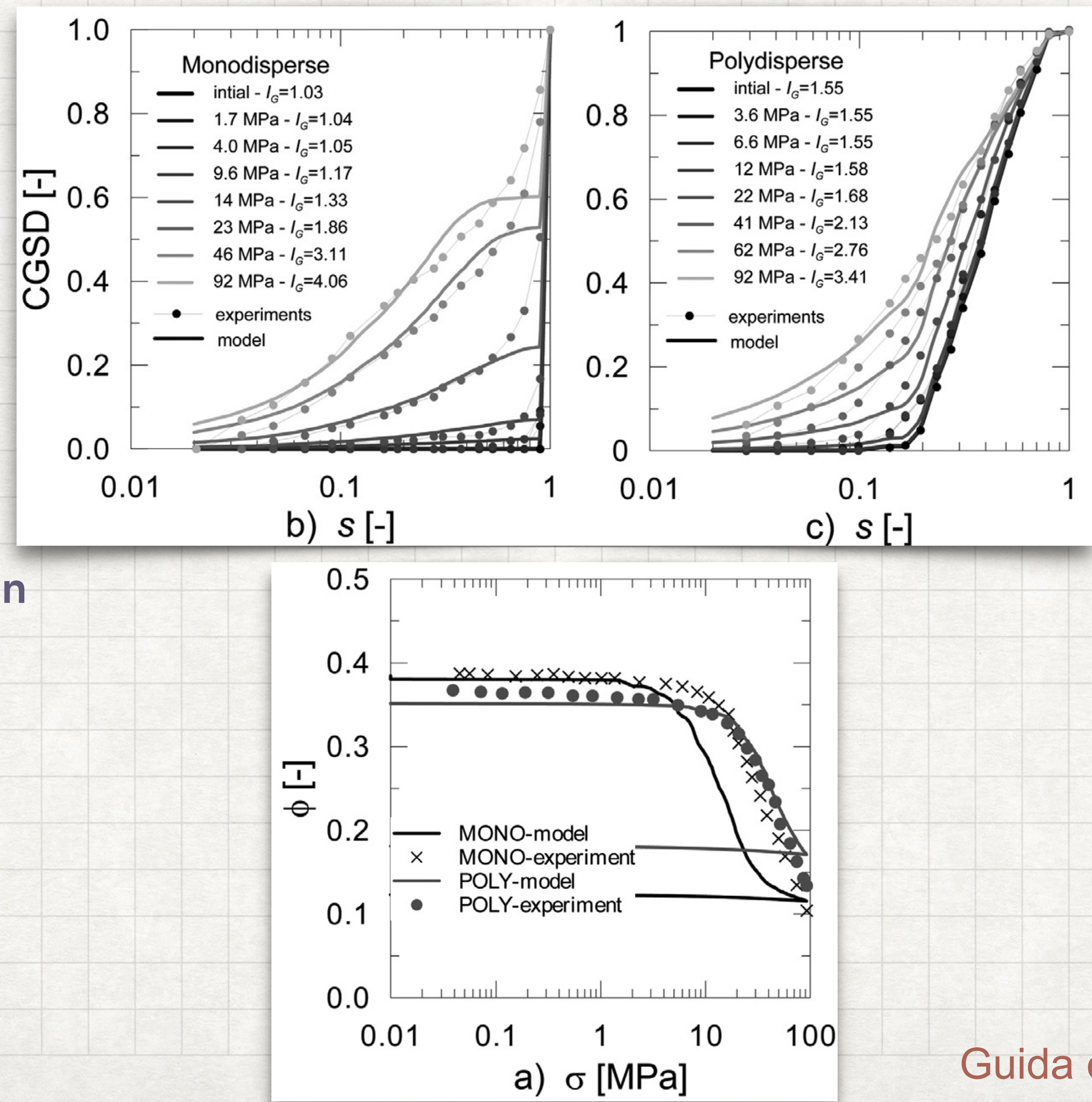
Fragment distribution
$$\rho_X(x) = \frac{\frac{\kappa}{\lambda}\left(\frac{x}{\lambda}\right)^{\kappa-1} e^{-(x/\lambda)^\kappa}}{1 - e^{-\lambda^{-\kappa}}}$$



Marks-Einav, GM, 2017

CRUSHING OF GRANULAR ASSEMBLY

- K0 COMPRESSION WITH POROSITY REDUCTION

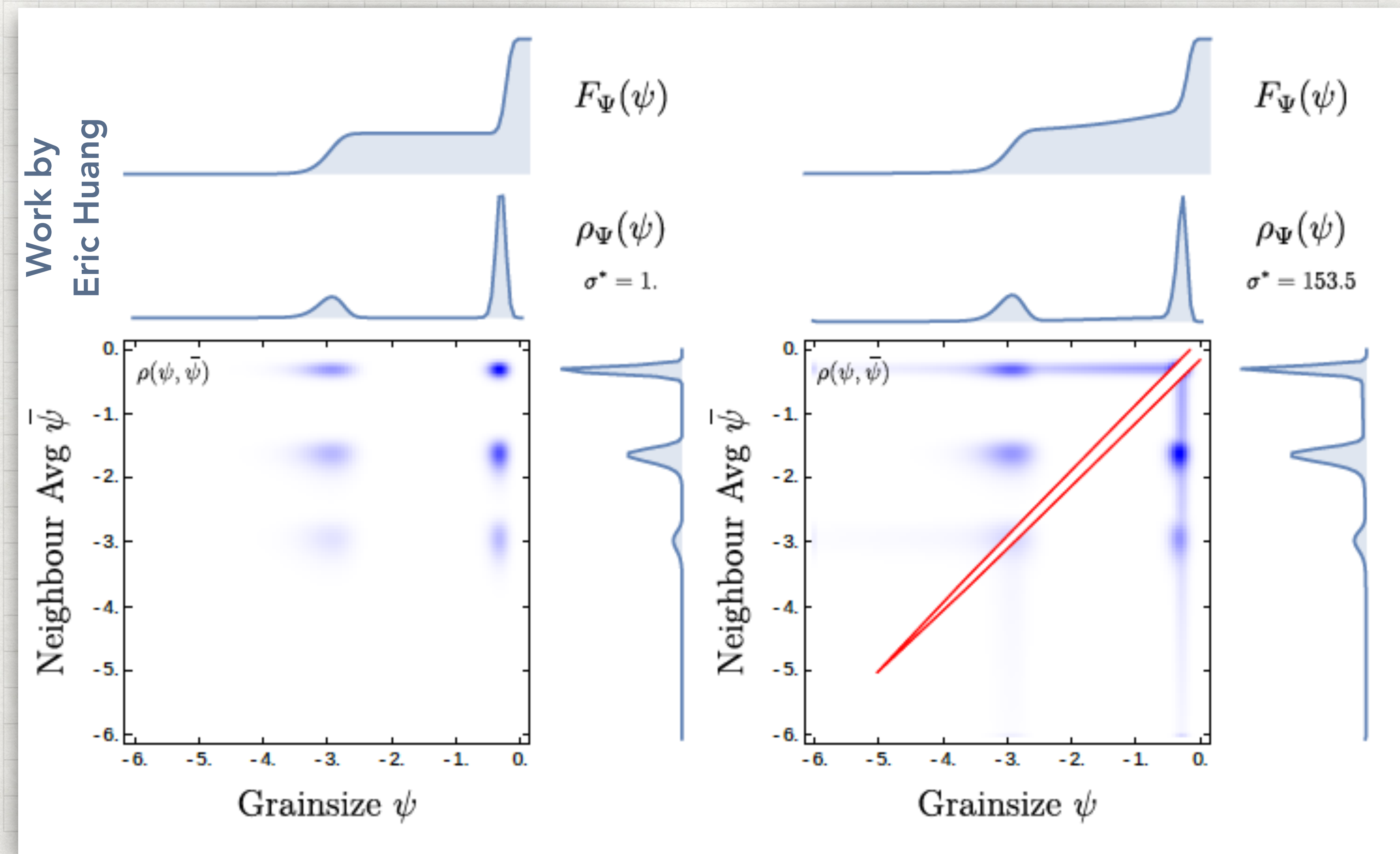


Link GSD to porosity evolution

Guida et al. IJSS in press

CRUSHING OF GRANULAR ASSEMBLY

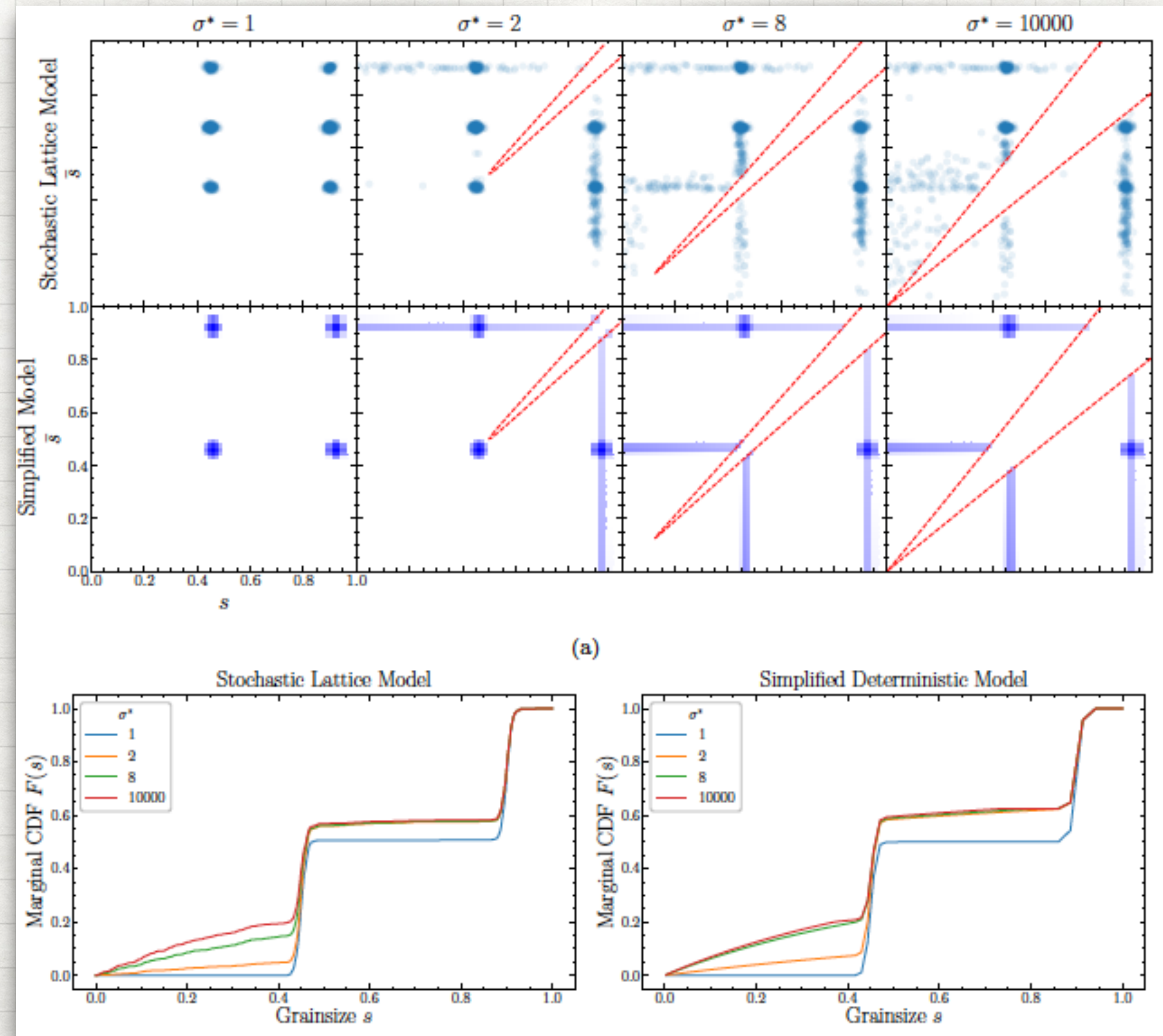
- HOMOGENISATION FROM STOCHASTIC TO CONTINUA



CRUSHING OF GRANULAR ASSEMBLY

- HOMOGENISATION FROM STOCHASTIC TO CONTINUA

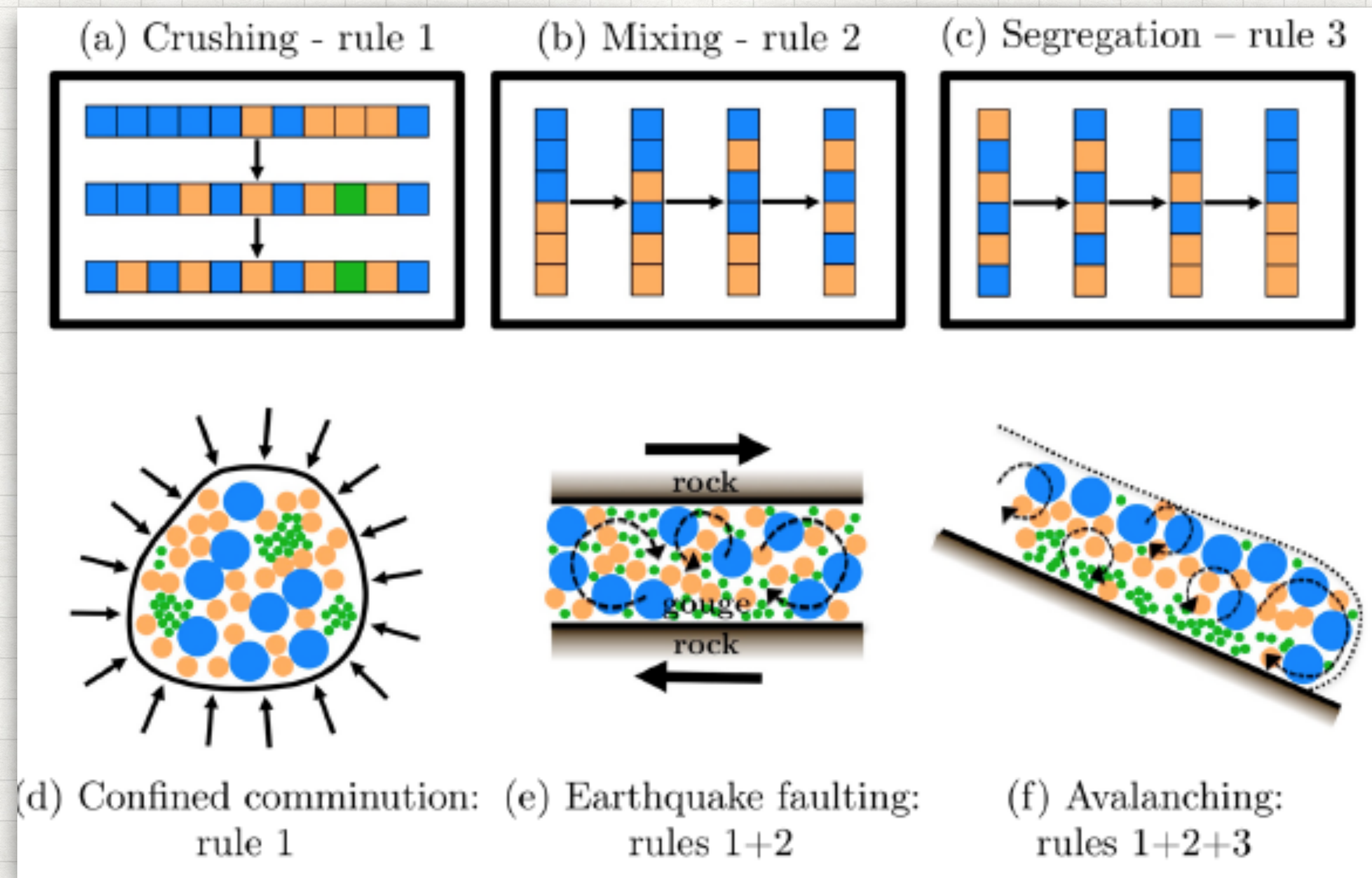
Work by
Eric Huang



COMBINED GRAINSIZE DYNAMICS

COMBINED DYNAMICS

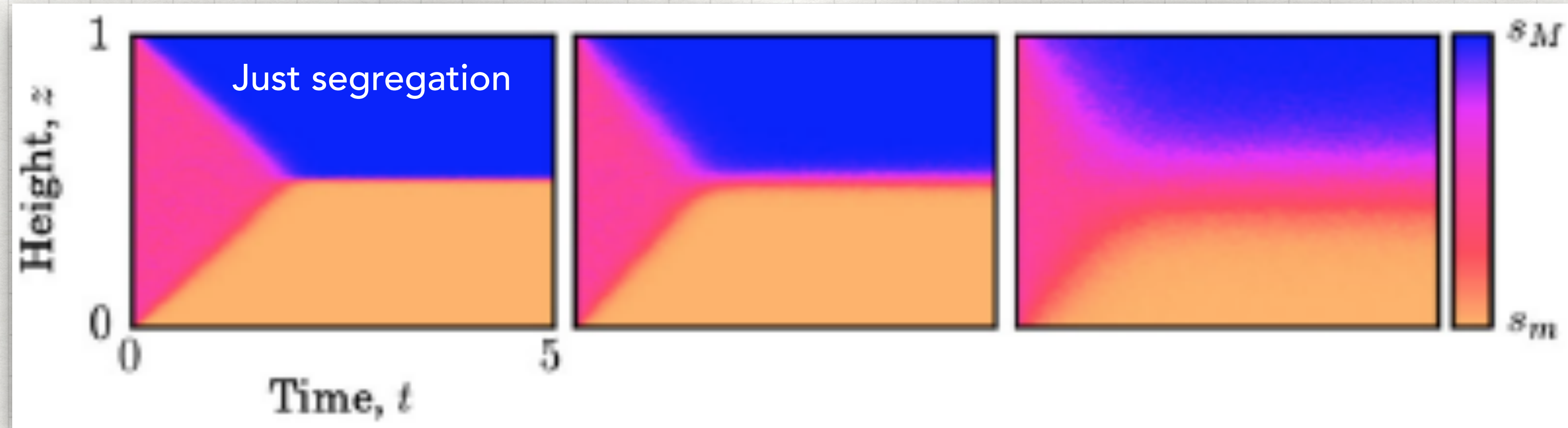
- STOCHASTIC LATTICE MODEL



SEGREGATION & MIXING

- STOCHASTIC LATTICE MODEL

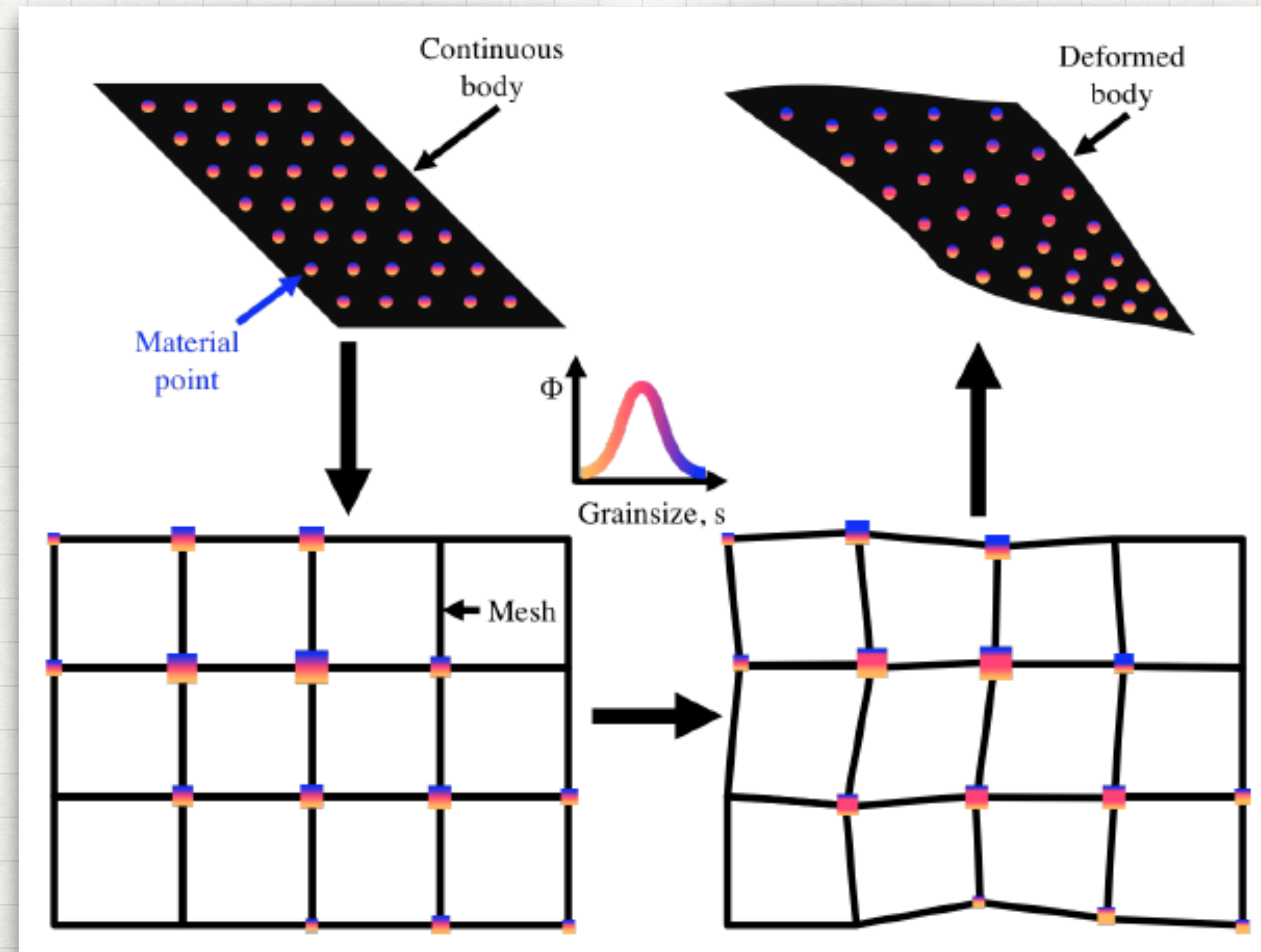
adding mixing
(increasing D)



Marks-Einav, GM, 2017

SEGREGATION & MIXING

- 'HETERARCHIAL' MATERIAL POINT METHOD



SEGREGATION & MIXING

- ‘HETERARCHIAL’ MATERIAL POINT POINT

see book chapter

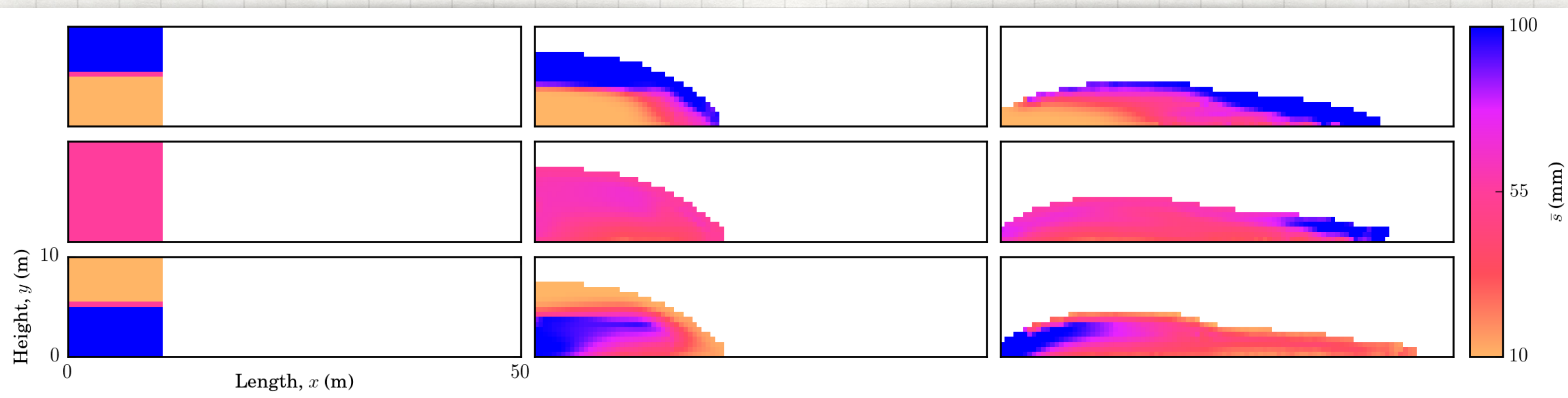
Mass balance $\partial_t \rho + \nabla \cdot (\rho \mathbf{u}) = 0$

Momentum balance $\partial_t (\rho \mathbf{u}) + \nabla \cdot (\rho \mathbf{u} \otimes \mathbf{u}) = \mathbf{F}_\rho$

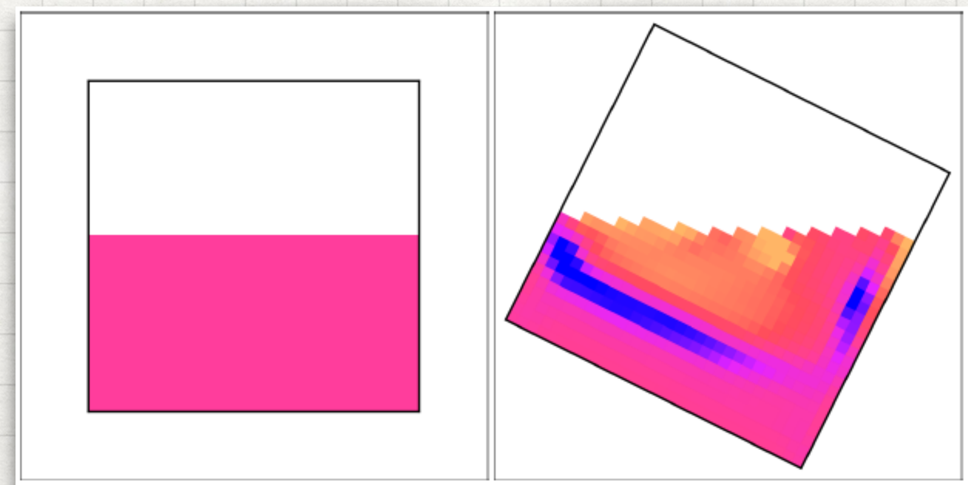
with segregation velocity $\hat{u}_s \propto \nabla p_k \propto \nabla (T_g^2)$

SEGREGATION & MIXING (& RHEOLOGY)

- 'HETERARCHIAL' MATERIAL POINT POINT



Breaking dam (inclined slope)

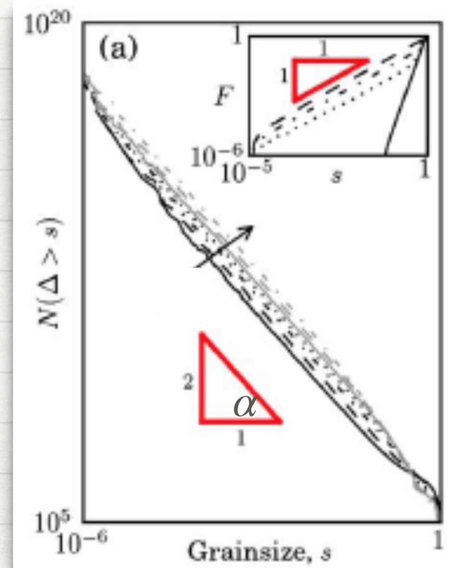


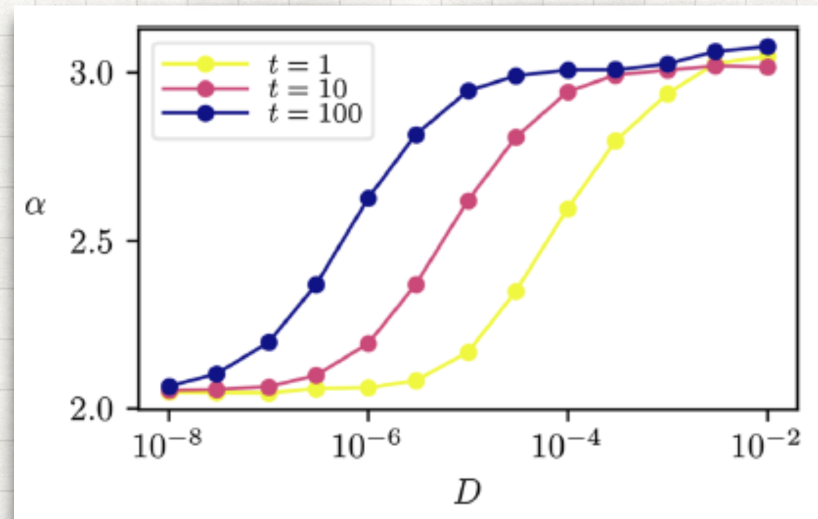
Rotating square drum

MIXING & CRUSHING

- STOCHASTIC LATTICE MODEL

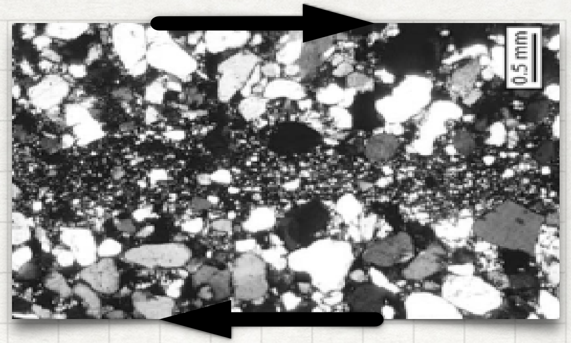
No mixing
$D = 0$

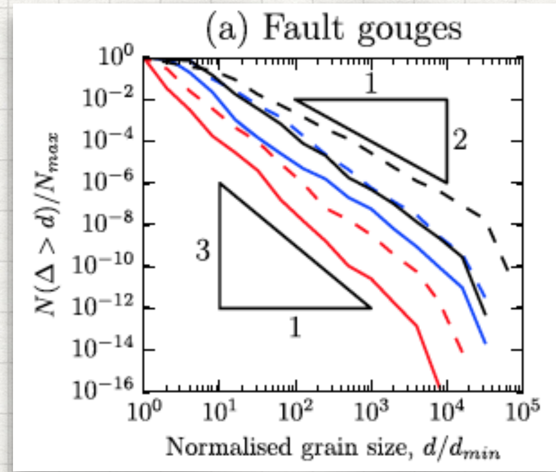


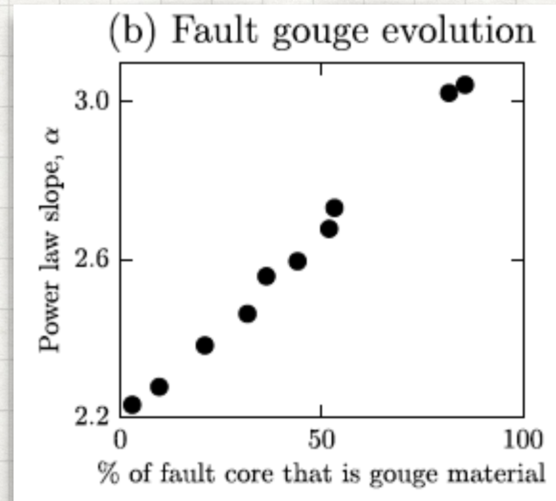


Model

Faults



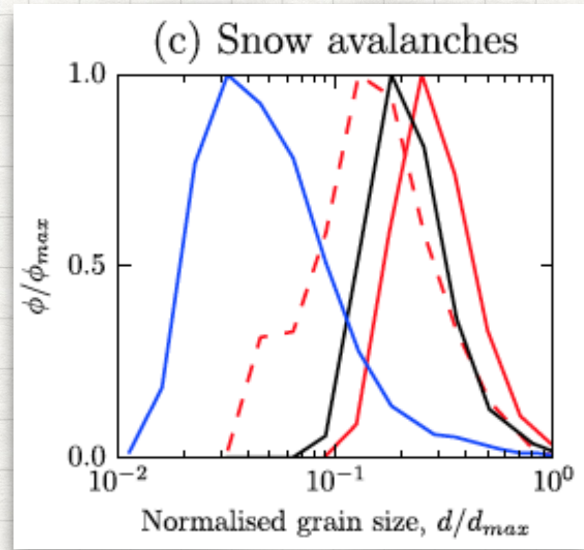


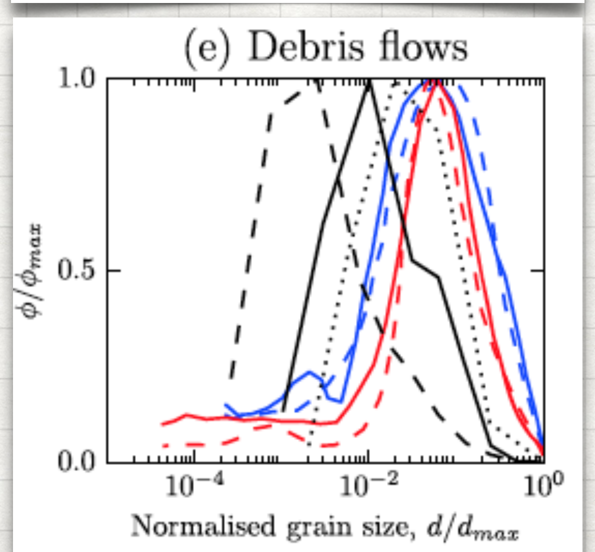


Marks-Einav, GRL, 2015

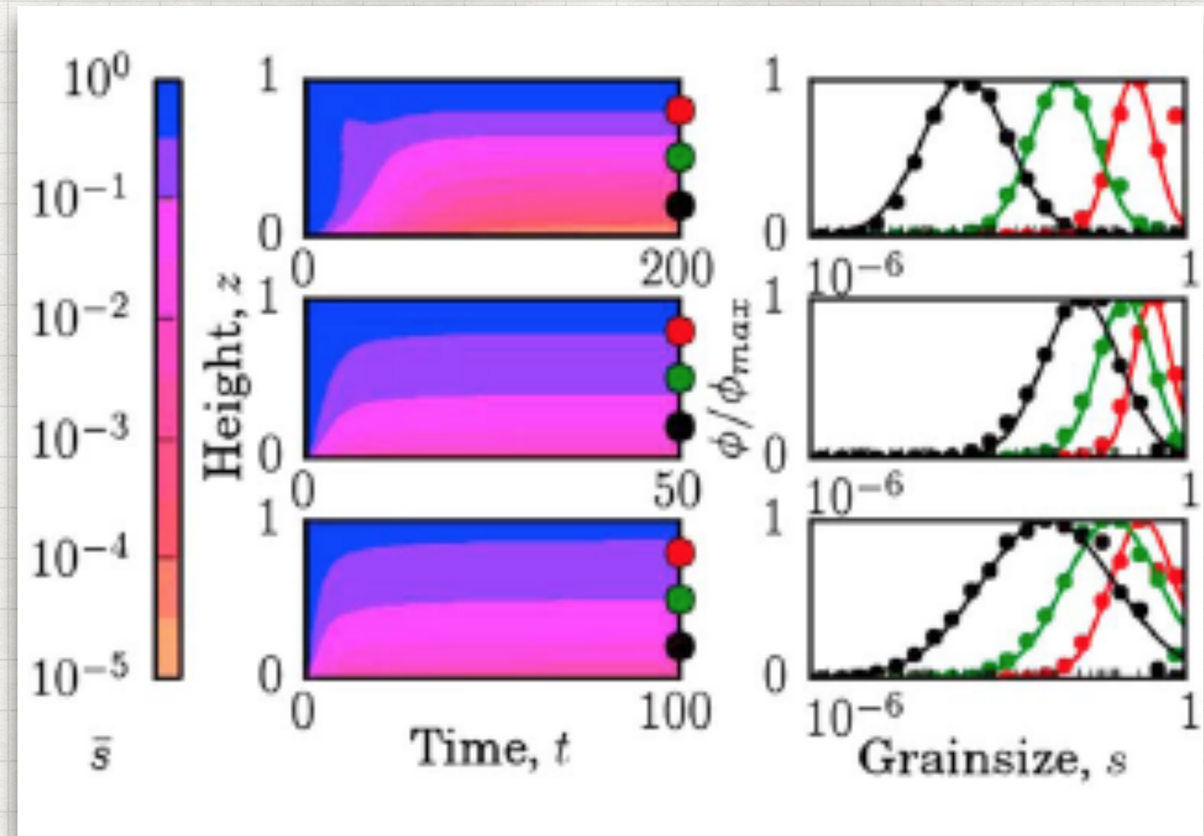
SEGREGATION, MIXING & CRUSHING

- STOCHASTIC LATTICE MODEL





Model



Marks-Einav, GRL, 2015

FINAL WORDS

FINAL WORDS ON VARDOULAKIS CHALLENGE

12 YEARS AFTER AND WE ARE NEARLY READY TO ADDRESS HIS CHALLENGE...

FINAL WORDS - WHAT ABOUT SHAPE?

1) SIZE — 1ST ORDER

+

2) ROUNDNESS
3) SPHERICITY
4) ANGULARITY
5) ELONGATION

2ND ORDER

+

7) SURFACE ROUGHNESS
8) SURFACE FRACTAL DIMENSION

3RD ORDER

+

9) ETC