CRITICAL STATE SOIL FABRIC IN CONSTITUTIVE MODELING OF CLAYS

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Most Relevant Publications

Dafalias, Y.F. (1986). "An anisotropic critical state soil plasticity model," **Mechanics Research Communications**, Vol. 13, pp. 341-347.

Wheeler, S.J., Naatanen, A., Karstunen, M., and Lojander, M. (2003). "An anisotropic elastoplastic model for soft clasy", **Can. Geot. J**., Vol. 40(2), pp.403-418

Dafalias, Y.F., and Taiebat, M. (2013). "Anatomy of rotational hardening in clay plasticity", **Geotechnique,** Vol. 63, pp. 1406-1418.

Dafalias, Y.F., and Taiebat, M. (2014). "Rotational hardening with and without anisotropic fabric at critical state", **Geotechnique,** doi.org/10.1680/geot.13.T.035.

Isotropic and Anisotropic Modified Cam-Clay Models

Isotropic MCC (Burland, 1965)

$$p\dot{\varepsilon}_v^p + q\dot{\varepsilon}_q^p = p\left[(\dot{\varepsilon}_v^p)^2 + (M\dot{\varepsilon}_q^p)^2\right]^{1/2}$$

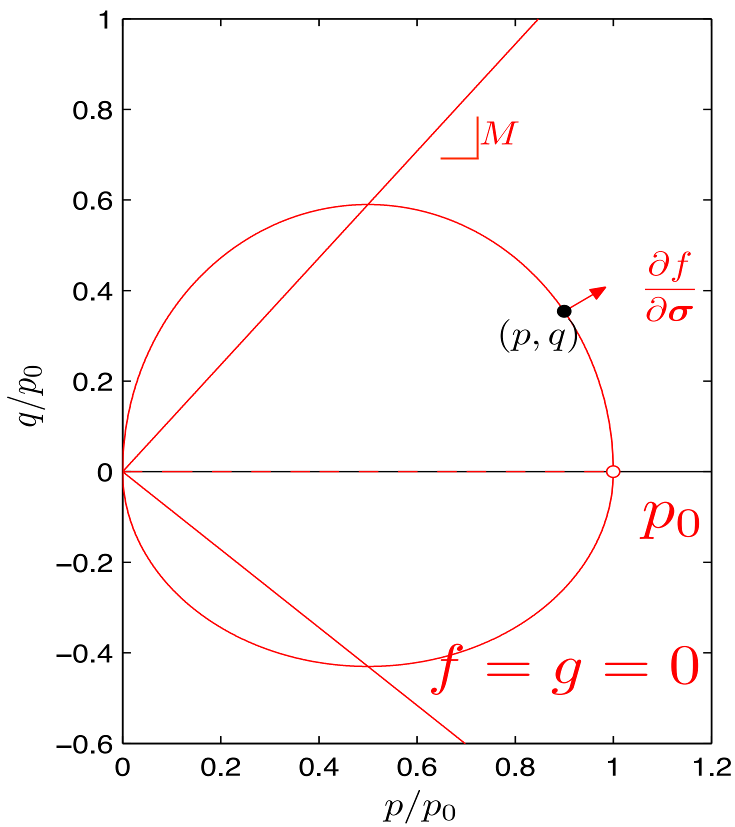
$$f = g = q^2 - M^2 p(p_0 - p) = 0$$

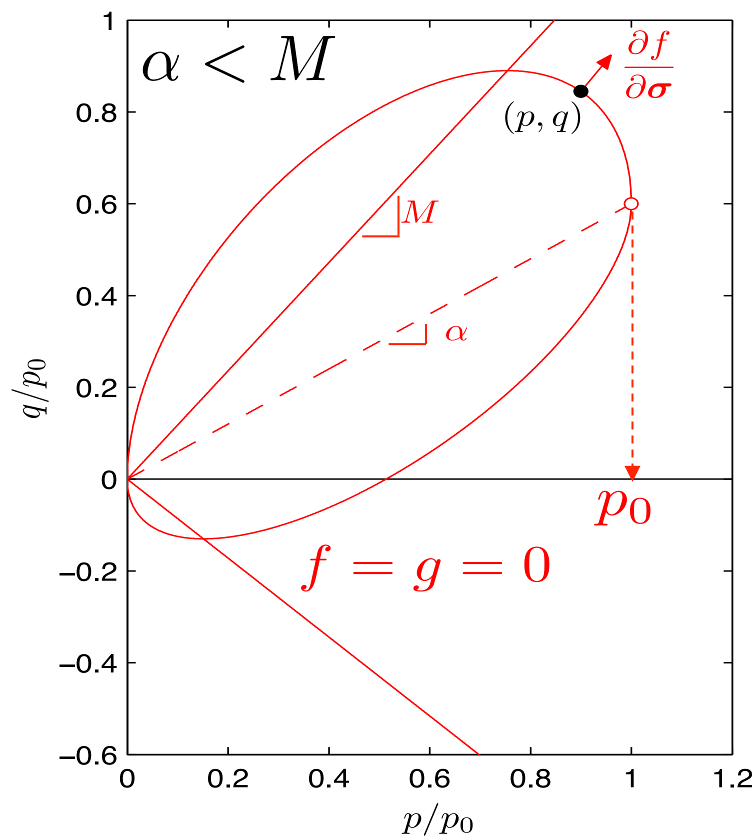
Anisotropic MCC (AMCC) (Dafalias, 1986)

$$p\dot{\varepsilon}_v^p + q\dot{\varepsilon}_q^p = p\left[(\dot{\varepsilon}_v^p)^2 + (M\dot{\varepsilon}_q^p)^2 + 2\alpha\dot{\varepsilon}_q^p\dot{\varepsilon}_v^p\right]^{1/2}$$

$$f = g = (q - p\alpha)^2 - (M^2 - \alpha^2)p(p_0 - p) = 0$$

Fabric Tensor : α **RH:** $\dot{\alpha} = ?$





SANICLAY: Plastic Potential Surface (PPS) and Yield Surface (YS)

Dafalias (1986), Dafalias *et al* (2006), Jiang *et al* (2012) Modification

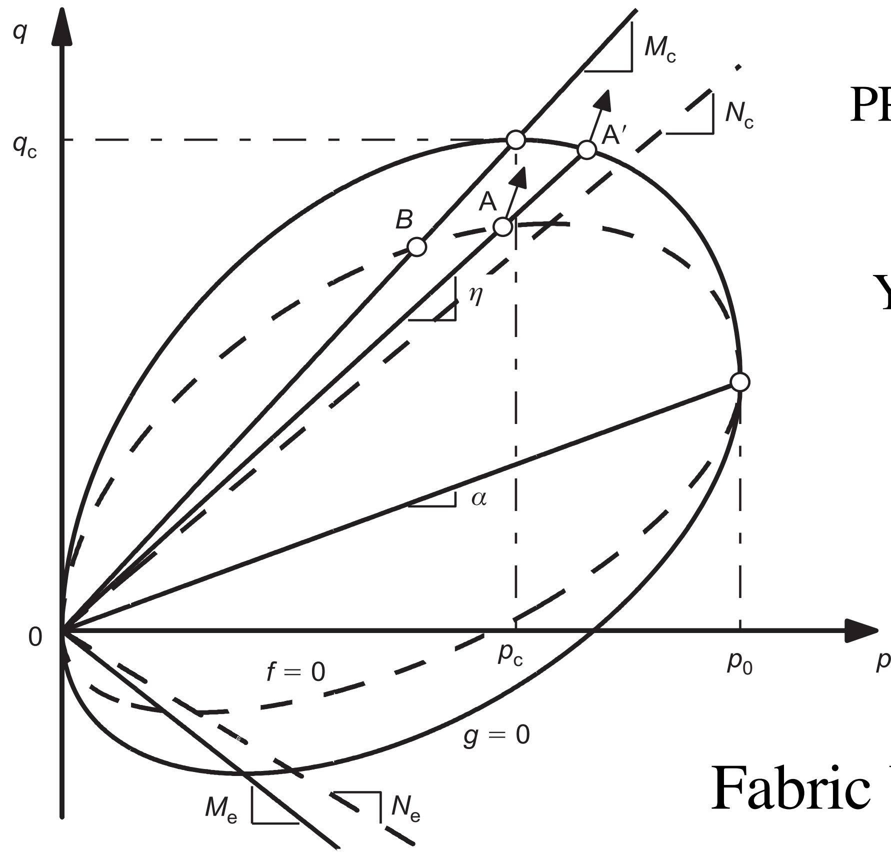


Fig. 1. Schematic diagram of the anisotropic yield and plastic potential surfaces in the p–q space; reproduced from Dafalias & Taiebat (2013)

PPS: $$g = (q - \alpha p)^2 - (M^2 - \alpha^2)p(p_0 - p) = 0$$

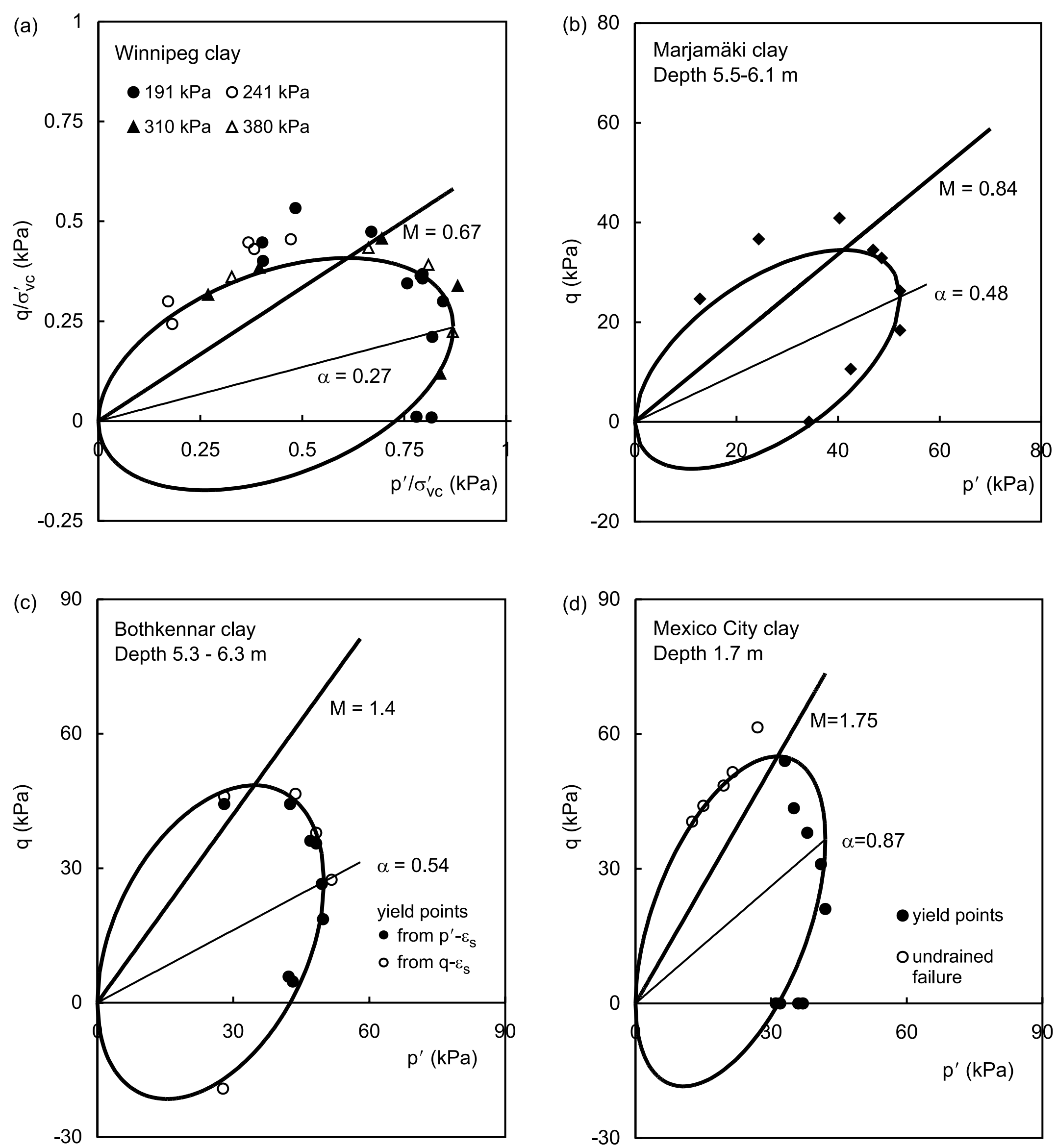
YS: $$f = (q - \alpha p)^2 - (N^2 - \alpha^2)p(p_0 - p) = 0$$

$M = M_c$ and $N = N_c$ when $\eta > \alpha$

$M = M_e$ and $N = N_e$ when $\eta < \alpha$

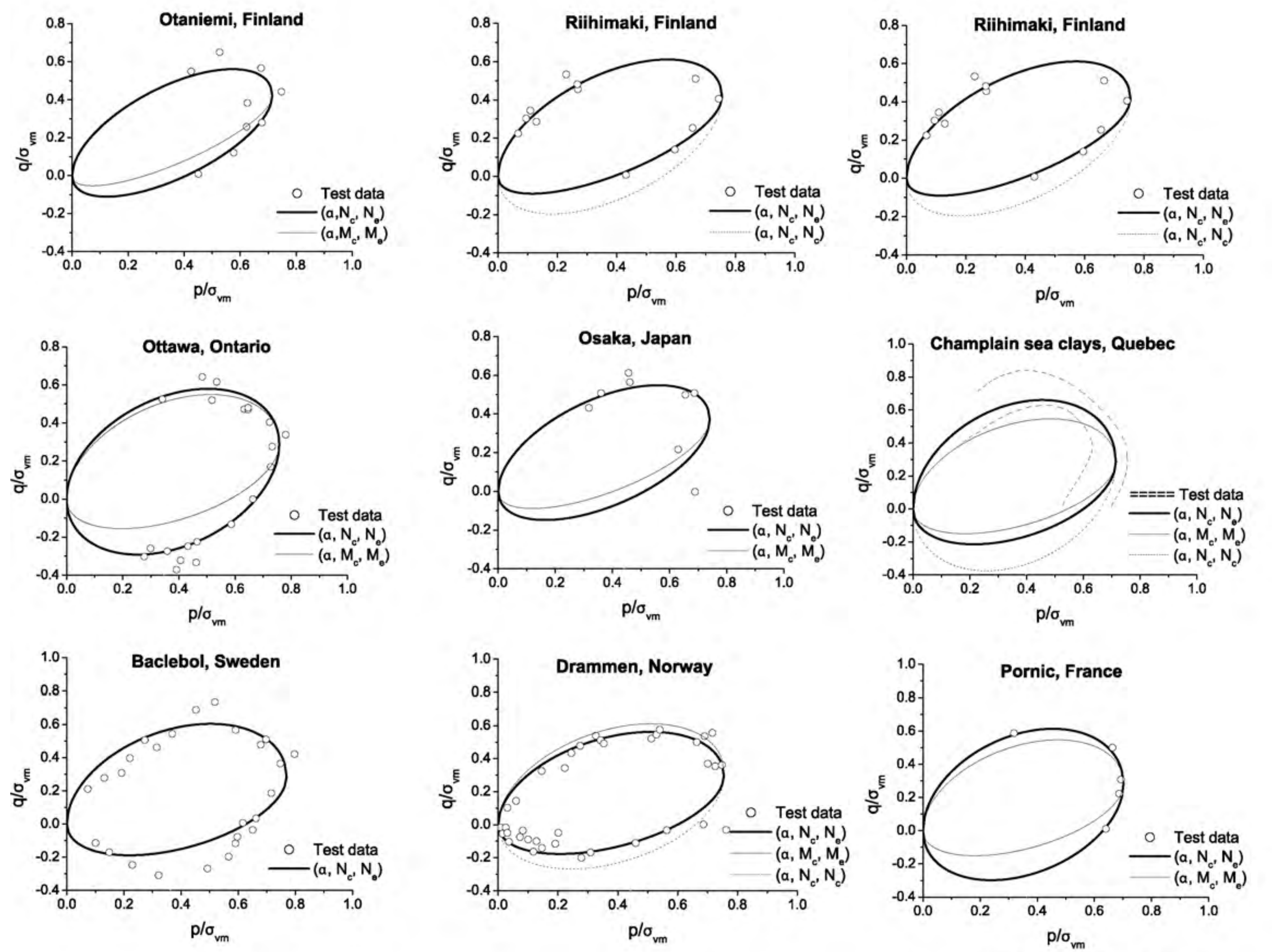
Fabric Variable α common to YS and PPS

Fitting YS of AMCC to data with N=M
(Wheeler et al., 2003)



Fitting YS of AMCC to data with N>M, N=M, N<M

(Jiang and Ling, 2010)



Forms of Rotational Hardening (RH)

Generic form: $\dot{\alpha} = \ <L> cp_{at}\frac{p}{p_0}[\alpha_b(\eta)-\alpha]$ $\qquad \eta = \frac{q}{p}$

Loading index *(plastic multiplier)*: $L \sim \sqrt{\dot{\varepsilon}^p : \dot{\varepsilon}^p}$

Bounding attractor: $\alpha_b(\eta)$ towards which α converges under fixed η

Dafalias (1986) : $\alpha_b(\eta) = \frac{\eta}{x}$

Dafalias and Taiebat (2013): $\alpha_b(\eta) = \pm\frac{M}{z}(1-\exp[-s\frac{|\eta|}{M}])$

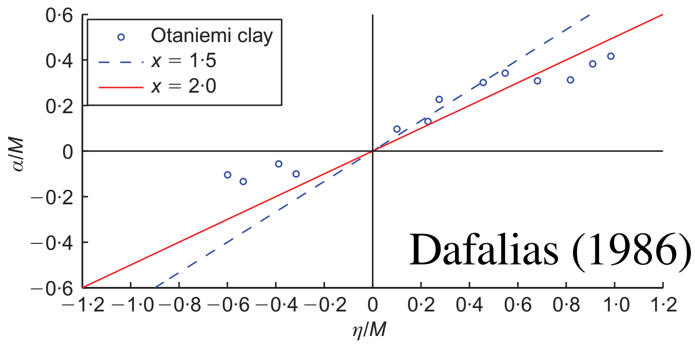
Dafalias and Taiebat (2014): $\alpha_b(\eta) = \eta\left(m\left\langle 1-\left(\frac{|\eta|}{M}\right)^n\right\rangle + \frac{\alpha_c}{M}\exp\left(-\mu\left\langle\frac{|\eta|}{M}-1\right\rangle\right)\right)$

Non generic form: (Wheeler et al, 2003)

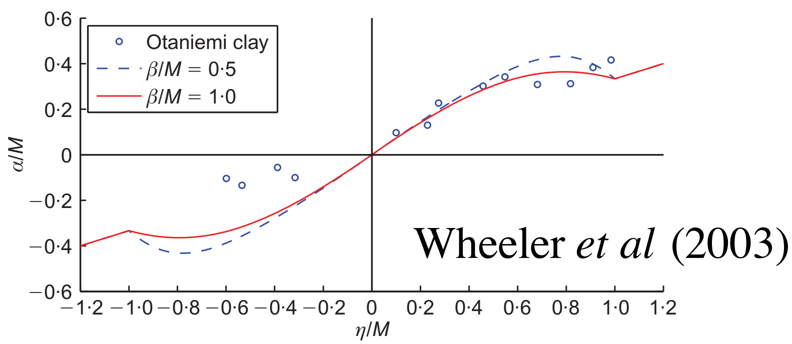
$$\dot{\alpha} = \mu[(\frac{3\eta}{4} - \alpha) < \dot{\varepsilon}_v^p > +\beta(\frac{\eta}{3} - \alpha)|\dot{\varepsilon}_d^p|]$$

Variation of **bounding attractor** $\alpha_b(\eta)/M$ under fixed η/M

Non-zero CS fabric $\Rightarrow \alpha_b(M) = \alpha_c \neq 0$



(a)



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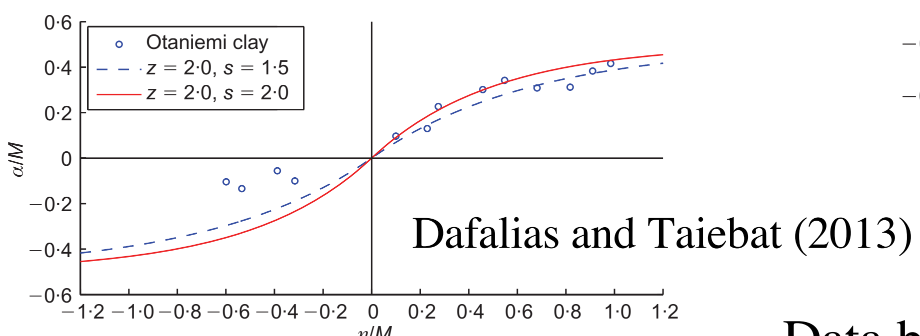
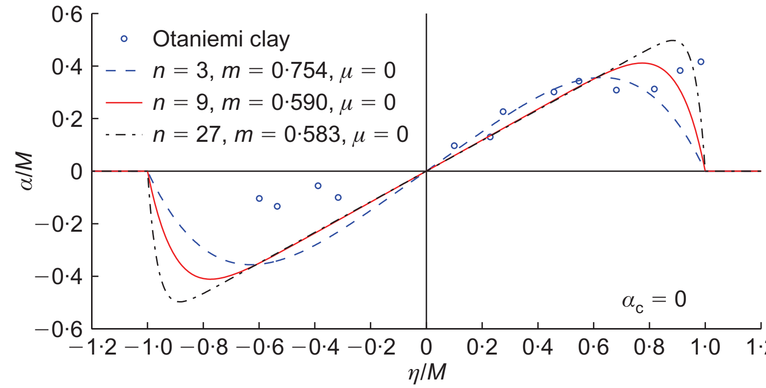


(c)

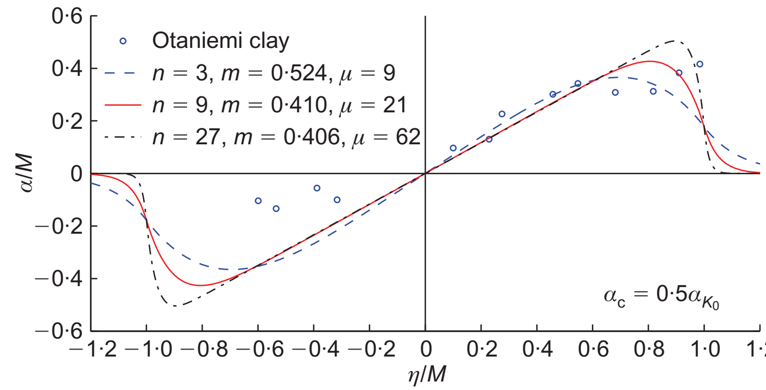
Zero CS fabric $\Rightarrow \alpha_b(M) = \alpha_c \neq 0$

Non-zero CS fabric $\Rightarrow \alpha_b(M) = \alpha_c \neq 0$

Note: $\alpha_c \to 0$ as $\eta \to \infty$



(a)



(b)

Dafalias and Taiebat (2014)

Data by Wheeler et al (2003)

Requirements for RH

- Unique Critical State Line (CSL) in e – p space
- Must be able to simulate K_0 loading for calibration
- Not excessive rotation (e.g. $\alpha_{\max} = \alpha_{b\max} < \mathrm{M}$)
- Must have $\alpha_b(\eta) < \eta$ for all η in order to avoid $\dot{\varepsilon}_q^p < 0$ for $\eta > 0$

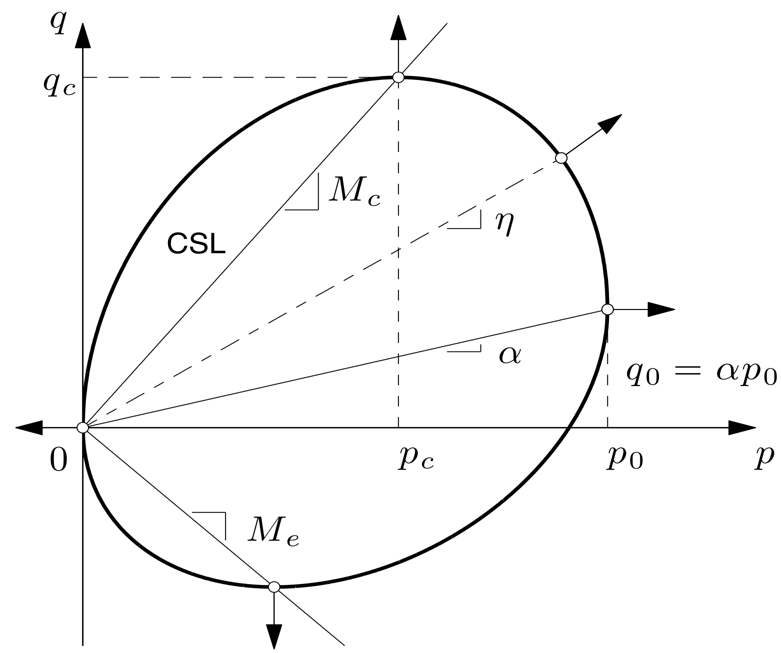
RH and Uniqueness of Critical State Line (CSL)

At $\eta = M \quad \Rightarrow \alpha_b(M) = \alpha_c$

For $N = M$ and $N \neq M$ one has

$$\frac{p_c}{p_0} = \frac{1}{2}\left(1 + \frac{\alpha_c}{M}\right)$$

$$\frac{p_c}{p_0} = \frac{1 - (\alpha_c / N)^2}{1 + (M / N)^2 - 2(M / N)(\alpha_c / N)}$$



If (α_c / M) is same for all Lode angles the ratio (p_c / p_0) is fixed and defines unique CSL in e-p space in regards to a unique Normal Consolidation Line (NCL) p_0 versus e

$NOTE: \quad \alpha_c / N = (\alpha_c / M)(M / N)$

Specification of Unique CSL by Various RH Rules

Recall $\frac{p_c}{p_0} = \frac{1}{2}(1 + \frac{\alpha_c}{M}) \rightarrow$ Unique CSL if $\frac{\alpha_c}{M}$ independent of Lode Angle

(i) RH of Dafalias (1986): $\frac{\alpha_c}{M} = \frac{1}{x}$

(ii) RH of Wheeler et al (2003): $\frac{\alpha_c}{M} = \frac{1}{3}$

(iii) RH of Dafalias and Taiebat (2013): $\frac{\alpha_c}{M} = \frac{1}{z}\left[1 - \exp(-s)\right]$

(iv) RH of Dafalias and Taiebat (2014): $\frac{\alpha_c}{M}$: input

$NOTE$: If $\alpha_c = 0 \rightarrow \frac{p_c}{p_0} = \frac{1}{2}$ (MCC)

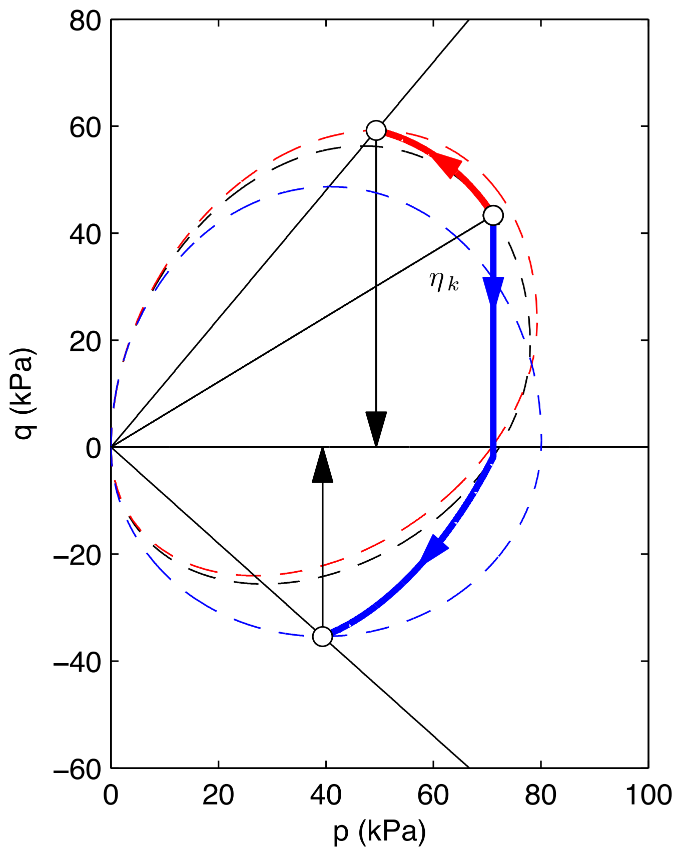
Comparison of RH Rule of Dafalias (1986) with and without $\left|\partial g / \partial p\right|$

$$\dot{\alpha} = < L > c\left|\frac{\partial g}{\partial p}\right|\frac{p}{p_0}\left[\frac{\eta}{x} - \alpha\right]$$

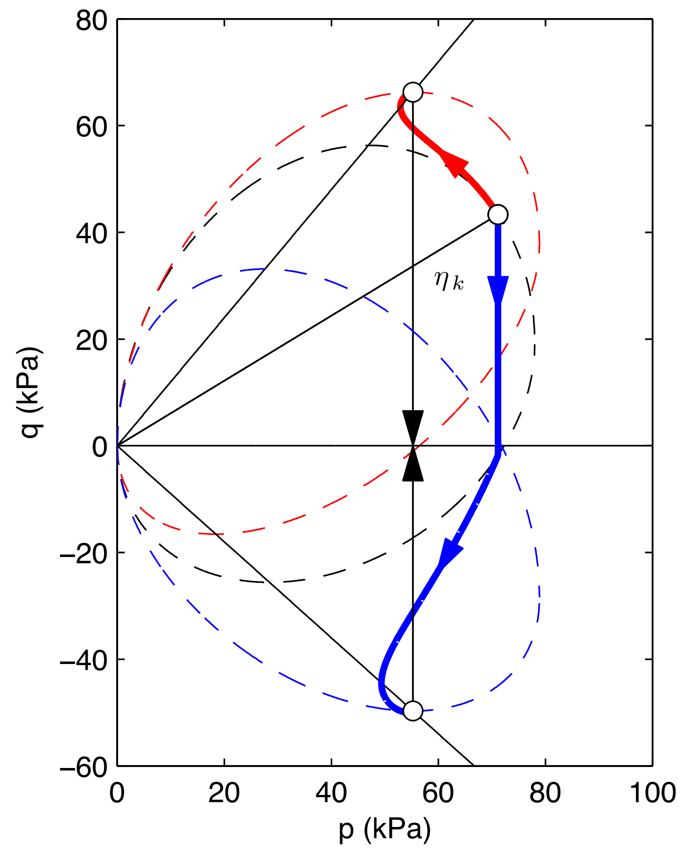
$$\dot{\alpha} = < L > cp_{atm}\frac{p}{p_0}\left[\frac{\eta}{x} - \alpha\right]$$

$$\frac{\alpha_b(M)}{M} = \frac{\alpha_c}{M} = \frac{1}{x}$$

Non-Unique CSL

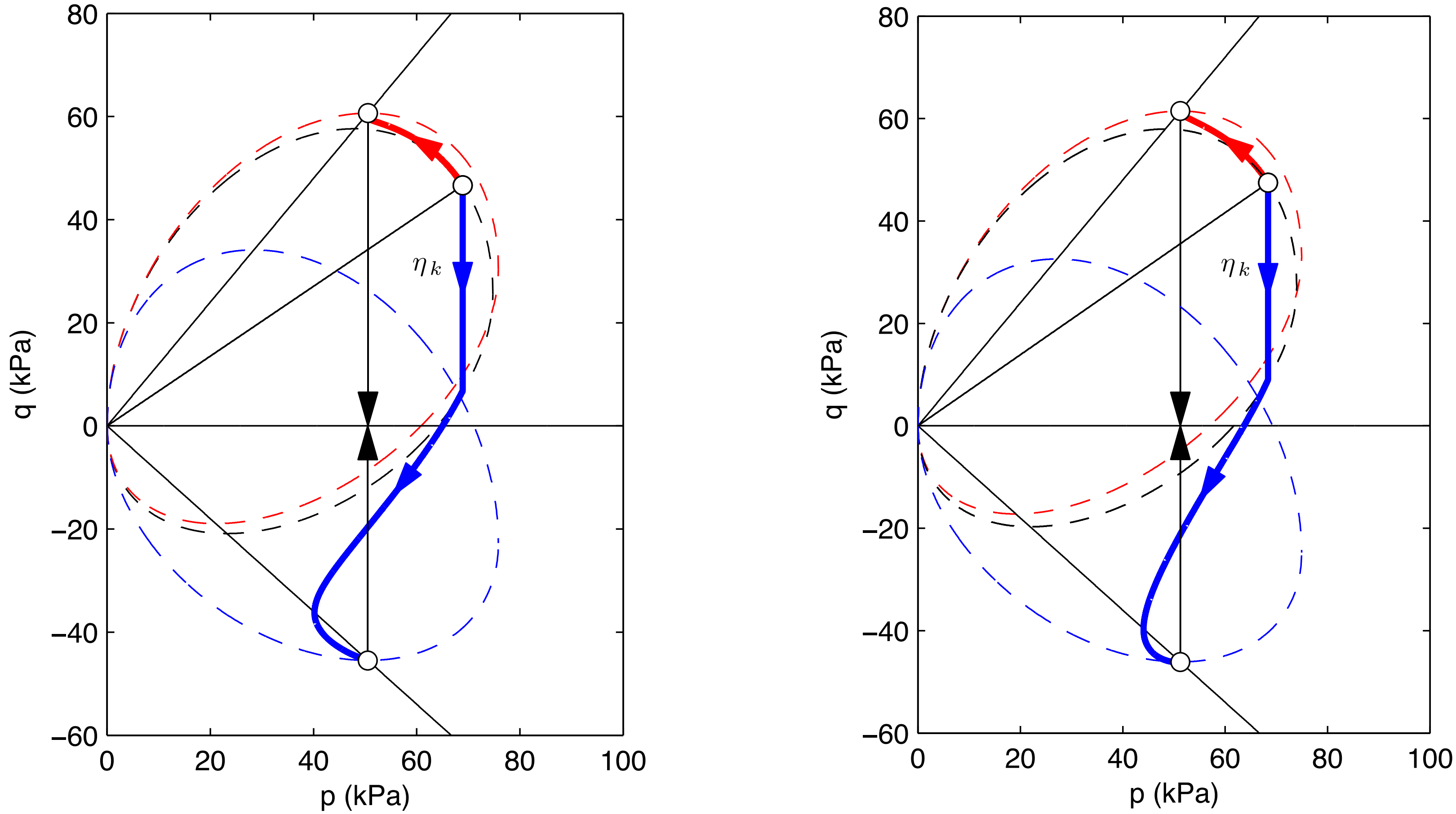


Unique CSL



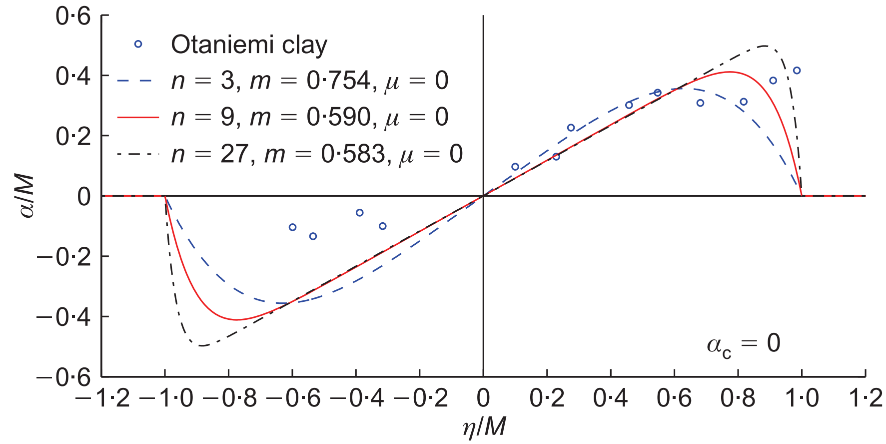
Comparison of Wheeler et al. (2003) and Dafalias and Taiebat (2013) RH Rules

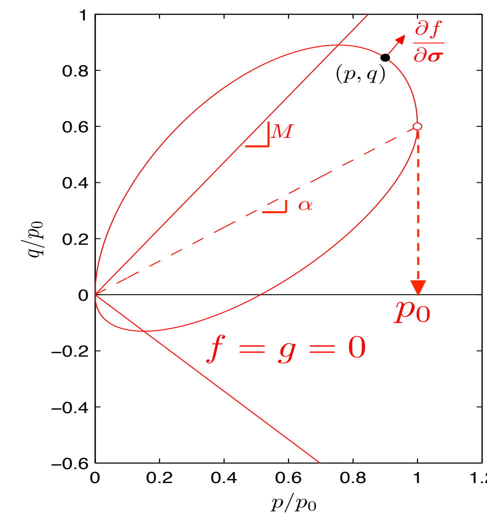
RH of Wheeler et al (2003) -> Unique CS <- RH of Dafalias and Taiebat (2013)



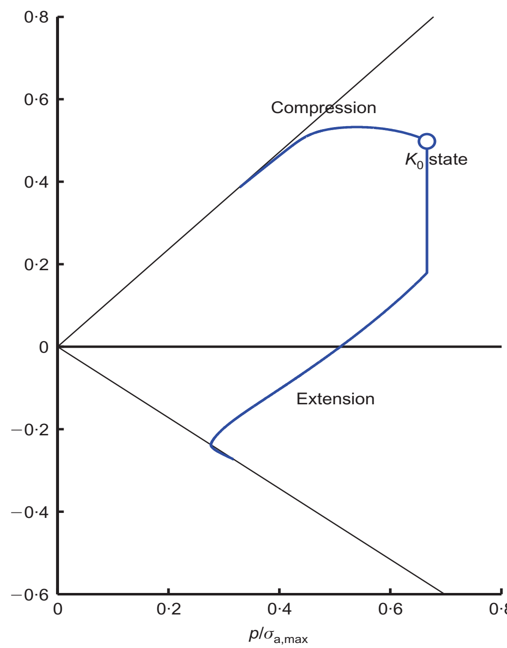
Softening when fabric at CS is small or zero

RH by Dafalias and Taiebat (2014)

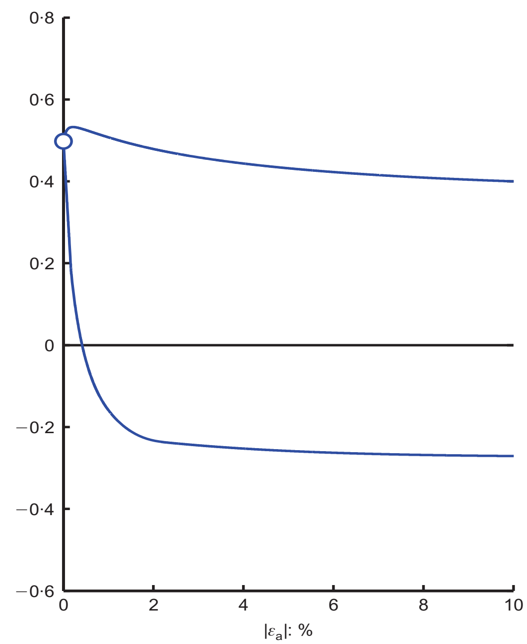




$$\dot{\alpha} = < L > c p_{at} \frac{p}{p_0} [\alpha_b(\eta) - \alpha]$$



(a)



(b)

Value of RH variable *α* under *Ko* Loading

Under $Ko \Rightarrow \ \eta = \eta_{K_0} = \dfrac{3(1-K_0)}{1+2K_0}$ and $\alpha = \alpha_{K_0}$ given by:

For $G = \infty \ \Rightarrow$ $$\alpha_{K_0} = \frac{\eta_{K_0}^2 + [3(1-(\kappa/\lambda))]\eta_{K_0} - M_c^2}{3(1-(\kappa/\lambda))}$$ (Dafalias, 1986)

For also $\kappa = 0 \ \Rightarrow$ $$\alpha_{K_0} = \frac{\eta_{K_0}^2 + 3\eta_{K_0} - M_c^2}{3}$$ (Wheeler et al, 2003)

NOTE: The value of α_{K_0} is **INDEPENDENT** of the RH rule used

Calibration of RH parameters under *Ko* loading

(i) RH of Dafalias (1986): $x = \dfrac{\eta_{K_0}}{\alpha_{K_0}}$

(ii) RH of Wheeler et al (2003): $\beta = \dfrac{3(3\eta_{K_0} - 4\alpha_{K_0})(M_c^2 - \eta^2{}_{K_0})}{8(3\alpha_{K_0} - \eta_{K_0})(\eta_{K_0} - \alpha_{K_0})}$

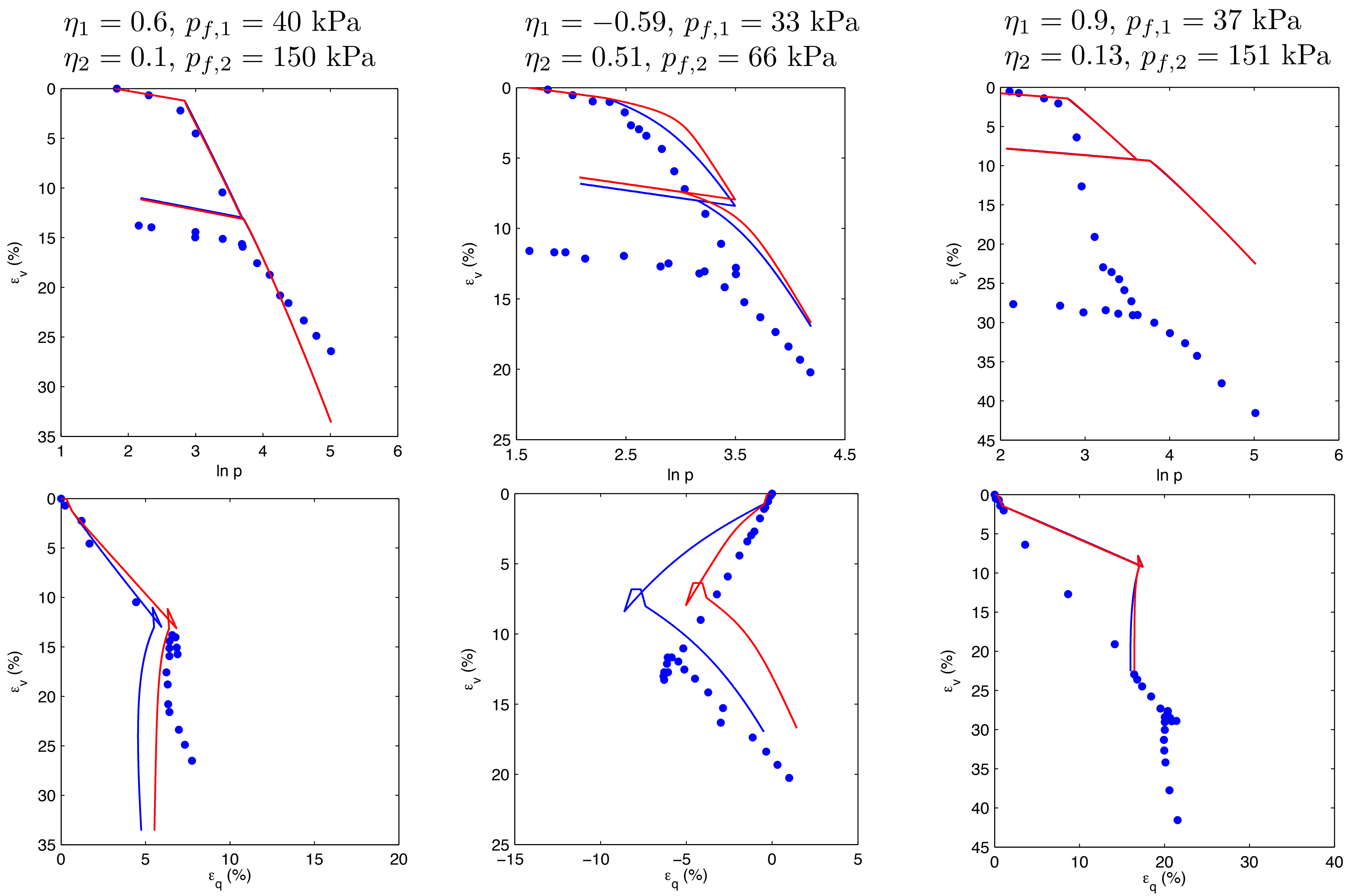
(iii) RH of Dafalias and Taiebat (2013): $\alpha_{K_0} = \pm \dfrac{M}{z}\left[1 - \exp\left(-s\dfrac{|\eta_{K_0}|}{M}\right)\right]$

(iv) RH of Dafalias and Taiebat (2014): $\alpha_{K_0} = \eta_{K_0}\left[\dfrac{\alpha_c}{M} + m\left(\dfrac{|\eta_{K_0}|}{M}\right)^n\right]$

***NOTE*:** K_0 fitting does not guarantee the correct fitting under other constant η loading

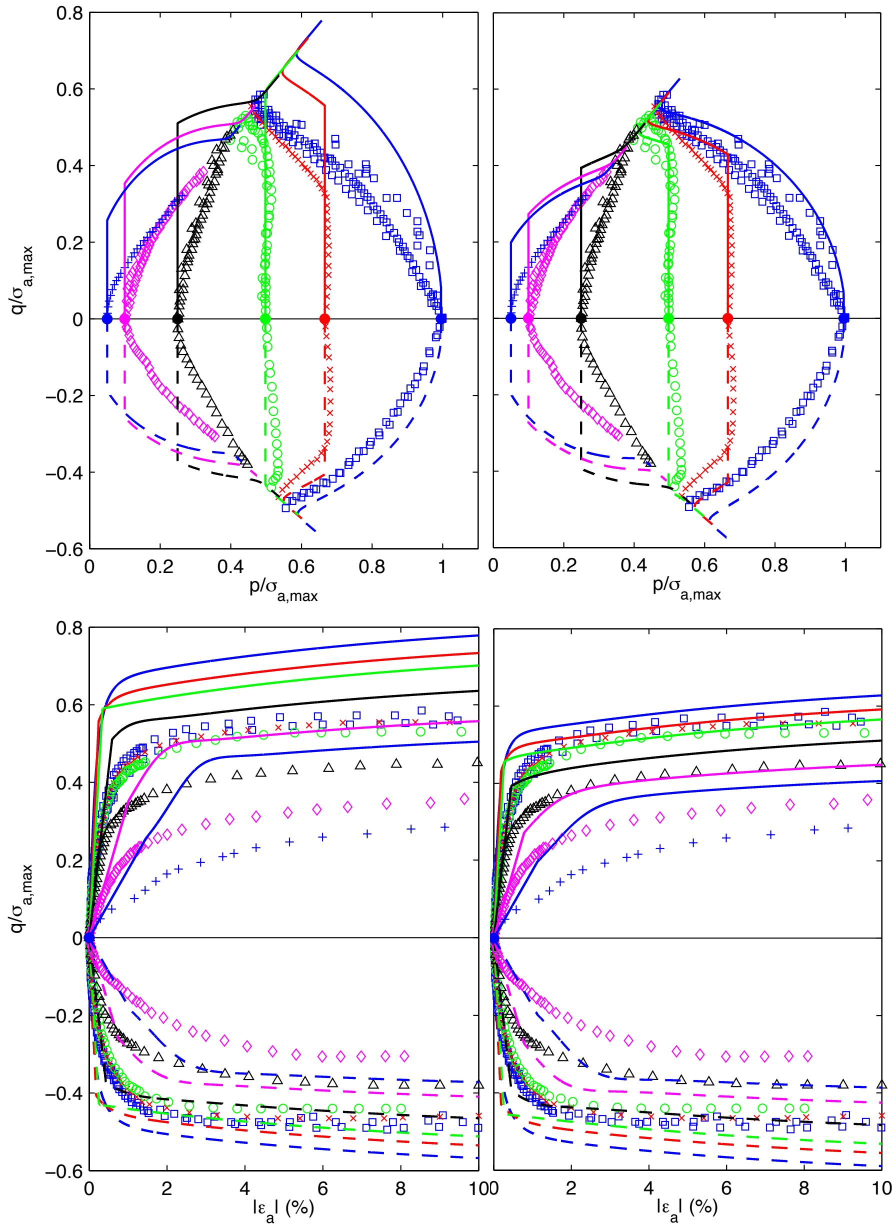
Drained Loading Simulation for Otaniemi clay

Wheeler et al (2003) RH & Dafalias and Taiebat (2013) RH



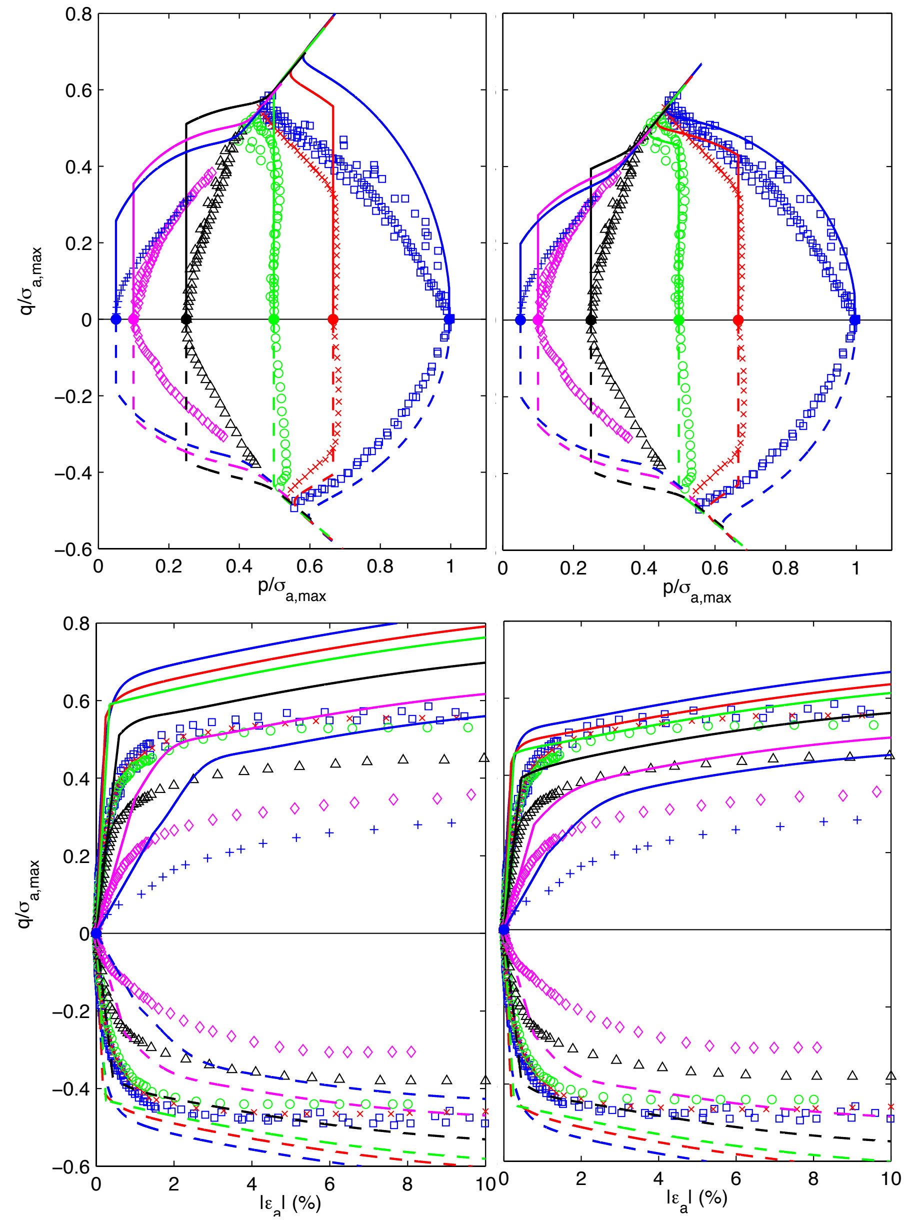
Undrained Simulation LCT Clay
(Wheeler et al, 2003)

N=M N<M



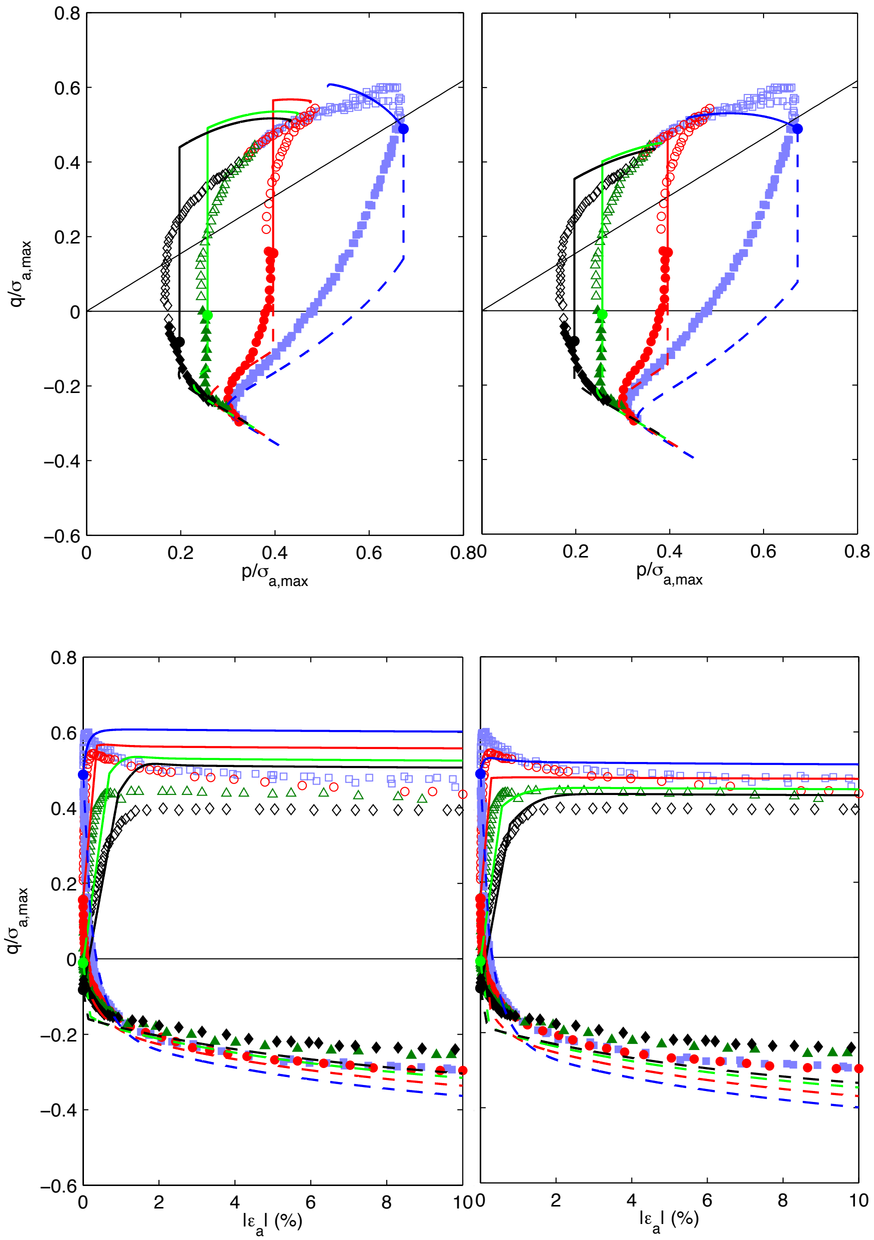
Undrained Simulation LCT Clay
(Dafalias and Taiebat, 2013)

N=M N<M



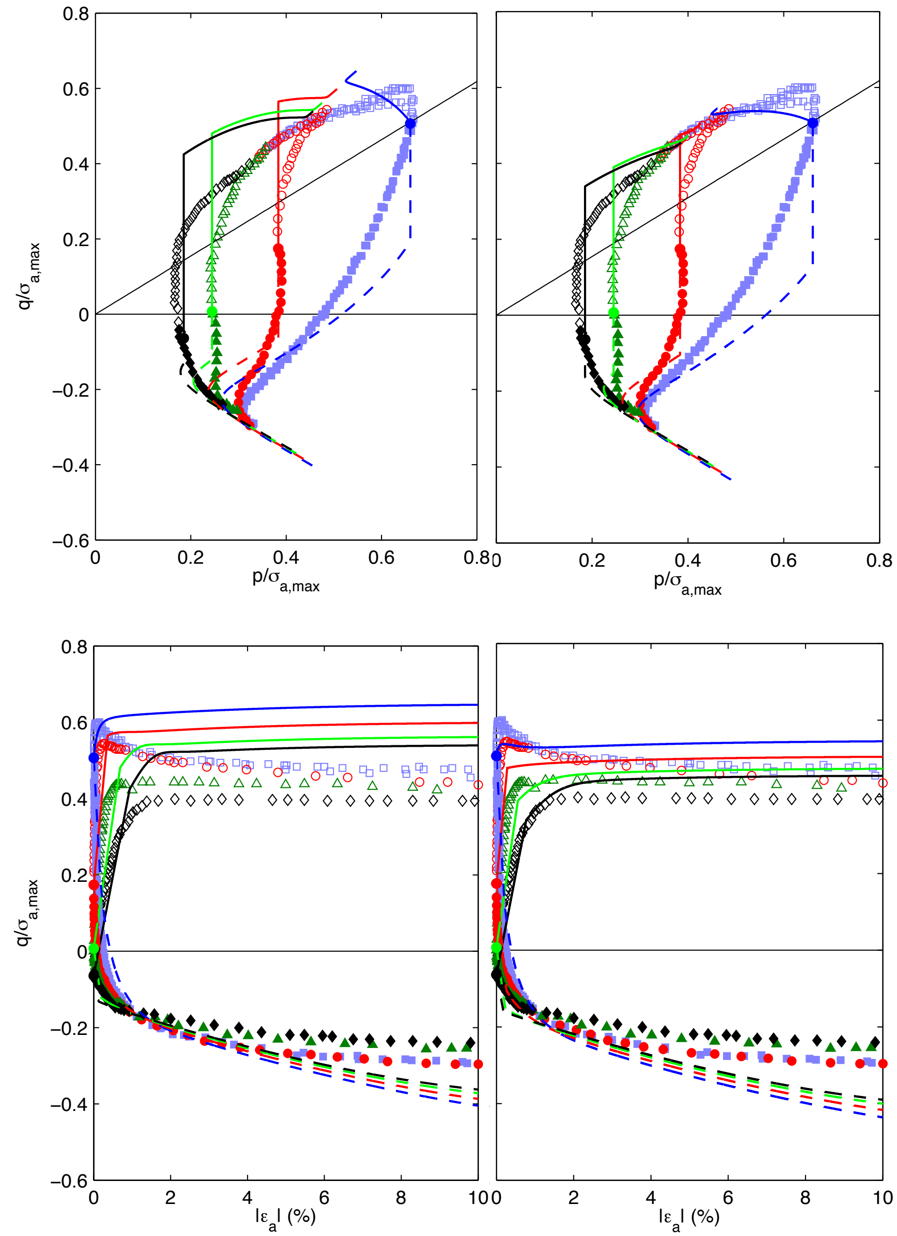
Undrained Simulation LCT Clay
(Wheeler et al, 2003)

N=M **N<M**



Undrained Simulation LCT Clay
(Dafalias and Taiebat, 2013)

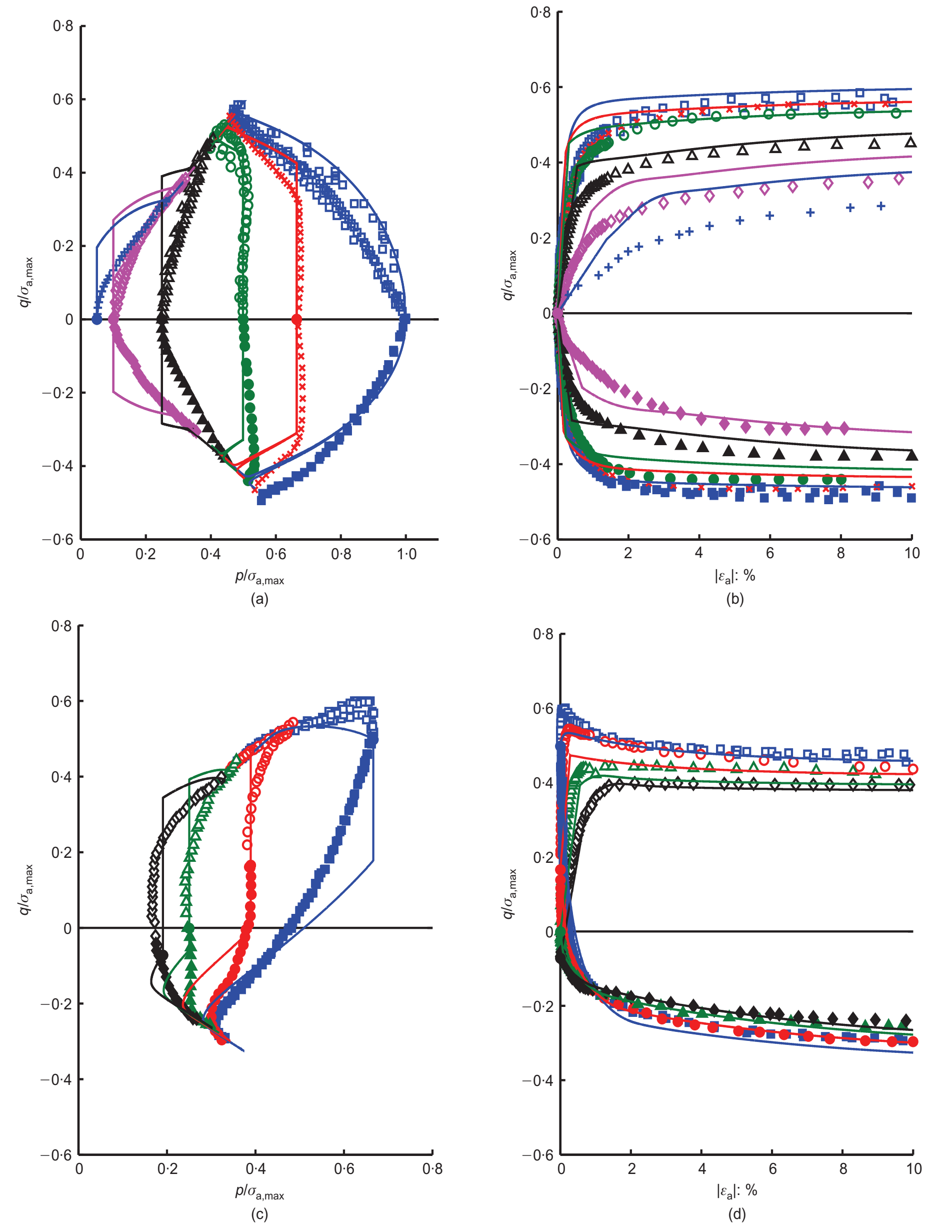
N=M **N<M**



Undrained Loading Simulation of LCT Clay with N<M

(Dafalias and Taiebat, 2014)

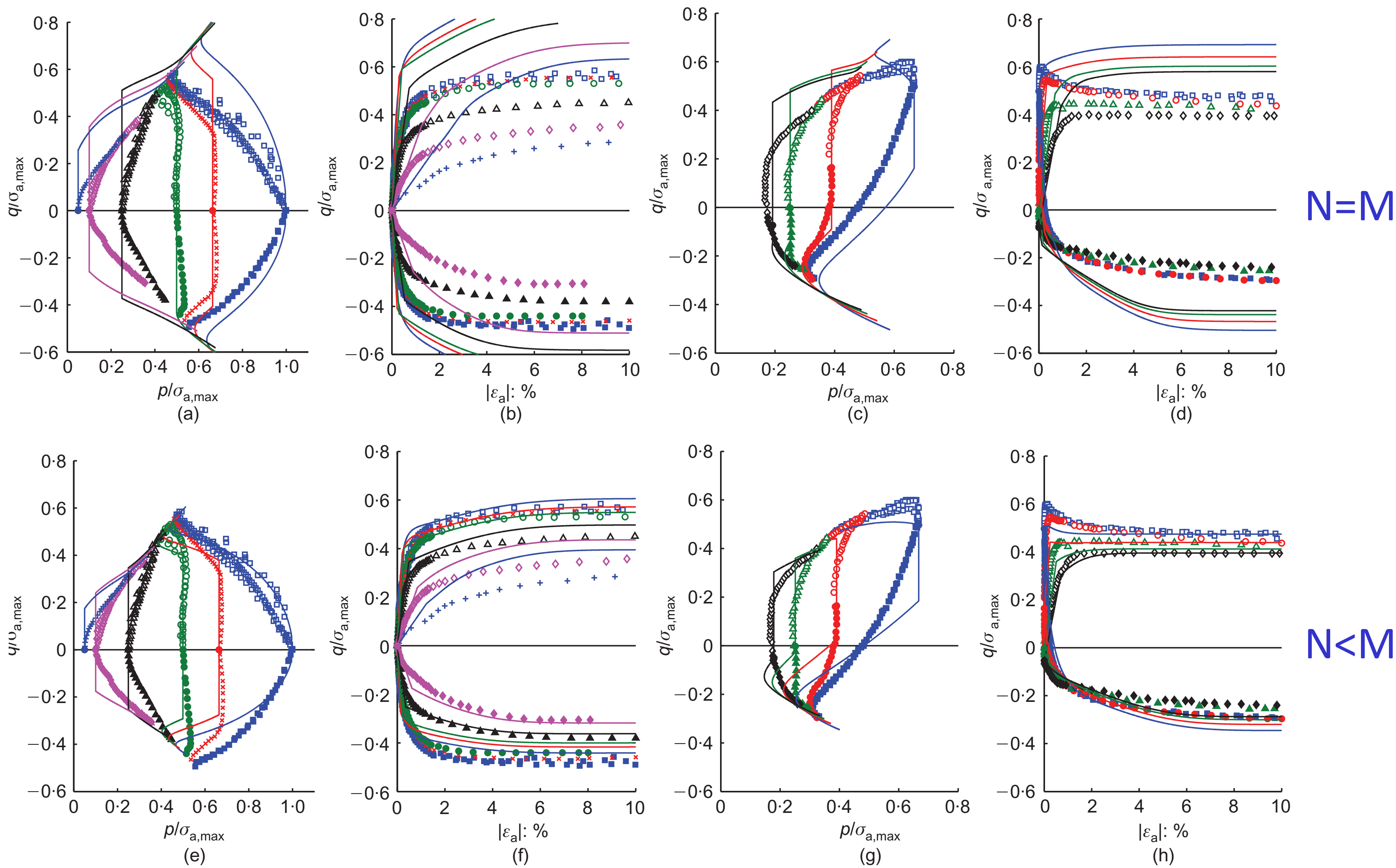
N<M



Undrained Loading Simulation of LCT Clay with the Simplest RH

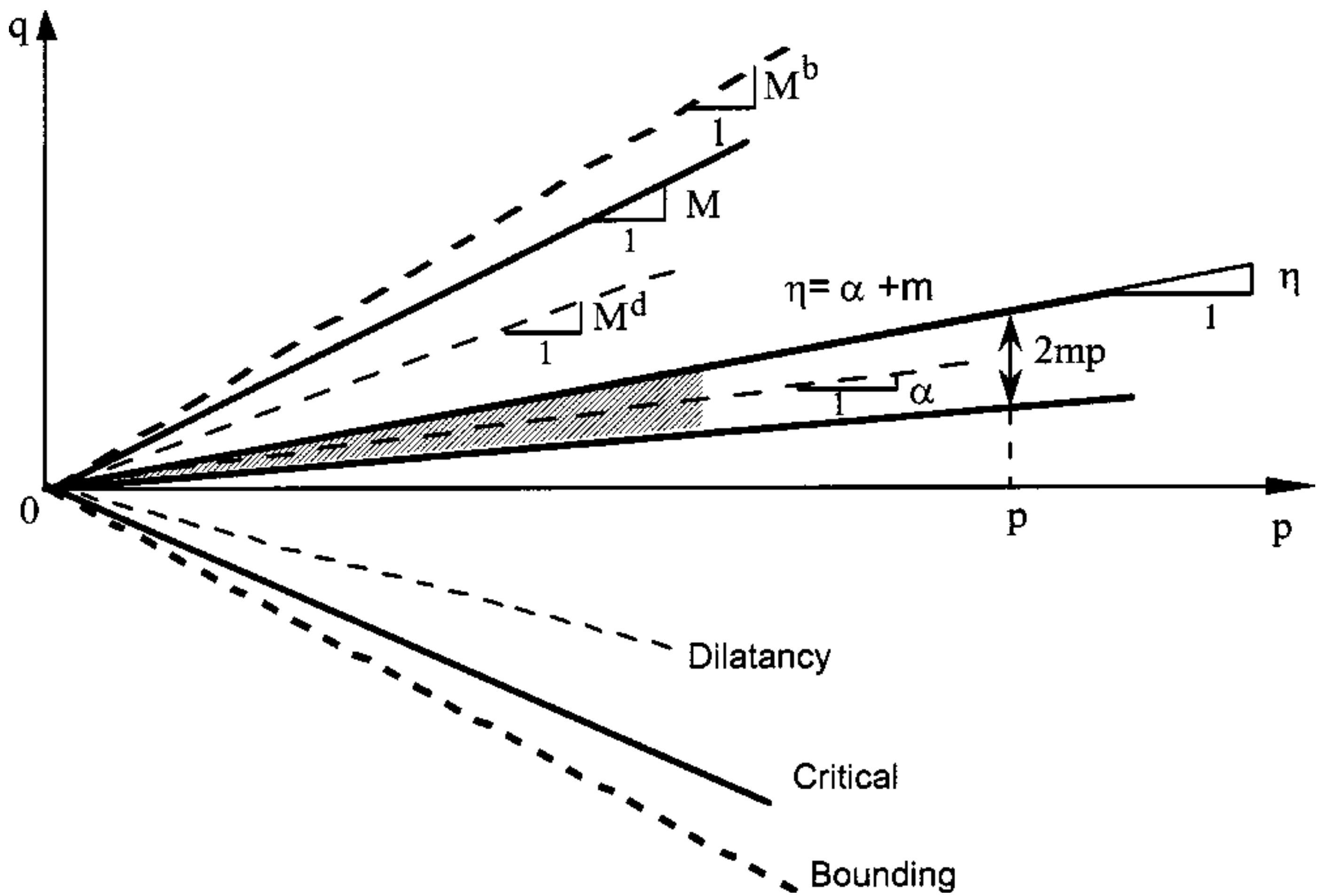
$$\dot{\alpha} = \langle L \rangle c p_{at} \frac{p}{p_0}[\frac{\eta}{x} - \alpha]$$

(Dafalias, 1986)



Does RH for sands suffices to characterize fabric ?

Answer: take α=0; yet sand can have anisotropic fabric by means of deposition => needs a Fabric Tensor



CONCLUSION

The RH variable *α* is the macroscopic manifestation of fabric in clays. Its evolution towards a unique CS value guarantees uniqueness of CSL in e-p space

RH Rule	Unique CSL	Restricted Rotation	Ko simulation/calibration
Dafalias (1986) (modified)	Yes	**No**	Yes
Wheeler et al (2003)	Yes	**No**	Yes
Dafalias/Taiebat (2013)	Yes	Yes	Yes
Dafalias/Taiebat (2014)	Yes	Yes	Yes

NOTE 1: With the exception of Dafalias/Taiebat (2014), all other RH rules provide necessarily non-zero anisotropic fabric at Critical State

NOTE 2: The NO's can be corrected