Barodesy - A new perspective of hypoplasticity

Aussois September 2014

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University of Innsbruck

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What is your constitutive home?

Ubi patria ibi bene

Embarcation pour Barodesie

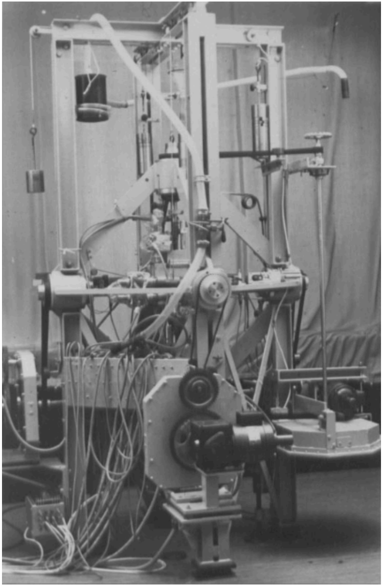
The exotic island of Barodesy

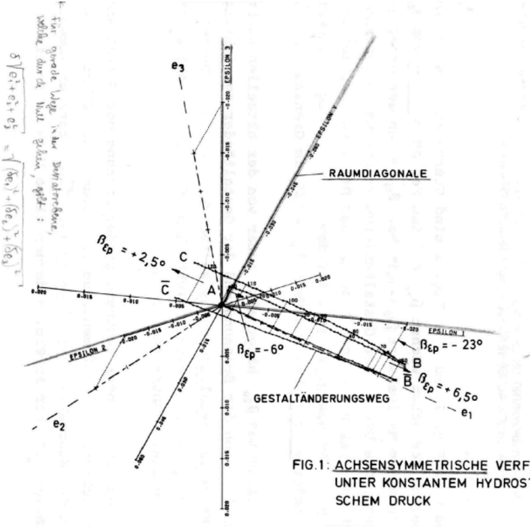
Packing

You don't need to take with you

- yield surface
- plastic potential
- normality rule
- ...

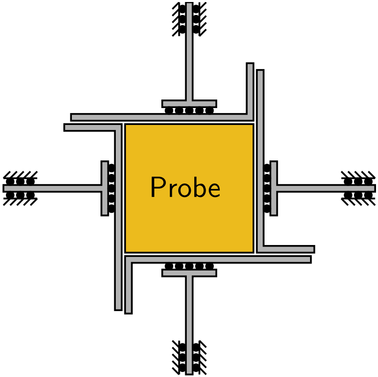
Empirical basis of barodesy: Experiments by Goldscheider




EPSILON 3
RAUMDIAGONALE
EPSILON V
EPSILON 1
EPSILON 2
e_3
e_2
e_1
$\beta_{\varepsilon p}$ = + 2,5°
$\beta_{\varepsilon p}$ = - 6°
$\beta_{\varepsilon p}$ = - 23°
$\beta_{\varepsilon p}$ = + 6,5°
A
B
$\bar{B}$
C
$\bar{C}$
GESTALTÄNDERUNGSWEG
FIG.1: ACHSENSYMMETRISCHE VERF
UNTER KONSTANTEM HYDROS
SCHEM DRUCK

Versuche von Goldscheider[1]

True triaxial tests



[1] M. Goldscheider. Grenzbedingung und Fließregel von Sand. Mechanics Research Communications, 3:463-468, 1976

1. Goldscheider rule

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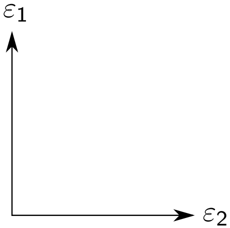

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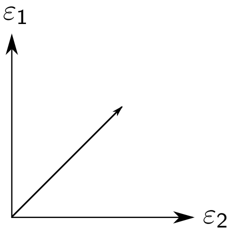

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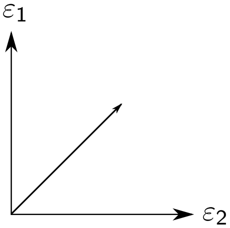

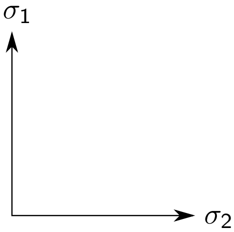

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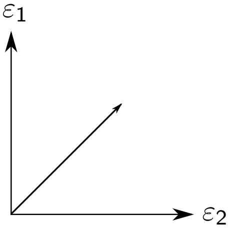

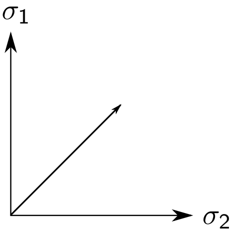

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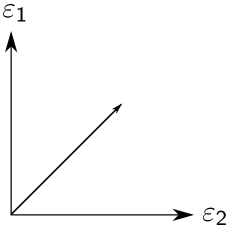

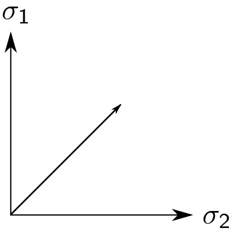

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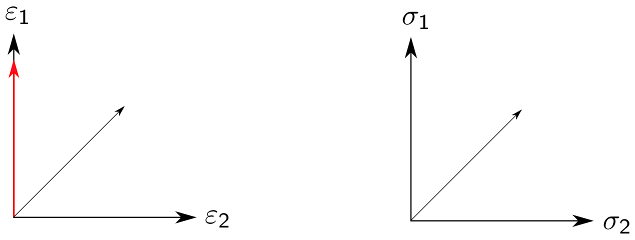

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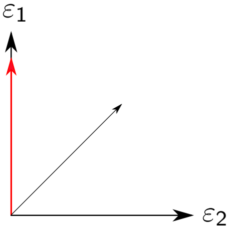

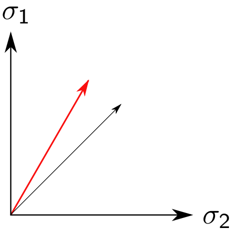

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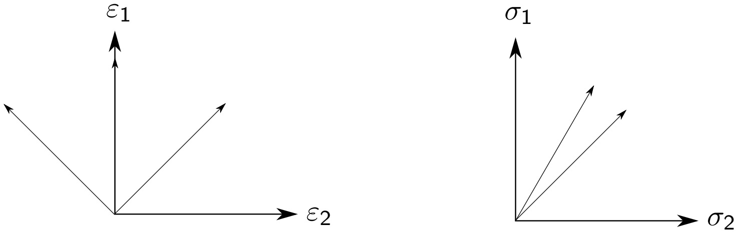

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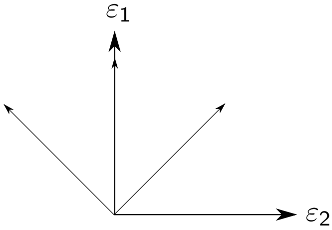

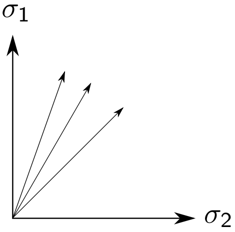

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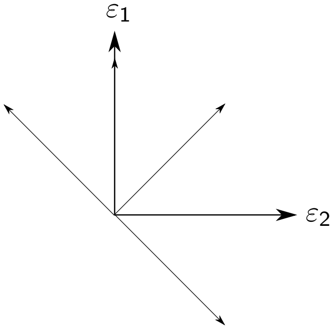

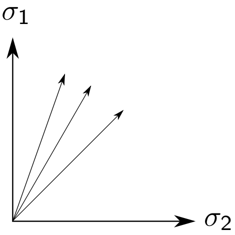

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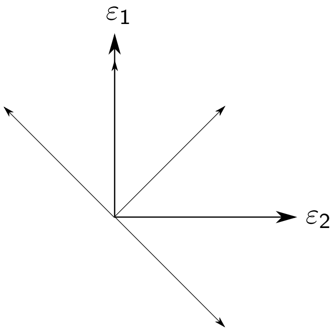

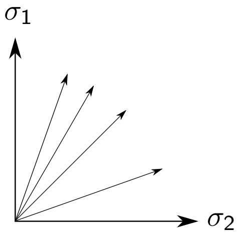

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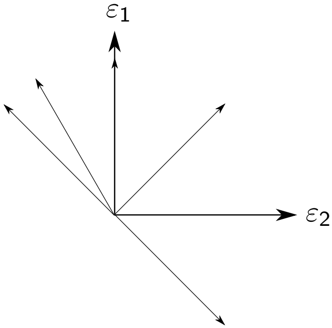

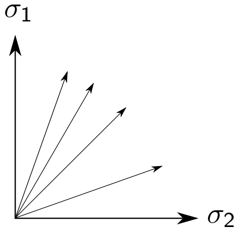

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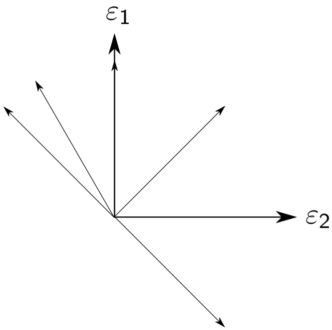

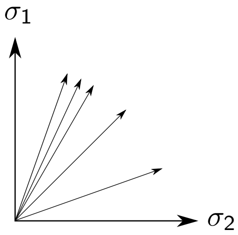

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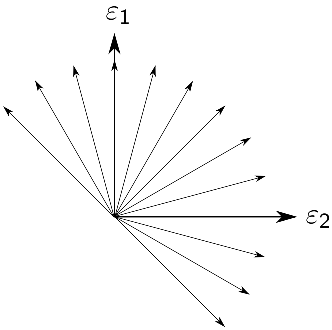

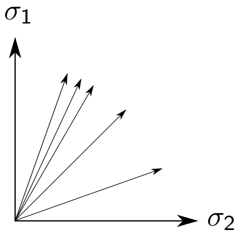

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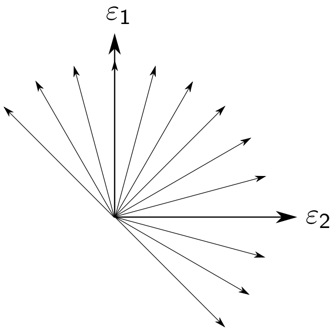

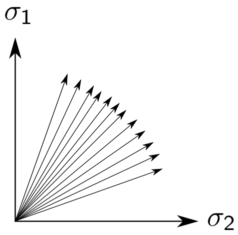

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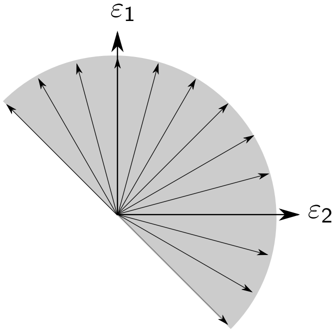

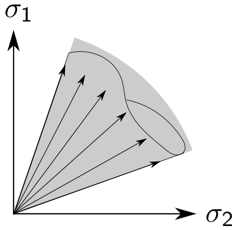

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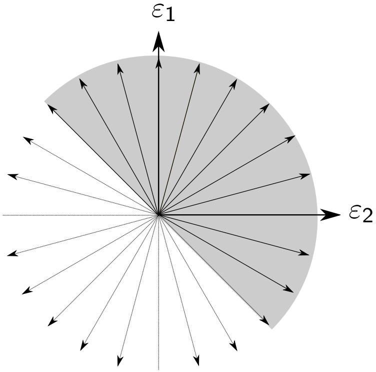

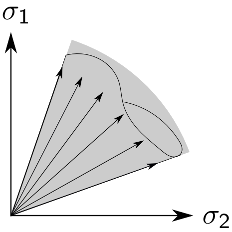

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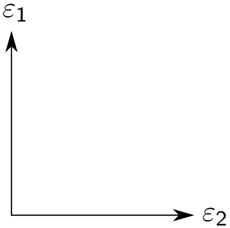

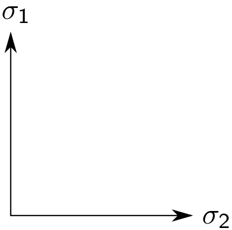

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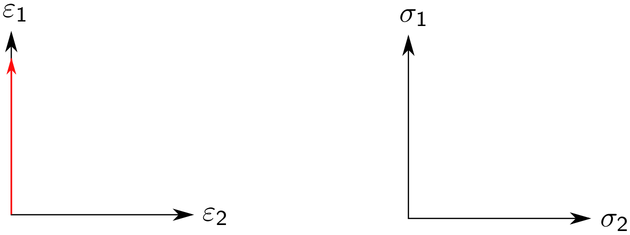

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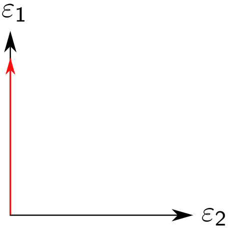

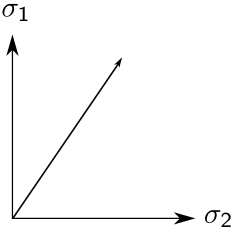

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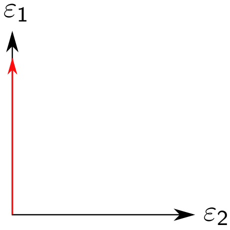

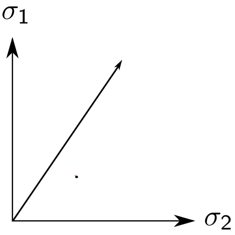

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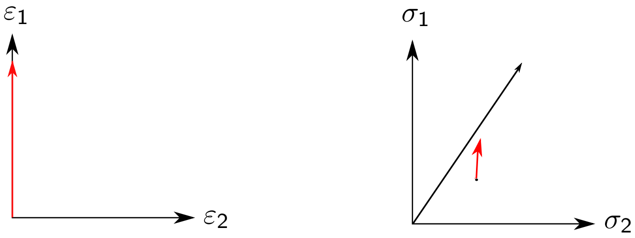

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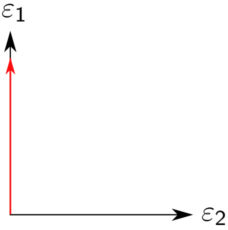

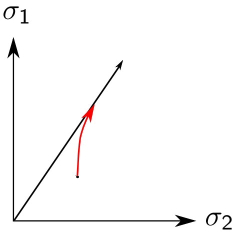

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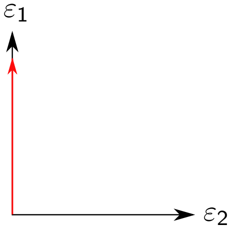

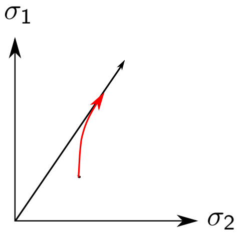

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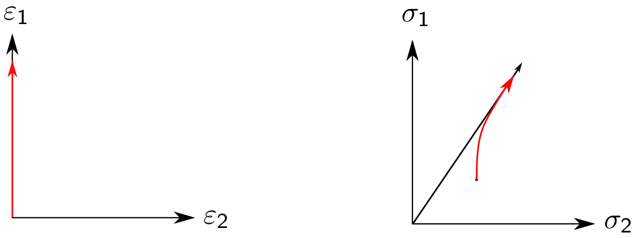

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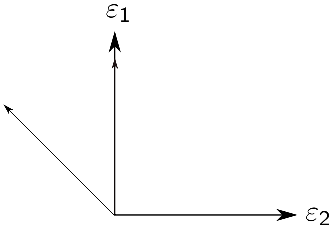

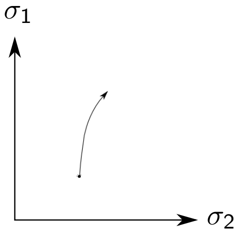

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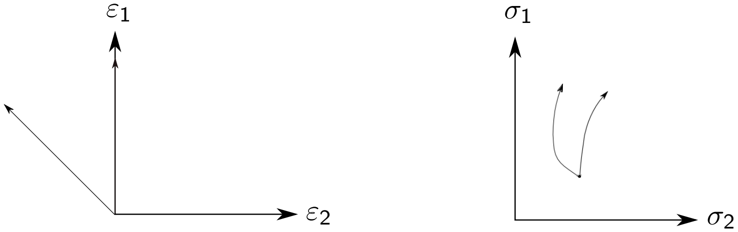

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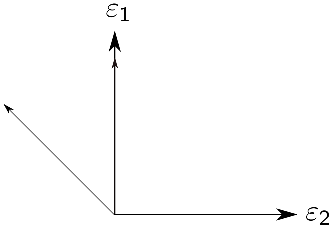

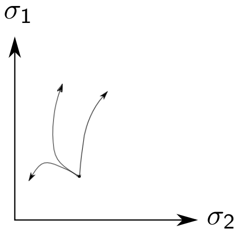

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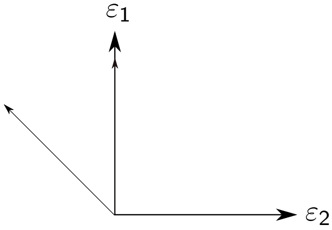

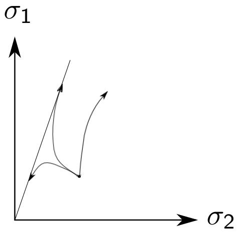

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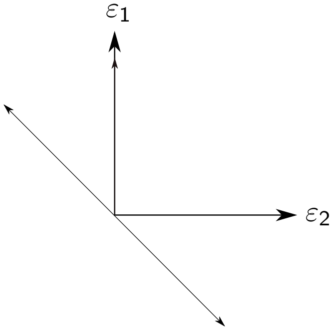

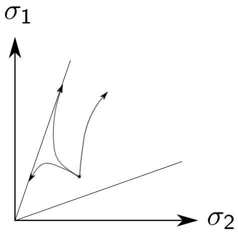

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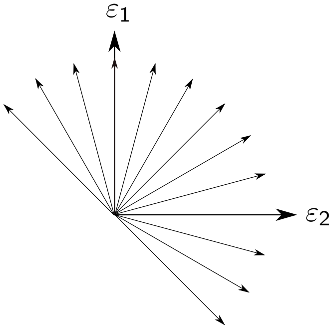

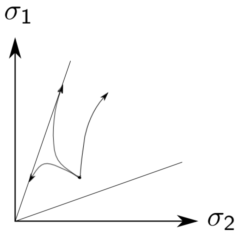

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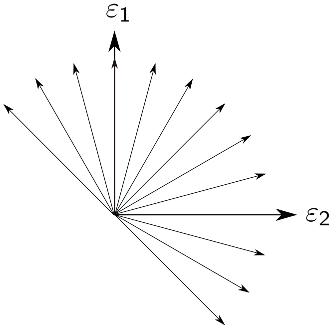

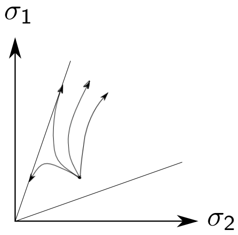

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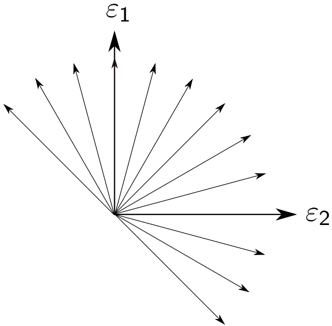

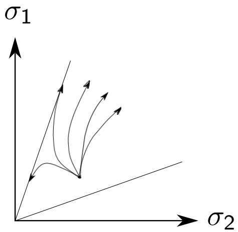

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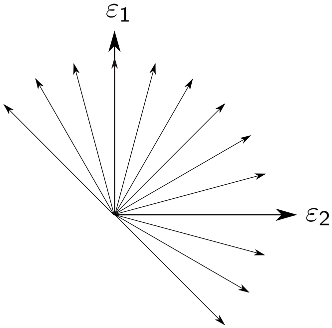

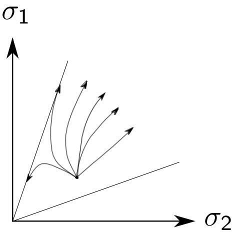

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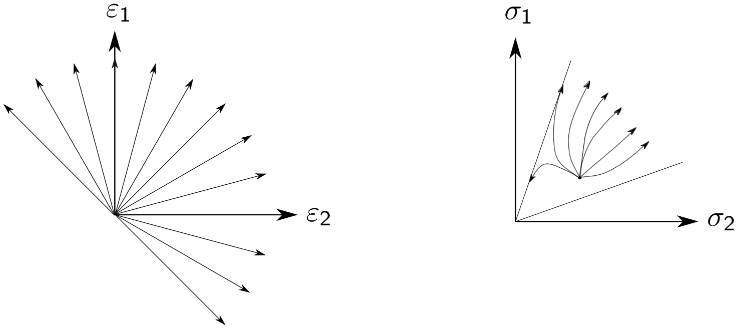

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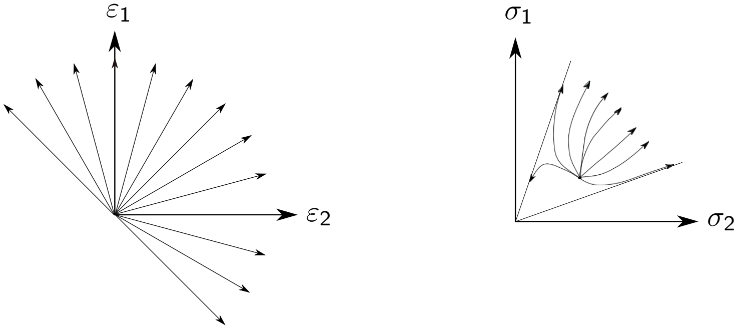

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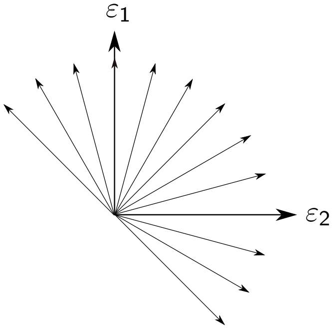

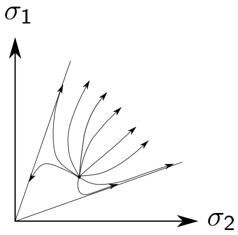

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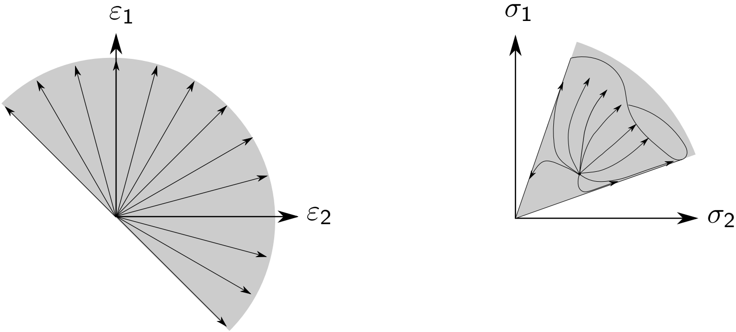

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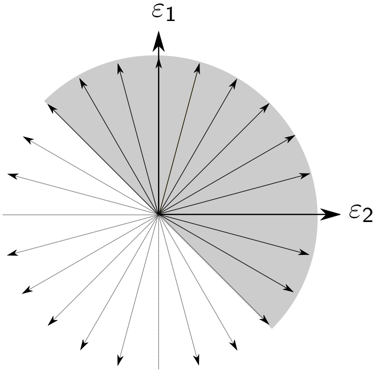

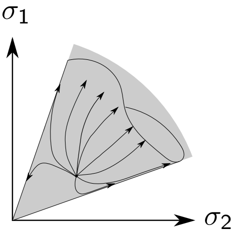

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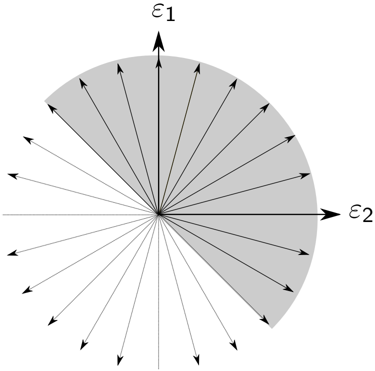

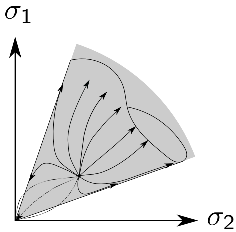

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- volume -preserving ('isochoric' or 'undrained'), tr$\mathbf{D}$$= 0$
- volume -increasing, tr$\mathbf{D}$$> 0$ (not feasible with cohesionless sand!)

Modelling Goldscheider's 1st rule

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$\mathbf{R}$: tensor with the direction of a proportional stress path.

How depends $\mathbf{R}$ on the direction $\mathbf{D}^0 (= \mathbf{D}/|\mathbf{D}|)$ of the corresponding proportional strain path?

How can we determine the relation $\mathbf{R}(\mathbf{D}^0)$?

Consolidations are mapped into the octant with $\sigma_i < 0$.

$$\leadsto \quad \sigma_1 \sigma_2 \sigma_3 < 0$$

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With $\sigma_i = \mu R_i(\mathbf{D}) = \mu R(D_i)$

we obtain:

$$R(D_1)R(D_2)R(D_3) < 0 \qquad \text{for} \quad \mathrm{tr}\mathbf{D} = D_1 + D_2 + D_3 < 0 \tag{1}$$

$\rightsquigarrow$ $R(D_1)R(D_2)R(D_3)$ is function of $D_1 + D_2 + D_3$

Recall: $f(x_1)f(x_2) = f(x_1 + x_2) \rightsquigarrow f(x) = \exp(ax)$

$\rightsquigarrow$ exponential mapping:

$$\mathbf{R}(\mathbf{D}) = -\exp(a\mathbf{D}^0) \tag{2}$$

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Equation (2) maps consolidations into a cone with apex at $\mathbf{T} = \mathbf{0}$

Its boundary (corresponds to paths with tr$\mathbf{D} = 0$):

critical state surface

The intersection of the $\mathbf{R}$-cone with a plane tr$\mathbf{T}$ = const can be derived from equation (2):

For isochoric deformations ($\mathrm{tr}\mathbf{D}^0 = 0$) we eliminate $\mathbf{D}^0$ from (2):

$$\mathbf{D}^0 = \frac{1}{a} \ln(-\mathbf{R}) \; . \tag{3}$$

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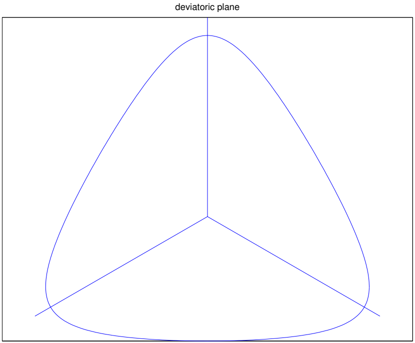

Cross section of the **R**-cone with a deviatoric plane.
Numerically obtained with equ. (2).

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Equation (2) can be calibrated if we know ϕ_c:

$$K_c := \frac{1 - \sin\varphi_c}{1 + \sin\varphi_c} \ .$$

Evaluating the relation $R_2/R_1 = K_c$ with equ. (2) $\rightsquigarrow$:

$$a = \sqrt{\frac{2}{3}} \ \ln K_c \tag{6}$$

Better simulations can be obtained with $a := c_1 \exp(c_2\epsilon)$ and $c_1 = \sqrt{\frac{2}{3}} \ \ln K_c$.

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Adding to **R** a term proportional to **T** will incorporate also rule 2 without breaking rule 1.

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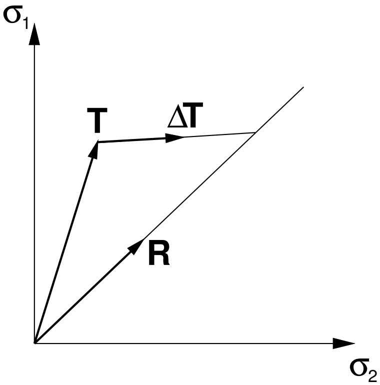

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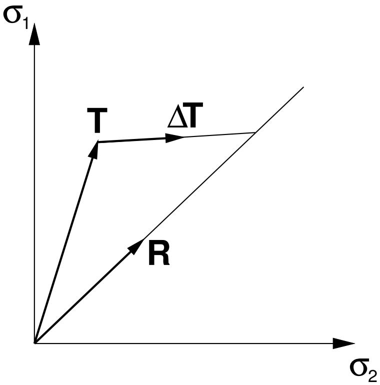

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Thus, $\dot{\mathbf{T}}$ will fulfil the equation $\mathbf{T} + \dot{\mathbf{T}}\ \Delta t = \mu \mathbf{R}^0$.

This equation can also be written as

$$\dot{\mathbf{T}} = \hat{f}\mathbf{R}^0 + \hat{g}\ \mathbf{T}\ . \tag{8}$$

To preserve homogeneity of the 1st degree with respect to $\mathbf{D}$, $\hat{f}$ and $\hat{g}$ must be homogeneous of the 1st degree with respect to $\mathbf{D}$.

It proves that

$$\begin{aligned} \hat{f} &:= f \cdot \dot{\varepsilon} \cdot h(\sigma) \\ \hat{g} &:= g \cdot \dot{\varepsilon} \cdot h(\sigma)/\sigma \end{aligned}$$

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The name 'barodesy' is motivated by the fact that granular materials gain their stiffness ($\delta \acute{\varepsilon} \sigma \iota \varsigma =$ bond, hence stiffening, hardening) from externally applied pressure ($\beta \acute{\alpha} \rho o \varsigma$).

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Limit states and peaks

Limit states: $\dot{\mathbf{T}} = \mathbf{0}$, i.e.

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$$f\mathbf{R}^0 + g\mathbf{T}^0 = \mathbf{0} \tag{10}$$

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Equation (11) can be interpreted as flow rule:
It contains a stress-dilatancy relation for peak states.

A simple way to fulfill equation (12) for limit states is to set

$$f + g = \epsilon + c_3(e_c - e) \tag{13}$$

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Sand: low compressibility at monotonic compression (jamming)
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For proportional paths: $\dot{\mathbf{T}} = \dot{\sigma}\mathbf{T}^0$, hence equation (9) reduces to

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Using equation (13) ($f + g = \epsilon + c_3(e_c - e)$) and

$$\epsilon\,\dot{\varepsilon} = \frac{\mathrm{tr}\mathbf{D}}{\dot{\varepsilon}}\,\dot{\varepsilon} = \mathrm{tr}\mathbf{D} = \frac{\dot{e}}{1+e} \ , \tag{16}$$

we obtain for consolidations:

$$\dot{\sigma} = h\,[\epsilon + c_3(e_c - e)] \cdot \frac{1}{\epsilon} \cdot \frac{\dot{e}}{1+e} \ . \tag{17}$$

or

$$\frac{de}{d\sigma} = \frac{1+e}{1+c_3\frac{e_c-e}{\epsilon}} \cdot \frac{1}{h} \tag{18}$$

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For consolidations: $\mathbf{T}^0 = \mathbf{R}^0$. Hence, equation (9) $\rightsquigarrow$

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Equation (18) expresses compressibility at consolidations.

h expresses the stress-dependence of stiffness.

Stiffness increases with σ but should not vanish for $\sigma = 0$.

e_{min}: lower bound of e.

e_{min} and e_{max} are prescribed by the geometry of the granulate, if we exclude grain crushing.

(they do not necessarily coincide with the conventionally obtained ones).

These requirements can be fulfilled e.g. by the function

$$h = -\frac{c_4 + c_5\sigma}{e - e_{min}} \ . \qquad (19)$$

$e \to e_{min}$ for $\sigma \to \infty \rightsquigarrow e \geq e_{min}$.

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The fraction in equation (17)

$$\frac{e_c - e}{\epsilon} \tag{20}$$

deserves particular attention.

It implies that compressibility depends also on dilatancy ϵ.

We distinguish between dense ($e < e_c$) and loose ($e > e_c$) sand.

Dense sand has a tendency to loosening and loose sand has a tendency to get denser $\rightsquigarrow$ a loose sand is more compressible than a dense one. In accordance, equation (18) implies:

- compressibility of dense sand increases with increasing ϵ
- compressibility of loose sand decreases with increasing ϵ

(Increasing ϵ means increasing deviatoric part of deformation).

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Distinguish between the following compression lines $e(\sigma)$:

red: hydrostatic compession

blue: compression with a large deviatoric component,

$$\mathbf{D} = \begin{pmatrix} -2 & 0 & 0 \\ 0 & 0.7 & 0 \\ 0 & 0 & 0.7 \end{pmatrix} \qquad (21)$$

green: critical state line, $e_c(\sigma)$

starting at a

dense state: $e = 0.75 < e_{c0} (= 0.90)$

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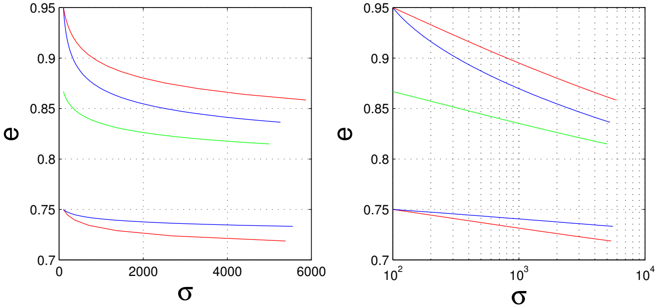
Consolidations with various deviatoric parts



Compression curves calculated with barodesy, calibrated as shown

Consolidations with various deviatoric parts

Deviatoric part increases the compressibility of contractant sand.
Deviatoric part decreases the compressibility of dilatant sand.

Interesting implication of equation (18) is obtained when the denominator vanishes:

This is obtained for a particular void ratio $e = \hat{e}$, that depends on ϵ.

For this void ratio, the slope $de/d\sigma$ becomes vertical, and for $e > \hat{e}$ this slope becomes negative.

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Special case of equation (18) for $e \to e_c$ and $\epsilon \to 0$:

The fraction then reads 0/0.

The compression curve is then the Critical State Line (CSL).

To determine the value of the fraction in the limit, we check, which limit transition prevails.

The transition $e \to e_c$ can occur before the transition $\epsilon \to 0$, and thus prevails.

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$$\lim_{e \to e_c, \epsilon \to 0} \frac{e_c - e}{\epsilon} = 0 \tag{22}$$

Introducing into equation (18) $\rightsquigarrow$ differential equation for the CSL:

$$\frac{d\sigma}{de_c} = -\frac{c_4 + c_5\sigma}{(1 + e_c)(e_c - e_{min})} \tag{23}$$

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The transition $e \to e_c$ can occur before the transition $\epsilon \to 0$, and thus prevails.

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$$\lim_{e \to e_c, \epsilon \to 0} \frac{e_c - e}{\epsilon} = 0 \tag{22}$$

Introducing into equation (18) $\rightsquigarrow$ differential equation for the CSL:

$$\frac{d\sigma}{de_c} = -\frac{c_4 + c_5\sigma}{(1 + e_c)(e_c - e_{min})} \tag{23}$$

Special case of equation (18) for $e \to e_c$ and $\epsilon \to 0$:

The fraction then reads $0/0$.

The compression curve is then the Critical State Line (CSL).

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"Isochoric consolidations"

Integration $\rightsquigarrow$ equation of the CSL :

$$e_c(\sigma) = \frac{e_{min} + B}{1 - B} \tag{24}$$

with the abbreviation

$$B := \frac{e_{c0} - e_{min}}{e_{c0} + 1} \left(\frac{c_4 + c_5 \sigma}{c_4} \right)^{-\frac{1+e_{min}}{c_5}} . \tag{25}$$

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Critical State Line

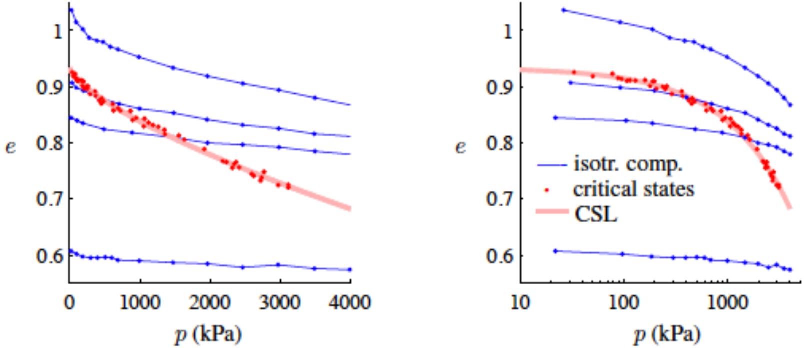

Figure 3.32: Isotropic compression curves starting from different relative densities and critical states for Toyoura Sand; data digitalized from Ishihara and Verdugo

Final step: partitioning equation (13) into f and g.

A possibility is:

$$f = \epsilon + c_3 e_c \tag{26}$$

$$g = -c_3 e \tag{27}$$

Hence, barodetic constitutive equation for sand reads:

$$\overset{\circ}{\mathbf{T}} = -\frac{c_4 + c_5\sigma}{e - e_{min}} \left[(\epsilon + c_3 e_c)\mathbf{R}^0 - c_3 e \mathbf{T}^0 \right] \dot{\varepsilon} \ , \tag{28}$$

where $\mathbf{R}$ is given be equation (7) and e_c is given by equation (24) .

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Calibration

Calibration is difficult because neither the experimental results nor the constitutive relation mirror perfectly the reality.

At that, experimental results are burdened by errors.

Therefore, calibration means optimization.

Also for barodesy, an optimized calibration procedure is still missing.

Here: only rough estimation of the material constants for Hostun sand:

$c_1 \approx -1.05$, $c_2 \approx -2.3$, $c_3 \approx 28$, $c_4 \approx 465$ kPa, $c_5 \approx 1$.

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Limit cycles and shake-down

Constitutive relations "of the rate type" are ODE and may exhibit limit cycles or cyclic orbits.

Equation 9 exhibits limit cycles at cyclic loading ('shake-down').

This means that stress cycles lead asymptotically to cyclic changes of void ratio.

In case of e.g. oedometric deformation (but not for conventional triaxial tests), this implies also cyclic strain, i.e. strains due to cyclic stress are bounded, i.e. they do not increase to infinity.

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Consider cyclic oedometric loading:

If σ_1 is periodically changed between a lower and an upper limit, then σ_2, which is bounded, will also become cyclic, i.e. a limit cycle will eventually be obtained in the stress space.

Cyclic stress, asymptotically obtained with strain cycles of infinitesimally small amplitude, is related with the void ratio $\check{e}$, which can be called the cyclic void ratio.

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$$(f^{+}\mathbf{R}^{0+} + f^{-}\mathbf{R}^{0-}) + 2g\mathbf{T}^{0} = \mathbf{0} \ . \tag{29}$$

This is a relation between the direction of the strain amplitude, $\mathbf{D}$, the cyclic void ratio $\check{e}$ and the stress $\sigma\mathbf{T}^{0}$, around which the stress oscillation occurs. Eliminating $\mathbf{T}^{0}$ from equation 29 yields:

$$\mathbf{T}^{0} = \frac{-1}{2g}(f^{+}\mathbf{R}^{0+} + f^{-}\mathbf{R}^{0-}) \ . \tag{30}$$

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$$(f^{+}\mathbf{R}^{0+} + f^{-}\mathbf{R}^{0-}) + 2g\mathbf{T}^{0} = \mathbf{0} \ . \tag{29}$$

This is a relation between the direction of the strain amplitude, $\mathbf{D}$, the cyclic void ratio $\check{e}$ and the stress $\sigma\mathbf{T}^0$, around which the stress oscillation occurs. Eliminating $\mathbf{T}^0$ from equation 29 yields:

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Limit cycles and shake-down

With this equation and the condition $|\mathbf{T}^0| = 1$ we can determine for a given $\mathbf{D}^0$ the stress direction $\mathbf{T}^0$ of the corresponding cyclic state and also the pertaining cyclic void ratio $\check{e}(\sigma)$.

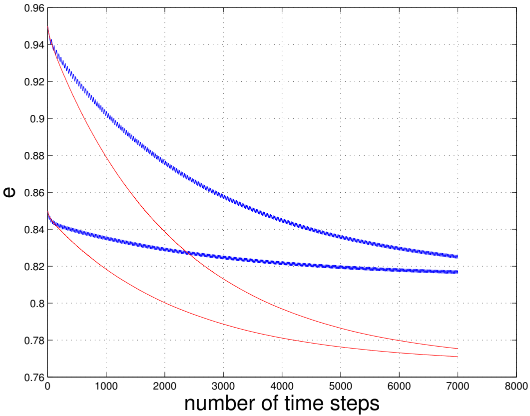

Figure : Evolution of void ratio with small stress cycles. Blue:

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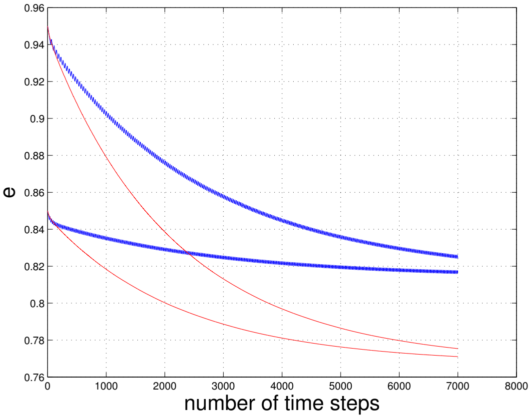

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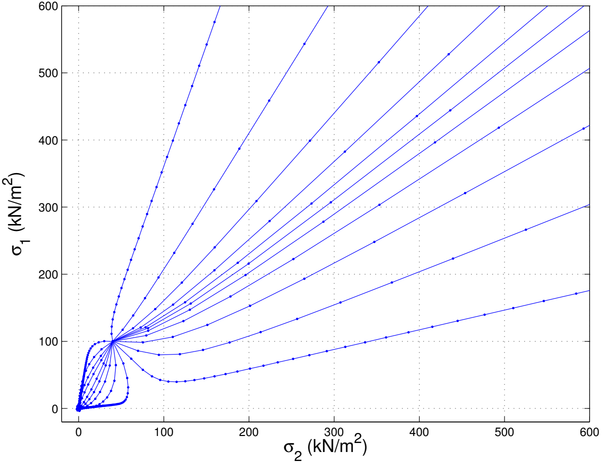
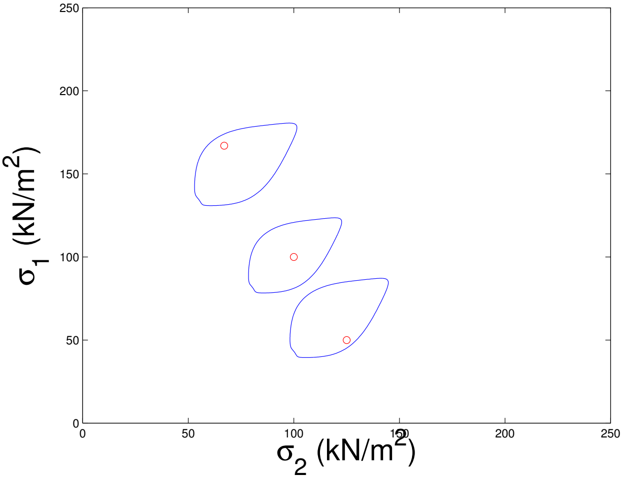
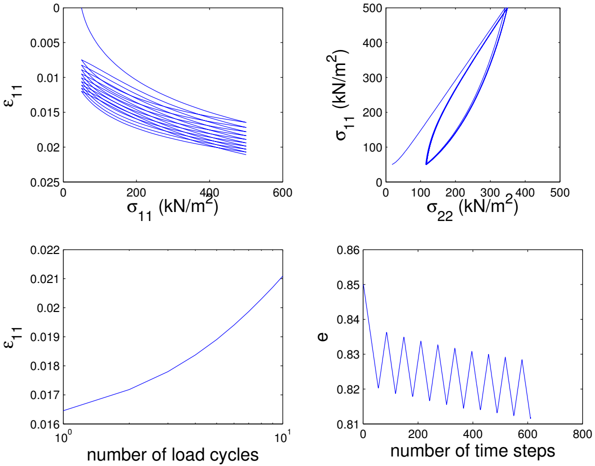
Stress paths for PεP



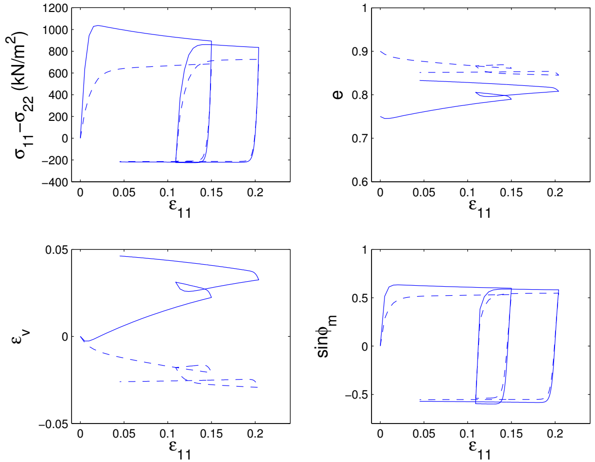
Response envelopes



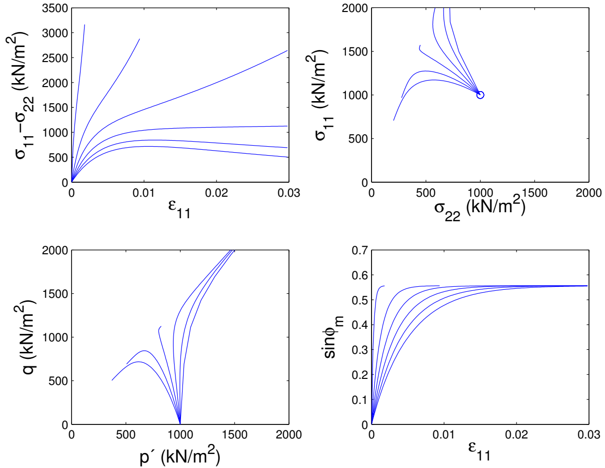
Simulation of oedometer test



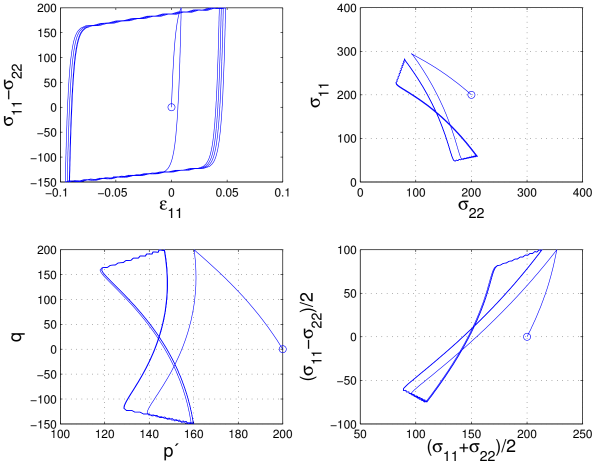
Simulation of drained triaxial test



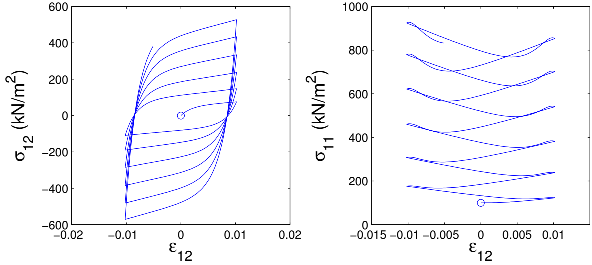
Simulation of undrained triaxial test



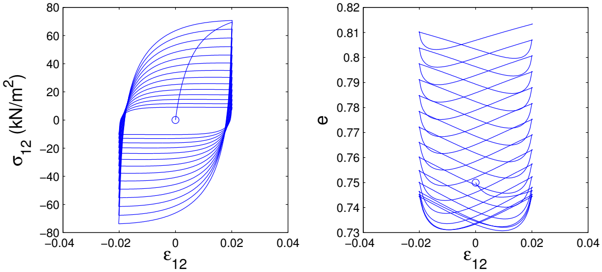
Simulation of cyclic undrained triaxial test



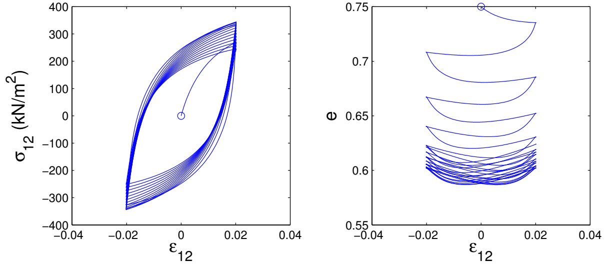
Simulation of undrained simple shear test



Simulation of drained simple shear test



Simulation of drained simple shear test



Thank you!