ALERT Geomaterials

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Multiphysics couplings and stability in geomechanics

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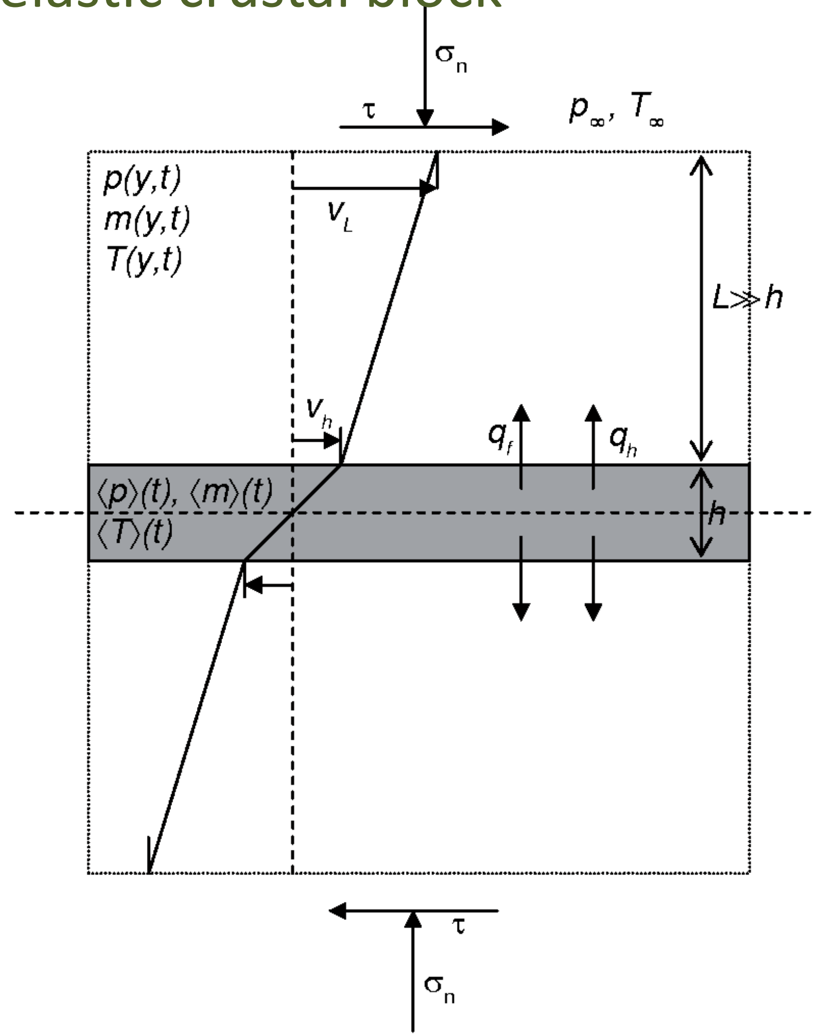
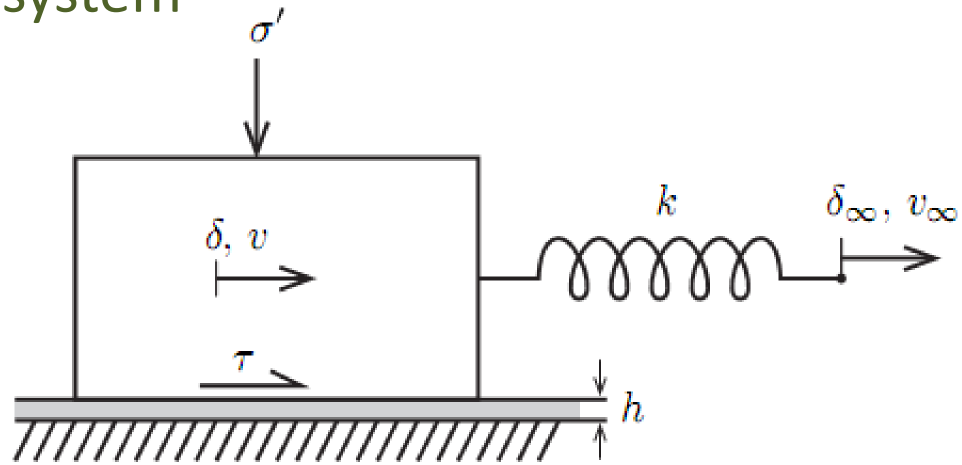
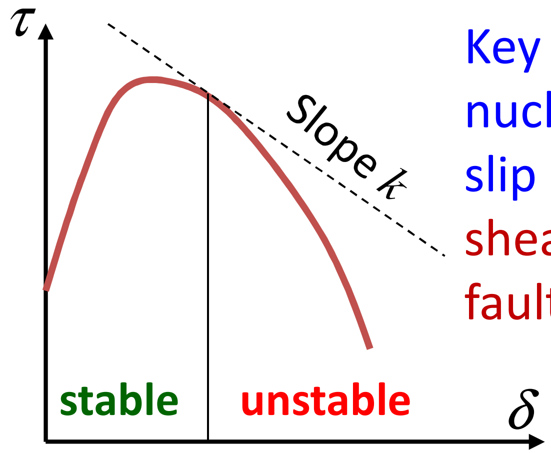
Seismic event : Unstable slip along a pre-existing mature fault (stick-slip instability)

Fault zone embedded in an elastic crustal block



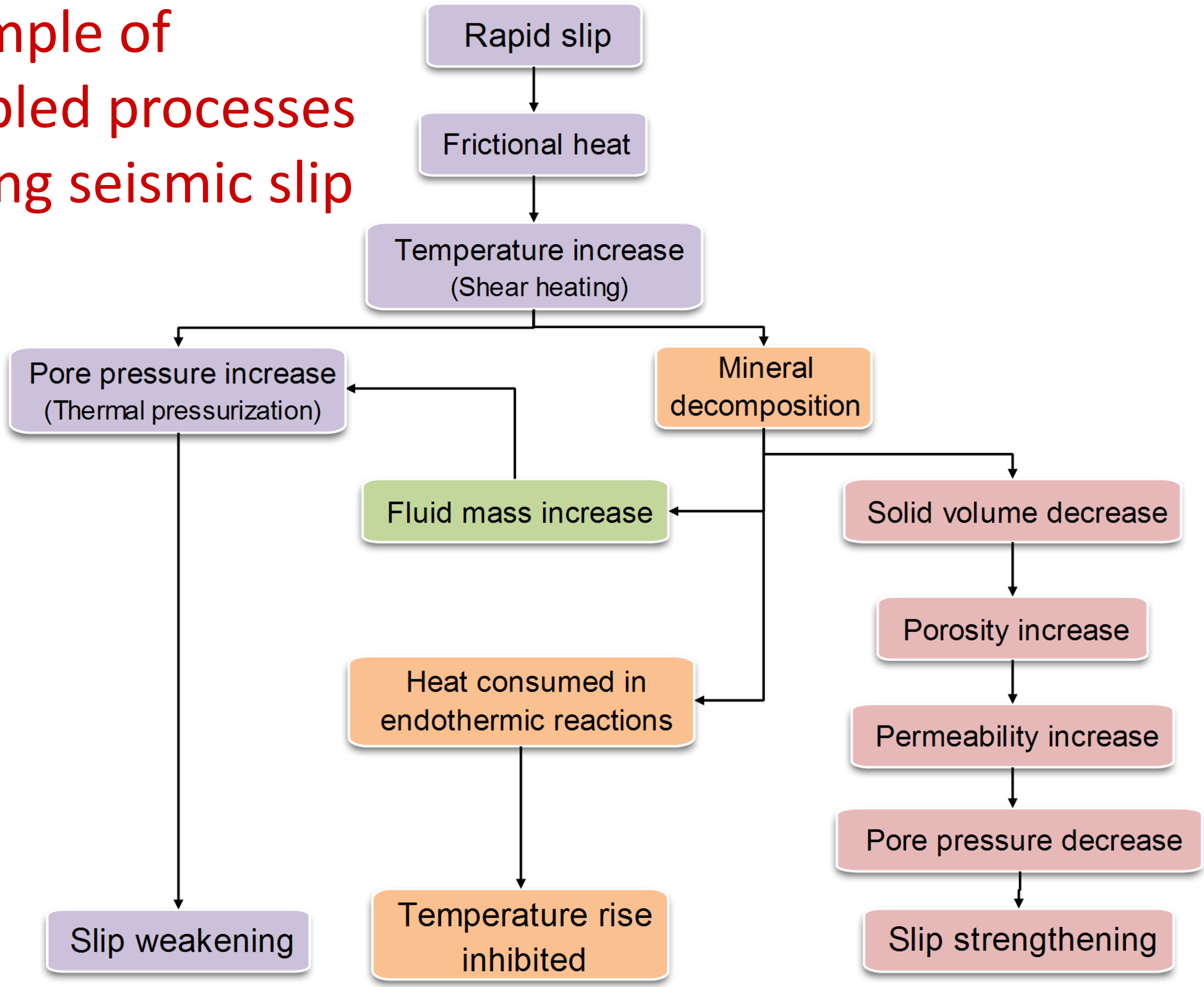
Conceptual model of a spring slider system

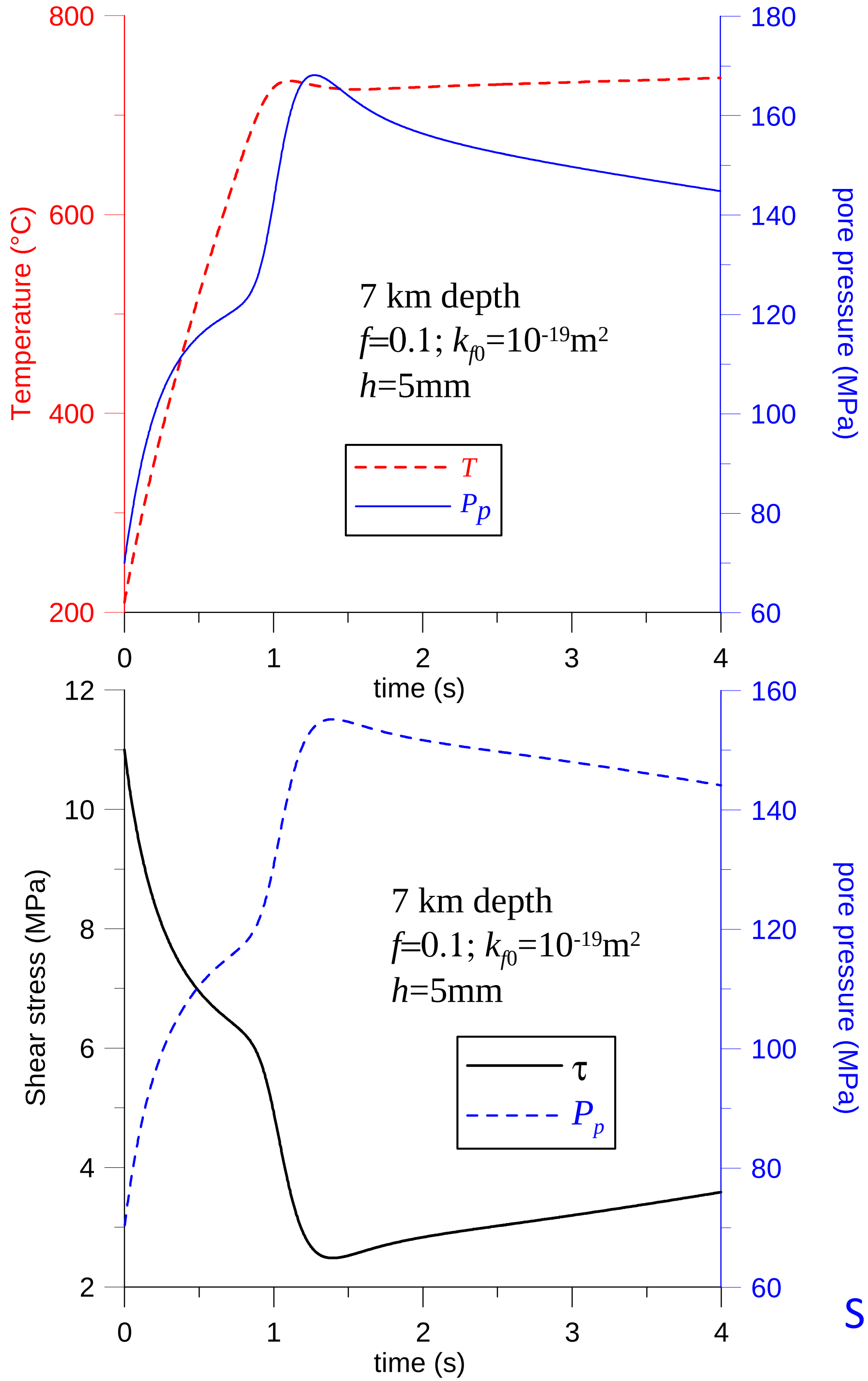




Key factor for the nucleation of seismic slip : Softening of the shear resistance of the fault zone

Example of coupled processes during seismic slip





Thermal decomposition of carbonates in fault zones: Slip-weakening and temperature-limiting effects

-The endothermic chemical reaction limits the co-seismic temperature increase

- Pore pressure exhibits a maximum and then decreases due to the reduction of solid volume (pore pressure pulse)

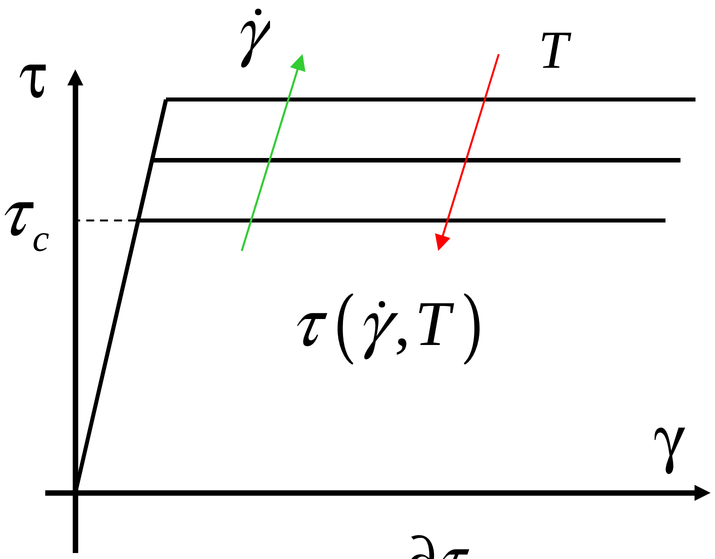
- Weakening/restrengthening of the shear stress

Sulem & Famin, (2009), *J. Geoph. Res.*

A key parameter :
Size of the localized zone

ADIABATIC SHEAR BANDING

Adiabatic shear-banding (1/3)



Strain rate hardening

and

Temperature softening

$$H = \frac{\partial \tau}{\partial \dot{\gamma}} \; (>0)$$: strain rate hardening modulus

$$\xi = \frac{\partial \tau}{\partial T} \; (<0)$$: temperature softening coefficient

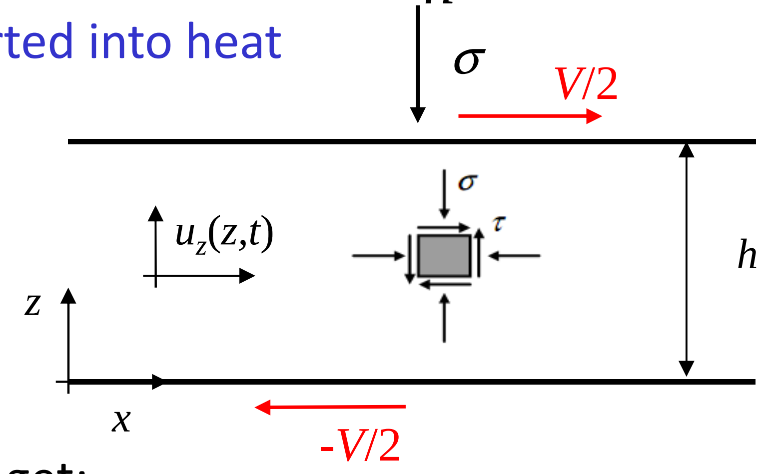
Simple model: $\tau = \tau_0 + H(\dot{\gamma} - \dot{\gamma}_0) + \xi(T - T_s)$

T_s : threshold for temperature softening

Adiabatic shear-banding (2/3)

We consider a shear experiment at constant strain rate $\dot{\gamma}_0 = \dfrac{V}{h}$

Assumption: all the frictional energy is converted into heat



$$\rho C\dot{T} = \tau\dot{\gamma}_0 = \left(\tau_0 + \xi\left(T - T_s\right)\right)\dot{\gamma}_0$$

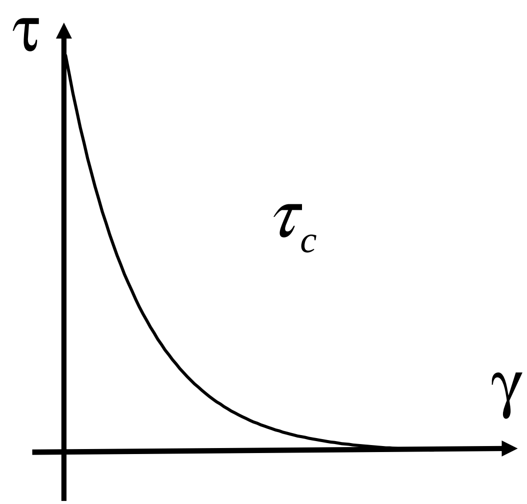
$$\text{at } t = 0 : T = T_s \text{ and } \tau = \tau_0$$

ρC : specific heat

Integrating the above differential equation we get:

$$T = T_s + \frac{\tau_0}{-\xi}\left(1 - \exp\left(\frac{\xi\dot{\gamma}_0}{\rho C}t\right)\right)$$

$$\tau_c = \tau_0 \exp\left(\frac{\xi\dot{\gamma}_0}{\rho C}t\right) = \tau_0 \exp\left(\frac{\xi}{\rho C}\gamma\right)$$



$$\text{for } t \to \infty : T \to T_s + \frac{\tau_0}{-\xi} \text{ and } \tau \to 0$$

Linear stability analysis of adiabatic shear-banding (1/5)

To study the stability of adiabatic shear banding we perform a linear perturbation analysis of the uniform solution

$$T(z,t) = \bar{T}(t) + T_1(z,t)$$

$$V(z,t) = \dot{\gamma}_0 z + V_1(z,t)$$

$$\tau(z,t) = \bar{\tau}(t) + \tau_1(z,t)$$

$$T_1(z,t) = A\exp(st)\exp\left(2\pi i\frac{z}{\lambda}\right)$$

$$V_1(z,t) = B\exp(st)\exp\left(2\pi i\frac{z}{\lambda}\right)$$

$$\tau_1(z,t) = D\exp(st)\exp\left(2\pi i\frac{z}{\lambda}\right)$$

s is the growth coefficient in time of the instability

$\lambda = h/N$ is the wave length of the instability (N=1,2,… wave number)

Stability analysis of adiabatic shear-banding (2/5)

Governing equations (with heat diffusion term)

Constitutive equation:

$$\tau = \tau_0 + H\left(\dot{\gamma} - \dot{\gamma}_0\right) + \xi\left(T - T_s\right)$$

Energy balance:

$$\rho C\left(\dot{T} - c_{th}\frac{\partial^2 T}{\partial z^2}\right) = \tau\dot{\gamma}$$

Momentum balance:

$$\frac{\partial \tau}{\partial z} = \rho\frac{\partial V}{\partial t}$$

$$\begin{pmatrix} \xi & H\dfrac{2\pi i}{\lambda} & -1 \\ \rho C\left(s + c_{th}\left(\dfrac{2\pi}{\lambda}\right)^2\right) & -\tau_0\dfrac{2\pi i}{\lambda} & -\dot{\gamma} \\ 0 & \rho s & -\dfrac{2\pi i}{\lambda} \end{pmatrix}\begin{pmatrix} A \\ B \\ D \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

c_{th} : thermal diffusivity

For non trivial solution : $(A,B,C) \neq (0,0,0)$

$$\rho C\rho s^2 + \left(\rho C\left(\rho c_{th} + H\right)\left(\frac{2\pi}{\lambda}\right)^2 - \xi\rho\dot{\gamma}_0\right)s + \xi\tau_0\left(\frac{2\pi}{\lambda}\right)^2 + \rho C c_{th} H\left(\frac{2\pi}{\lambda}\right)^4 = 0$$

Stability condition: All the roots of the characteristic equation have a negative real part

Stability analysis of adiabatic shear-banding (3/5)

For strain rate and temperature softening ($H < 0$ and $\xi < 0$), the uniform solution is unstable for all wave lengths of the perturbation

For strain rate hardening and temperature softening ($H > 0$ and $\xi < 0$)

Stability condition:

$$\xi\tau_0 + c_{th} H\rho C\left(\frac{2\pi}{\lambda}\right)^2 > 0 \Leftrightarrow \lambda < \lambda_{cr}, \text{ with } \lambda_{cr} = 2\pi\sqrt{\frac{c_{th} H\rho C}{-\xi\tau_0}}$$

Perturbations with wavelengths shorter than the critical value will decay exponentially; those greater than the critical value will grow exponentially.

Only shear zones with a thickness $h < \lambda_{cr}/2$ will support stable homogeneous shear.

Stability analysis of adiabatic shear-banding (4/5)

Numerical example

$$c_{th} = 1\text{mm}^2 / \text{s}$$

$$\rho C = 2.7\text{MPa/°C}$$

$$\tau_0 = 70\text{MPa}$$

$$H = 0.05\text{MPa.s}$$

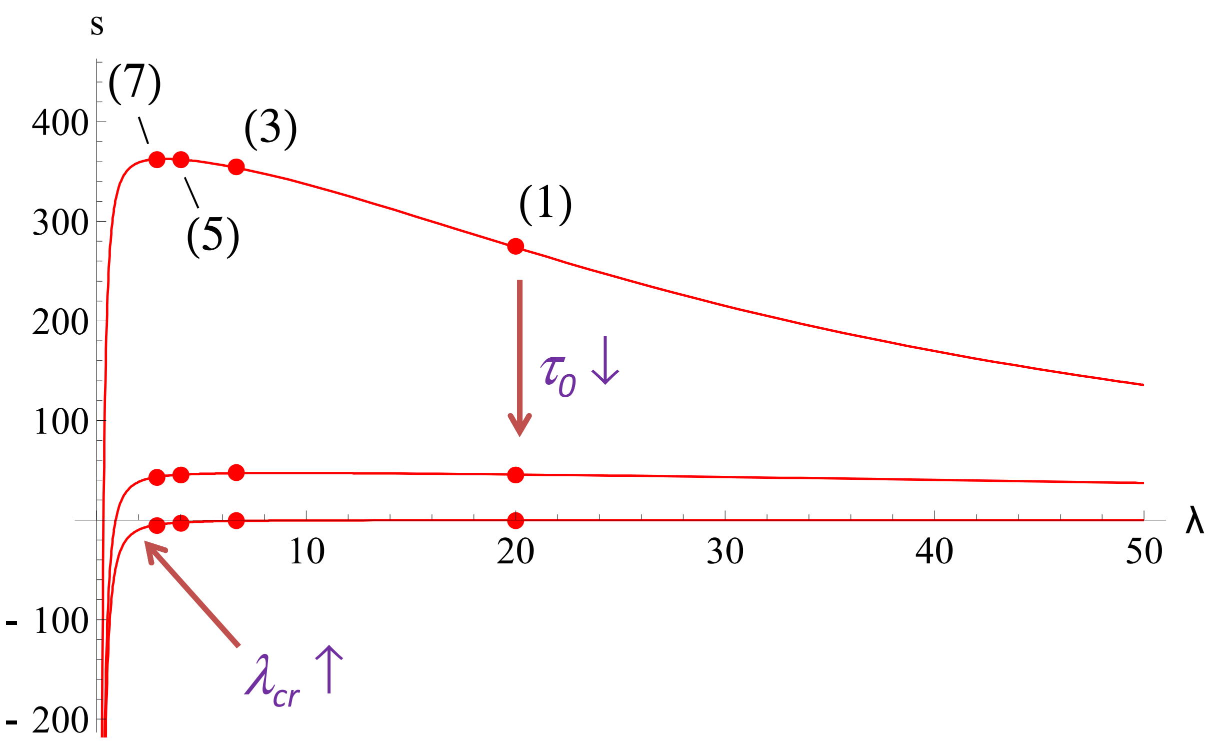
$$\xi = -0.5\text{MPa/°C}$$

$$\lambda_{cr} = 2\pi\sqrt{\frac{c_{th} H \rho C}{-\xi \tau_0}}$$

$$\lambda_{cr}\left(\tau_0 = 70\text{MPa}\right) = 0.39\text{mm}$$

$$\lambda_{cr}\left(\tau_0 \to 0\right) = \infty$$

Growth coefficient *s*

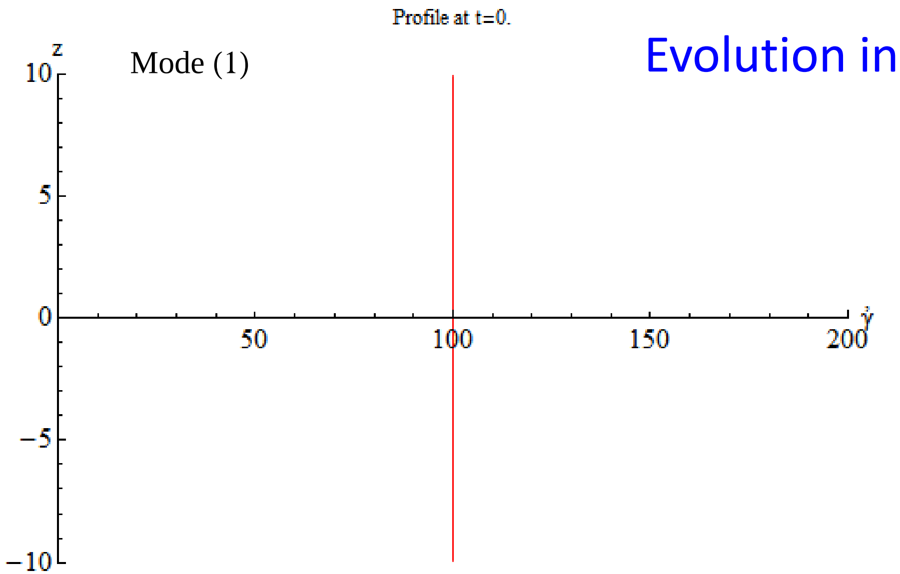


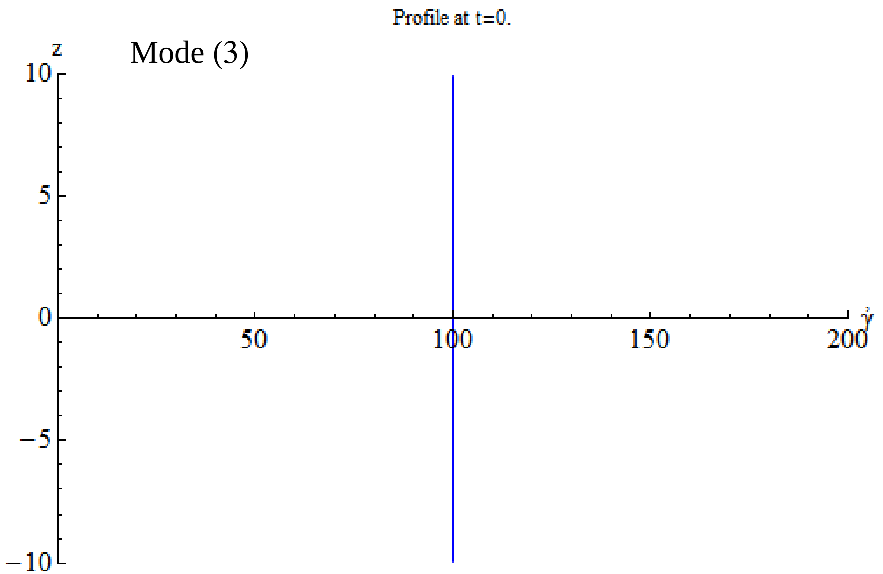
For $\lambda \geq \lambda_{cr}$ the growth coefficient *s* of the instability reaches a maximum for a particular wave length λ

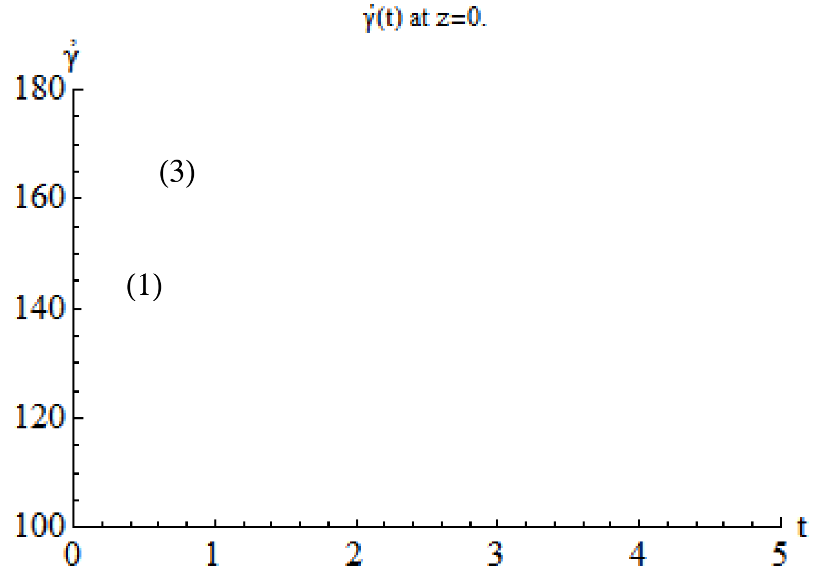
Shear will localize in narrow zones with a size controlled by the wave length with fastest growth coefficient of the instability

Stability analysis of adiabatic shear-banding (5/5)

Evolution in time and space of the strain rate

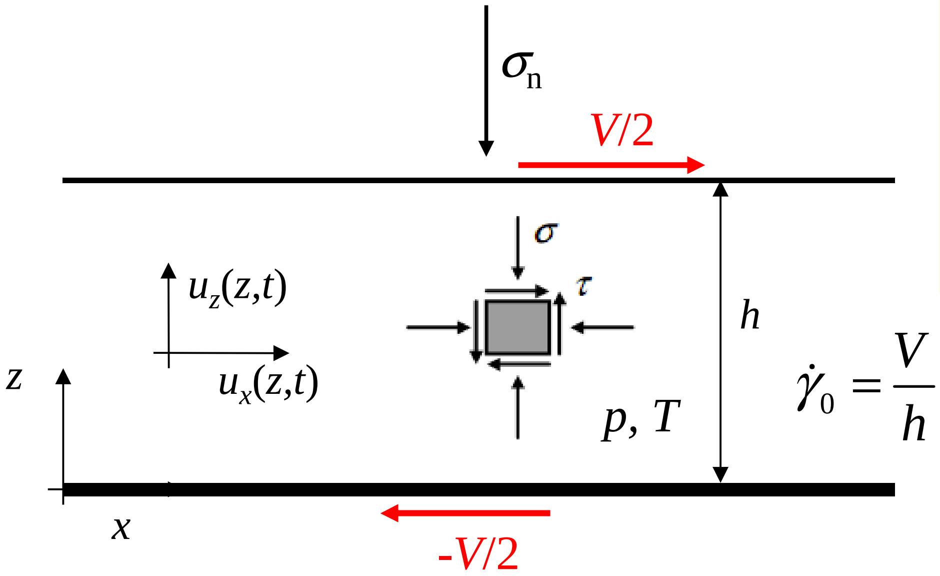






THERMAL PRESSURIZATION OF THE PORE FLUID : A MECHANISM OF THERMAL SOFTENING

Undrained adiabatic shearing of a saturated rock layer



mass balance: $\frac{\partial p}{\partial t} = \Lambda \frac{\partial T}{\partial t}$

energy balance: $\frac{\partial T}{\partial t} = \frac{1}{\rho C}(\sigma_n - p)\, f \frac{V}{h/2}$

$$f = f_0 + H \log \frac{\dot\gamma}{\dot\gamma_0}$$

is the rate-dependent friction coefficient

$$\Lambda = \frac{\lambda_f - \lambda_n}{\beta_n + \beta_f}$$

is the coefficient of thermal pressurization

(typical values: 0.1 to 1 MPa/°C)

Solution:

$$p = p_0 + \left(\sigma_n - p_0\right)\left(1 - \exp\left(-\frac{f\Lambda}{\rho C}\dot\gamma_0 t\right)\right)$$

$$T = T_0 + \frac{\left(\sigma_n - p_0\right)}{\Lambda}\left(1 - \exp\left(-\frac{f\Lambda}{\rho C}\dot\gamma_0 t\right)\right)$$

The pore-pressure increases towards the imposed normal stress σ_n .

In due course of the shear heating and fluid pressurization process, the shear strength τ is reduced towards zero.

Linear stability analysis of undrained adiabatic shearing of a saturated rock layer

Stability condition:

Rice et al., (2014), *J. Geoph. Res.*

$$\lambda < \lambda_{cr}, \text{ with } \lambda_{cr} = 2\pi \sqrt{\frac{H \rho C}{f_0 \Lambda} \frac{\left(c_{th} + c_{hy}\right)}{\left(f_0 + 2H\right)\dot{\gamma}_0}}$$

Only shear zones with a thickness $h < \lambda_{cr}/2$ will support stable homogeneous shear.

Competing processes: *Fluid and thermal diffusion and rate-dependent frictional strengthening tend to expand the localized zone, while thermal pressurization tends to narrow it*.

For representative material parameters at a seismogenic depth of 7km, V=1m/s, h=10mm

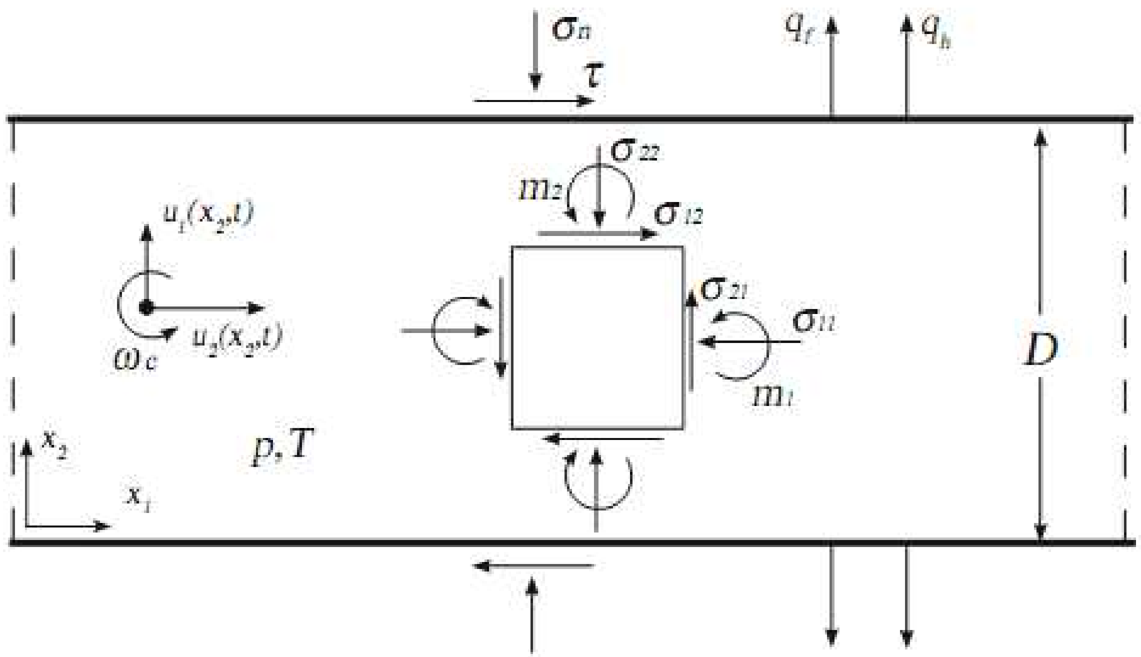
$$\lambda_{cr} / 2 = 3\mu\text{m for intact material, } 23\ \mu\text{m for damaged material}$$

The localized zone thickness may be comparable with the gouge grain size

LOCALIZED ZONE THICKNESS AND MICROSTRUCTURE

Stability analysis of undrained adiabatic shearing of a rock layer with Cosserat microstructure

Sulem, Stefanou, Veveakis, (2011), *Granular Matter*



Strain hardening elasto-plasticity for 2D Cosserat continuum

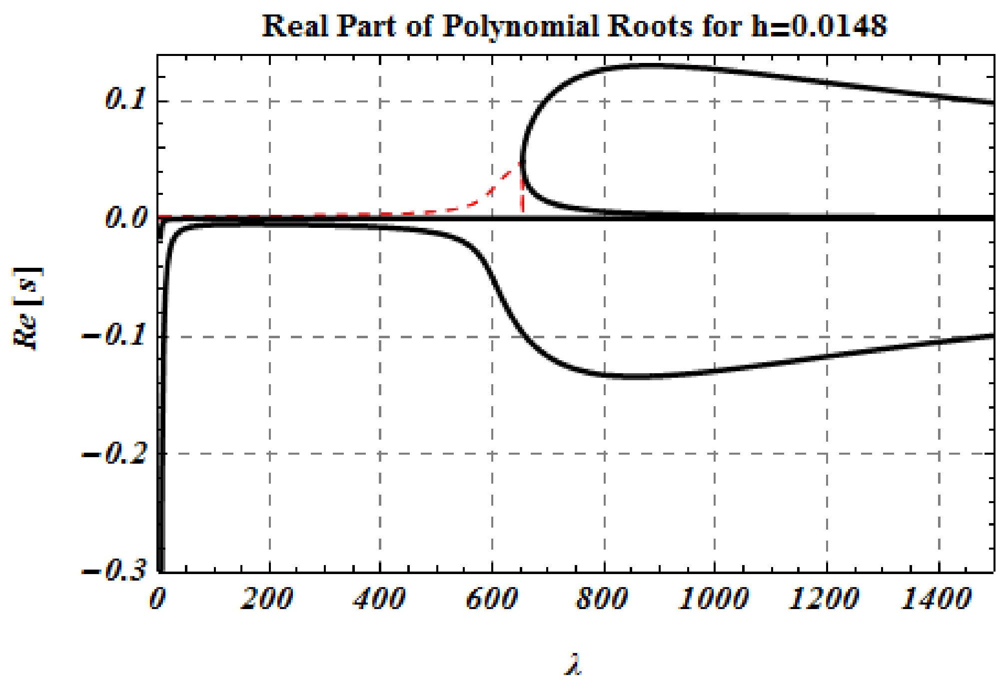
Rock layer at great depth (7km)

Cosserat continuum

– Cosserat terms are active only in the post localization regime

– Microinertia

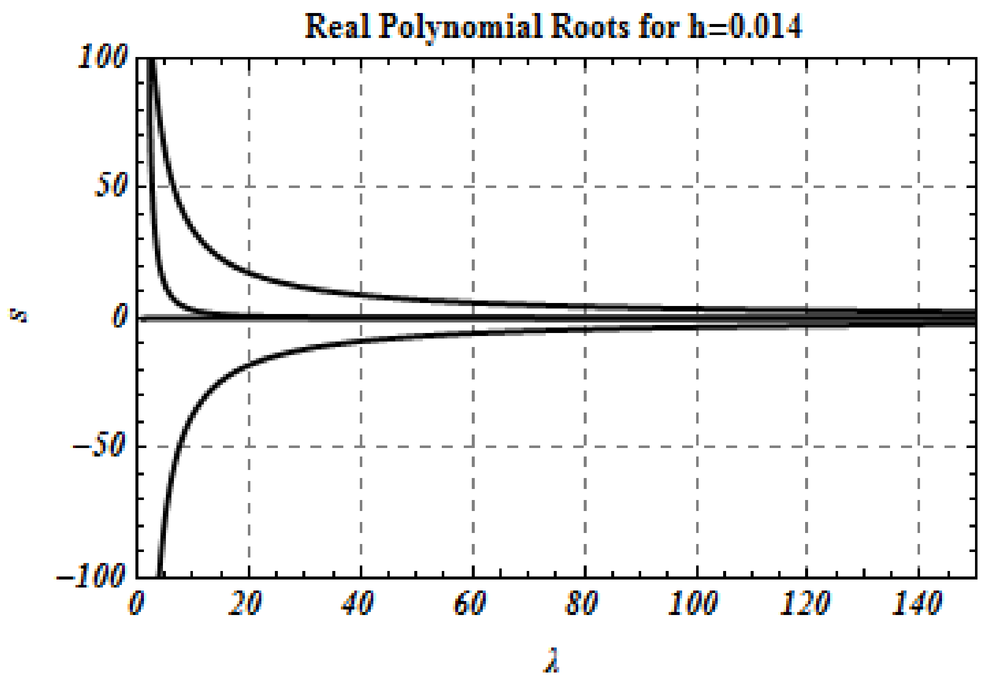
– Wave length selection is obtained (wave length with fastest growth in time)

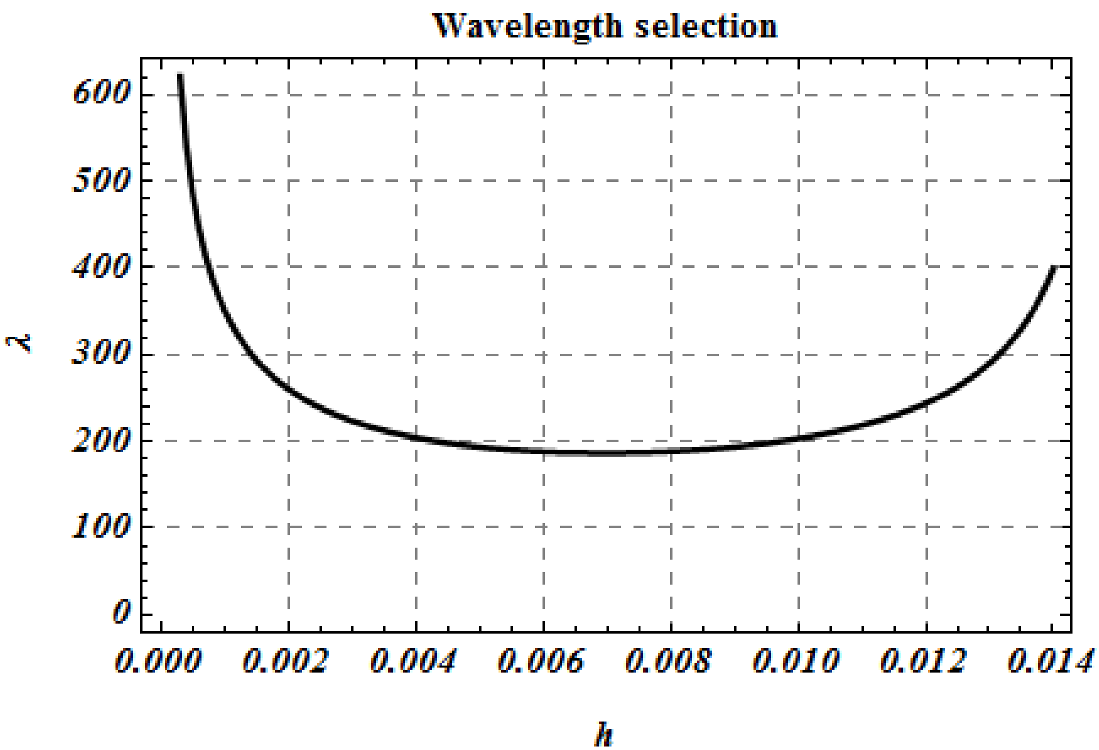


Cauchy continuum

– The critical hardening modulus for instability is unchanged (h_{cr} = 0.015).

–The growth coefficient tends to infinity for the infinitely small wave length limit (ill-posedness).





•The selected wave length decreases with decreasing hardening modulus and reaches a minimum (for λ~200).

λ is the wave length normalized by the internal length of the Cosserat model *R* (grain size).

•With *R*=10μm (grain size for highly finely granulated fault core) the obtained localized zone thickness is about 1mm which is compatible with field observations of localized shear zones in broader damaged fault zones.

Effect of chemical reactions such as thermal decomposition of minerals on the localized zone thickness

Veveakis, Sulem, Stefanou, (2012), *J. Stuct. Geol.*

Mass balance : Pore pressure diffusion and generation

$$\frac{\partial p}{\partial t} = \Lambda \frac{\partial T}{\partial t} + c_{hy} \frac{\partial^2 p}{\partial z^2} + \frac{1}{\rho_f \beta^*} \frac{\partial m_d}{\partial t} - \frac{1}{\beta^*} \frac{\partial n_d}{\partial t}$$

Thermal pressurization

Fluid diffusion

Fluid production

Porosity increase (solid decomposition)

Energy balance

$$\frac{\partial T}{\partial t} = c_{th} \frac{\partial T^2}{\partial z^2} + \frac{1}{\rho C} \tau \frac{\partial v}{\partial z} - \frac{1}{M_{CaCO_3} \rho C} \Delta H^0 \frac{\partial m_d}{\partial t}$$

Heat diffusion

Frictional heat

Heat consumed in the chemical reaction

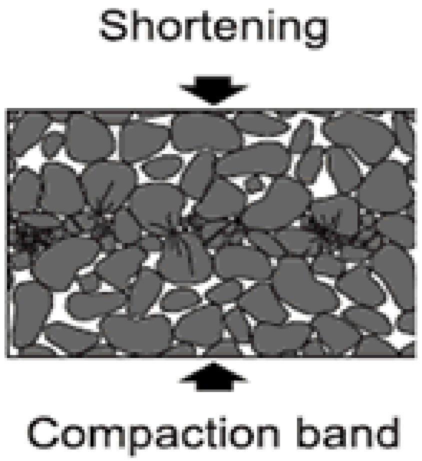
Example: Thermal decomposition of Carbonate

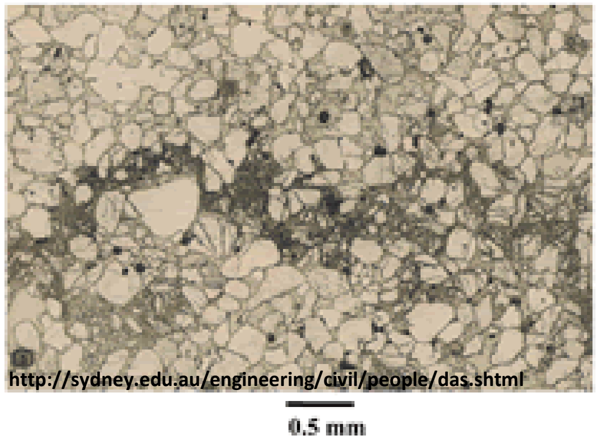
$$\underset{\text{calcite}}{CaCO_3} \longrightarrow \underset{\text{lime}}{CaO} + CO_2$$

For representative material parameters at a seismogenic depth of 7km, a selected wave length of 4-5 * Cosserat length scale is obtained

The shear band thickness is initially controlled by the thermal pressurization and can be reduced when the temperature of the slipping zone reaches the critical temperature of mineral decomposition (about 700°C for carbonates)

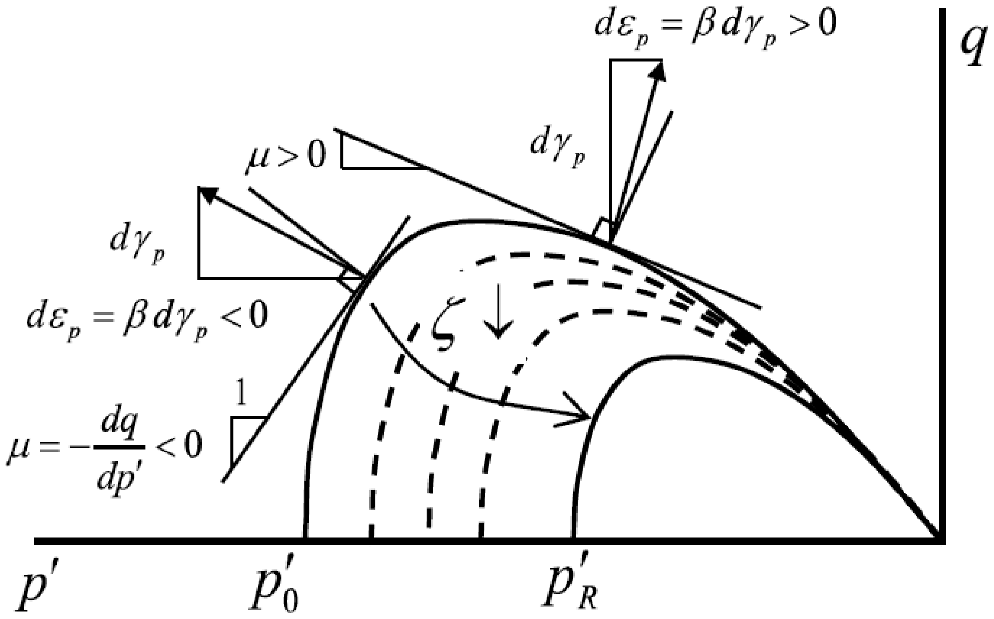
CHEMICAL DEGRADATION AND COMPACTION INSTABILITIES IN GEOMATERIALS





Stefanou & Sulem, (2014), *J. Geoph. Res*

Theoretical framework for deformation bands formation



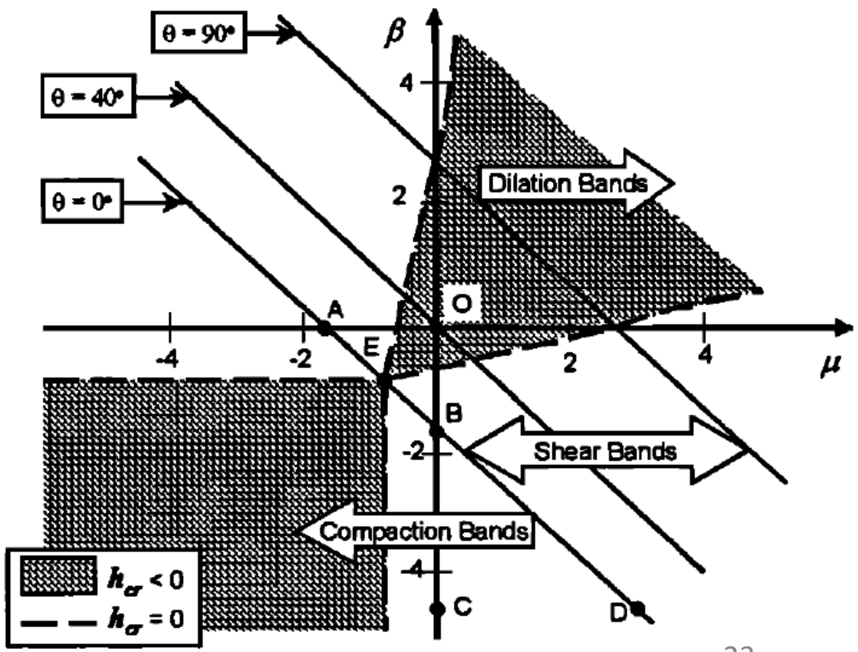
ζ is a chemical softening parameter (e.g. Grain Dissolution)

e.g. Nova et al. (2003), *Int.J.Num.An.Meth.Geom.*
Xie et al. (2011) *Int.J.Rock Mech. Min. Sc,*
Hu & Hueckel (2007), *Int.J.Num.An.Meth.Geom.*
…

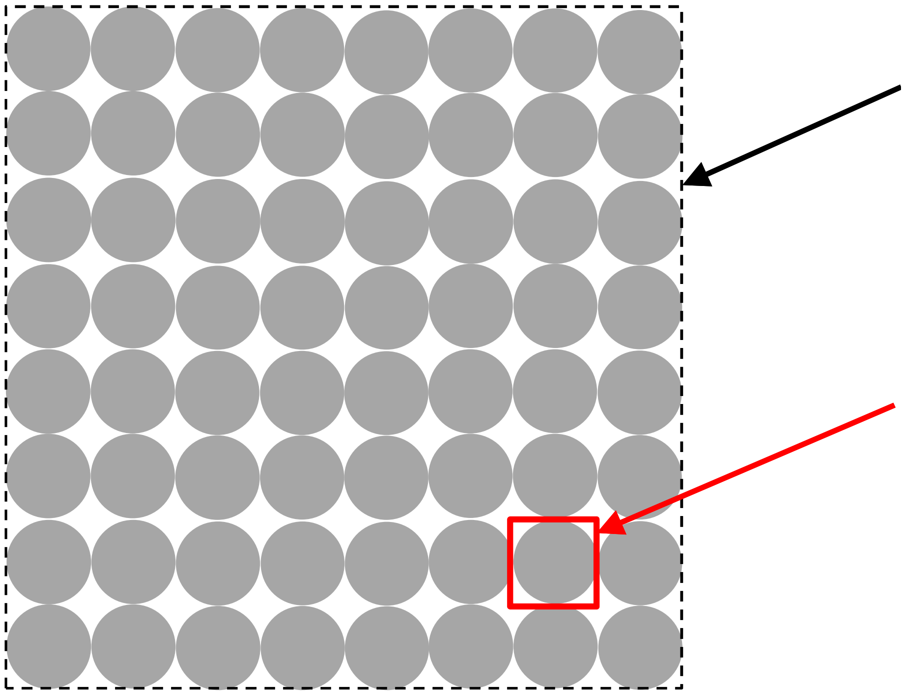
Criterion for compaction bands without dissolution: $\beta + \mu \le -\sqrt{3}$

Associate plasticity: $\beta = \mu = -\frac{\sqrt{3}}{2}$

Issen & Rudnicki (2000)



Distinction of scales



Macro-scale / Elementary volume (REV)

- Constitutive behavior
- Momentum balance
- Mass balance

Micro-scale / Single Grain

- Reaction kinetics of dissolution
- Grain crushing

Reaction kinetics (micro-scale)

$$\underset{(3)}{\text{solid}} + \underset{(1)}{\text{solvent}} \rightleftharpoons \underset{(2)}{\text{solution}}$$

e.g. dissolution of quartz $SiO_2\text{(solid)}+2H_2O\text{(liquid)} \rightleftharpoons H_4SiO_4\text{(aqueous solution)}$

or carbonate $CaCO_3\text{(solid)}+H_2CO_3\text{(aqueous solution)} \rightleftharpoons Ca\left(HCO_3\right)_2\text{(aqueous solution)}$

$$\frac{\partial w_2}{\partial t} = k^* \frac{S}{e}\left(1 - \frac{w_2}{w_2^{eq}}\right)$$

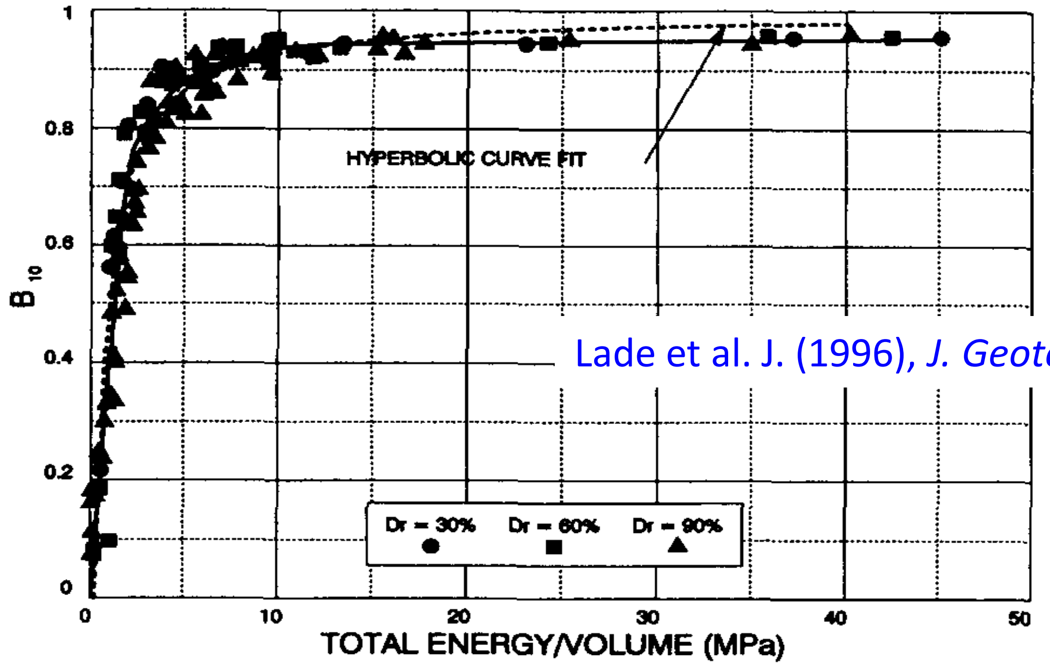
w_2 is the mass fraction of the dissolution product in the fluid

k^* is a reaction rate coefficient

e is the void ratio

$S \propto \frac{1}{D}$ is the specific area of a single grain of diameter *D*

Evolution of the effective grain size
Grain crushing (micro-scale)

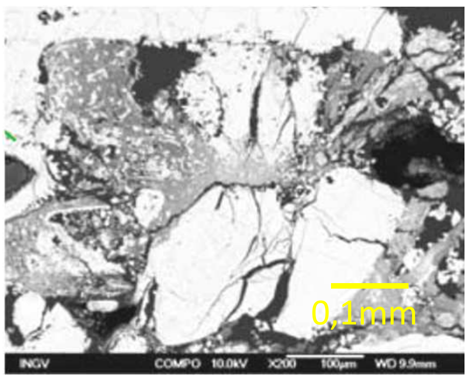


Lade et al. J. (1996), *J. Geotech. Engrg.*

$$D = D_0\left(\frac{a}{a+E_T}\right)$$

or

$$S = S_0\left(1+\frac{E_T}{a}\right)$$

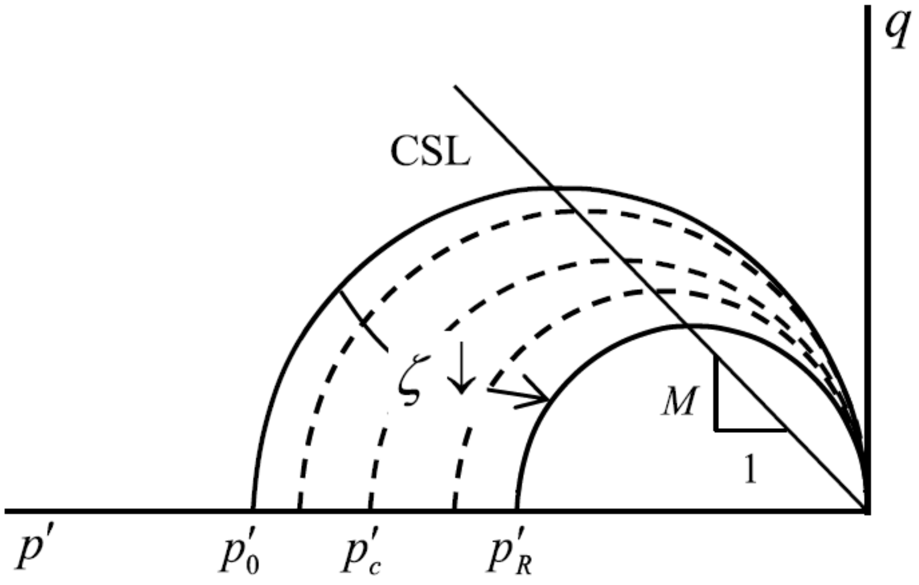


Baud et al. (2009)

a is a material constant which describes grain crushability

E_T is the total energy density given to the system

Constitutive behavior (macro-scale)



Modified Cam-Clay plasticity model

$$f \equiv q^2 + M^2 p'(p' - p'_c) = 0$$

$$p'_c \equiv p'_R - (p'_R - p'_0)\,\zeta^{\kappa}$$

Non local chemical softening

$$\frac{\partial \zeta}{\partial t} = -\frac{\mu_3}{\mu_2}\frac{\rho_f}{\rho_s} e\,\zeta \frac{\partial w_2^M}{\partial t}$$

$$w_2^M = \frac{1}{V_T}\int_{V_T} w_2 dV \approx w_2 + \ell_c^2 \frac{\partial^2 w_2}{\partial z^2}$$

ℓ_c characteristic length

Mass balance (macro-scale)

$$\frac{\partial p_f}{\partial t} = c_{hy} \nabla_{\mathbf{x}}^2 p_f - \frac{1}{n\beta_f} \frac{\partial \varepsilon}{\partial t} - c_{p,ch} \frac{\partial w_2}{\partial t}$$

c_{hy} is the hydraulic diffusivity

n is the porosity

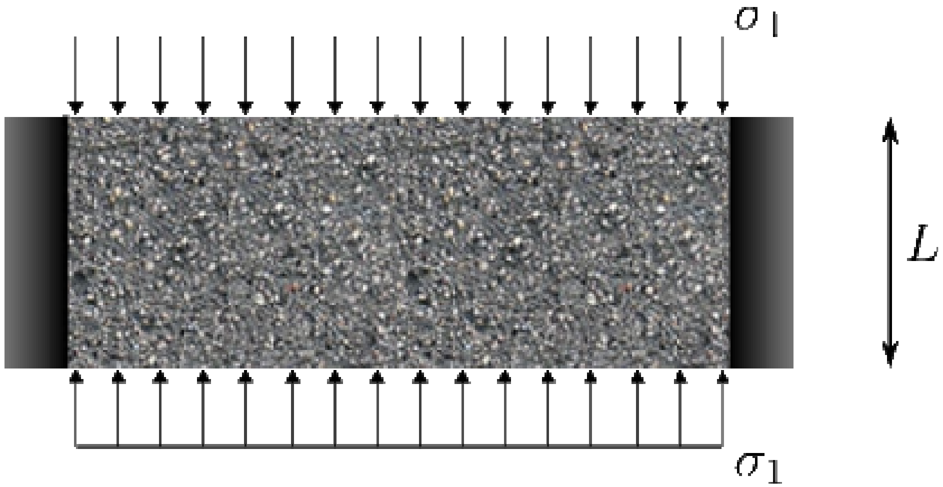
β_f is the fluid compressibility

$c_{p,ch}$ is the chemical pressurization coefficient

p_f is pressure of the fluid

ε is the volumetric strain

Linear stability analysis of oedometric compaction



$$u_z(z,t) = u_z^h + \tilde{u}_z(z,t) \qquad \tilde{u}_z(z,t) = Ue^{st}\cos\left(\frac{z}{\lambda}\right)$$

$$p_f(z,t) = p_f^h + \tilde{p}_f(z,t) \qquad \tilde{p}_f(z,t) = Pe^{st}\sin\left(\frac{z}{\lambda}\right)$$

$$w_2(z,t) = w_2^h + \tilde{w}_2(z,t) \qquad \tilde{w}_2(z,t) = We^{st}\sin\left(\frac{z}{\lambda}\right)$$

s is the growth coefficient of the perturbation (Lyapunov exponent)

Numerical example : Compaction banding in a reservoir

Carbonate grainstone

Elastic constants

$K = 5\text{GPa}$

$G = 5\text{GPa}$

Cam clay yield surface

$p'_R \approx 0\text{MPa}$

$p'_0 = 30\text{MPa}$

$M = 0.9$

Physical properties

$c_{hy} = 10^{-3}\text{m}^2\,\text{s}^{-1}$

$D_0^{50} = 0,2\text{mm}$

$n = 0.25$

Chemical parameters

$k'_- = 10^{-6}\text{mol}\,\text{s}^{-1}\text{m}^{-2}$

Grain crushing parameter:

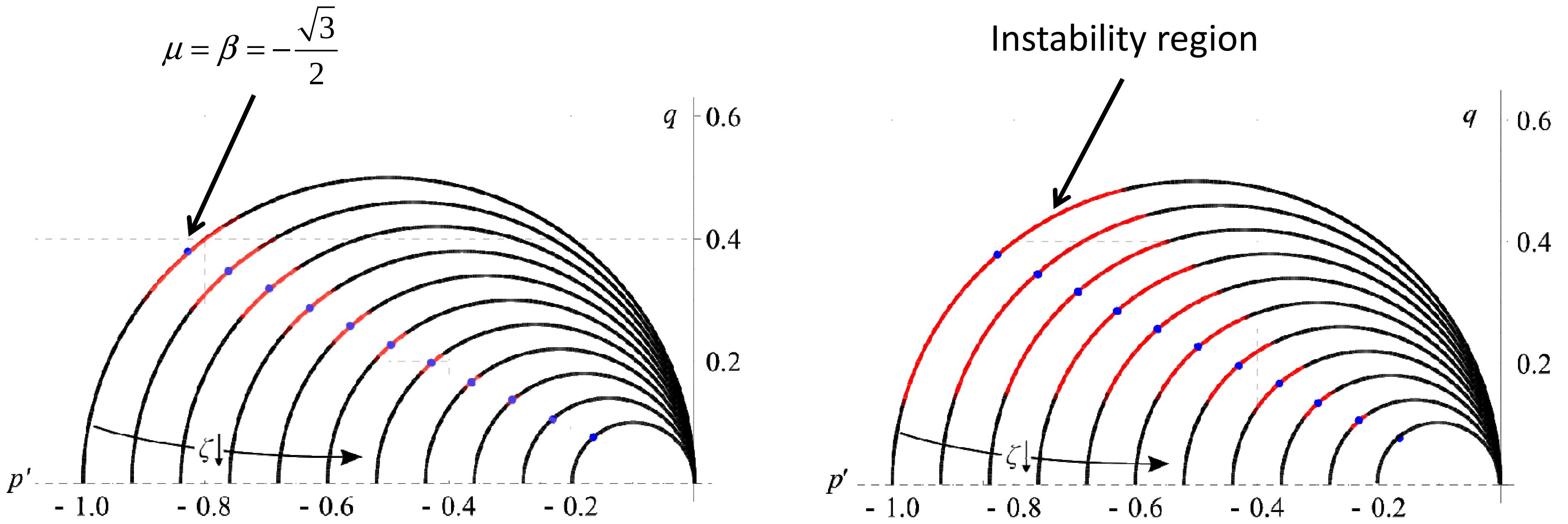
$a = 0.5\text{ MPa}$

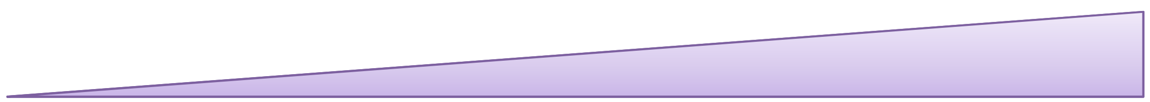
Initial stress state at 1.8km (oedometric)

$\sigma_n \approx 45\text{MPa}$

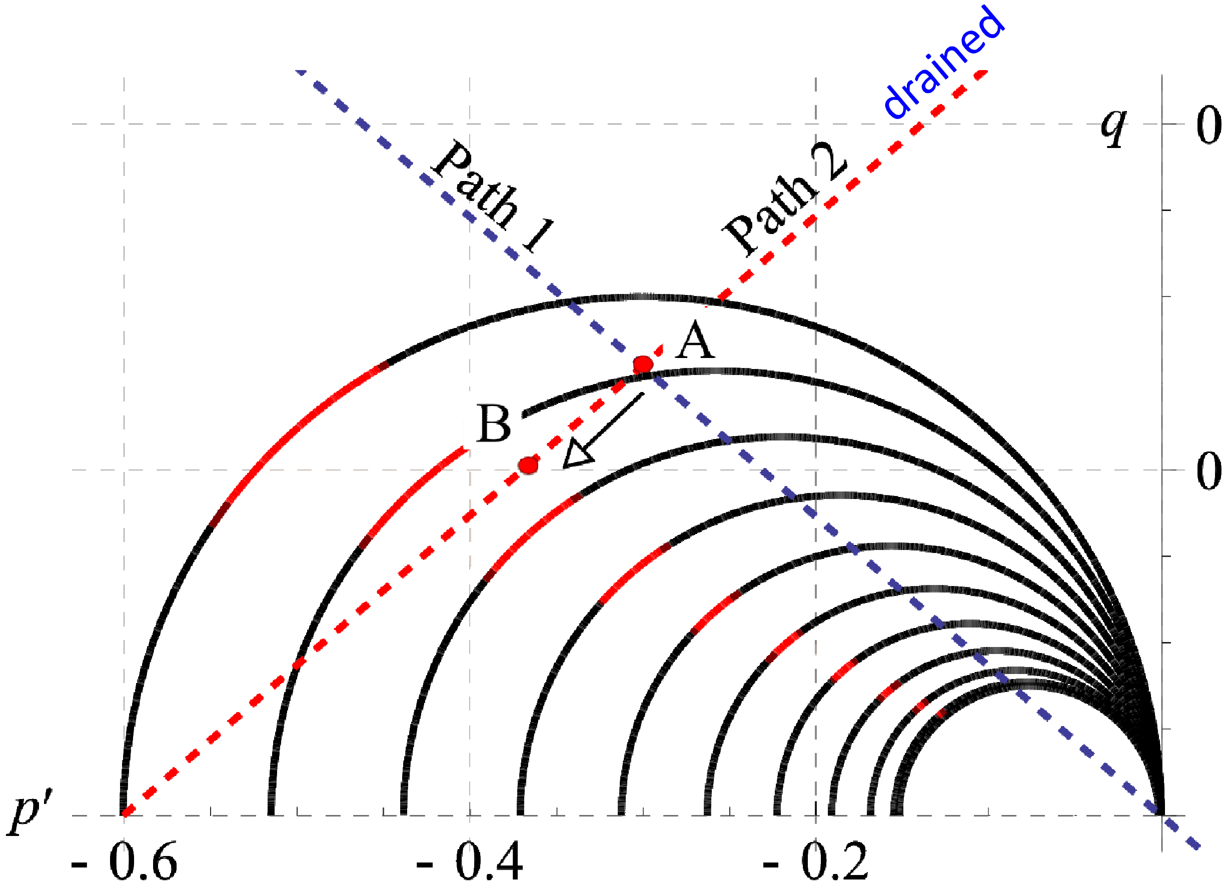
$p_f \approx 18\text{MPa}$

Linear Stability Analysis & zones of instability

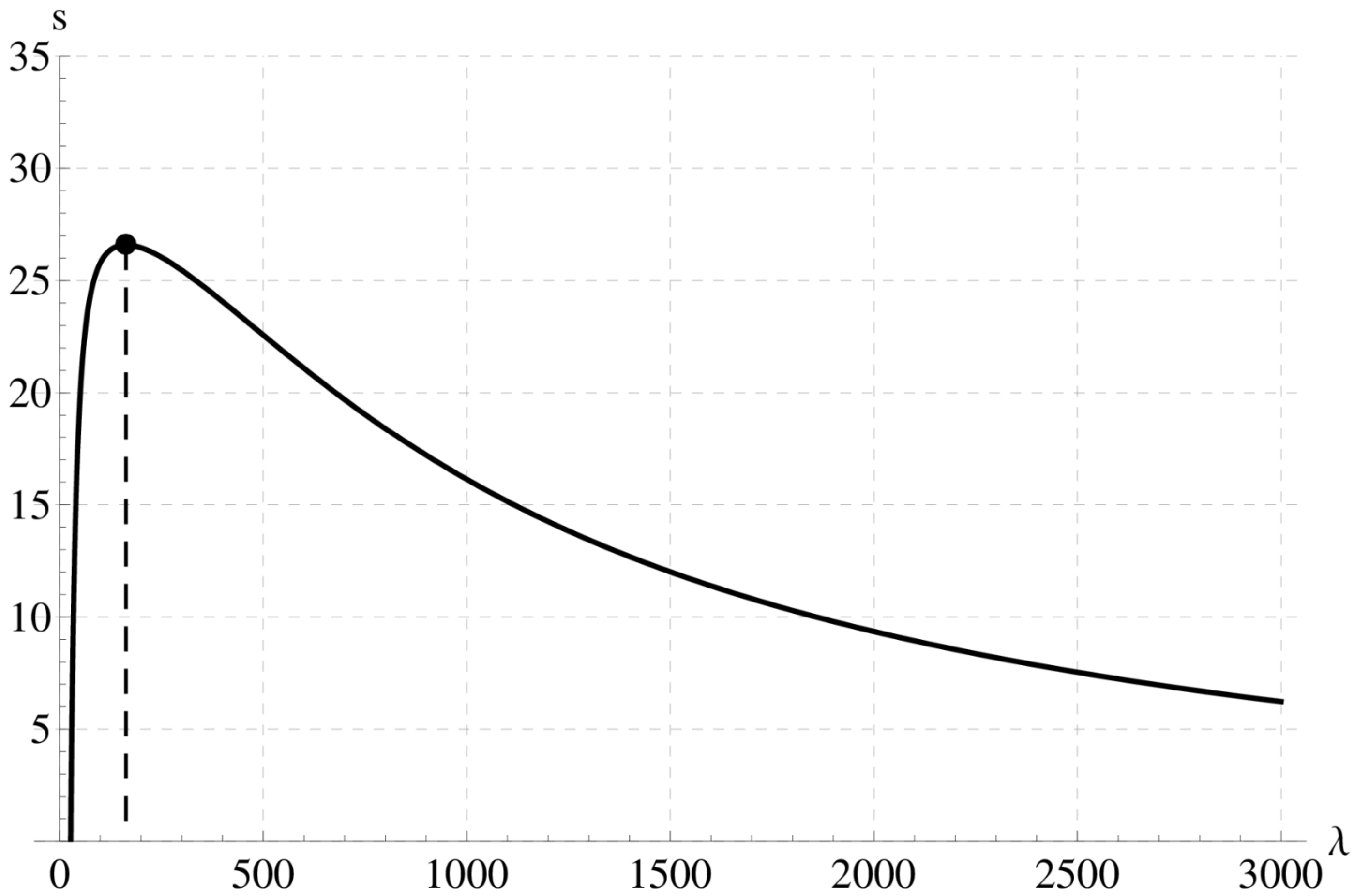




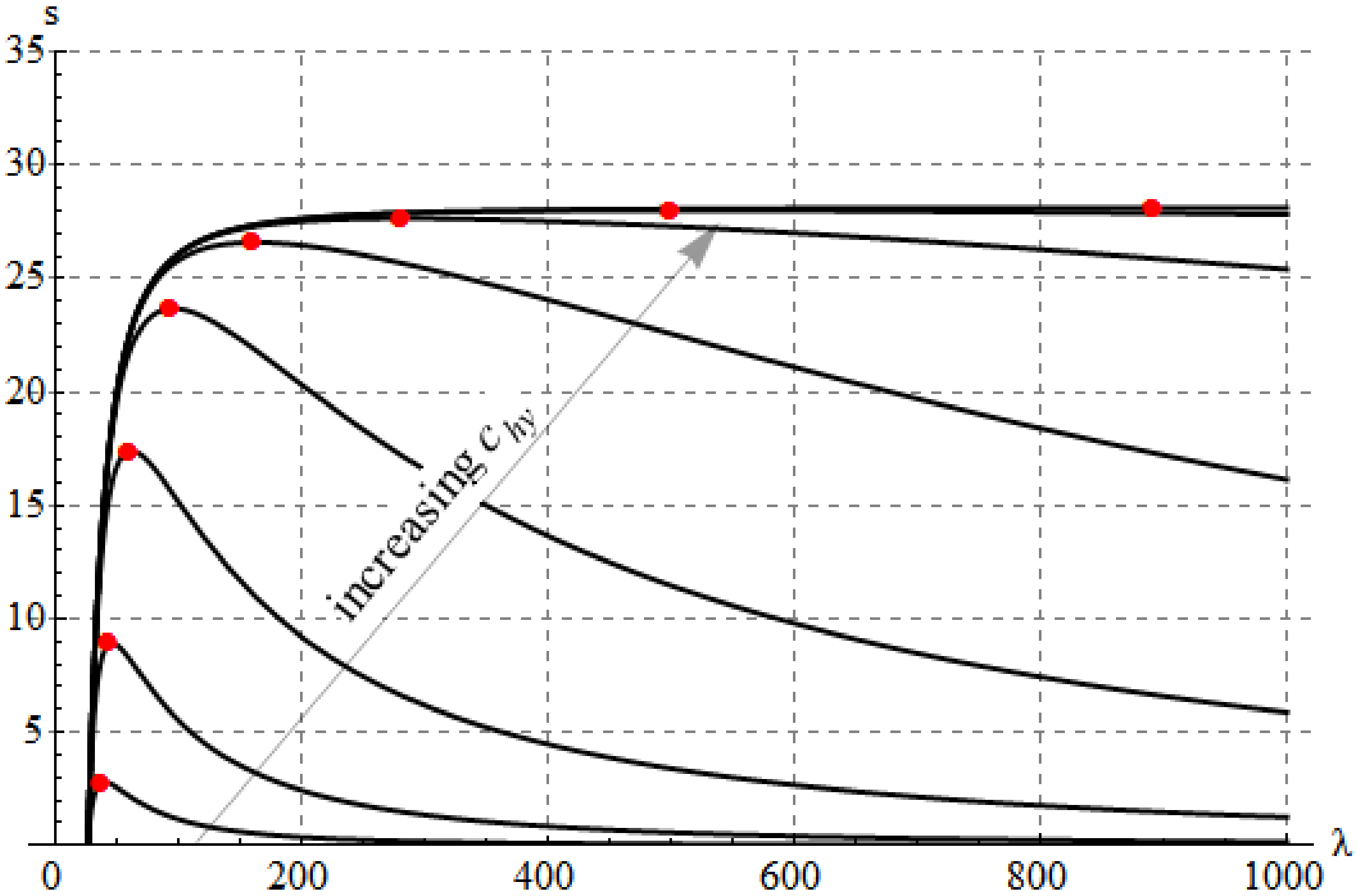
Oedometric stress path



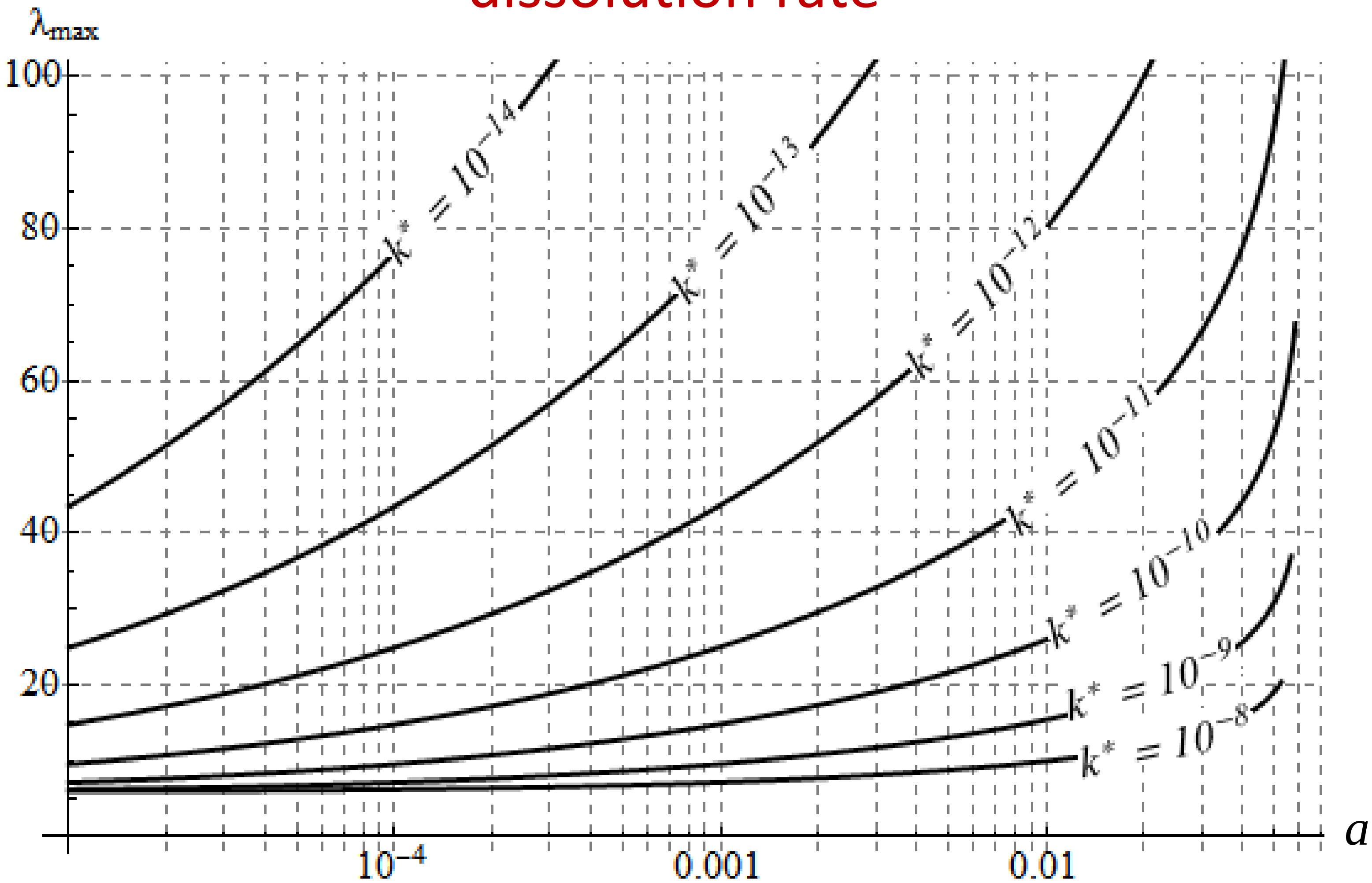
Wave length selection



Wave length selection – influence of the hydraulic diffusivity



Wave length selection – influence of grain crushing and dissolution rate



Conclusions

- Importance of coupled processes in seismic slip: shear heating, pore fluid pressurization, thermal decomposition of minerals…

- Thermal decomposition of minerals can explain the lack of pronounced heat outflow along major tectonic faults

- A key parameter : thickness of the localized zone (competition of several length scales)

- Thickness and periodicity of compaction bands: a new chemo-mechanical model with strong coupling that accounts for the increase of dissolution kinetics rate because of grain crushing