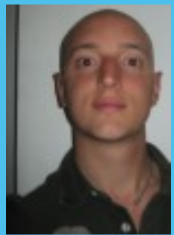
BREAKAGE MECHANICS

statistical constitutive approach from micro to macro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PROPOSED PHILOSOPHY

› Energy considerations at **micro-scale**

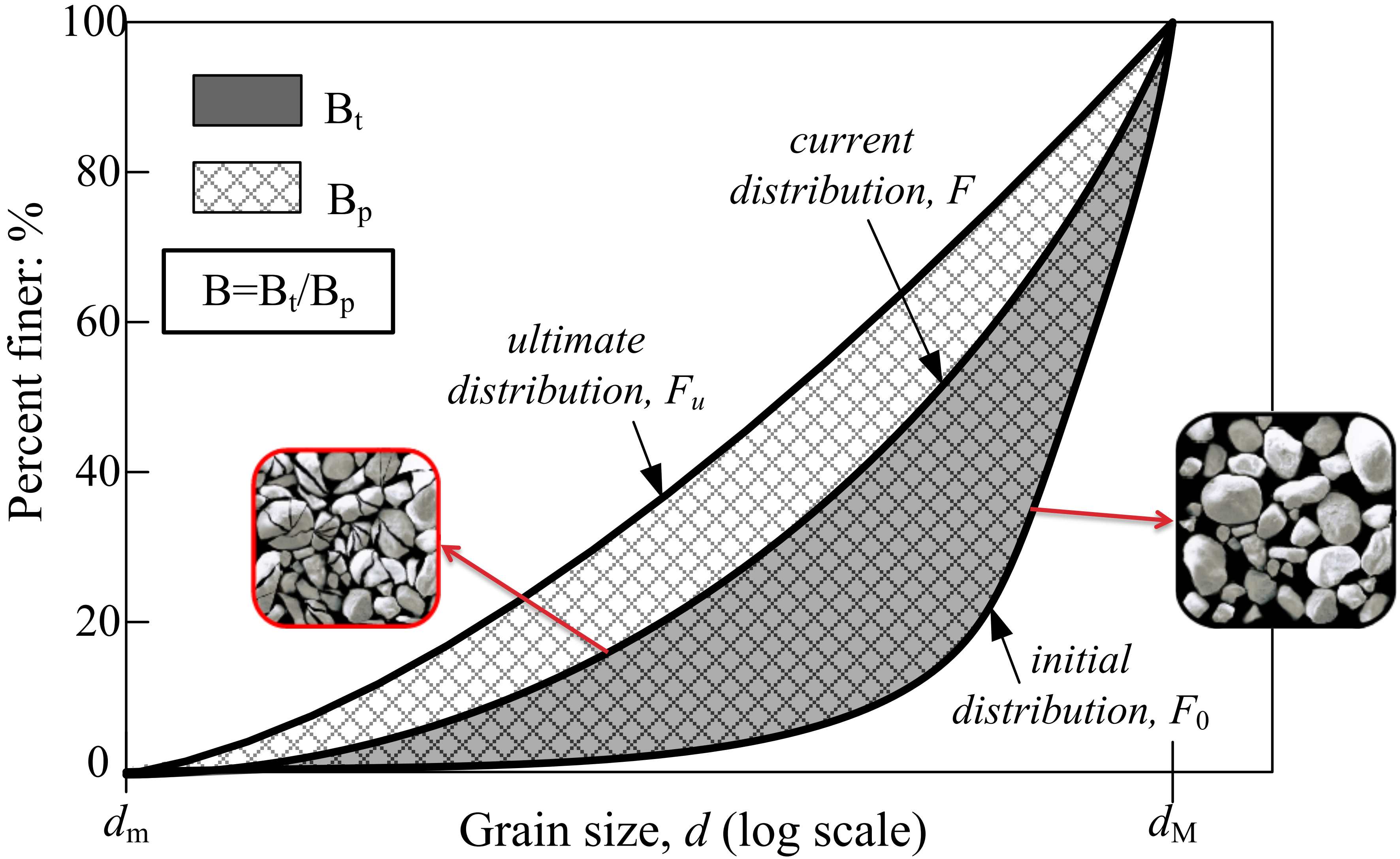
- *energy **scaling** (e.g. with grain sizes)*
- *rate of dissipation via **redistribution of strain energy***

› **Statistical homogenisation**

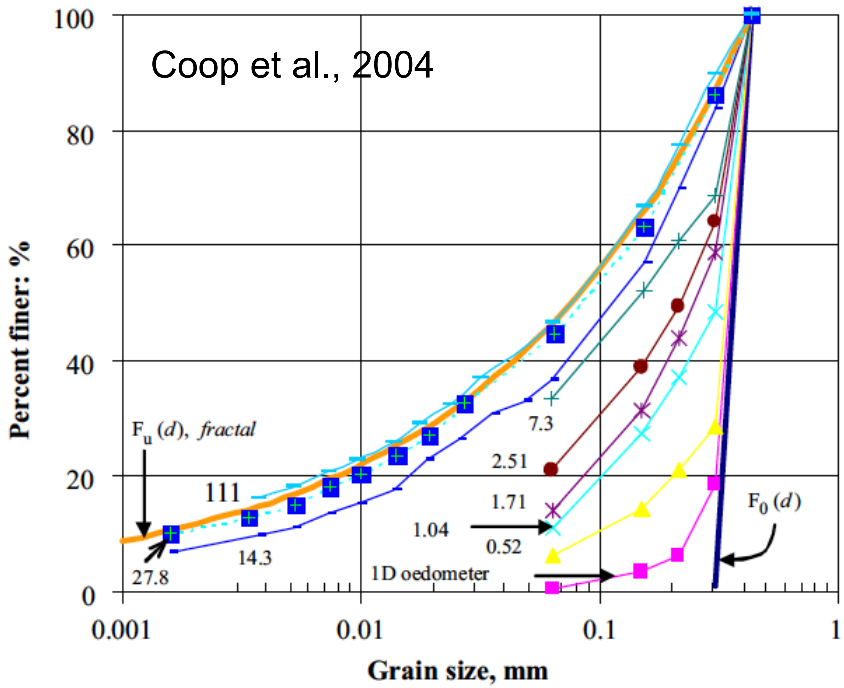
- *use of **grainsize distribution** for averaging strain energy*
- *simplified grainsize distribution via scalar **Breakage***

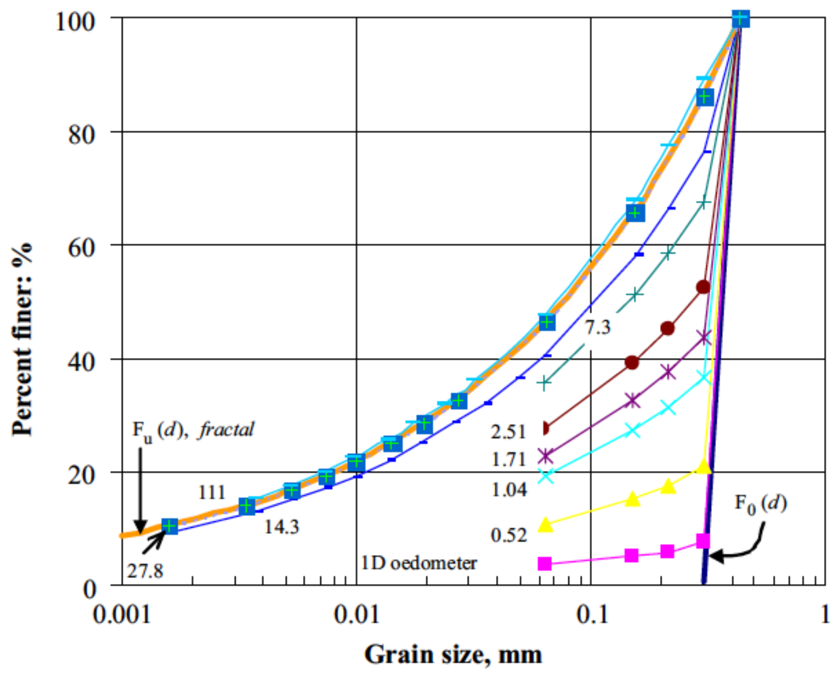
› Use thermodynamics to derive **macro-scale** constitutive laws

BREAKAGE DEFINITION



equivalence hypothesis



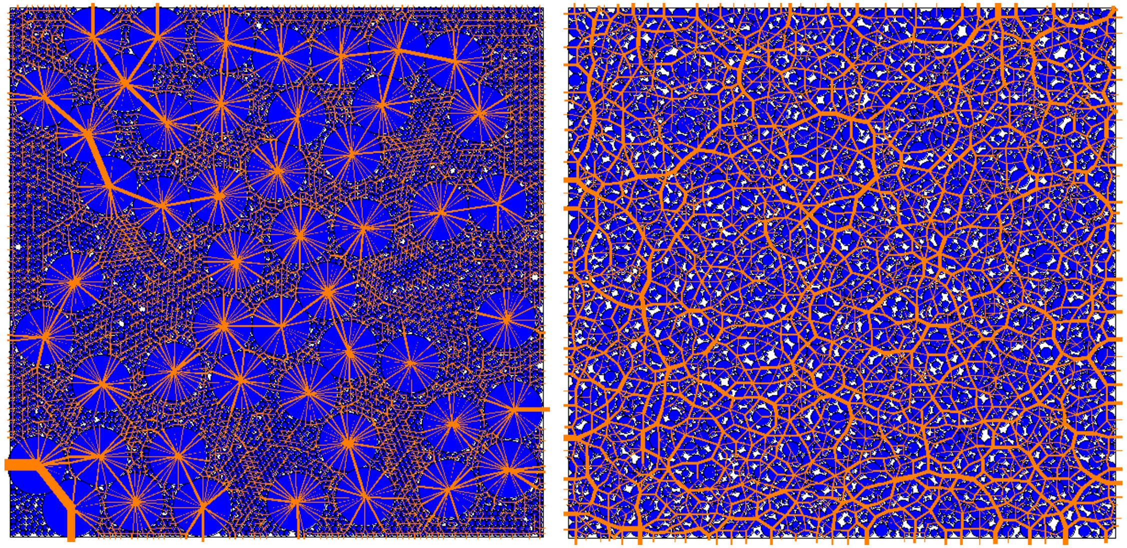


$$\boxed{p(B,d) = p_0(d)(1-B) + p_u(d)B}$$

energy scaling

$$\Psi \equiv \int p(B,x)\psi(x,state)\,dx = \left(\int p(B,x) f_{\psi}(x)\,dx\right)\cdot \psi_r(state)$$

$$\Psi \equiv (1-\vartheta B)\psi_r(state)$$



Force chains in compacted granular assemblies using DEM:
(left) bi-modal distribution (d_{large}=10.d_{small}), and (right) uniform distribution (by size).

energy scaling

$$\Psi \equiv \int p(B,x)\psi(x,state)\,dx$$
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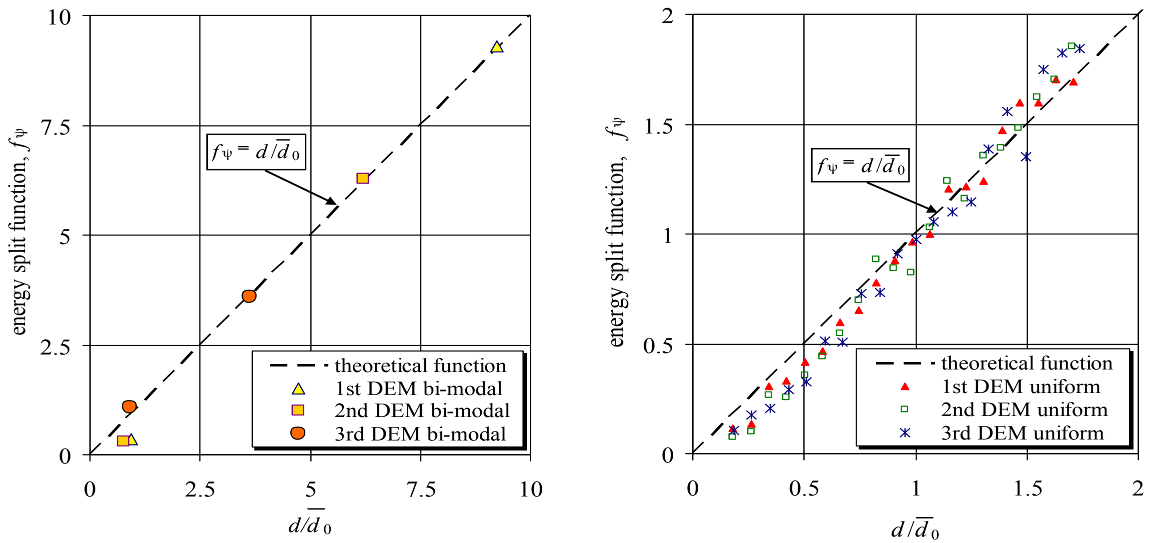
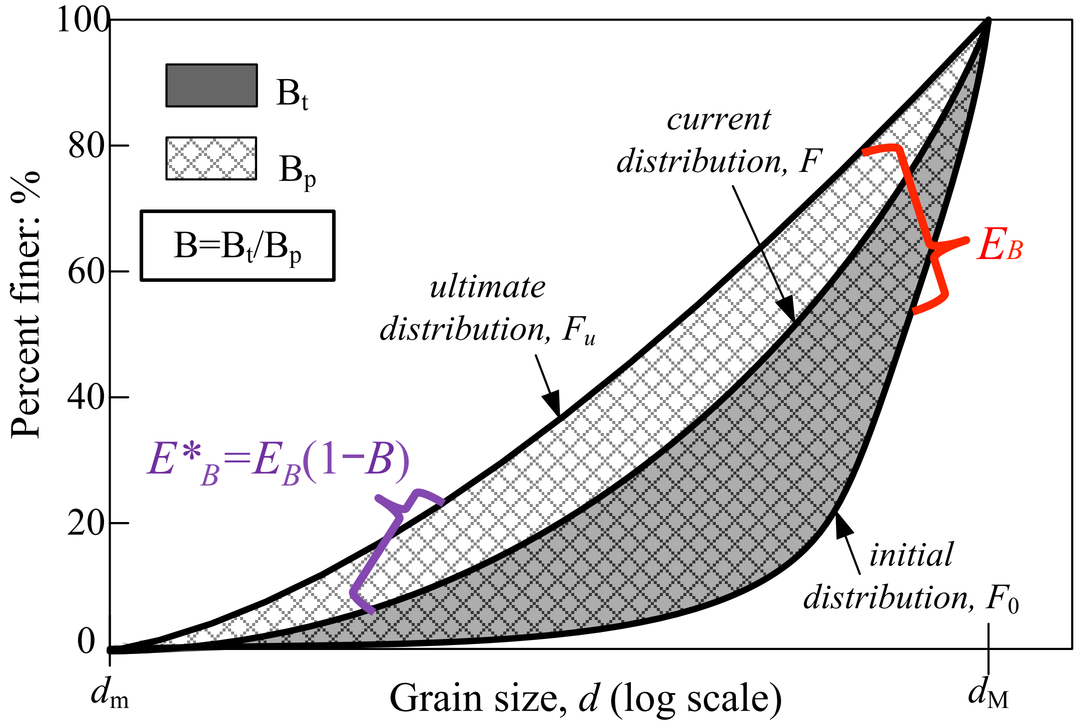


Force chains in compacted granular assemblies using DEM:
(left) bi-modal distribution (d_{large}=10.d_{small}), and (right) uniform distribution (by size).

and residual breakage energy



stress – elastic strain

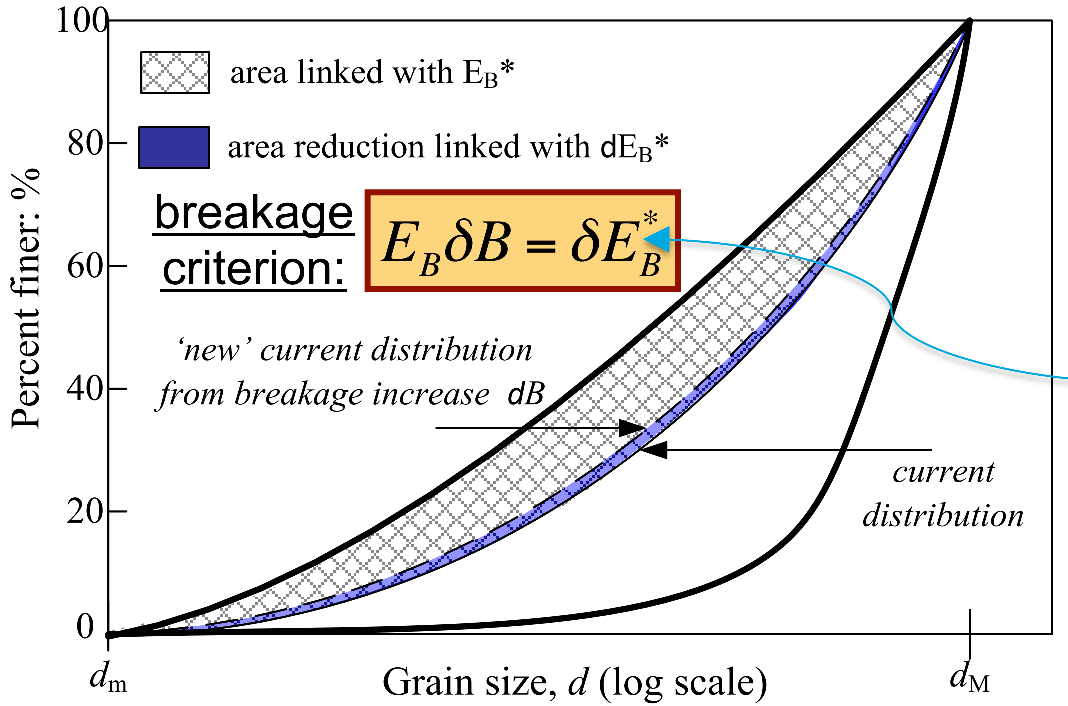
$$\boxed{\sigma_{ij} = \frac{\partial \Psi}{\partial \varepsilon_{ij}^{e}}}$$

breakage energy – breakage

$$\boxed{\begin{aligned} E_B &= -\frac{\partial \Psi}{\partial B} \\ &= \Psi_0 - \Psi_u \end{aligned}}$$

Residual Breakage Energy: $E_B^* = E_B(1-B) = \Psi - \Psi_u$

BREAKAGE CRITERION



breakage criterion: $\boxed{E_B \delta B = \delta E_B^*}$

$$E_B^* = E_B(1-B)$$

$$\boxed{y_B(B, E_B) = E_B(1-B)^2 - E_c \le 0}$$

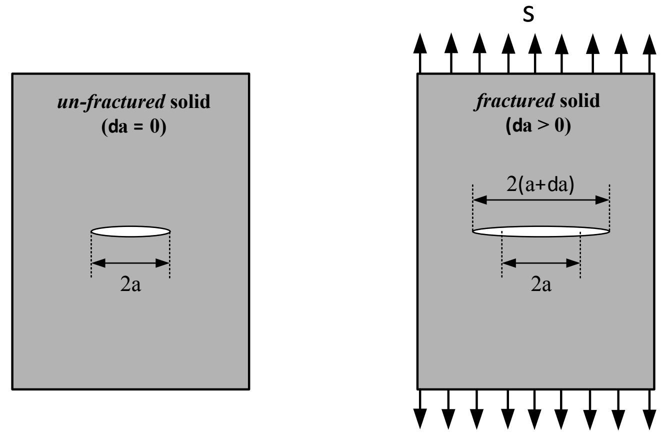
E_c = critical breakage energy

$$\boxed{\delta B = \delta\lambda \frac{\partial y_B}{\partial E_B}}$$

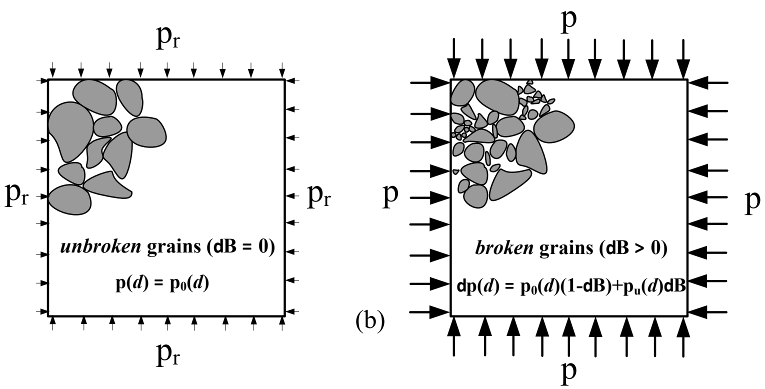
$E_B = \partial\Psi / \partial B$ and $p = \partial\Psi / \partial\varepsilon_v^e$ so e.g. when $\Psi = \frac{1}{2}(1-\vartheta B)K\varepsilon_v^{e2}$ ⇒ $p_{cr} = \sqrt{\frac{2KE_c}{\vartheta}}$

ONSET OF BREAKAGE

connection to established theory



(a) fracture propagation in tensioned brittle solids (left: before, right: after)



(b) fracture propagation in compressed brittle granular-agglomerates (left: before, right: after)

fracture mechanics

$$\sigma_{cr} = \sqrt{\frac{EG_c}{\pi a}}$$

breakage mechanics

$$p_{cr} = \sqrt{\frac{2KE_c}{\vartheta}}$$

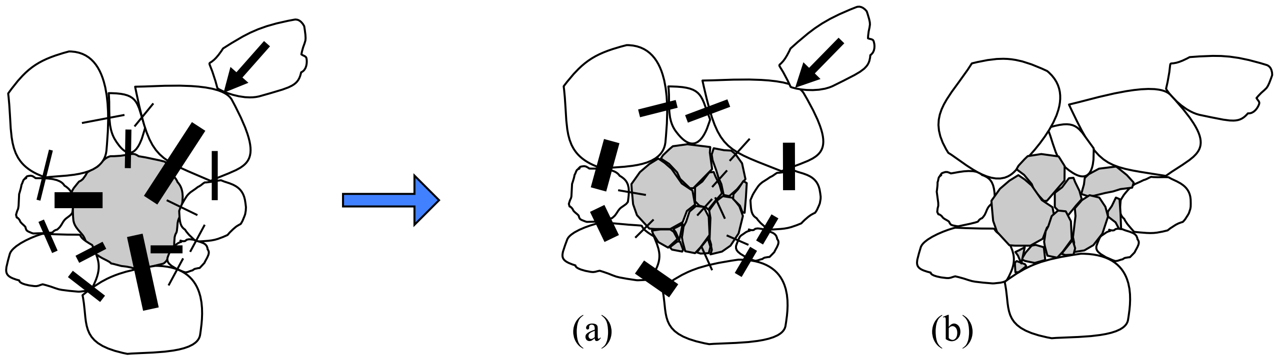
$$p_{cr} = p_r \left(\frac{3\bar{K}E_c}{4\vartheta p_r} \right)^{2/3}$$

E_c = critical breakage energy constant

ϑ = normalised surface area

MODES OF DISSIPATION

compression



(a) Active: dissipation by freeing surface energy
Passive: redistribution of locked-in strain energy
$$D_B = E_B \delta B$$

(b) Passive: Configurational dissipation (volumetric plastic dissipation) D_v

distortion

Active: plastic dissipation (Coulomb/distortional $D_d \equiv Mp\left|\dot{\varepsilon}_s^p\right| \geq 0$)

Passive: breakage/abrasion $\rightsquigarrow D_B = E_B \delta B$

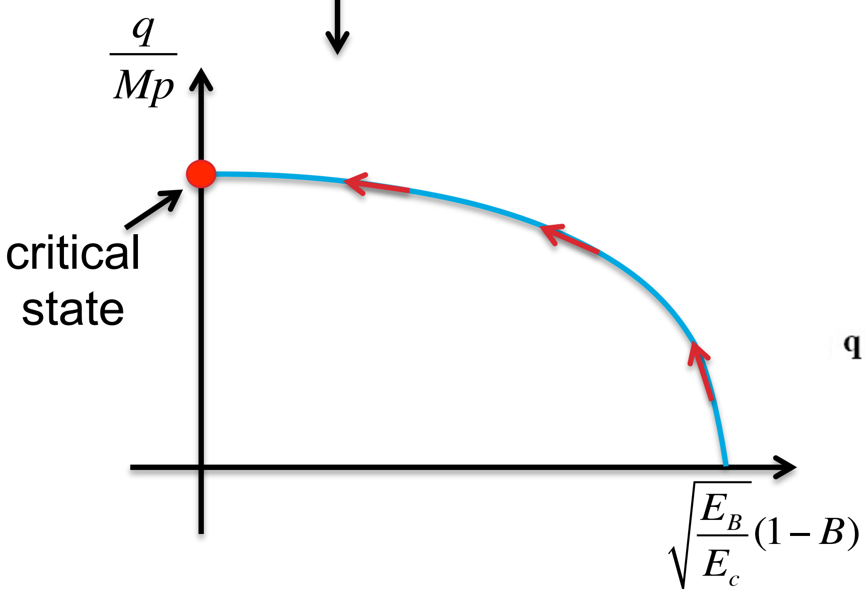
dissipation, yield, flow, and critical state (1st go)

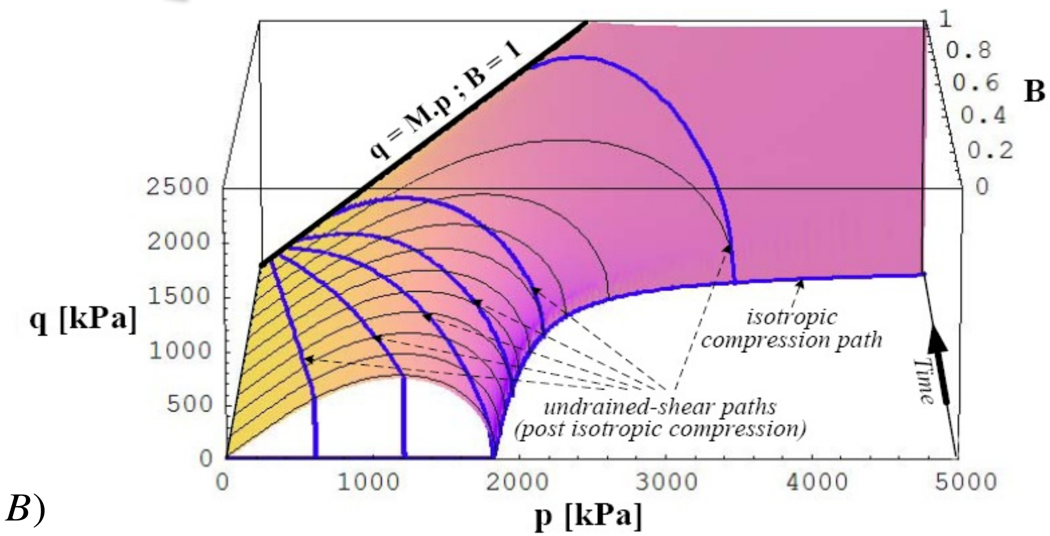
$$y_B = \frac{E_B}{E_c}(1-B)^2 + \left(\frac{q}{Mp}\right)^2 - 1 \leq 0$$

$$\Phi \equiv \sqrt{D_B^2 + D_d^2 + D_v^2}$$

$$\dot{B} = \lambda(1-B)^2$$

$$\Psi = \tfrac{1}{2}(1-\vartheta B)\left(K\varepsilon_v^{e2} + 3G\varepsilon_s^{e2}\right)$$





MULTI-COMPONENT THEORY

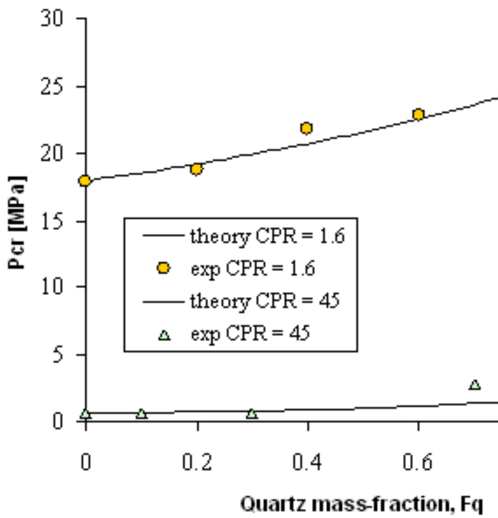
effect on crushing strength

$$\sum_{i=1}^{N} f^{(i)} = 1$$

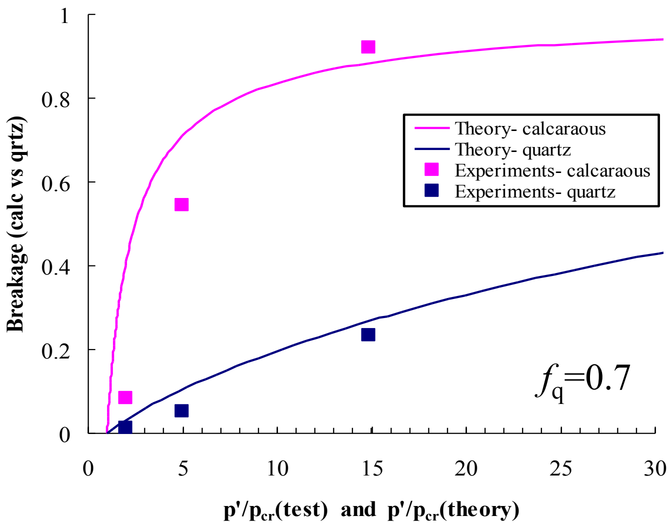
$$p^{(i)}(D) = p_0^{(i)}(D)(1 - B^{(i)}) + p_u^{(i)}(D)B^{(i)}$$

$$\boxed{\Psi \equiv \sum_{i=1}^{N} \psi_r^{(i)}(\varepsilon) f^{(i)} (1 - \vartheta^{(i)} B^{(i)})}$$

$$\boxed{\Phi_B = \sum_{i=1}^{N} f^{(i)} E_B^{(i)} \delta B^{(i)} \geq 0}$$

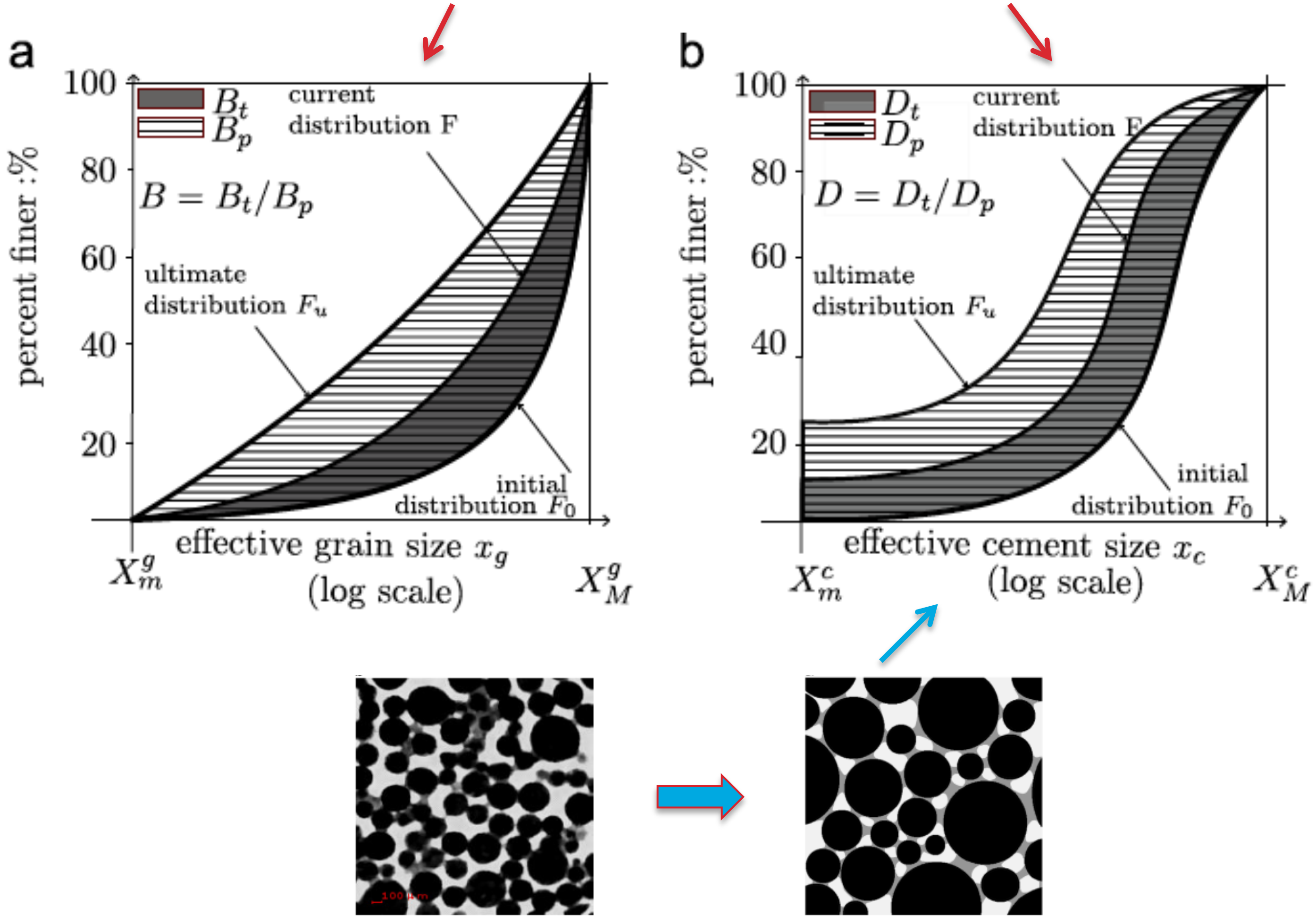


yielding pressure against the (strong) quartz mass fraction



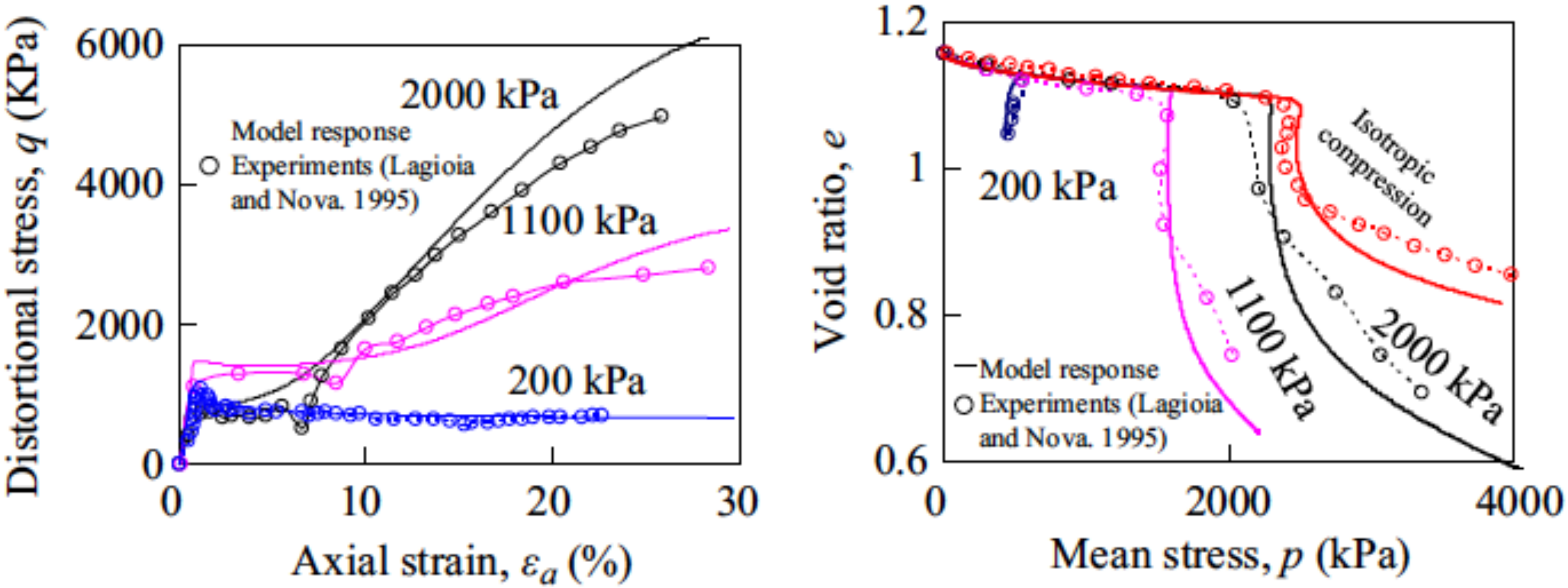
breakage evolution in the weak (calcareous) and strong (quartz) phases , against pressure

Bi-mixture of grains (breakage) and cement (damage)



Bi-mixture of grains (breakage) and cement (damage)

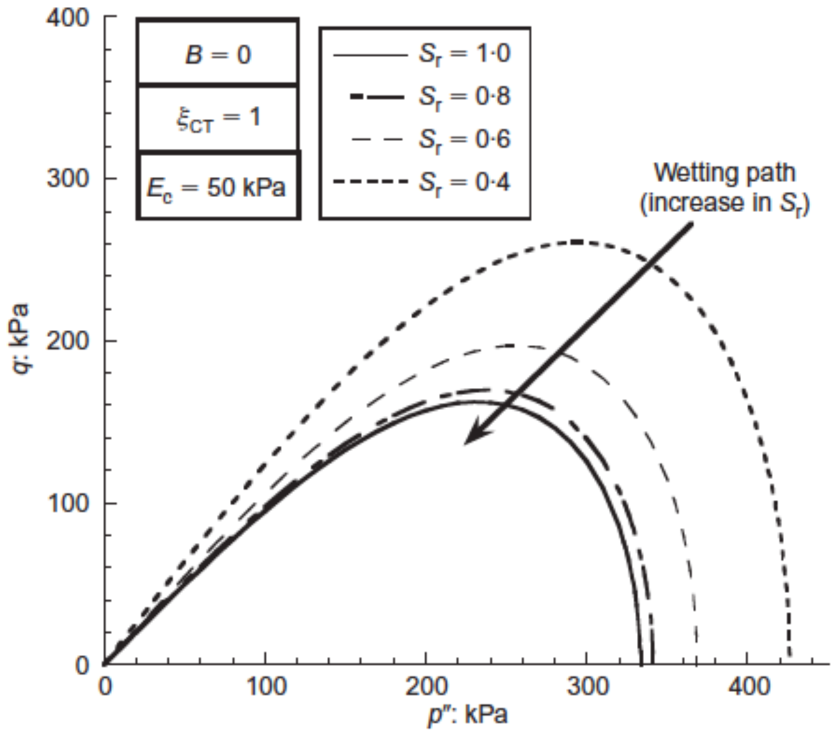
Parameters[a]	Adamswiller	Bentheim	Calcarenite
G_g and G_c	3200 MPa	7588 MPa	75,500 kPa
$\overline{K}_g$ and $\overline{K}_c$	20,000	42,000	3470
M	1.5	1.7	1.62
E_{BC}	2.6 MPa	3.85 MPa	15 kPa
E_{DC}	1.6 MPa	3.0 MPa	18 kPa
c	40 MPa	60 MPa	300 kPa
ω_B	40°	70°	78°

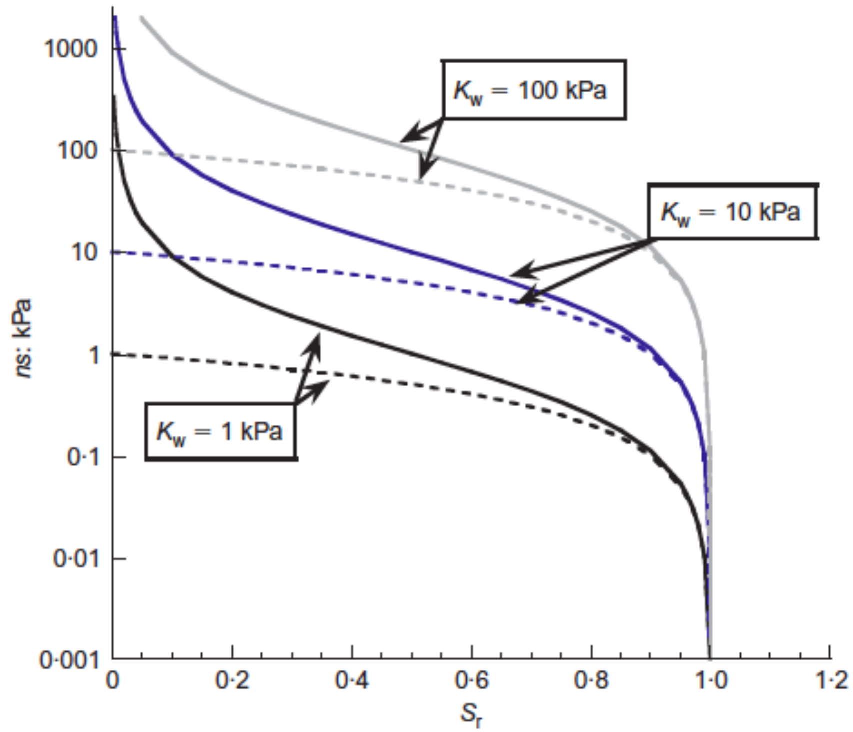


Bi-mixture of grains (mechanics) and partial saturation (hydro)

$$W \equiv \sigma''_{ij}\dot{\varepsilon}_{ij} - \phi s \dot{S}_r$$

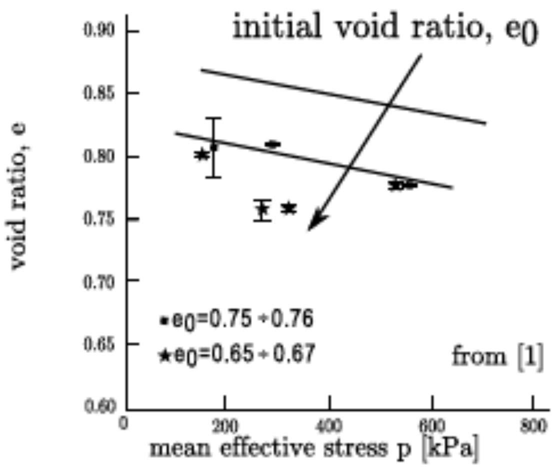
$$\Psi \equiv (1-\vartheta_M B)\psi_r(\varepsilon^e_{ij}) + (1+\vartheta_H B)\psi_r(S_r)$$



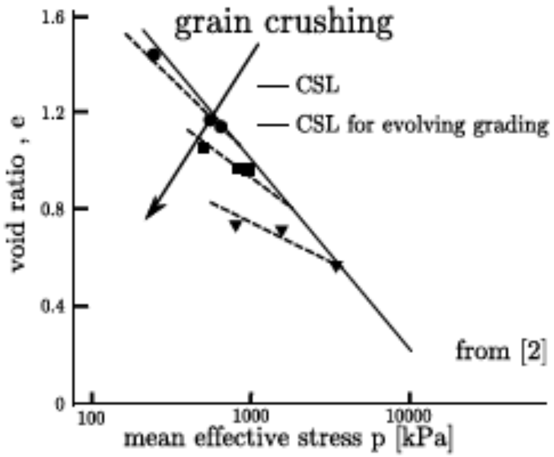


‘NON-UNIQUE CRITICAL STATE’

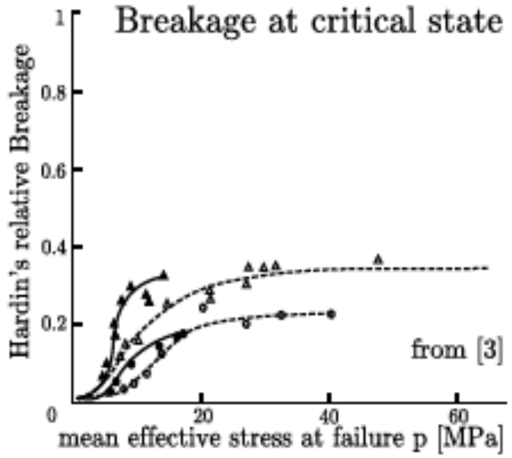
The problem (phenomenologically)



[1] Finno-Rechenmacher (2003)

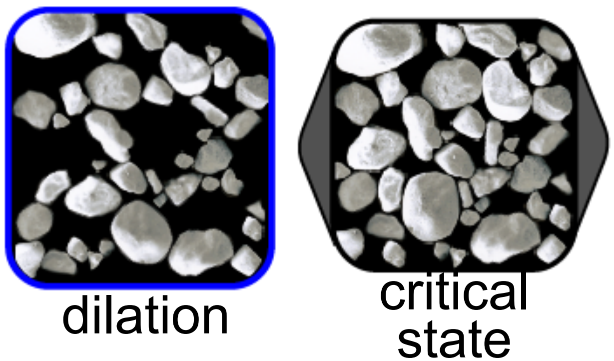


[2] Bandini-Coop (2012)

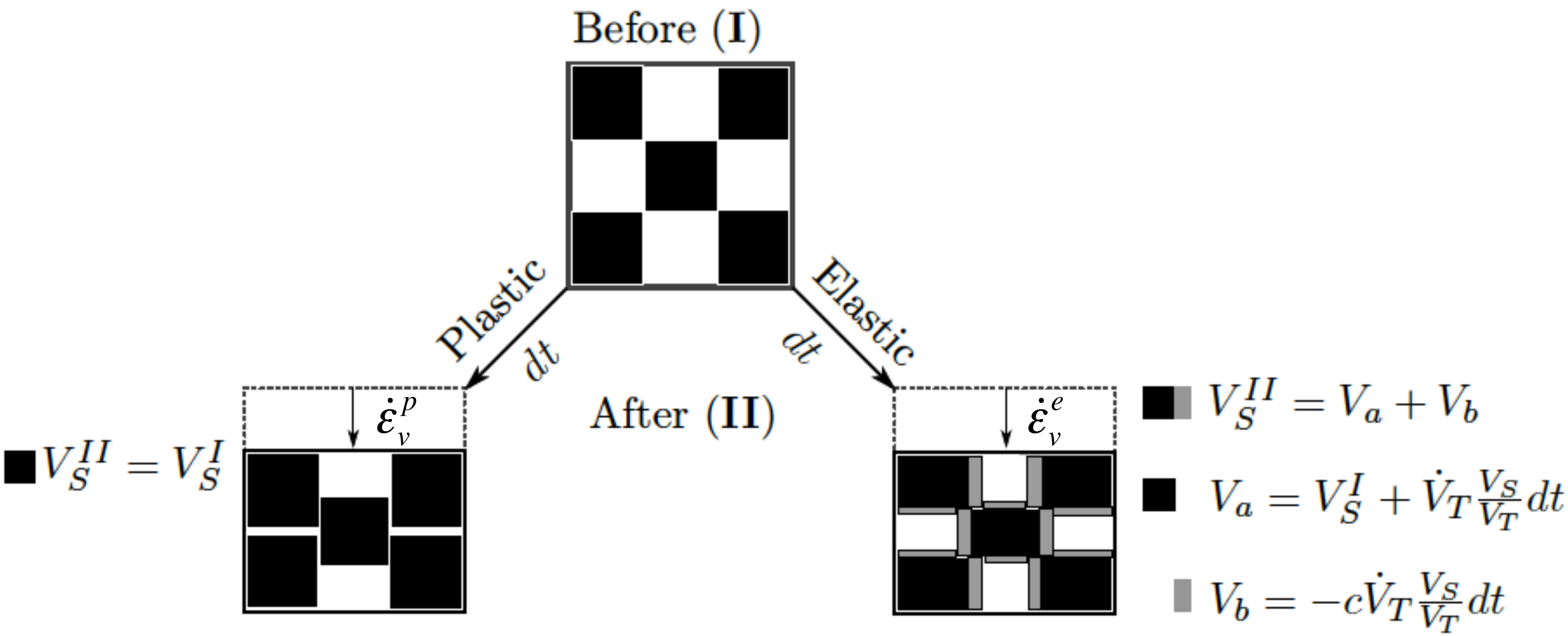


[3] Lade-Yamamuru (1996)

The reason (micromechanically)



POROSITY AND STRAINS



$$\boxed{\begin{aligned} \dot{\phi}^e &= -c(1-\phi)\dot{\varepsilon}_v^e \\ \dot{\phi}^p &= -(1-\phi)\dot{\varepsilon}_v^p \end{aligned}}$$

$$c = (2\nu_g)^{-\frac{1}{\phi}}$$

$\nu_g = 0.5 \rightarrow$

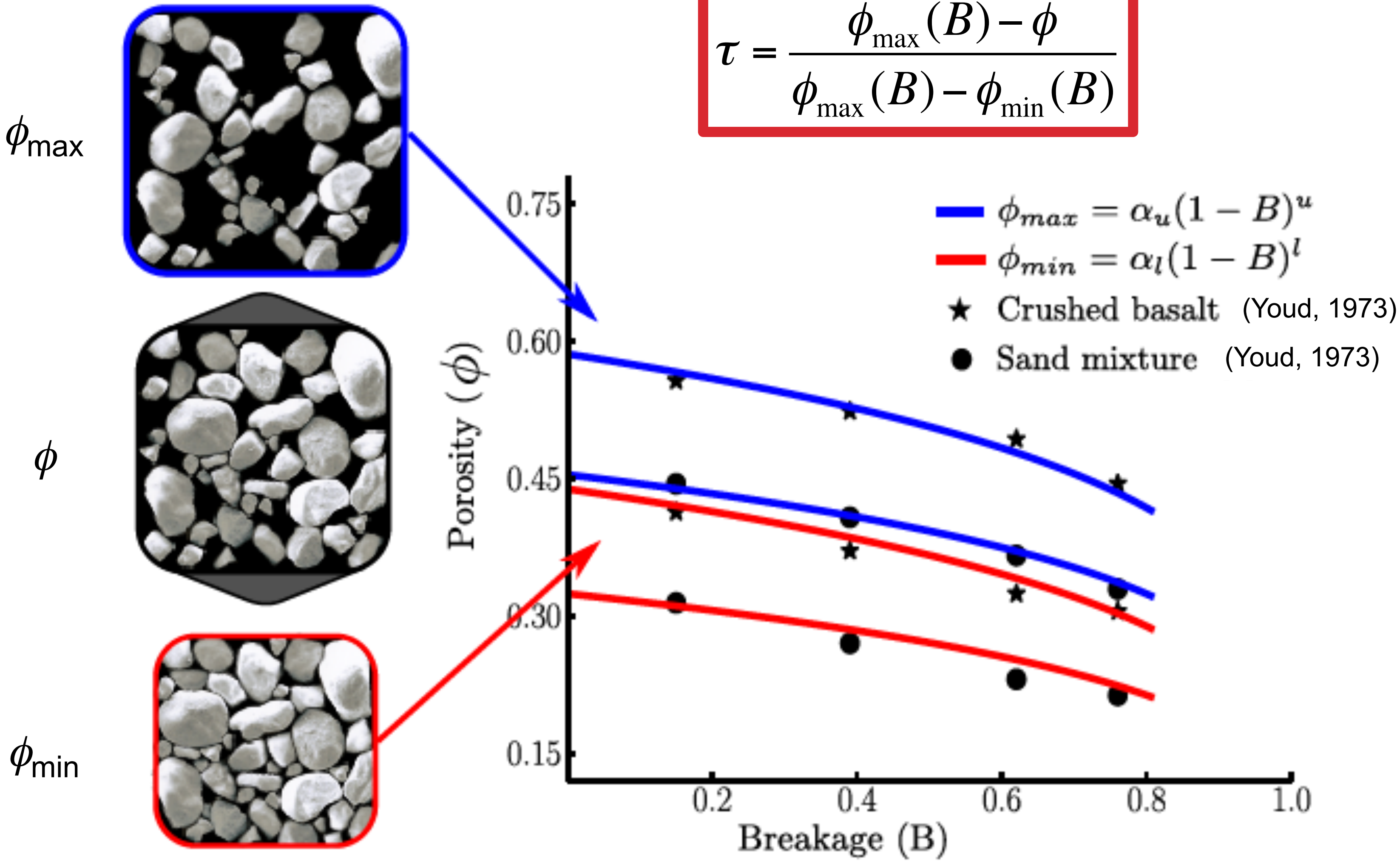
$$\dot{\varepsilon}_v = -\frac{\dot{\phi}}{(1-\phi)} \quad \text{(soil mechanics)}$$

$\nu_g = 0 \rightarrow$

$$\dot{\varepsilon}_v^p = -\frac{\dot{\phi}}{(1-\phi)} \quad \left(\text{and } \dot{\phi}^e = 0\right)$$

RELATIVE POROSITY

$$\tau = \frac{\phi_{\max}(B) - \phi}{\phi_{\max}(B) - \phi_{\min}(B)}$$



NEGATIVE WORK AND DISSIPATION

$$\boxed{\Phi \equiv \sqrt{D_B^2 + D_\phi^2 + D_s^2} + r_B D_B + r_\phi D_\phi}$$

$$\left\{\begin{aligned} D_B &= \frac{\sqrt{E_B E_C}}{(1-B)\cos(\omega)} \frac{\dot{B}^2}{\langle \dot{B} \rangle} \\ D_\phi &= \frac{\sqrt{E_B E_C}}{(1-B)\sin(\omega)} \frac{E_\phi}{E_B} \dot{\phi} \\ D_s &= M p \dot{\varepsilon}_s^p \end{aligned}\right.$$

during breakage $\dot{B}$>0:

$$\frac{E_\phi \dot{\phi}}{E_B \dot{B}} = \tan(\omega)$$

$$\frac{\text{Dissip. } \textit{via} \text{ pore collapse}}{\text{Dissip. } \textit{via} \text{ breakage}} = \tan(\omega)$$

$$r_B = \gamma \tau \cos(\omega)$$

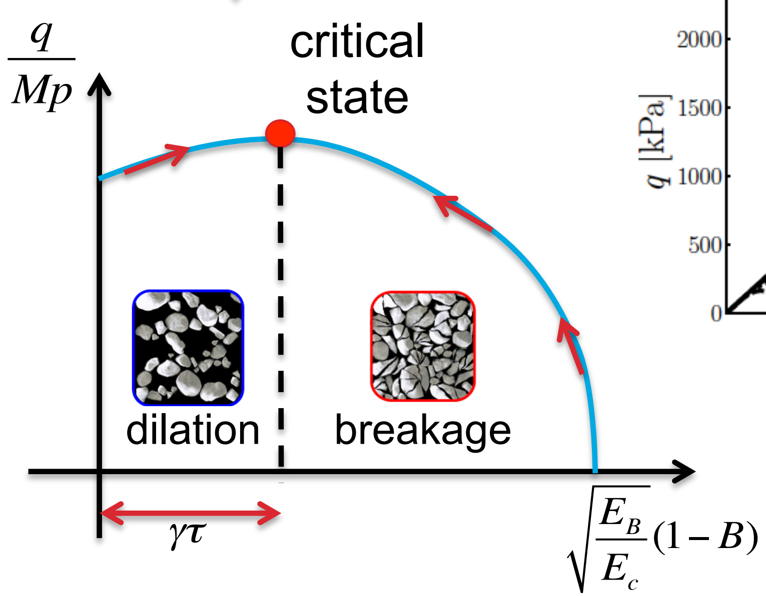
$$r_\phi = \gamma \tau \sin(\omega)$$

MODEL OF BREAKAGE AND POROSITY

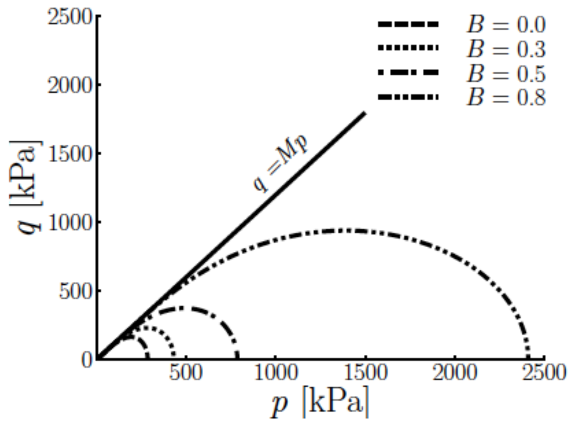
dissipation, yield, flow, and critical state (2nd go)

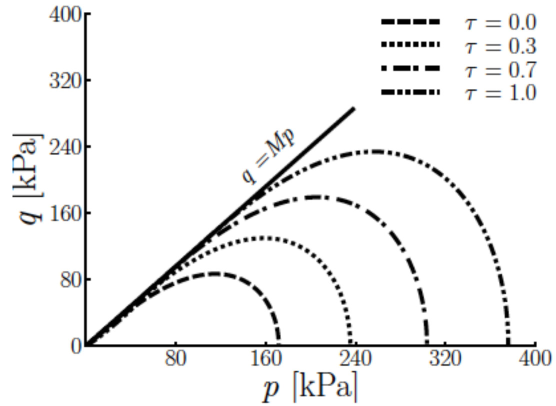
$$y_B = \left(\sqrt{\frac{E_B}{E_c}}(1-B) - \gamma\tau\right)^2 + \left(\frac{q}{Mp}\right)^2 - 1 \leq 0$$

$$\Phi \equiv \sqrt{D_B^2 + D_\phi^2 + D_s^2} + r_B D_B + r_\phi D_\phi$$



elastic potential





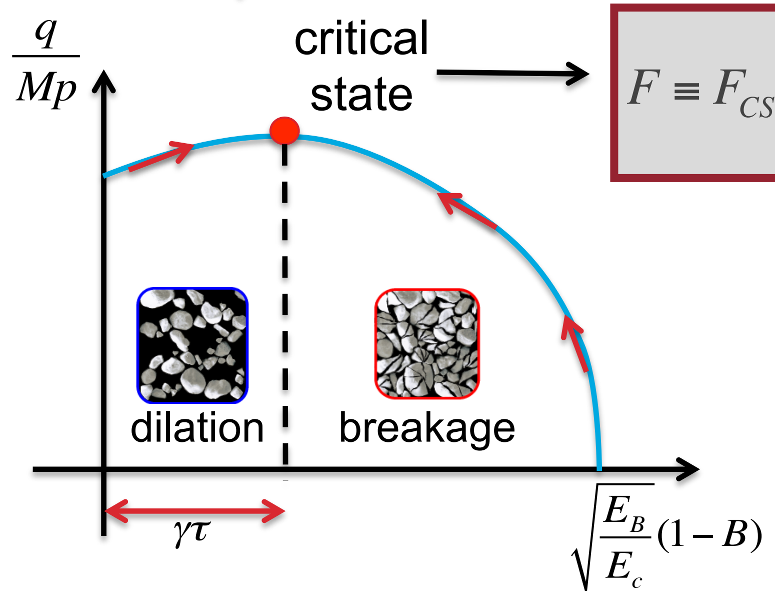
MODEL OF BREAKAGE AND POROSITY

dissipation, yield, flow, and critical state (2nd go)

$$\boxed{y_B = \left(\sqrt{\frac{E_B}{E_c}}(1-B) - \gamma\tau\right)^2 + \left(\frac{q}{Mp}\right)^2 - 1 \le 0}$$

$$\Phi \equiv \sqrt{D_B^2 + D_\phi^2 + D_s^2} + r_B D_B + r_\phi D_\phi$$

$$\dot{\varepsilon}_p^s = 2\lambda \frac{q}{(Mp)^2} \qquad \dot{B} = 2\lambda \langle F \rangle \frac{1-B}{\sqrt{E_B E_C}} \cos^2(\omega) \qquad \dot{\phi} = 2\lambda F \frac{1-B}{\sqrt{E_B E_C}} \frac{E_B}{E_\phi} \sin^2(\omega)$$



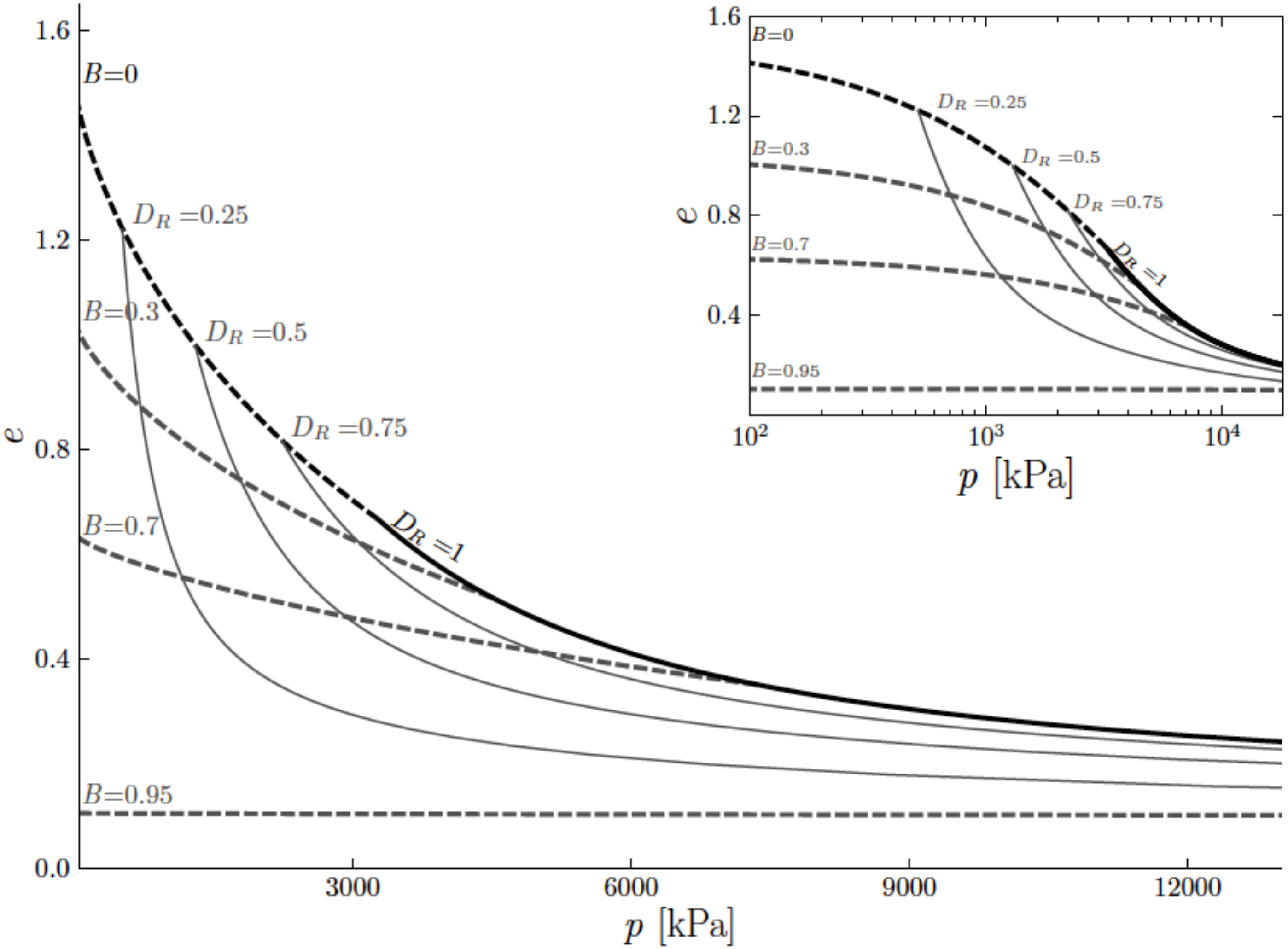
$$\boxed{F \equiv F_{CS} = \sqrt{\frac{E_B}{E_c}}(1-B) - \gamma\tau = 0}$$

elastic potential

critical state 'for humans'

MODEL OF BREAKAGE AND POROSITY

‘non-unique’ critical state

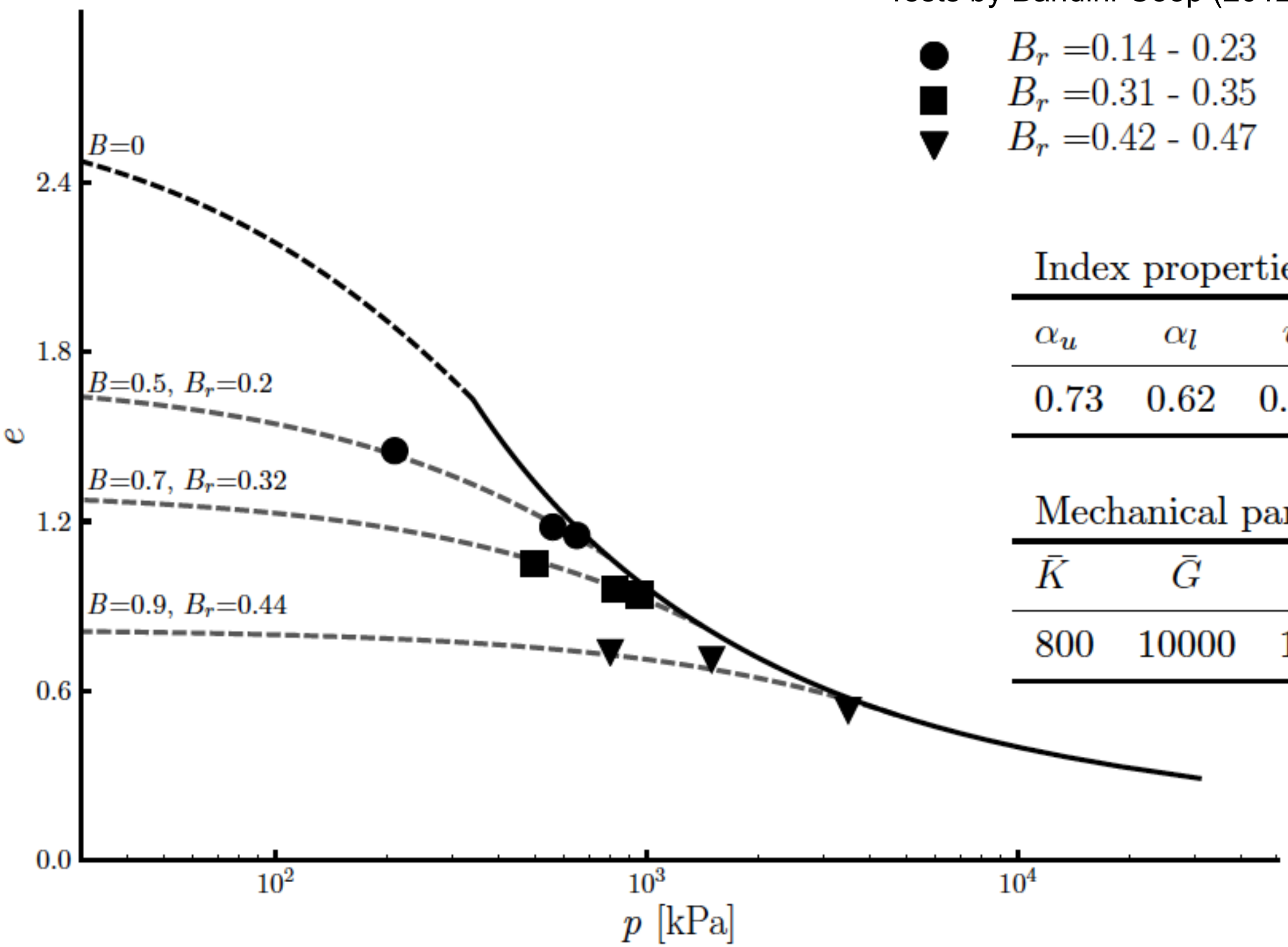


MODEL OF BREAKAGE AND POROSITY

‘non-unique’ critical state

Tests by Bandini-Coop (2012):

● B_r =0.14 - 0.23
■ B_r =0.31 - 0.35
▼ B_r =0.42 - 0.47



Index properties

α_u	α_l	u	l	θ
0.73	0.62	0.21	0.26	0.83

Mechanical parameters

$\bar{K}$	$\bar{G}$	M	E_C [kPa]	γ
800	10000	1.65	5	0.95

only ‘free’ parameter

CONCLUSIONS

- Micro to Macro *via* energy scaling and redistribution, and statistical homogenisation
- Breakage mechanics used to get GSD in space and time
- Porosity is internal variable (not total plastic strain!)
- Clearer link of porosity rate to strain rate
- Introducing relative porosity dependent on GSD
- Predicting critical state, preconsolidation pressure, yield surface and wetting collapse rather than imposing them