The flow of saturated geomaterials simulated with a DEM-fluid coupling

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01 - 10 - 2014
Introduction

Context

Large movements of saturated geomaterials (e.g. debris flow) need advanced constitutive models, often including a so-called solid-fluid transition (and fluid-solid transition ?)

Objectives

A micromechanical model for saturated materials capturing both solid and fluid regimes in a unified framework
Tools

DEM-fluid coupled model.

www.yade-dem.org
1. Poromechanical coupling
2. Short range hydrodynamical interactions
3. Rheology of saturated geomaterials
4. Conclusions
1. Poromechanical coupling

2. Short range hydrodynamical interactions

3. Rheology of saturated geomaterials

4. Conclusions
Coupled model

Resolution of the fluid problem

Contact law

- Computation of particles' positions and velocities
- Computation of contact forces

Motion law

- Triangulation
- Computation of pore pressure
- Computation of fluid forces

Conclusions
Regular triangulation

1 tetrahedron = 1 pore
vertices = centers of particles

**Coupled model**

**Continuity:**

\[ \Delta \dot{V} + \int_{S_{ij}} (u^f - u^s) \cdot n \, ds = 0 \]

**Conductance:**

\[ \int_{S_{ij}} (u^f - u^s) \cdot n \, ds = K_{ij} (p_i - p_j) \]

\[ K_{ij} = f(R_{ij}^h). \]

\[ \dot{V} = \sum_{j=j_1}^{j=j_4} \int_{S_{ij}} (u^s - u^f) \cdot n \, ds = \sum_{j=j_1}^{j=j_4} K_{ij} (p_i - p_j) \]

Fluid forces

\[ F^k = \int_{\partial \Gamma_k} p \, n \, ds + \int_{\partial \Gamma_k} \tau \, n \, ds \]

\[ F^k = \underbrace{F_{p,k}}_{\text{pressure}} + \underbrace{F_{v,k}}_{\text{viscous stress}} \]

*Chareyre et al, Transport in porous media (2012)*
Plan

1. Poromechanical coupling
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Normal motion

\[ F_{i,n}^L = -F_{j,n}^L = \frac{3}{2} \pi \, \eta \, \frac{a^2}{h} \, u_n \]

\[ u_n = (u_j - u_i) \cdot n \]

Frankel & Acrivos (Chemical Engineering Science, 1967): the hydrodynamic interactions are developed from the balance equation of the energy dissipation in small gaps between particles.
(up): Normal approach and Poiseuille flow in the gap, (bottom): contact between a few solid surface asperities for rough grains

Rognon et al, JFM, 2011
Shear motion

\[
F_{i,s}^L = -F_{j,s}^L = \frac{\pi \eta}{2} (-2a + (2a+h) \ln\left(\frac{2a + h}{2a}\right)) u_t
\]

\[
C_{i,j}^s = (a_{i,j} + \frac{h}{2}) F_{i,j,s}^L
\]

\[
u_t = ((u_j - u_i) \cdot t)t + (a\omega_i + a\omega_j)n \times t
\]

Frankel & Acrivos (Chemical Engineering Science, 1967): the hydrodynamic interactions are developed from the balance equation of the energy dissipation in small gaps between particles
Shear motion

Comparaison with the FEM solution (Stokes solver of Comsol)

Frankel & Acrivos (Chemical Engineering Science, 1967): the hydrodynamic interactions are developed from the balance equation of the energy dissipation in small gaps between particles

Jeffrey & Onishi (Stokesian Dynamics)
Shear motion

Comparaison with the FEM solution (Stokes solver of Comsol)

- J&O expression is negative for $h > a$: False!
- F&A expression suits well with the FEM solution and tends to zero for high $h$.

Frankel & Acrivos (Chemical Engineering Science, 1967): the hydrodynamic interactions are developed from the balance equation of the energy dissipation in small gaps between particles

Jeffrey & Onishi (Stokesian Dynamics)
Rolling motion

\[
C_i^r = -C_j^r = \pi \eta a^3 f^r \left( \frac{h}{a} \right) (\omega_i - \omega_j) \cdot \mathbf{tt}
\]

\[
f^r \left( \frac{h}{a} \right) = \frac{3}{2} \ln \frac{a}{h} + \frac{63}{500} \frac{h}{a} \ln \frac{a}{h}
\]

Jeffrey & Onishi (JFM, 1984): Stokesian Dynamics for small gaps between particles
Twist motion

\[ C_i^t = -C_j^t = \pi \eta a^2 f^t\left(\frac{h}{a}\right) (\omega_i - \omega_j) \cdot n \]

\[ f^t\left(\frac{h}{a}\right) = \frac{h}{a} \ln \frac{a}{h} \]

*Jeffrey & Onishi (JFM, 1984): Stokesian Dynamics for small gaps between particles*
The evolution of $F^L_{n}$ vs $\frac{h}{2a}$ is faster than that of $F^L_s$ when $h \to 0$.

$\implies F^L_s$ is usually neglected compared to $F^L_n$.

Is $F^L_s$ really negligible?

Does it participate to the rheology of the material?

The contribution of the other terms (rolling and twist) will be studied, too.
Coupled model

Resolution of the fluid problem

- Contact law
  - Computation of particles' positions and velocities
  - Computation of contact forces

- Motion law

- Triangulation
- Computation of pore pressure
- Computation of fluid forces
  + lubrication forces
Plan

1. Poromechanical coupling
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Configuration

Numerical configuration

Experimental configuration of Boyer et al

Poromechanical coupling
Short range hydrodynamical interactions
Rheology of saturated geomaterials
Conclusions
**Phenomenological laws from the experiments of Boyer et al.**


![Diagram](image)

\[ \frac{\eta}{I_v} = \frac{\dot{\gamma}}{I_v} \]
Shear stress

\[ l_v = 0.223 \]

\[ T_x = \frac{F_x}{S} \]
Shear stress

\[ T_x = \sigma_{xy}^C + \sigma_{xy}^{LN} + \sigma_{xy}^{LS} + \sigma_{xy}^I \]

For \( I_v = 0.223 \):

\[ \sigma^I = \sum_i m_i \mathbf{v}_i \otimes \mathbf{v}_i \]

\[ \sigma_{xy}^I < 2\% \ T_x \]

→ Non inertial regime
Shear stress

\[ T_x = \sigma^C_{xy} + \sigma^{LN}_{xy} + \sigma^{LS}_{xy} + \sigma^I_{xy} \]

For \( l_v = 0.223 \):

\[ \sigma^C = \frac{1}{V} \sum_i F^C_i \otimes l_i \]

\[ \sigma^C_{xy} \approx 50\% T_x \]

→ Contacts are dominant
Shear stress

\[ T_x = \sigma_{xy}^C + \sigma_{xy}^{LN} + \sigma_{xy}^{LS} + \sigma_{xy}^I \]

For \( I_v = 0.223 \):

\[ \sigma_{xy}^{LN} = \frac{1}{V} \sum_i F_{i,n}^L \otimes I_i \]
\[ \sigma_{xy}^{LS} = \frac{1}{V} \sum_i F_{i,s}^L \otimes I_i \]
\[ \sigma_{xy} \simeq 30\% \ T_x \]
\[ \sigma_{xy} \simeq 20\% \ T_x \]

→ The shear lubrication is not negligible compared to the normal one.
$I_v = 0$: dry case

Comparaison with the phenomenological laws from the experiments of Boyer et al:

* For $I_v = 0$, the result is that of the dry case.
* $\mu = 0.31$ and $\phi = 0.585$: the results match those measured experimentally on glass beads without any fit.
Normal motion

* $\sigma^{LN}$ contributes to the shear stress.

* The simulation of only the normal motion overestimates the solid fraction: $\phi$ is almost constant for increasing $I_v$
* $\sigma^{LS}$ contributes to the shear stress.
* $\sigma^{LS}$ plays a key role to the dilatancy.
* By combining $\sigma^{LS}$ and $\sigma^{LN}$, the behavior gets closer to that of the phenomenological laws from the experiments of Boyer.
The effect of the rolling motion is very negligible.
The effect of the twist motion is very negligible.
Normal + shear + rolling + twist motion + poromechanical coupling

The poromechanical coupling doesn’t contribute to $\mu$ and $\phi$ (in the steady state).
Contact forces play a significant role. They saturate for larger $I_v$.

Lubrication stress increase linearly. For $I_v > 0.15$, it exceeds the contact stress.

Two regimes are observed:

- Low $I_v$, contacts are important.
- At high $I_v$, lubrication is important.

→ Consistent with Boyer law:

$$\mu(I_v) = \mu_1 + \frac{\mu_2 - \mu_1}{l_0/I_v + 1} + I_v + \frac{5}{2} \phi_m l_v^{1/2}$$
The poromechanical coupling has an effect on the transient regime and not on the steady regime.
Plan

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3. Rheology of saturated geomaterials
4. Conclusions
* A complete micro-hydrodynamical framework has been developed, including solid contact interactions, short range hydrodynamic interactions, and poromechanical couplings.

* The numerical model matches very well rheometer experiments on model materials.

* This framework is general and it lets one analyse, namely:
  1- the transition between a dense stable material and a flowing material
  2- the rheology at very large deformations

* Potential applications range from triggering, runoff, and stabilisation of debris flows, to sediments morphodynamics.
Application to granular avalanches: dependance on the fluid viscosity and the angle of inclination: collaboration with P. Dutto (UPM Madrid)
Thanks for your attention