Is Critical State a utopia ? A Barodesy answer

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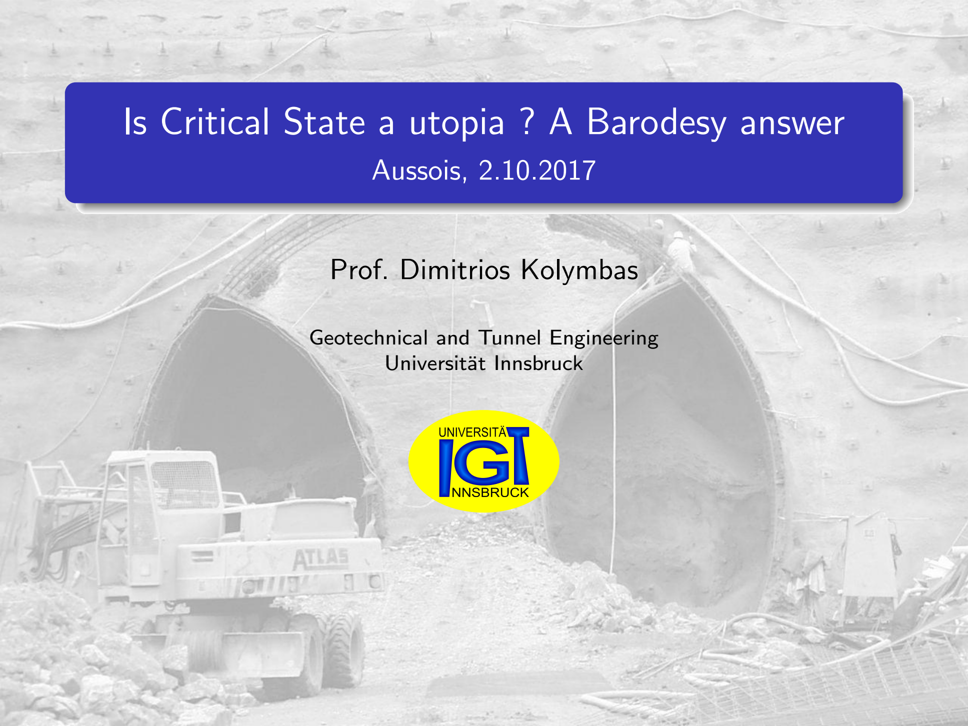

Introduction

Gases at a given temperature: p-ρ-relation is unique.

Granulates: p-ρ-relation is not unique.

Dilatancy & contractancy $\rightarrow \ldots$ critical state.

Large deformation $\rightsquigarrow$ loss of controllability

$\rightsquigarrow$ critical state is difficult to obtain.

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They can be only asymptotically (i.e., strictly speaking, never) reached.

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Summary of Barodesy

Stress: **T**, stretching: **D**.
Exponent 0: normalisation ($\mathbf{T}^0 := \mathbf{T}/|\mathbf{T}|, \mathbf{D}^0 := \mathbf{D}/|\mathbf{D}|$).

Abbreviations: $\sigma := |\mathbf{T}|, \dot{\varepsilon} := |\mathbf{D}|$, $\delta := \mathrm{tr}\mathbf{D}^0$;
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Proportional strain paths ($\mathbf{D}^0 = \text{const}$) starting from $\mathbf{T} = \mathbf{0}$ lead to proportional stress paths with direction **R**.

Dependence of **R** on **D** is given by

$$\mathbf{R} = -\exp(a\mathbf{D}^0),$$

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$$\dot{\mathbf{T}} = h \cdot (f\mathbf{R}^0 + g\mathbf{T}^0) \cdot \dot{\varepsilon} \quad . \tag{1}$$

h: stiffness, depends on σ:

$$h = -\frac{c_4 + c_5\sigma}{e - e_{min}}.$$

At limit states the stiffness vanishes:

$$\dot{\mathbf{T}} = \mathbf{0}.$$

$$\rightsquigarrow f\mathbf{R}^0 + g\mathbf{T}^0 = \mathbf{0} \rightsquigarrow \mathbf{R}^0 = \mathbf{T}^0 \quad \text{and} \quad f + g = 0$$

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Limit states

Critical limit states: $\delta = 0$ **and** $e = e_c$
Peak limit states: $\delta > 0$ **and** $e < e_c$

To obtain both limit states with $f + g = 0$ we set:

$$f + g = \delta + c_3(e_c - e) \tag{2}$$

e_c: critical void ratio.

We partition equation (2) into f and g:

$$f = \delta + c_3 e_c \tag{3}$$

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Critical states—mathematics

e_c is obtained when a critical (or 'steady') state is reached,
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A critical state is obtained asymptotically.

Critical states are attractors

Attractors are typical for evolution equations ('dynamical systems').

Kolmogorof-Arnold-Moser theorem

Special attractors: limit cycles

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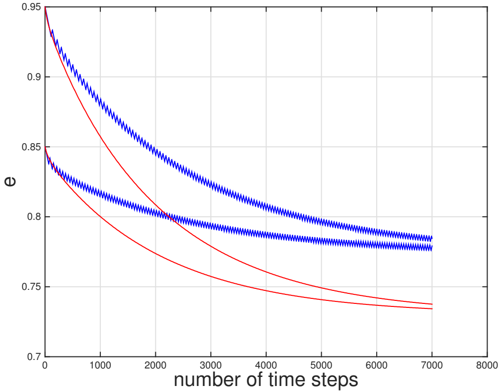

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Oedometric stress cycles converge to the upper limit,
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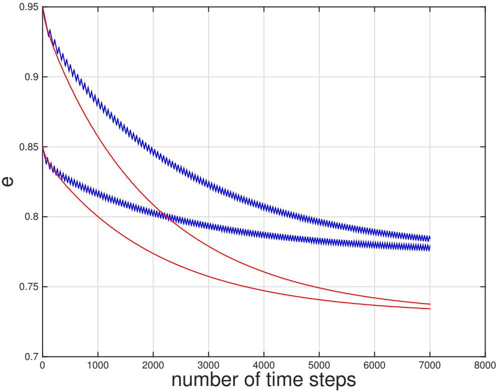

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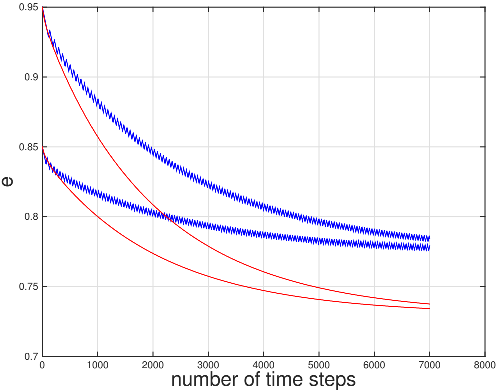

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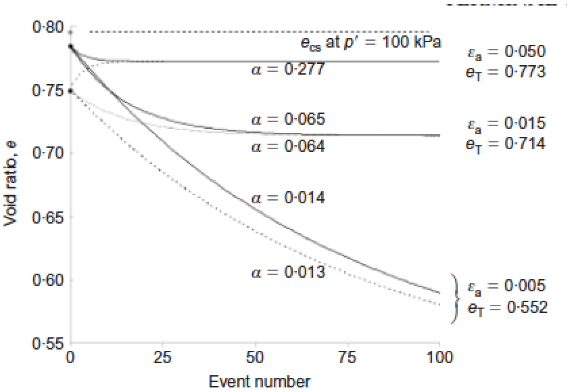
Cyclic void ratios with *NorSand*



Called 'terminal densities' by *Narsilio & Santamarina*

Géotechnique **58**, No. 8, 669-674, 2008

Critical State Line (CSL)

e_c depends on mean stress $p := -\mathrm{tr}\mathbf{T}/3$

(dashes are omitted, as exclusively effective stresses are addressed).

The experimental determination of the CSL (the relation $e_c(p)$) is difficult.

D. Muir Wood: *The paths of tests on loose and dense samples head towards a somewhat diffuse, but clearly pressure dependent, zone of critical void ratios.*

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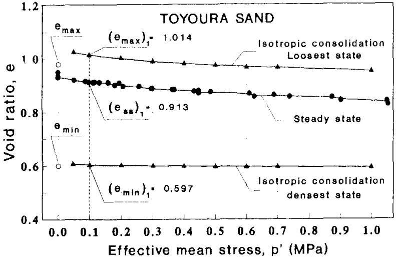

CSL according to Verdugo and Ishihara

Experimental difficulties to determine $e_c(p) \rightsquigarrow$ does e_c depend on more variables?

e_c plays a crucial role even at states far off the critical ones.

Is e_c a physical entity or merely an internal variable in the frame of a particular constitutive relation?

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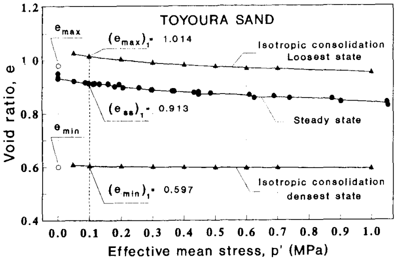

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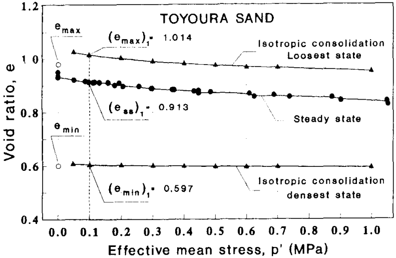

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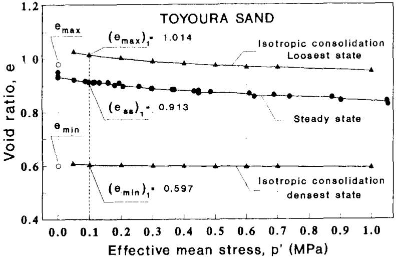

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$\delta := \mathrm{tr}\mathbf{D}^0$

Observe $\mathbf{D}$ with $\mathrm{tr}\mathbf{T} = \mathrm{const}$ (or $\mathrm{tr}\dot{\mathbf{T}} = 0$, i.e. $\dot{p} = 0$)

Two types of behaviour:

- *dilatant* behaviour, if $\delta > 0$,
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This distinction can also be applied if we relax the requirement $\dot{p} = 0$ and allow for $\dot{p} \neq 0$.

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Second definition of dilatancy

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Consider $\dot{p}$ at undrained deformations ($\mathrm{tr}\mathbf{D} = 0$ or $\delta = 0$).

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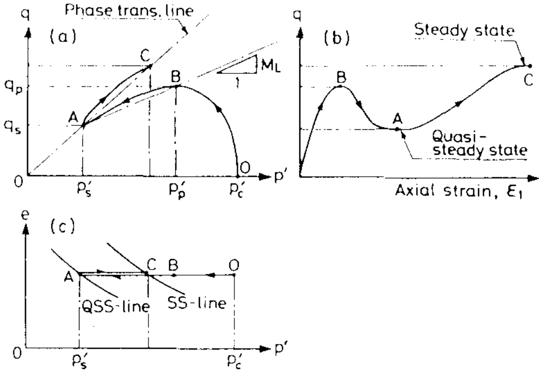
CU-tests schematically , Ishihara



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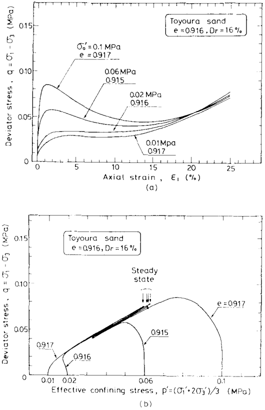
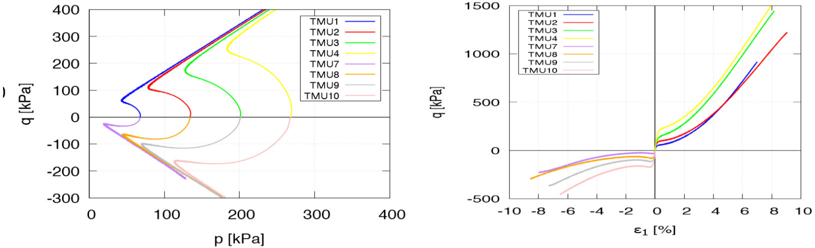
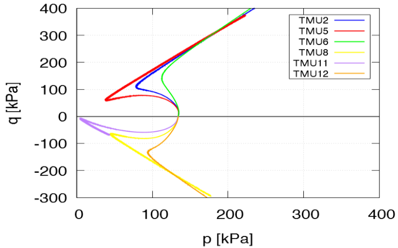


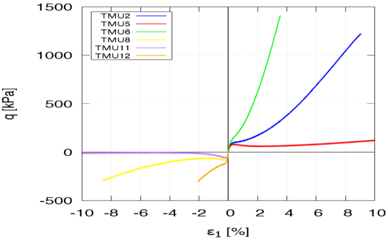
Stiffness does not vanish at 'steady states'!

CU-tests, Wichtmann, medium dense sand

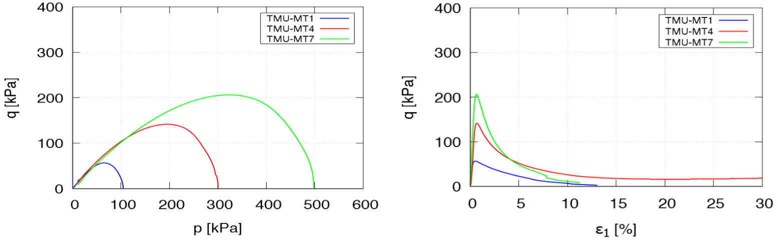


CU-tests, Wichtmann, sand at various densities





CU-tests, Wichtmann, very loose sand



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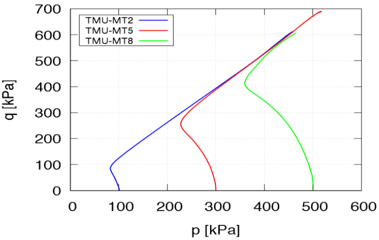

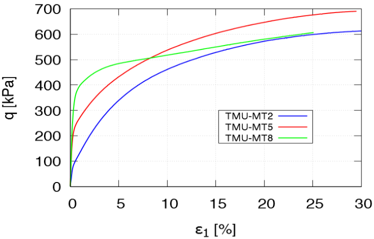

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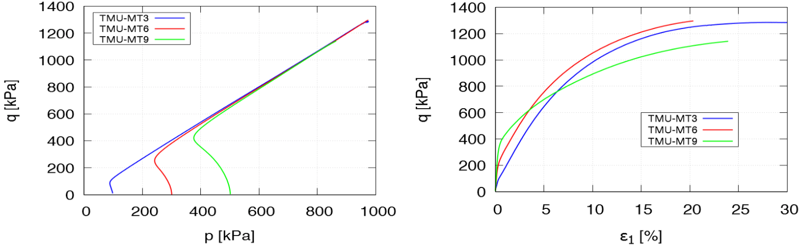

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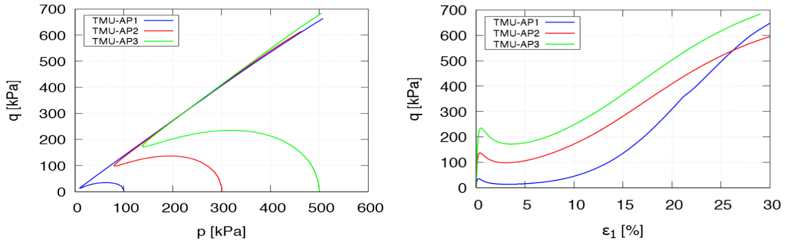
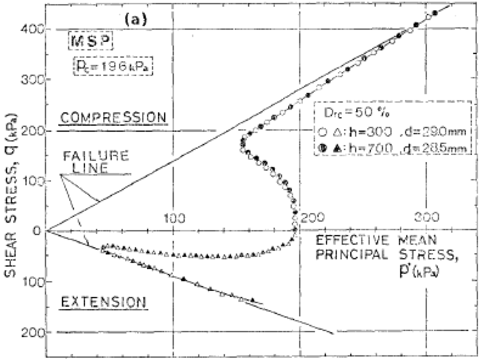


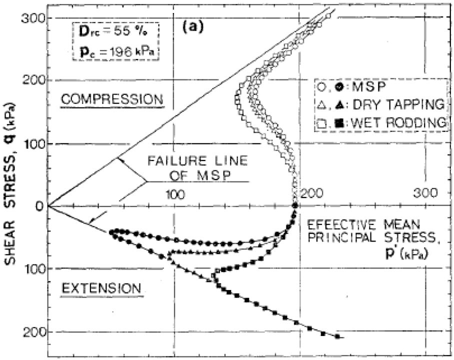
CU-tests, MSP method, Miura



MSP: Multiple sieving pluviation

The final (straight) part of the compression stress paths is not identical to the 'failure line' (=critical state line?)!

CU-tests, preparation: various methods, Miura



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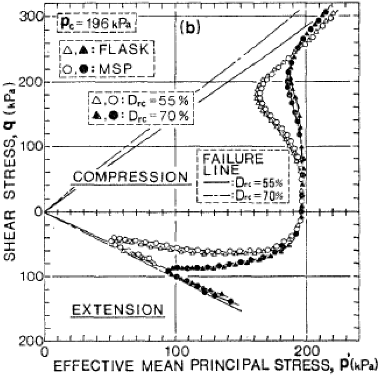

Stress paths at CU-tests

In this sense: An undrained stress path can be initially contractant and then dilatant!

Transition, characterised by $\dot{p} = 0$, is called: phase transformation.

$\dot{\mathbf{T}}$ does not vanish at phase transformations.

Thus, phase transformations are not critical states.

Continuation of the undrained deformation $\rightsquigarrow$ isochoric proportional path.

$\rightsquigarrow$ limit state $\dot{\mathbf{T}} = \mathbf{0}$, where $e = e_c$.

In experiments: critical state is often not achieved.

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Scalar ϵ serves as a criterion (discriminant) for dilatancy or contractancy:

- $e < \epsilon$ implies dilatancy,
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ϵ depends on $\mathbf{T}$ and $\dot{p}$, in case of the first definition,
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For $\dot{\mathbf{T}} = \mathbf{0}$ we have: $\epsilon = e_c$.
Cf. 'critical states' in Physics (states in which two phases have the same temperature, pressure, and volume).
Associate critical with criterion (for dilatancy or contractancy)
$\rightsquigarrow \epsilon =$ critical void ratio.
Thus, 'critical void ratio' e_c merely denotes particular values of ϵ, which serves as criterion for dilatancy/contractancy
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Cf. 'critical states' in Physics (states in which two phases have the same temperature, pressure, and volume).

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Thus, 'critical void ratio' e_c merely denotes particular values of ϵ, which serves as criterion for dilatancy/contractancy
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Scalar ϵ serves as a criterion (discriminant) for dilatancy or contractancy:

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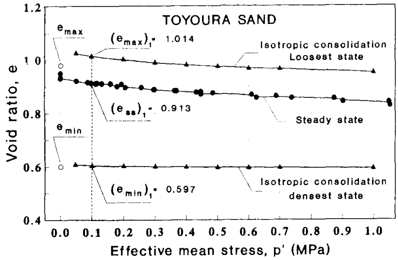
Variability of critical void ratio

Along proportional stress paths corresponding to tr$\mathbf{D} = 0$ (e.g. the line $q = mp$): $\sigma := |\mathbf{T}|$ increases, i.e. $\dot{\sigma} > 0$, as long as $e < e_c$, i.e. for 'dense' states.

With increasing σ the critical void ratio e_c reduces.

Eventually, at $e = e_c$: a steady state is reached ($\dot{\sigma} = 0$).

However, reduction of e_c with σ is small.



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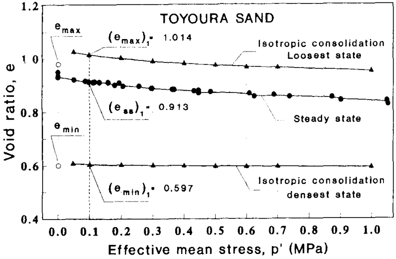

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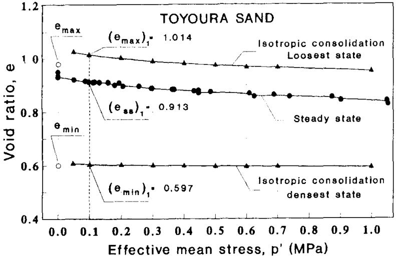

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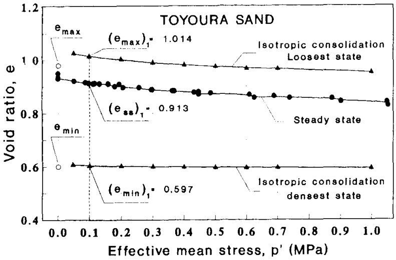

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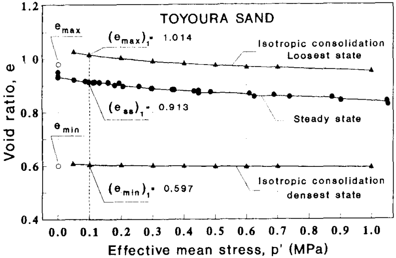

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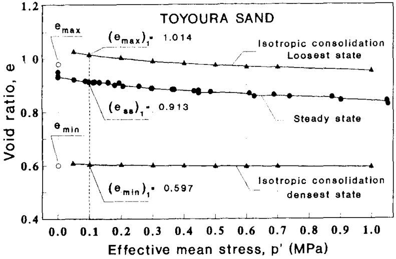

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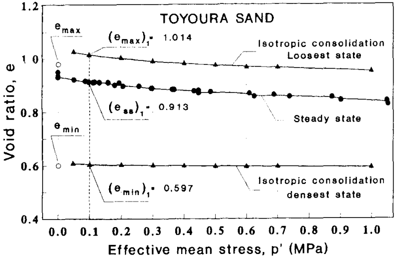

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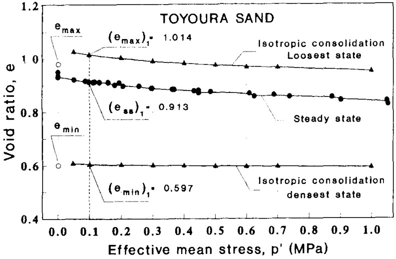

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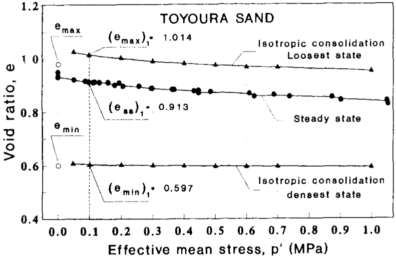

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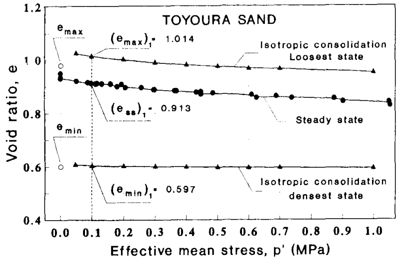

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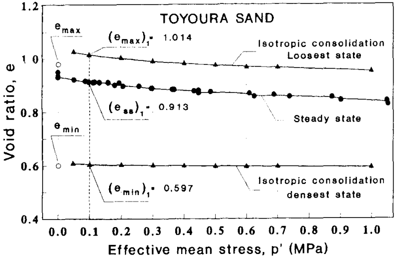

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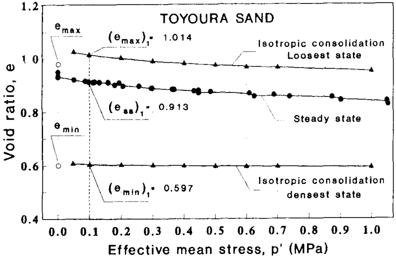

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Hence, states with considerably high void ratio are still 'dense'.

But: results of Wichtmann show that undrained 'dense' samples behave initially as loose ones (i.e. with $\dot{p} < 0$).

Conflict can be resolved if we assume a particular distribution of e_c in the p-q- space with a sharp change of e_c within a small transition zone (see next slide).

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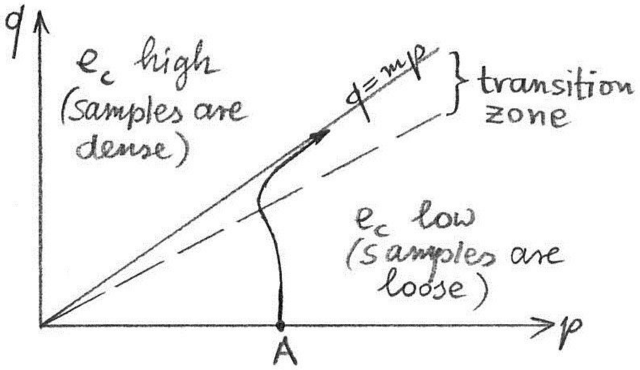

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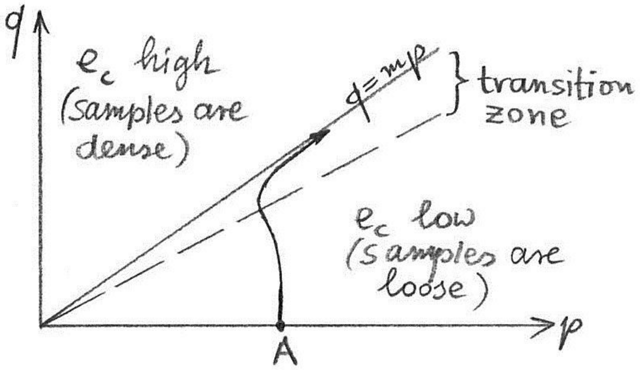

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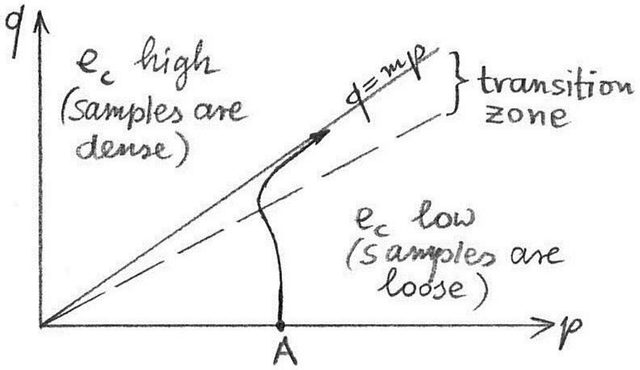

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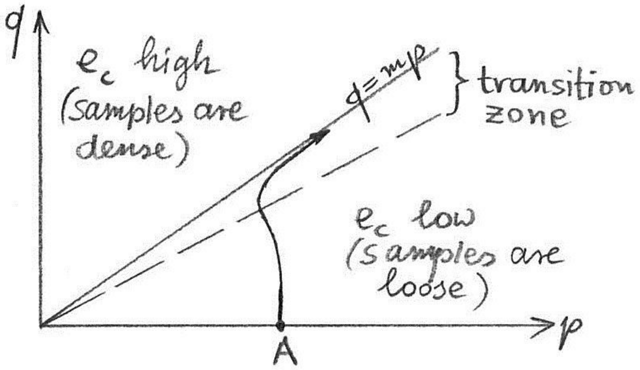

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Dependence of e_c on q and δ

Regarding $e_c(p)$, barodesy sticks to a whatsoever defined CSL (critical state line), e.g.

$$e_c = (1 + e_{c0}) \left(\frac{p}{p_0} \right)^{\lambda} - 1 \ , \qquad (5)$$

but accepts that e_{c0} depends on **T** (or, more specific, on the ratio q/p) and on the dilatancy δ.

The rather sharp bend of undrained paths at the phase transformation from loose or contractant ($\dot{p} < 0$) to dense or dilatant ($\dot{p} > 0$) behaviour points to a sharp increase of e_{c0} there.

To model this we use the symbol $m := q/p$ and set

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$$m_c = m_{c0} \exp(c_\delta \delta). \qquad (9)$$

Dependence of e_c on q and δ

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m_c is the value of m at critical states.
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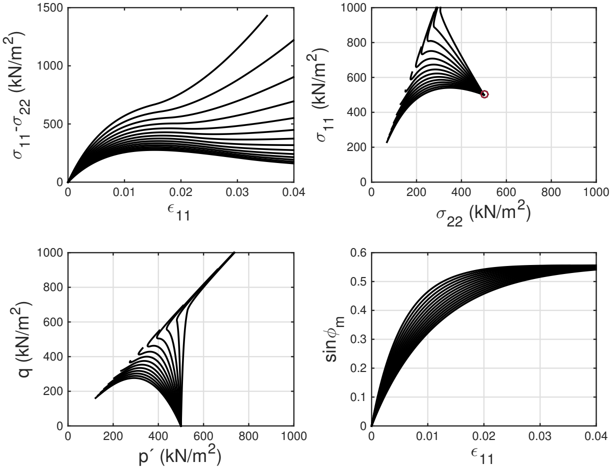
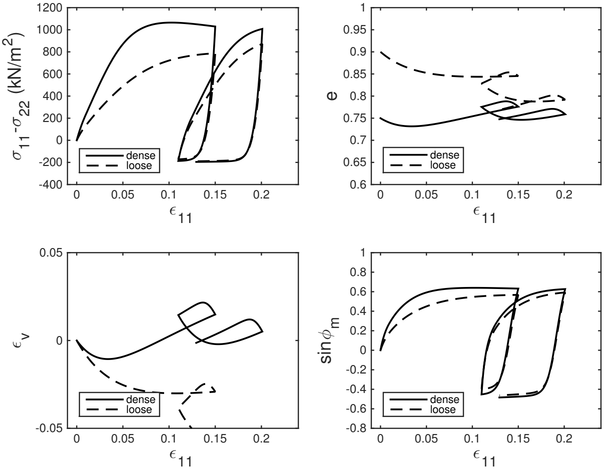
Simulations of CU-tests with barodesy



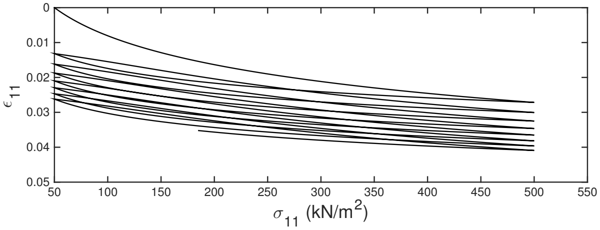
$$e_{c0o} = 0,85; e_{c0u} = 0,70; m_c = 1,36; m_1 = 1,10$$

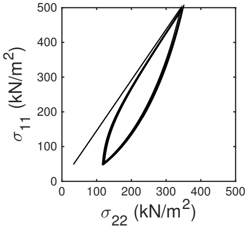
Simulations of CD-tests with barodesy



$$e_{c0o} = 0,85; e_{c0u} = 0,70; m_c = 1,36; m_1 = 1,10$$

Simulations of oedometer test with barodesy





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To conclude

Is 'critical state' an objective notion?

Yes, because it can be objectively defined.

Is e_c a physical variable?

Yes, because it has a value at every state. FALK: A physical variable has a particular value in every state. Cf. definition of 'state' in quantum mechanics: "A state is a unit vector of a vector space."

Can e_c be measured?

Not directly! MAX PLANCK said that what we can't measure, it doesn't exist! Note, however, that also the friction angle can't be directly measured.

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What is the physical meaning of material constants?

This question arises from the erroneous expectation that a material constant be directly measurable, i.e. the outcome of a particular experiment.
Similarly, a material function, like $e_c(\ldots)$ is expected to be directly measurable.

Is the friction angle a material constant?

We assign values to it, but it depends on σ and on process (compression or extension or shear). So, does it exist in the sense of PLANCK?

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- outcomes of experiments
- functions (or constitutive relations) of physical variables Their values are also physical variables
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The relation between physical variables and outcomes of experiments can only be stablished in the frame of a theory (constitutive relation).

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"If you do not know what you should be looking for in a site investigation, you are not likely to find much of value."

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