ALERT Workshop 2017
Must Critical State Theory for Granular Mechanics be Revisited?
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CRITICAL STATE OF FINE-GRAINED SOILS UNDER "ENVIRONMENTAL LOADS"

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Outline

- Introduction: “environmental processes” and their driving variables
- Constitutive modeling in presence of “environmental loads”: plasticity with generalized hardening
- Definitions of Critical State: extension to EP and consequences on constitutive functions
- Experimental evidences of CS for soils under environmental loads

Introduction: environmental loading

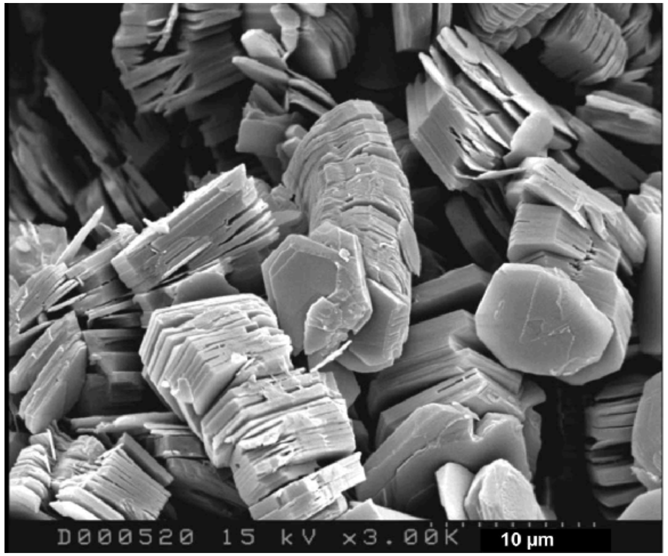
Several recent developments in soil mechanics have been focused on the description the effects of the interactions between the soil and the environment (Gens 2010).

The notion of “environmental actions” is intended in a rather broad sense to include:

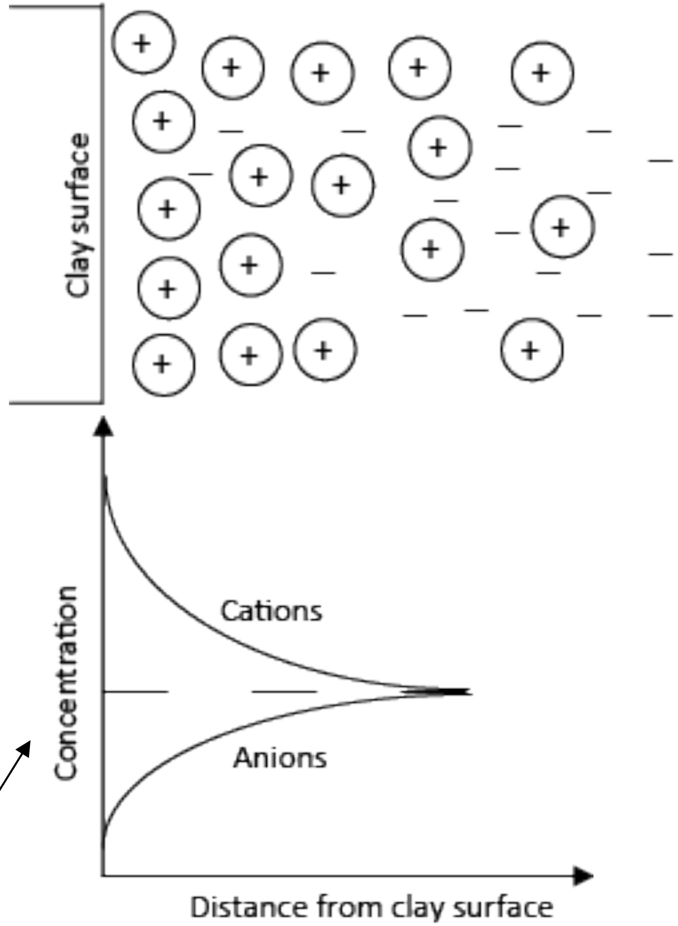
- changes in pore fluid pressures and degree of saturation (hydraulic coupling);
- changes in temperature (thermal coupling);
- changes in chemical composition of the liquid phase (chemical interaction).

Introduction: environmental loading

Coupled mechanical-environmental processes



Microstructure of kaolin clay
(from Wilson et al., Clay Minerals, 2014)



Diffuse double layer (from Mitchell & Soga, 2005)

Assumptions:

- linearized kinematics (small strains)

- environmental process variables:

$$\boldsymbol{\alpha} = \left\{ S_r,\ T,\ c_1,\ c_2,\ c_3,\ \ldots \right\}$$

- constitutive stress for the solid skeleton:

$$\boldsymbol{\sigma}^* := \boldsymbol{\sigma}' = \boldsymbol{\sigma} - p_w \mathbf{1} \qquad (S_r = 1)$$

$$\boldsymbol{\sigma}^* := \boldsymbol{\sigma}'' = \boldsymbol{\sigma} - \left[S_r p_w + (1 - S_r) p_g \right] \mathbf{1} \qquad (S_r \leq 1)$$

$$\boldsymbol{\sigma}^* := \bar{\boldsymbol{\sigma}} = \boldsymbol{\sigma} - p_g \mathbf{1} \qquad (S_r \leq 1)$$

Plasticity with Generalized Hardening (Tamagnini & Ciantia, 2016)

1. Strain decomposition:

$$\dot{\boldsymbol{\varepsilon}} = \dot{\boldsymbol{\varepsilon}}^e + \dot{\boldsymbol{\varepsilon}}^p$$

2. (hypo)Elastic behavior:

$$\dot{\boldsymbol{\sigma}}^* := \boldsymbol{D}^e(\dot{\boldsymbol{\varepsilon}} - \dot{\boldsymbol{\varepsilon}}^p) + \boldsymbol{M}\dot{\boldsymbol{\alpha}}$$

3. Elastic domain:

$$\mathcal{E}_\sigma := \left\{(\boldsymbol{\sigma}^*, \boldsymbol{q}, \boldsymbol{\alpha}) \mid f(\boldsymbol{\sigma}^*, \boldsymbol{q}, \boldsymbol{\alpha}) \leq 0\right\}$$

4. Flow rule:

$$\dot{\boldsymbol{\varepsilon}}^p = \dot{\gamma}\frac{\partial g}{\partial \boldsymbol{\sigma}^*} = \dot{\gamma}\boldsymbol{Q} \qquad\qquad g = g(\boldsymbol{\sigma}^*, \boldsymbol{q}, \boldsymbol{\alpha})$$

5. Generalized hardening law:

$$\dot{\boldsymbol{q}} := \boldsymbol{h}(\boldsymbol{\sigma}^*, \boldsymbol{q}, \boldsymbol{\alpha})\dot{\gamma} + \boldsymbol{N}(\boldsymbol{\sigma}^*, \boldsymbol{q}, \boldsymbol{\alpha})\dot{\boldsymbol{\alpha}}$$

6. Kuhn-Tucker complementarity conditions:

$$f(\boldsymbol{\sigma}^*, \boldsymbol{q}, \boldsymbol{\alpha}) \leq 0 \qquad \dot{\gamma} \geq 0 \qquad \dot{\gamma} f(\boldsymbol{\sigma}^*, \boldsymbol{q}, \boldsymbol{\alpha}) = 0$$

7. Plastic multiplier from consistency condition:

$$\dot{\gamma} = \frac{1}{K_p}\left\langle \nabla_{\boldsymbol{\sigma}^*} f \cdot \boldsymbol{D}^e \dot{\boldsymbol{\varepsilon}} + \nabla_{\boldsymbol{\sigma}^*} f \cdot \boldsymbol{M} \dot{\boldsymbol{\alpha}} + \nabla_{\boldsymbol{q}} f \cdot \boldsymbol{N} \dot{\boldsymbol{\alpha}} \right\rangle$$

$$K_p = \nabla_{\boldsymbol{\sigma}^*} f \cdot \boldsymbol{D}^e \nabla_{\boldsymbol{\sigma}^*} g - \nabla_{\boldsymbol{q}} f \cdot \boldsymbol{h} > 0$$

NOTE: Plastic multiplier split

$$\dot{\gamma} = \dot{\gamma}_{\text{mech}} + \dot{\gamma}_{\text{env}} \geq 0$$

$$\dot{\gamma}_{\text{mech}} := \frac{1}{K_p}\left(\nabla_{\boldsymbol{\sigma}^*} f \cdot \boldsymbol{D}^e \dot{\boldsymbol{\varepsilon}} \right) \qquad \dot{\gamma}_{\text{env}} := \frac{1}{K_p}\left(\nabla_{\boldsymbol{\sigma}^*} f \cdot \boldsymbol{M} \dot{\boldsymbol{\alpha}} + \nabla_{\boldsymbol{q}} f \cdot \boldsymbol{N} \dot{\boldsymbol{\alpha}} \right)$$

Evolution equations for the state variables

$$\dot{\boldsymbol{\sigma}}^* = \boldsymbol{D}^{ep}\dot{\boldsymbol{\varepsilon}} + \boldsymbol{M}^{ep}\dot{\boldsymbol{\alpha}}$$

$$\dot{\boldsymbol{q}} = \boldsymbol{H}^{p}\dot{\boldsymbol{\varepsilon}} + \boldsymbol{N}^{ep}\dot{\boldsymbol{\alpha}}$$

$$\dot{s} = \boldsymbol{k}_{\varepsilon} \cdot \dot{\boldsymbol{\varepsilon}} + \boldsymbol{k}_{\alpha} \cdot \dot{\boldsymbol{\alpha}}$$

$$\boldsymbol{D}^{ep} = \boldsymbol{D}^{e} - \frac{1}{K_p}\left\{\boldsymbol{D}^{e}\nabla_{\boldsymbol{\sigma}^*} g\right\} \otimes \left\{(\nabla_{\boldsymbol{\sigma}^*} f)^T \boldsymbol{D}^{e}\right\}$$

$$\boldsymbol{H}^{p} = \frac{1}{K_p}\boldsymbol{h} \otimes \left\{(\nabla_{\boldsymbol{\sigma}^*} f)^T \boldsymbol{D}^{e}\right\}$$

$$\boldsymbol{M}^{ep} = \boldsymbol{M} - \frac{1}{K_p}\left\{\boldsymbol{D}^{e}\nabla_{\boldsymbol{\sigma}^*} g\right\} \otimes \left\{(\nabla_{\boldsymbol{\sigma}^*} f)^T \boldsymbol{M} + (\nabla_{\boldsymbol{q}} f)^T \boldsymbol{N}\right\}$$

$$\boldsymbol{N}^{ep} = \boldsymbol{N} + \frac{1}{K_p}\boldsymbol{h} \otimes \left\{(\nabla_{\boldsymbol{\sigma}^*} f)^T \boldsymbol{M} + (\nabla_{\boldsymbol{q}} f)^T \boldsymbol{N}\right\}$$

Purely mechanical processes

a) Asymptotic states in which the soil deforms at constant stress and constant volume:

$$\dot{\boldsymbol{e}}^p \neq \mathbf{0} \qquad \dot{\varepsilon}_v^p = 0 \qquad \dot{v} = 0 \qquad \dot{\boldsymbol{\sigma}}^* = \mathbf{0}$$

b) Asymptotic states in which the soil deforms at constant stress and constant volume, and reaches a unique critical specific volume function of p^*:

$$\dot{\boldsymbol{e}}^p \neq \mathbf{0} \qquad \dot{\varepsilon}_v^p = 0 \qquad v = v_c(p^*) \qquad \dot{\boldsymbol{\sigma}}^* = \mathbf{0}$$

c) Asymptotic states in which the soil deforms at constant stress, constant volume and constant fabric:

$$\dot{\boldsymbol{e}}^p \neq \mathbf{0} \qquad \dot{\varepsilon}_v^p = 0 \qquad \dot{\boldsymbol{\sigma}}^* = \mathbf{0} \qquad \dot{\boldsymbol{f}} = \mathbf{0}$$

Coupled mechanical-environmental processes

For coupled mechanical-environmental processes, the changes in state variables may depend on the changes of $\boldsymbol{\alpha}$, as well as of $\boldsymbol{\varepsilon}$.

The previous definitions of CS could still be applied, provided that they refer to processes which occur at constant $\boldsymbol{\alpha}$:

$$\dot{\boldsymbol{\alpha}} = \mathbf{0} \qquad \dot{\boldsymbol{e}}^p \neq \mathbf{0} \qquad \dot{\varepsilon}_v^p = 0 \qquad \dot{\boldsymbol{\sigma}}^* = \mathbf{0}$$

The loci of CS in the space of state variables can be different for different $\boldsymbol{\alpha}$s.

Requirements of CE to incorporate Critical States

In order to reach an asymptotic state at constant environmental loads, we must have:

$$\dot{\boldsymbol{\varepsilon}} = \dot{\boldsymbol{\varepsilon}}^p \qquad Q_v = \frac{\partial g}{\partial p^*}(\boldsymbol{\sigma}^*, \boldsymbol{q}, \boldsymbol{\alpha}) = 0 \qquad \boldsymbol{h}(\boldsymbol{\sigma}^*, \boldsymbol{q}, \boldsymbol{\alpha}) = \boldsymbol{0}$$

This can be achieved in two ways:

a) assuming that $\boldsymbol{h}$ is a homogeneous function of Q_v

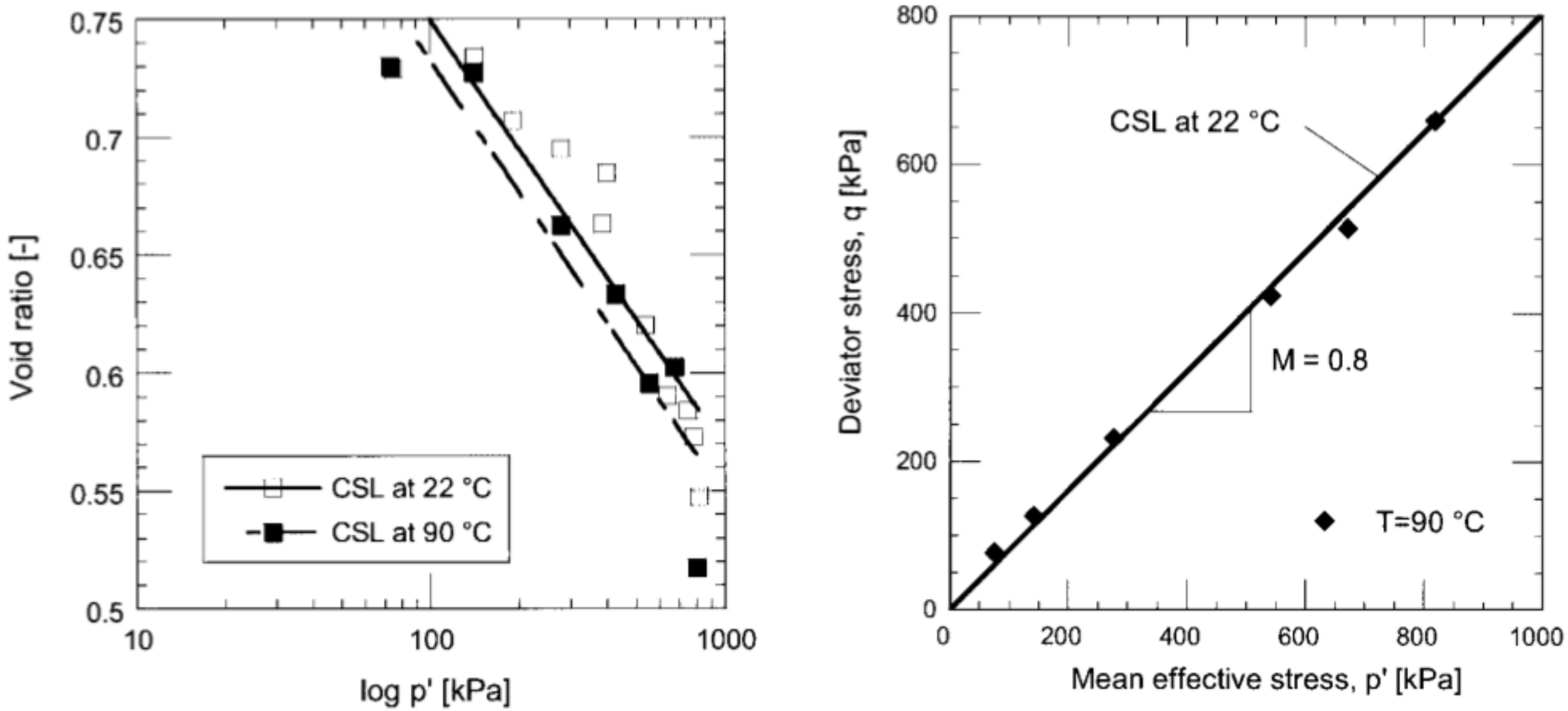
b) Assuming that both Q_v and $\boldsymbol{h}$ depend on a state variable:

$$\psi = v - v_c \qquad Q_v\Big|_{\psi=0} = 0 \qquad \boldsymbol{h}\Big|_{\psi=0} = \mathbf{0}$$

In both cases, the functions defining the CS locus may incorporate environmental process variables as coefficients, making the CS locus dependent of $\boldsymbol{\alpha}$

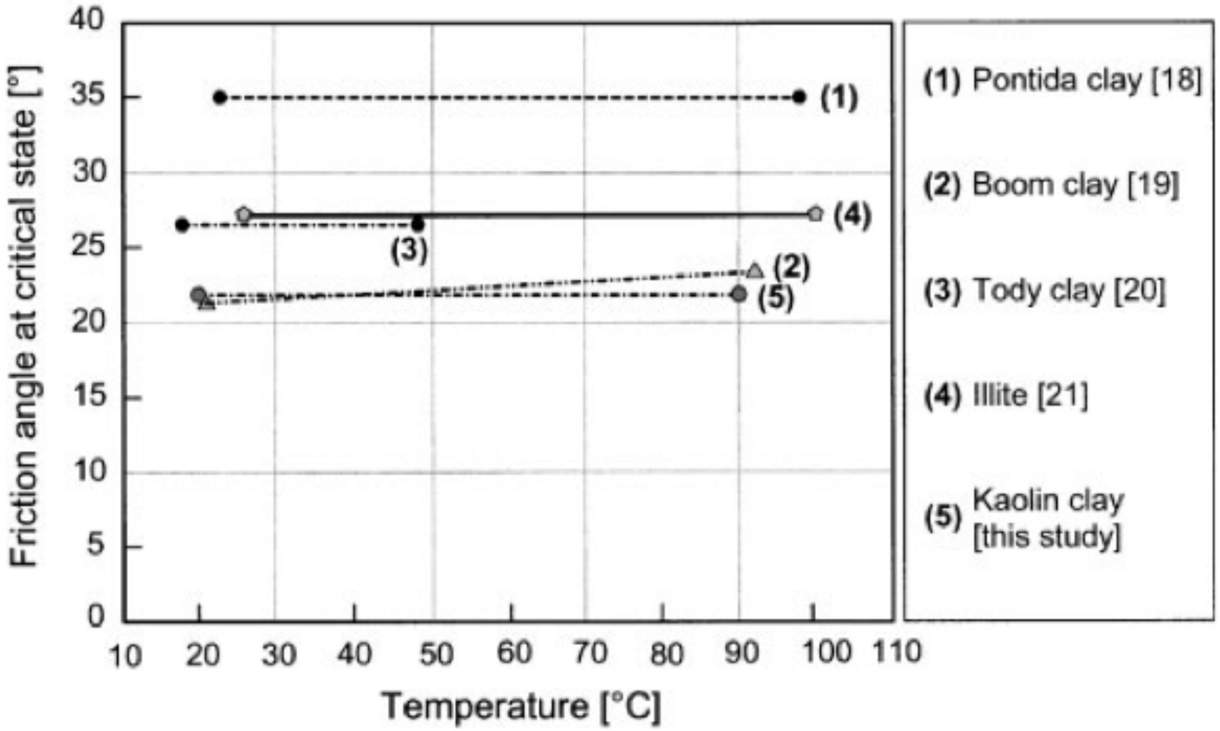
Exptl. evidences of CS: effect of T

Drained TX tests on kaolin (Cekerevac & Laloui, 2004)



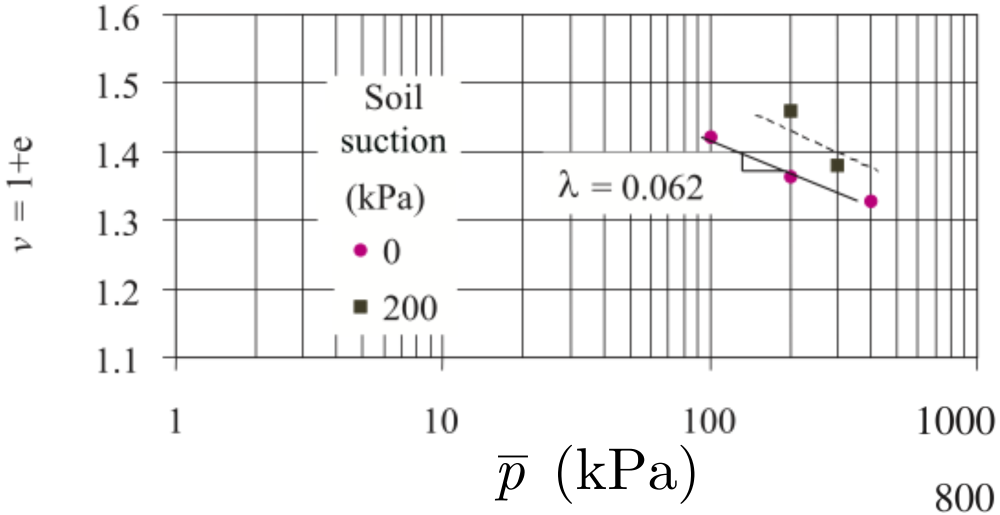
Exptl. evidences of CS: effect of T

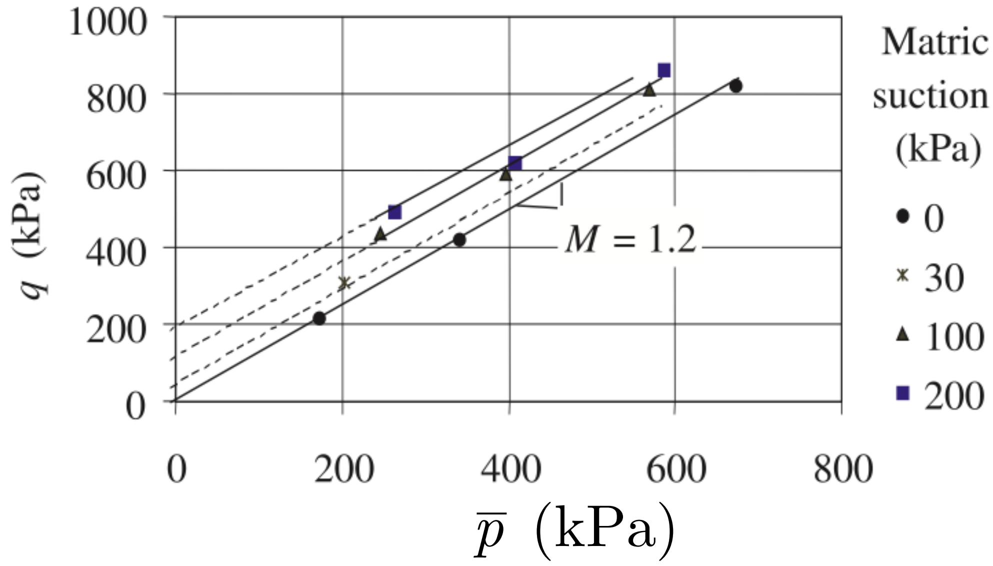
Observations from other studies (from Cekerevac & Laloui, 2004)



[18] Hueckel T., Baldi G., ASCE J. Geotech. Engng. (1990)
[19] Hueckel T., Pellegrini R., Proc. NUMOG III (1989)
[20] Burghignoli et al., Canadian Geotech. J. (2000)
[21] Graham et al., Canadian Geotech. J. (2001)

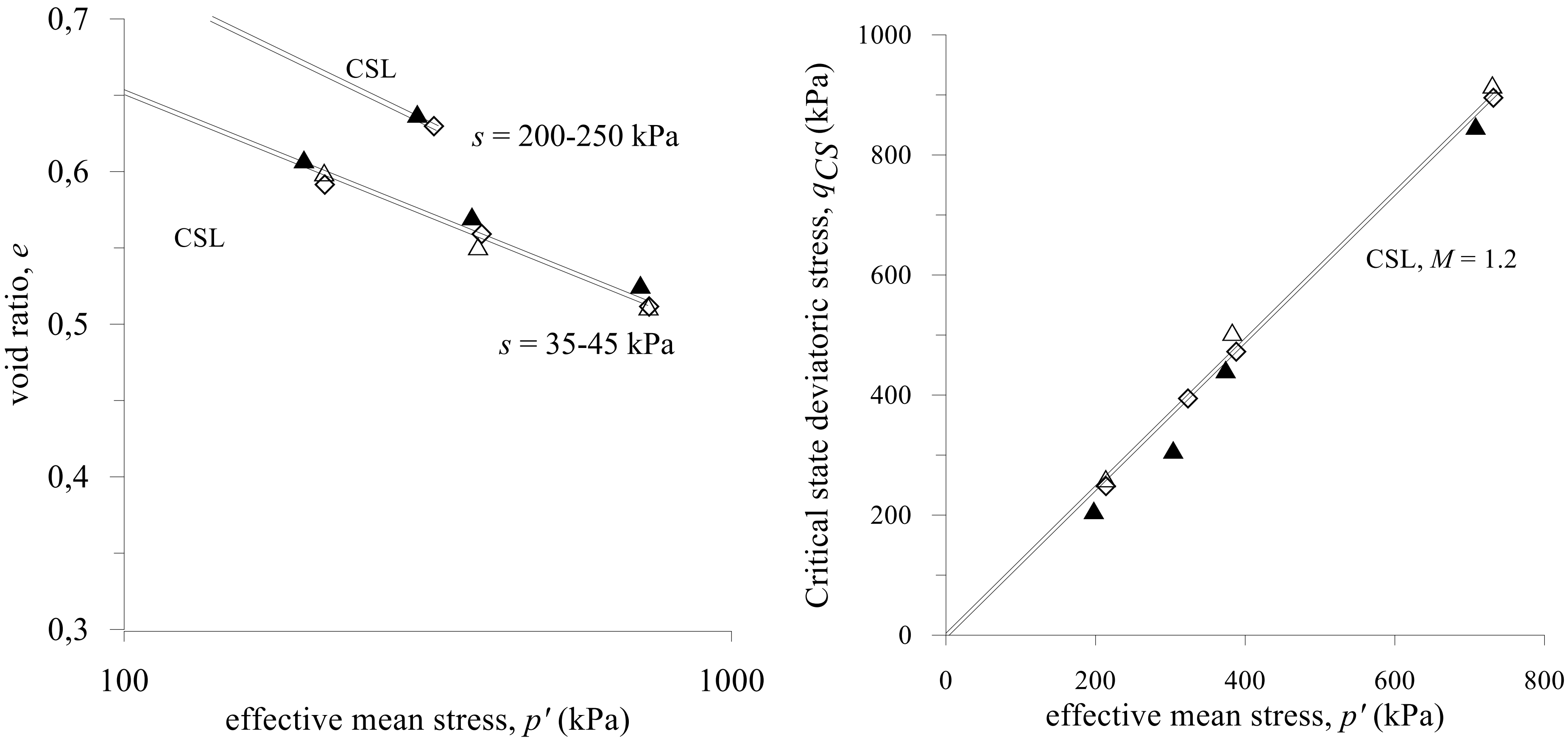
Suction controlled TX tests on Botkin silt (Wang et al., 2002)





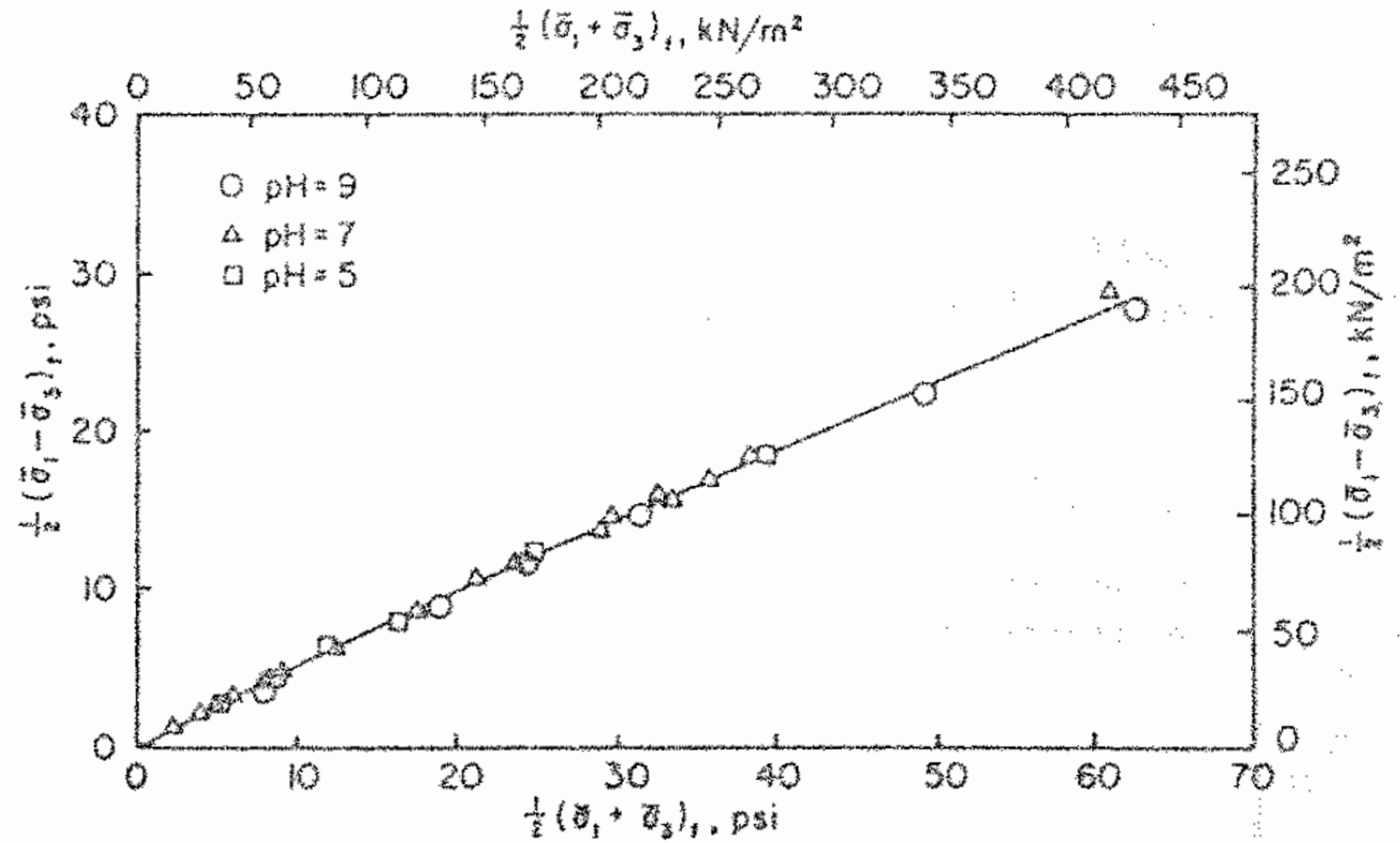
Exptl. evidences of CS: effect of S_r

Suction controlled TX tests on Viadana silt (Azizi et al., 2017)



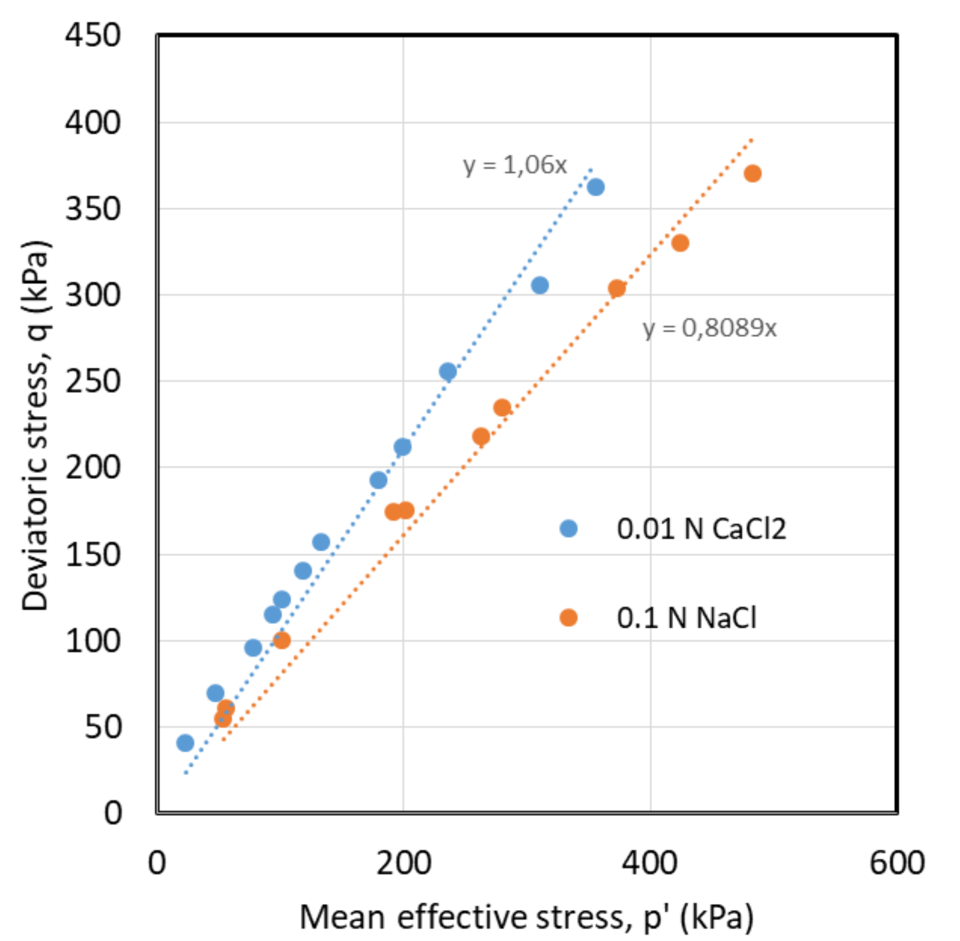
Exptl. evidences of CS: effect of p.w. chemistry

TX-CD and TX-CU tests on Kaolin (Olson, 1974)



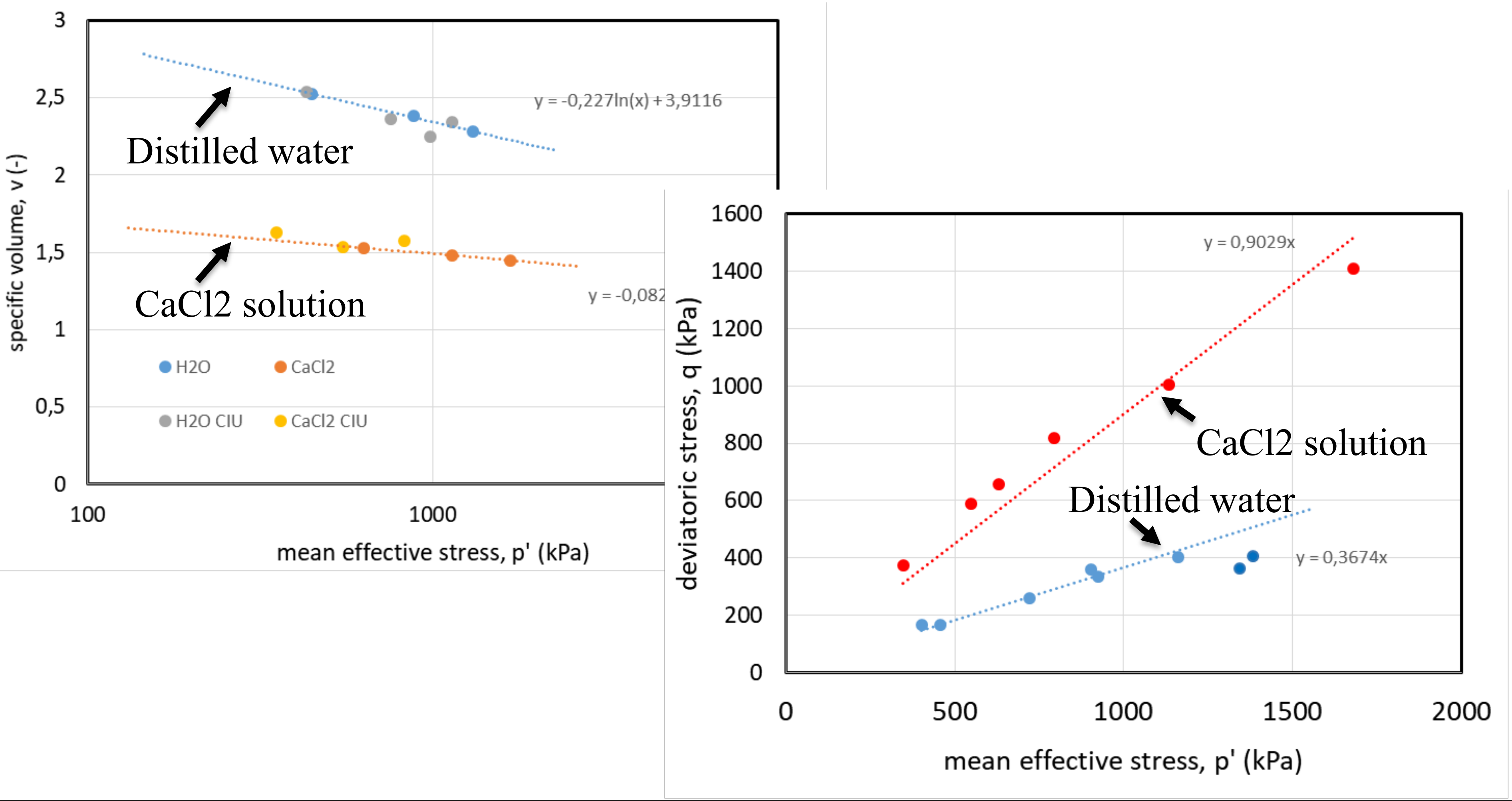
Exptl. evidences of CS: effect of p.w. chemistry

TX-CD and TX-CU tests on Illite (Olson, 1974)



Exptl. evidences of CS: effect of p.w. chemistry

TX-CD and TX-CU tests on Sand Bentonite mixture (50%-50%, Siddiqua et al., 2014)



Concluding remarks

- In presence of environmental loading process – of thermal, hydraulic, chemical type – the definition of critical state provided for purely mechanical processes needs to be revisited.

- Different sets of CS loci in state variables space (i.e. CS lines) can in principle be found for different environmental conditions.

- The main reason for that is the effects that environmental conditions may have on the electrochemical interactions between particles, and thus on the soil fabric.

Concluding remarks

- For clays of low to medium activity, the effects of T and S_r are clearly visible in the shift of the CS locus on the v:p* space. This is not always the case for the CS locus in stress space.

- For active clays, like bentonites, an important influence of T and S_r on the critical friction angle is observed as a consequence of the significant modifications induced by the environmental loads on the soil fabric.

Concluding remarks

- Experimental evidences are still quite limited and in most cases not conclusive. Temperature and chemical composition of pore water have larger effects on CS for active clays.

- The proper experimental characterization of CS is made quite difficult by a number of limitations in the measurement techniques and by the tendency of the material to loose homogeneity during intense shearing processes (e.g., strain localization).