Response to the comment by David Muir Wood with title: Critical states and critical state theory

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I would like firstly to sincerely thank David for his interest to participate by his comment to our workshop, to which he was invited but could not participate due to prior obligations.

In regards to the opening paragraph of David's statement, we have defined the relevant terms among the participants in order to avoid any misunderstanding by giving different meanings to the same names; in particular the following definition was proposed and sent to participants before the workshop:

Critical State Theory (CST): It is the theory that, among other things, proposes necessary and sufficient conditions on stress and density to reach and maintain Critical State (CS).

The classical CST defined by Schofield and Wroth (1968) explicitly proposed two such conditions: $q=Mp$ and $e=e_c=e_c(p)$; when CST is used herein it will refer to this classical CST. Similar definitions for other concepts can be found in the opening set of slides with title "Why the Workshop?".

During the workshop I have stressed the importance of distinguishing a physical event like CS from the theory developed to describe it, and promptly named CST. Subsequently I have specifically emphasized the fact that when considering CS, it is implicitly accepted that the shear strain rate maintains a fixed direction **n**. I have also emphasized that no corresponding condition about fixity of **n** was included in the two conditions on CS stress and void ratios, proposed by the classical CST as being necessary and sufficient to reach and maintain CS. It is this missing link on the fixity of **n** between CS and CST that constituted the theme of my presentation.

When I considered the imposition of rotation of principal stress directions at CS keeping the principal stress values fixed, I have explicitly stated that according to the CS premises such stress rotation implies also rotation of **n**, thus, violation of one of the observed attributes of CS, hence, it is expected that CS will be abandoned. On the other hand I stated that CST does not provide any condition that precludes such rotation to take place, and since during the rotation the stress invariants q and p remain fixed while the void ratio e is at its CS value for the given fixed p, according to the theory the sample must remain at CS. This contradicted both what CS would suggest and what in fact was shown to happen during the DEM simulations, proving that the two conditions of classical CST are necessary but not sufficient to reach and maintain CS. At this junction I should point out that the example given by David on reversal of shear strain rate, implies automatically reduction of stress ratio from its CS value M, hence, it is expected that CS will be abandoned; contrary to this, the stress rotation with fixed principal values in my presentation, maintains the critical state stress ratio at value M as well as the critical void ratio for the given p, and yet

CS is abandoned; they are two totally different examples in regards to the message they send.

Subsequently, in proposing an enhancement of the CST to include a condition on the fixity of **n** that was missing, we were led in a logical way (see slides 20-23 of my presentation), by making use of invariance requirements under superposed rigid body rotation, that such fixity condition must be imposed in relation to the sample, hence, necessarily introducing the requirement for describing a sample orientation that is achieved only by use of a fabric tensor. The rest follows, including the completion of CST's two conditions by a third on fabric anisotropy as proposed by the recently developed Anisotropic Critical State Theory (ACST). These three conditions of ACST are now both necessary and sufficient for CS to be reached and maintained and can fully explain the observed DEM results of abandonment of CS when the stress rotation was imposed, as well as the results for monotonic loading back to CS, following the stress rotation.

Therefore there is no basic disagreement with what David commented upon, as long as one can distinguish CS as a physical event from its theory, namely the CST.

As a closing comment I would consider a nice addition to the subject matter if other ALERT members and audience of the workshop could submit for uploading their opinion.