The role of constitutive models in large deformation MPM simulations

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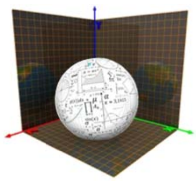
ENGINEERING FLOW CHART

Observe → **Understand** → **Model** → **Simulate**



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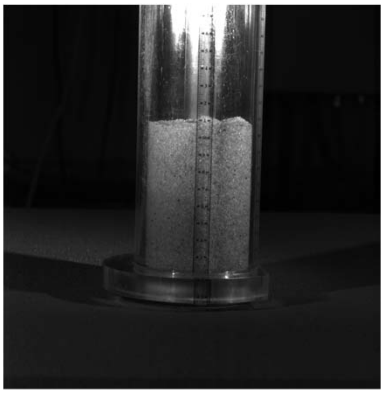
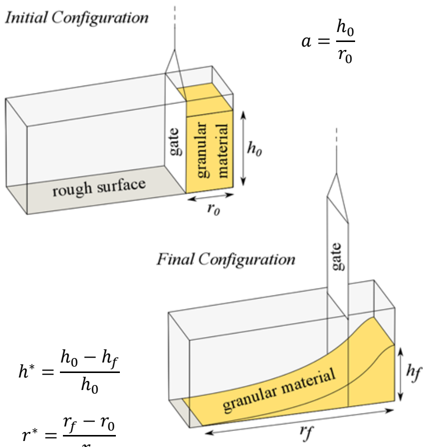
Granular Column Collapse





$$a = \frac{h_0}{r_0}$$

$$h^* = \frac{h_0 - h_f}{h_0}$$

$$r^* = \frac{r_f - r_0}{r_0}$$

(Fern and Soga 2016)

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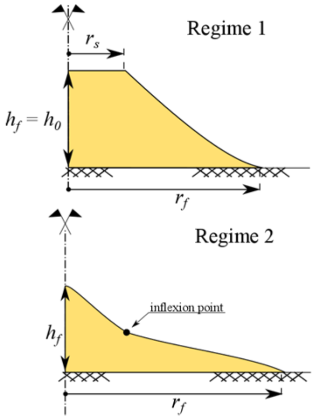

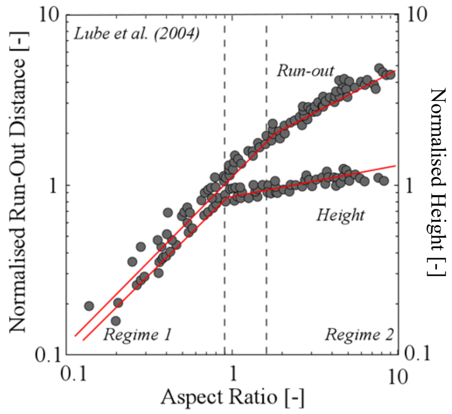

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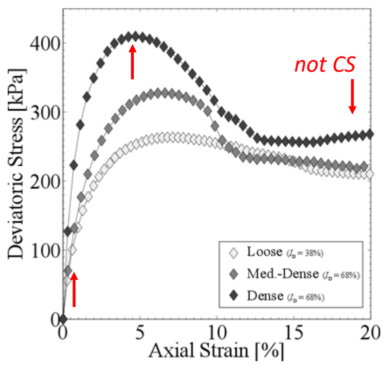
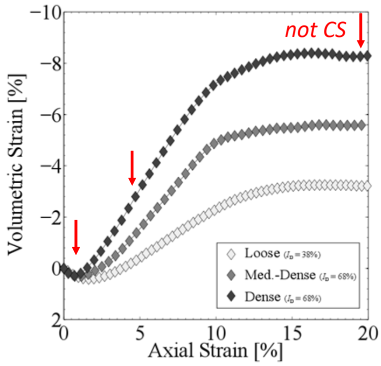
MECHANICS OF SAND





Dry pluviated Chiba sand (Fern 2016)

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MOHR-COULOMB *(Terzaghi 1943)*

$$\tau = c' + \sigma'_N \cdot \tan\varphi'$$

Option 1: Small Deformation

$$\varphi' = \varphi'_{max}$$

$$\psi = \psi_{max}$$

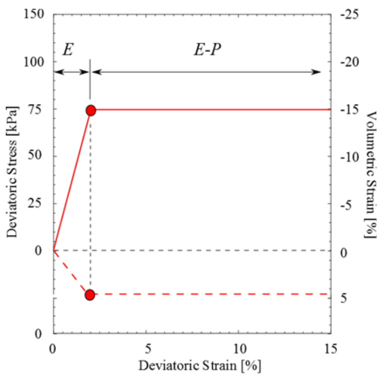
$\psi \neq 0$ causes unrealistic results

Option 2: Large Deformation

$$\varphi' = \varphi'_{cs}$$

$$\psi = 0$$

$$(e \neq e_{cs})$$



Does the small strain behaviour play a role in the large deformation simulations?

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MOHR-COULOMB STRAIN SOFTENING *(Terzaghi 1943, Abbo and Sloan 1995)*

$$\tau = c' + \sigma'_N \cdot \tan\varphi'$$

$$\varphi'_{max} \xrightarrow[E_d^p \to \infty]{} \varphi'_{res}$$

$$\psi_{max} \xrightarrow[E_d^p \to \infty]{} \psi_{res}$$

$$c'_{max} \xrightarrow[E_d^p \to \infty]{} c'_{res}$$

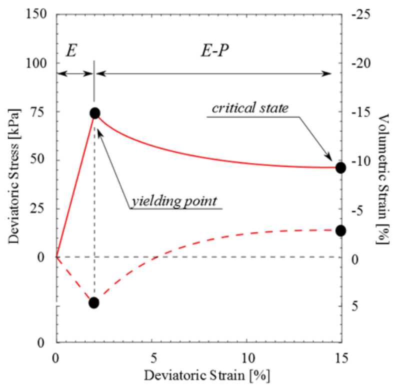
Small Deformation

$$\psi = \psi_{max} \rightarrow \varphi' = \varphi'_{max}$$

Large Deformation

$$\psi = 0 \rightarrow \varphi' = \varphi'_{cs}$$

$$e \neq e_{cs}$$



Does the absence of energy dissipation in the 'elastic hardening' phase play a role?

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Nor-Sand *(Jefferies 1993)*

Stress-Dilatancy rule

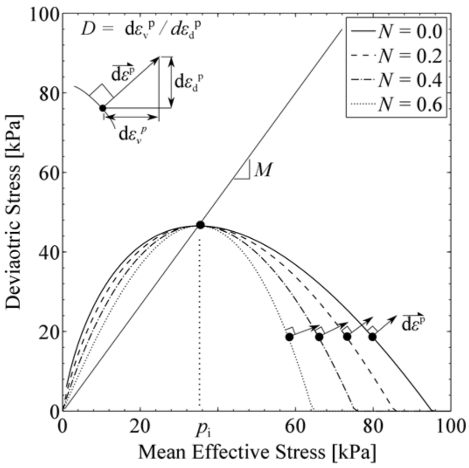
$$\eta' = M + (N-1)D$$

Yield Surface

$$F = \eta' - \frac{M_\theta}{N}\left[1 + (N-1)\left(\frac{p_i}{p'}\right)^{\frac{N}{1-N}}\right]$$

Hardening rule

$$\frac{\partial p_i}{\partial \varepsilon_d} = H \cdot \ldots \cdot (p_{i,max} - p_i)$$



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Nor-Sand *(Jefferies 1993)*

Maximum Yield Surface

$$p_{i,max} = p'\left(1 + D_{min}\frac{N}{M}\right)^{\frac{N-1}{N}}$$

$$D_{min} = \chi \cdot \Psi$$

$$\Psi = e - e_c$$

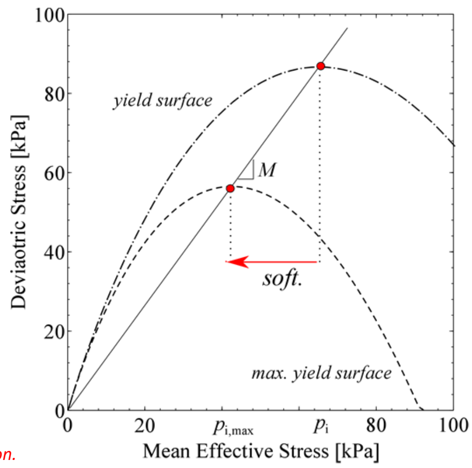
hardening $\rightarrow p_i < p_{i,max}$

softening $\rightarrow p_{i,max} < p_i$

critical state $\rightarrow p' = p_i = p_{i,max}$

$\Psi = 0 \rightarrow e = e_{cs}$

Causes instabilities at large deformation.



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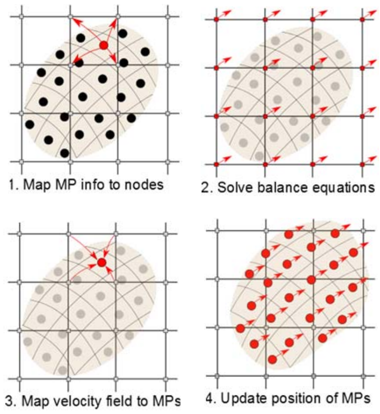
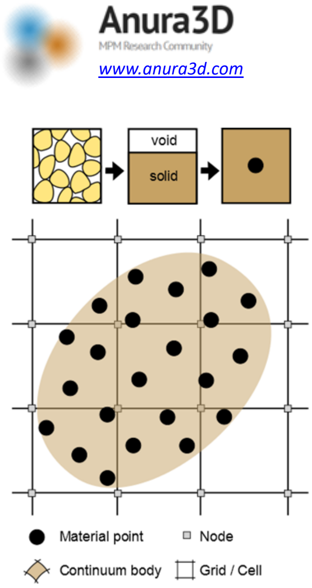
Material Point Method





(Fern and Soga, 2016a)

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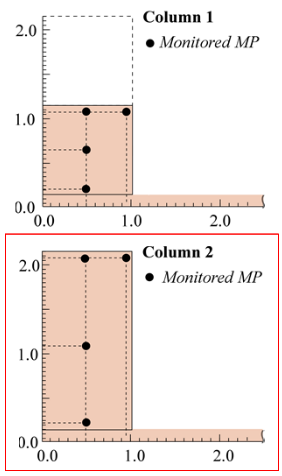

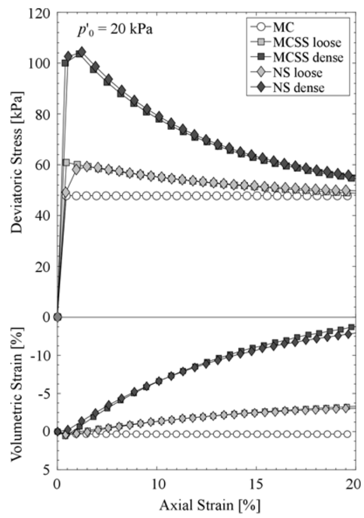

(Fern and Soga, 2016a)

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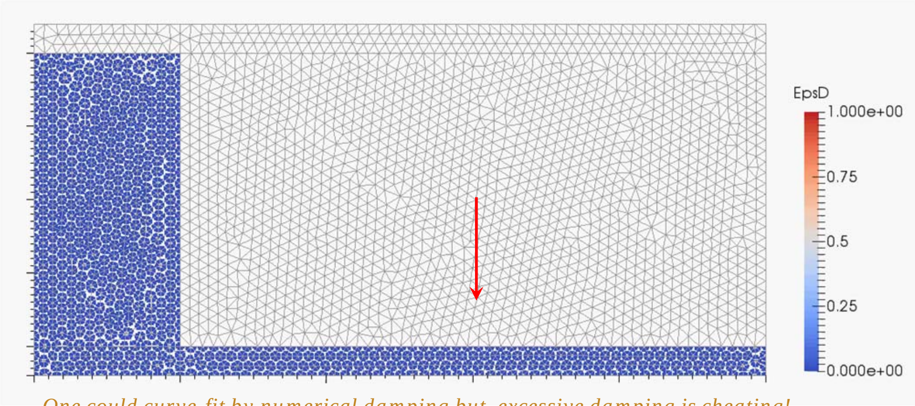
Granular Column Collapse

Mohr-Coulomb (critical state parameters)



One could curve-fit by numerical damping but, excessive damping is cheating!

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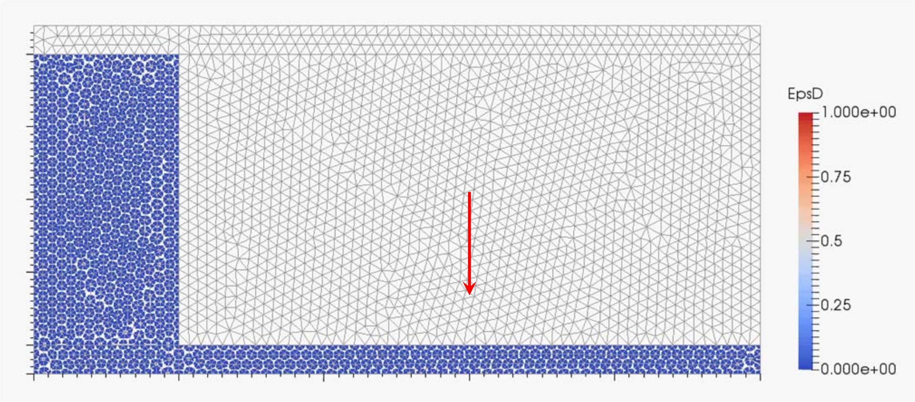
Granular Column Collapse

Mohr-Coulomb Strain Softening – loose sand



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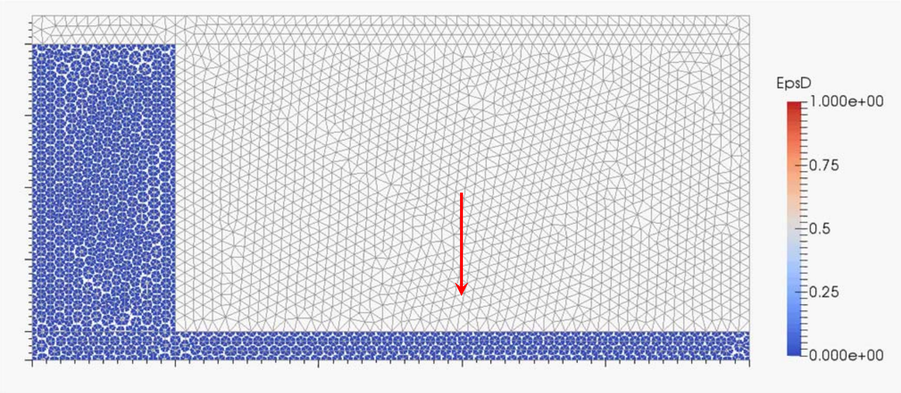
Mohr-Coulomb Strain Softening – dense sand



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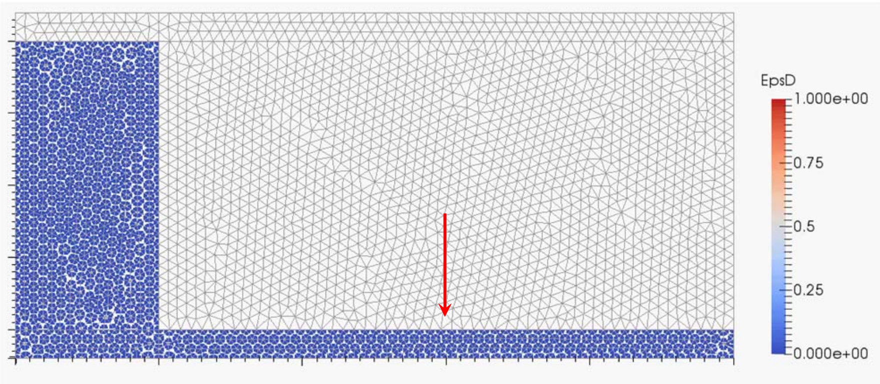
Nor-Sand – loose sand



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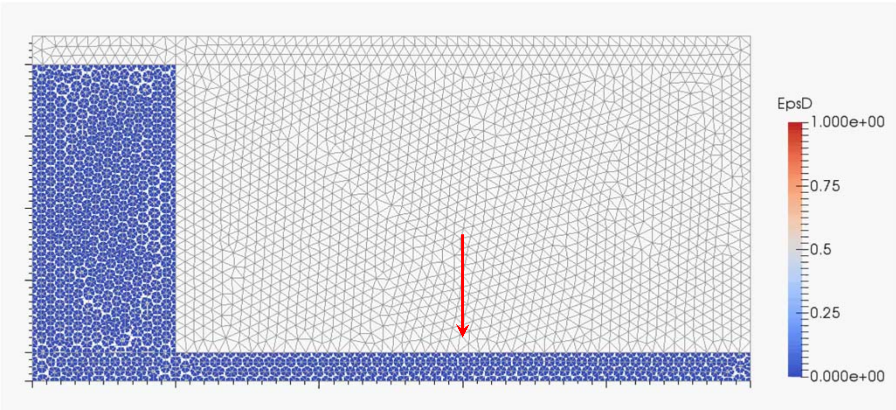
Nor-Sand – dense sand



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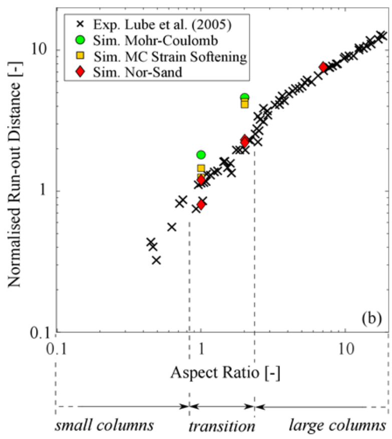

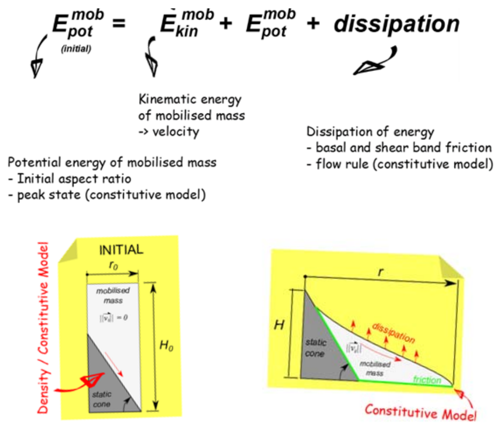

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Energy Considerations

Small columns

$$E_{pot}^{mob} = m_{mob} \cdot g \cdot h_{mob}^{CG}$$
$$= \frac{1}{3} h_0^3 \cot \varphi' \, (1-n)\rho_s g$$

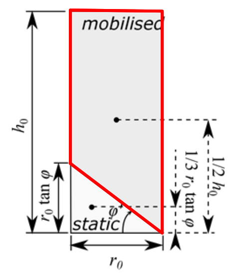
Large columns

$$E_{pot}^{mob} = m_{tot} \cdot g \cdot h_{tot}^{CG} - m_{stat} \cdot g \cdot h_{stat}^{CG}$$
$$= \left(\frac{1}{2} h_0^2 r_0 - \frac{1}{6} r_o^3 \tan^2 \varphi'\right)(1-n)\rho_s g$$

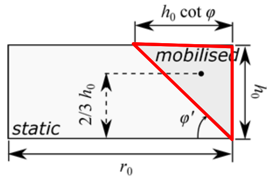
Failure angle *(Bolton, 1986)*

$$\varphi' = \varphi'_{cs} + 0.8\,\psi$$
$$\psi = \alpha \cdot I_R$$
$$I_R = \frac{e_{max} - e}{e_{max} - e_{min}} \ln\left(\frac{Q}{p'}\right) - R$$

Large Aspect Ratios



Small Aspect Ratios



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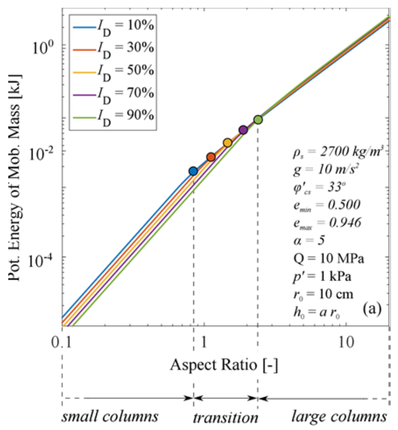

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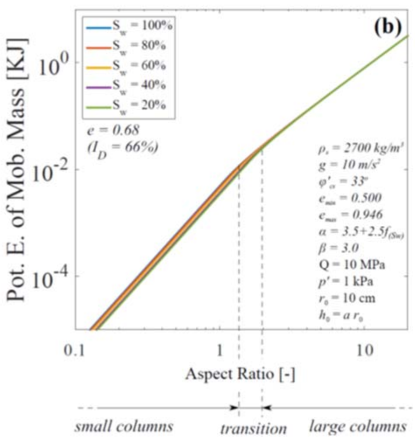

(Fern and Soga, 2017)

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Conclusion

- **The constitutive model has a two-fold influence.**
  - **It defines the failure surface** (hence the mobilised energy)
    - Therefore, the small strain behaviour cannot be neglected from 'large deformation simulations'
  - **and it controls the energy dissipation** (hence the run-out distance)
    - Which becomes increasingly important with large deformation
- The **initial density influences the constitutive model**
  - indirectly through an enhancement of the model parameters (i.e. Mohr–Coulomb)
  - or directly through its inclusion as model variable (i.e. Nor-Sand.and)
- **The requirements and implementations of constitutive models for large deformation MPM simulations can differ from small deformation FE simulations**.
- **The choice of the constitutive model plays a major role in the predictions of the failure** (i.e. type of failure, run-out distance) and its choice is not trivial.

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