Local Max-Ent meshfree method applied to large deformation problems in saturated soils

Dynamic approach

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Outline

1. Governing equations
2. Spatial discretization: OTM
3. Constitutive law
4. Benchmark examples
5. Embankment application
6. Conclusions and future work

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1. Governing equations
2. Spatial discretization: OTM
3. Constitutive law
4. Benchmark examples
5. Embankment application
6. Conclusions and future work

1. Governing equations

Biot's equations

- **Formulation**
  - Reduced formulation → **u-p_w**: Borja 95, Ehlers 96, Armero 99, Sanavia 01
  - Complete formulation
    - **u-U**: Dafalias 08 , Muraleetharan 09
    - **u-w**: López-Querol 11, Gioda 13

- **Time integration scheme**
  - Implicit
    - **u-p_w**: Borja 95, Ehlers 96, Armero 99, Sanavia 01
    - **u-U**: Dafalias 08 , Muraleetharan 09
    - **u-w**: **?**
  - Explicit
    - **u-U**: Ye 10
    - **u-w**: **?**

1. Governing equations

Biot's equations

Lewis & Schrefler (1998)

Linear momentum of the mixture

$$\text{div}\ \left[\boldsymbol{\sigma}' - p_w\,\mathbf{I}\right] - \rho\ddot{\boldsymbol{u}} - \rho_w\ddot{\boldsymbol{w}} + \rho\boldsymbol{g} = \mathbf{0}.$$

Linear momentum of the fluid phase

$$-\text{grad}\ p_w - \frac{\mu_w}{k}\dot{\boldsymbol{w}} + \rho_w\left(\boldsymbol{g} - \ddot{\boldsymbol{u}} - \frac{\ddot{\boldsymbol{w}}}{n}\right) = \mathbf{0}.$$

Mass conservation

$$\frac{\dot{p}_w}{Q} + \text{div}\ \dot{\boldsymbol{u}} + \text{div}\ \dot{\boldsymbol{w}} = 0$$

where:

Solid phase displacement	$\boldsymbol{u}$	$\rho = nS_w\rho_w + (1-n)\rho_s$
Total water displacement	$\boldsymbol{U}$	$n = \dfrac{V_h}{V_T} = \dfrac{V_h}{V_h + V_s}$
Relative water displacement	$\boldsymbol{w} = nS_w\,(\boldsymbol{U} - \boldsymbol{u})$	$Q = \left[\dfrac{1-n}{K_s} + \dfrac{n}{K_w}\right]^{-1}$

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Mass conservation

$$\frac{\dot{p}_w}{Q} + \text{div}\ \dot{\boldsymbol{u}} + \text{div}\ \dot{\boldsymbol{w}} = 0$$

deriving on time…

$$p_w = -Q\left(\text{div}\ \boldsymbol{u} + \text{div}\ \boldsymbol{w}\right) + p_{w0}.$$

1. Governing equations

Biot's equations Weak forms

LMBm

$$
-\int_B \boldsymbol{\sigma}' : \mathrm{grad}(\delta \boldsymbol{u})\, dv \; - \int_B Q\, \mathrm{div}(\boldsymbol{u}) \boldsymbol{I} : \mathrm{grad}(\delta \boldsymbol{u})\, dv
$$

$$
- \int_B Q\, \mathrm{div}(\boldsymbol{w}) \boldsymbol{I} : \mathrm{grad}(\delta \boldsymbol{u})\, dv \; + \int_B [-\rho \ddot{\boldsymbol{u}} - \rho_w \ddot{\boldsymbol{w}} + \rho \boldsymbol{g}] \cdot \delta \boldsymbol{u}\, dv + \int_{\partial B} \bar{\boldsymbol{t}} \cdot \delta \boldsymbol{u}\, ds = \boldsymbol{0}.
$$

LMBw

$$
-\int_B Q\, \mathrm{div}(\boldsymbol{u}) \mathrm{div}(\delta \boldsymbol{w})\, dv - \int_B Q\, \mathrm{div}(\boldsymbol{w}) \mathrm{div}(\delta \boldsymbol{w})\, dv \; - \int_B \frac{\mu_w}{k} \dot{\boldsymbol{w}} \cdot \delta \boldsymbol{w}\, dv
$$

$$
- \int_B \ddot{\boldsymbol{w}} \frac{\rho_w}{n} \cdot \delta \boldsymbol{w}\, dv \; + \int_B \rho_w (\boldsymbol{g} - \ddot{\boldsymbol{u}}) \cdot \delta \boldsymbol{w}\, dv - \int_{\partial B} \bar{\boldsymbol{t}}_w \cdot \delta \boldsymbol{w}\, ds = \boldsymbol{0}.
$$

1. Governing equations

Explicit scheme

Newmark central differences

$$\Delta \dot{\boldsymbol{w}}_{k+1} = \ddot{\boldsymbol{w}}_k \Delta t + \gamma \Delta t \Delta \ddot{\boldsymbol{w}}_{k+1}$$

LMBw - LMEm : incremental form

$$\rho_w \quad \left[\boldsymbol{\nabla}\left(\Delta \boldsymbol{\sigma'} - \Delta p_w\right)_k - \rho_w \, \Delta \ddot{\boldsymbol{w}}_{k+1} + \rho \, \Delta \boldsymbol{g}_{k+1}\right] =$$

$$\rho \left[-\boldsymbol{\nabla} \Delta p_{w_k} - \frac{1}{k} \ddot{\boldsymbol{w}}_k \Delta t \; - \left(\frac{1}{k}\gamma \Delta t + \frac{\rho_w}{n}\right) \Delta \ddot{\boldsymbol{w}}_{k+1} + \rho_w \, \Delta \boldsymbol{g}_{k+1}\right]$$

… after rearranging terms:

$$\left[\boldsymbol{M}^w \boldsymbol{M}^w - \gamma \Delta t \boldsymbol{M}^s \boldsymbol{C} - \frac{\boldsymbol{M}^s \boldsymbol{M}^w}{n}\right]^{-1} \left[\Delta \boldsymbol{R}_k^* + \Delta \boldsymbol{P}_{k+1}^* + \Delta t \boldsymbol{M}^s \boldsymbol{C} \ddot{\boldsymbol{w}}_k\right] = \Delta \ddot{\boldsymbol{w}}_{k+1}$$

$$\left[\boldsymbol{M}^s\right]^{-1} \left[\Delta \boldsymbol{R}_k^s + \Delta \boldsymbol{R}_k^w + \Delta \boldsymbol{P}_{k+1}^s - \boldsymbol{M}^w \Delta \ddot{\boldsymbol{w}}_{k+1}\right] = \Delta \ddot{\boldsymbol{u}}_{k+1}$$

where:

$$\begin{aligned} \Delta \boldsymbol{R}_k^s &= \boldsymbol{\nabla} \Delta \boldsymbol{\sigma'}_k, \\ \Delta \boldsymbol{R}_k^w &= \boldsymbol{\nabla} \Delta p_{w_k}, \\ \Delta \boldsymbol{R}_k^* &= \rho_w \Delta \boldsymbol{R}_k^s - (\rho_w - \rho) \Delta \boldsymbol{R}_k^w. \\ \Delta \boldsymbol{P}_{k+1}^* &= \rho_w \Delta \boldsymbol{P}_{k+1}^s - \rho \Delta \boldsymbol{P}_{k+1}^w. \end{aligned}$$

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LMBw - LMEm : incremental form

$$\rho_w \quad \left[\boldsymbol{\nabla} \left(\Delta \boldsymbol{\sigma'} - \Delta p_w \right)_k - \rho_w \, \Delta \ddot{\boldsymbol{w}}_{k+1} + \rho \, \Delta \boldsymbol{g}_{k+1} \right] =$$

$$\rho \left[-\boldsymbol{\nabla} \Delta p_{w_k} - \frac{1}{k} \ddot{\boldsymbol{w}}_k \Delta t \; - \left(\frac{1}{k} \gamma \Delta t + \frac{\rho_w}{n} \right) \Delta \ddot{\boldsymbol{w}}_{k+1} + \rho_w \, \Delta \boldsymbol{g}_{k+1} \right]$$

… after rearranging terms:

$$\left[\boldsymbol{M}^w \boldsymbol{M}^w - \gamma \Delta t \boldsymbol{M}^s \boldsymbol{C} - \frac{\boldsymbol{M}^s \boldsymbol{M}^w}{n} \right]^{-1} \left[\Delta \boldsymbol{R}^*_k + \Delta \boldsymbol{P}^*_{k+1} + \Delta t \boldsymbol{M}^s \boldsymbol{C} \ddot{\boldsymbol{w}}_k \right] = \Delta \ddot{\boldsymbol{w}}_{k+1}$$

$$\left[\boldsymbol{M}^s \right]^{-1} \left[\Delta \boldsymbol{R}^s_k + \Delta \boldsymbol{R}^w_k + \Delta \boldsymbol{P}^s_{k+1} - \boldsymbol{M}^w \Delta \ddot{\boldsymbol{w}}_{k+1} \right] = \Delta \ddot{\boldsymbol{u}}_{k+1}$$

where:

$$\boldsymbol{M}^w = \rho_w \boldsymbol{I}, \quad \boldsymbol{M}^s = \rho \boldsymbol{I}, \quad \boldsymbol{C} = \frac{1}{k} \boldsymbol{I}.$$

1. Governing equations

Explicit scheme

Explicit algorithm

1. Explicit Newmark Predictor ($\gamma = 0.5,\ \beta = 0$)

$$\begin{aligned} u_{k+1} &= u_k + \Delta t \dot{u}_k + 0.5 \Delta t^2\, \ddot{u}_k = u_k + \Delta u_{k+1} \\ w_{k+1} &= w_k + \Delta t \dot{w}_k + 0.5 \Delta t^2\, \ddot{w}_k = w_k + \Delta w_{k+1} \\ \dot{u}_{k+1} &= \dot{u}_k + (1-\gamma)\Delta t\, \ddot{u}_k \\ \dot{w}_{k+1} &= \dot{w}_k + (1-\gamma)\Delta t\, \ddot{w}_k \\ x_{k+1} &= x_k + \Delta u_{k+1} \end{aligned}$$

2. Material points position update
3. Deformation gradient calculation
4. Logarithmic strain and Pore pressure: $\mathbf{C} = \mathbf{F}^T\mathbf{F}$

$$\begin{aligned} \text{div}\ (\boldsymbol{u}) &= \text{tr}(\boldsymbol{\varepsilon}_{k+1}) = \text{tr}\left(\frac{1}{2}\log \mathbf{C}_{k+1}\right) \\ \text{div}\ (\boldsymbol{w}) &= \text{tr}(\boldsymbol{\varepsilon}^w_{k+1}) = \text{tr}\left(\frac{1}{2}\log \mathbf{C}^w_{k+1}\right) \\ p_w &= -Q\,(\text{div}\ \boldsymbol{u} + \text{div}\ \boldsymbol{w}) + p_{w_0} \end{aligned}$$

5. Remapping loop, reconnect the nodes with their new material neighbors
6. Update density and recompute lumped mass

$$\rho_{k+1} = n_{k+1}\rho_w + (1 - n_{k+1})\rho_s$$

7. Constitutive relations from the Elasto-Plastic model: $\boldsymbol{\sigma'}_{k+1}$ and $\boldsymbol{R}_{k+1}$
8. Computation of $\ddot{\boldsymbol{u}}_{k+1}$ and $\ddot{\boldsymbol{w}}_{k+1}$
9. Explicit Newmark Corrector

$$\dot{u}_{k+1} = \dot{u}_{k+1} + \gamma \Delta t\, \ddot{u}_{k+1}$$

$$\dot{w}_{k+1} = \dot{w}_{k+1} + \gamma \Delta t\, \ddot{w}_{k+1}$$

1. Governing equations

Implicit scheme

Wriggers (2008)

Newmark

$$\ddot{\boldsymbol{u}}_{k+1} = \alpha_1 \Delta \boldsymbol{u}_{k+1} - \alpha_2 \dot{\boldsymbol{u}}_k - \alpha_3 \ddot{\boldsymbol{u}}_k$$

$$\dot{\boldsymbol{u}}_{k+1} = \alpha_4 \Delta \boldsymbol{u}_{k+1} + \alpha_5 \dot{\boldsymbol{u}}_k + \alpha_6 \ddot{\boldsymbol{u}}_k$$

$$\alpha_1 = \frac{1}{\beta \Delta t^2} \qquad \alpha_2 = \frac{1}{\beta \Delta t}$$

$$\alpha_3 = \frac{1}{2\beta} - 1 \qquad \alpha_4 = \frac{\gamma}{\beta \Delta t}$$

$$\alpha_5 = 1 - \frac{\gamma}{\beta} \qquad \alpha_6 = \left(1 - \frac{\gamma}{2\beta}\right) \Delta t$$

$$\alpha_7 = 1 \qquad \alpha_8 = 1$$

$$\boldsymbol{R}_{k+1} + \boldsymbol{C}\,\dot{\boldsymbol{u}}_{k+1} + \boldsymbol{M}\,\ddot{\boldsymbol{u}}_{k+1} = \boldsymbol{P}_{k+1};$$

$$\begin{aligned} \boldsymbol{G}_{k+1} &= \boldsymbol{M}\left[\alpha_1 \Delta \boldsymbol{u}_{k+1} - \alpha_2 \dot{\boldsymbol{u}}_k - \alpha_3 \ddot{\boldsymbol{u}}_k\right] \\ &+ \boldsymbol{C}\left[\alpha_4 \Delta \boldsymbol{u}_{k+1} + \alpha_5 \dot{\boldsymbol{u}}_k + \alpha_6 \ddot{\boldsymbol{u}}_k\right] \\ &+ \alpha_7 \boldsymbol{R}_{k+1} - \boldsymbol{P}_k - \alpha_8 \Delta \boldsymbol{P}_{k+1} = \boldsymbol{0} \end{aligned}$$

$$\begin{aligned} \boldsymbol{G}(\overline{\boldsymbol{\chi}}, \boldsymbol{\eta}, \Delta \boldsymbol{u}^*)^{i+1}_{k+1} &\cong \\ \boldsymbol{G}(\overline{\boldsymbol{\chi}}, \boldsymbol{\eta})^{i}_{k+1} + D\boldsymbol{G}(\overline{\boldsymbol{\chi}}, \boldsymbol{\eta})^{i}_{k+1} \cdot \Delta \boldsymbol{u}^{*i+1}_{k+1} &\cong \boldsymbol{0}, \end{aligned}$$

$$D\boldsymbol{G} \cdot \Delta \boldsymbol{u}^* = \begin{bmatrix} DG_{LMS} \cdot \Delta \boldsymbol{u} + DG_{LMS} \cdot \Delta \boldsymbol{w} \\ DG_{LMW} \cdot \Delta \boldsymbol{u} + DG_{LMW} \cdot \Delta \boldsymbol{w} \end{bmatrix}$$

linearization

iteratively:

$$\begin{aligned} \left[\alpha_1 \boldsymbol{M} + \alpha_4 \boldsymbol{C} + \alpha_7 \boldsymbol{K}^i_{k+1}\right] \Delta \boldsymbol{u}^{i+1}_{k+1} &= -\boldsymbol{G}(\boldsymbol{u}^i_{k+1}), \\ \text{where} \;\; \boldsymbol{u}^{i+1}_{k+1} &= \boldsymbol{u}^i_{k+1} + \Delta \boldsymbol{u}^{i+1}_{k+1}. \end{aligned}$$

Newton-Raphson

1. Governing equations

Implicit scheme Weak forms after time integration scheme

LMBm

$$-\alpha_7 \int_B \boldsymbol{\sigma'} : \mathrm{grad}(\delta \boldsymbol{u}) \, dv \; - \alpha_7 \int_B Q \, \mathrm{div}(\boldsymbol{u}) \mathrm{div}(\delta \boldsymbol{u}) \, dv$$

$$-\alpha_7 \int_B Q \, \mathrm{div}(\boldsymbol{w}) \mathrm{div}(\delta \boldsymbol{u}) \, dv \; - \alpha_1 \int_B [\rho \boldsymbol{u} + \rho_w \boldsymbol{w}] \cdot \delta \boldsymbol{u} \, dv$$

$$+\alpha_8 \int_B \rho \boldsymbol{g} \cdot \delta \boldsymbol{u} \, dv + \alpha_8 \int_{\delta B} \bar{\boldsymbol{t}} \cdot \delta \boldsymbol{u} \, ds = \boldsymbol{0}$$

LMBw

$$- \int_B \alpha_7 Q \, \mathrm{div}(\boldsymbol{u}) \mathrm{div}(\delta \boldsymbol{w}) \, dv - \int_B \alpha_7 Q \, \mathrm{div}(\boldsymbol{w}) \mathrm{div}(\delta \boldsymbol{w}) \, dv$$

$$-\alpha_4 \int_B \frac{\mu_w}{k} \boldsymbol{w} \cdot \delta \boldsymbol{w} \, dv - \alpha_1 \int_B \frac{\rho_w}{n} \boldsymbol{w} \cdot \delta \boldsymbol{w} \, dv$$

$$-\alpha_1 \int_B \rho_w \boldsymbol{u} \cdot \delta \boldsymbol{w} \, dv + \alpha_8 \int_B \rho_w \boldsymbol{g} \cdot \delta \boldsymbol{w} \, dv$$

$$-\alpha_8 \int_{\delta B} \bar{\boldsymbol{t}}_w \cdot \delta \boldsymbol{w} \, ds = \boldsymbol{0}.$$

1. Governing equations

Implicit scheme

Linearization

DGLMS

$$
\begin{aligned}
&- \alpha_7 \int_B \operatorname{grad}(\delta \boldsymbol{u}) : \boldsymbol{c}^{ep} : \operatorname{grad}(\Delta \boldsymbol{u}) \, dv \\
&- \alpha_7 \int_B \boldsymbol{\sigma}' : \operatorname{grad}^T(\delta \boldsymbol{u}) \operatorname{grad}(\Delta \boldsymbol{u}) \, dv \\
&- \alpha_7 \int_B \operatorname{grad}(\delta \boldsymbol{u}) : (Q \left[\operatorname{div}(\Delta \boldsymbol{u}) + \operatorname{div}(\Delta \boldsymbol{w})\right] \boldsymbol{I}) \, dv \\
&- \alpha_7 \int_B \operatorname{grad}(\delta \boldsymbol{u}) : p_w \operatorname{grad}^T(\Delta \boldsymbol{u}) dv \\
&- \alpha_7 \int_B \operatorname{grad}(\delta \boldsymbol{u}) : p_w \frac{1-n}{n} \operatorname{div}(\Delta \boldsymbol{u}) \boldsymbol{I} dv \\
&- \alpha_1 \int_B \delta \boldsymbol{u} \cdot \left[\rho \Delta \boldsymbol{u} + \rho_w \Delta \boldsymbol{w} + \rho_w \operatorname{div}(\Delta \boldsymbol{u}) \, (\boldsymbol{u} + \boldsymbol{w})\right] dv \\
&+ \alpha_8 \int_B \rho_w \delta \boldsymbol{u} \cdot \boldsymbol{g} \operatorname{div}(\Delta \boldsymbol{u}) \, dv
\end{aligned}
$$

DGLMW

$$
\begin{aligned}
&- \alpha_7 \int_B \operatorname{grad}(\delta \boldsymbol{w}) : (Q \left[\operatorname{div}(\Delta \boldsymbol{u}) + \operatorname{div}(\Delta \boldsymbol{w})\right] \boldsymbol{I}) \, dv \\
&- \alpha_7 \int_B \operatorname{grad}(\delta \boldsymbol{w}) : p_w \operatorname{grad}^T(\Delta \boldsymbol{u}) dv \\
&- \alpha_7 \int_B \operatorname{grad}(\delta \boldsymbol{w}) : p_w \frac{1-n}{n} \operatorname{div}(\Delta \boldsymbol{u}) \boldsymbol{I} dv \\
&- \alpha_4 \int_B \frac{\mu_w}{k} \delta \boldsymbol{w} \cdot \left[\Delta \boldsymbol{w} + \operatorname{div}(\Delta \boldsymbol{u}) \left(1 - \frac{1-n}{k} \frac{\partial k}{\partial n}\right) \boldsymbol{w}\right] dv \\
&- \alpha_1 \int_B \frac{\rho_w}{n} \delta \boldsymbol{w} \cdot \left[\Delta \boldsymbol{w} + \frac{2n-1}{n} \operatorname{div}(\Delta \boldsymbol{u}) \boldsymbol{w}\right] dv \\
&- \alpha_1 \int_B \rho_w \delta \boldsymbol{w} \cdot \left[\Delta \boldsymbol{u} + \operatorname{div}(\Delta \boldsymbol{u}) \boldsymbol{u}\right] dv \\
&+ \alpha_8 \int_B \rho_w \delta \boldsymbol{w} \cdot \boldsymbol{g} \operatorname{div}(\Delta \boldsymbol{u}) \, dv
\end{aligned}
$$

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1. Governing equations
2. Spatial discretization: OTM
3. Constitutive law
4. Benchmark examples
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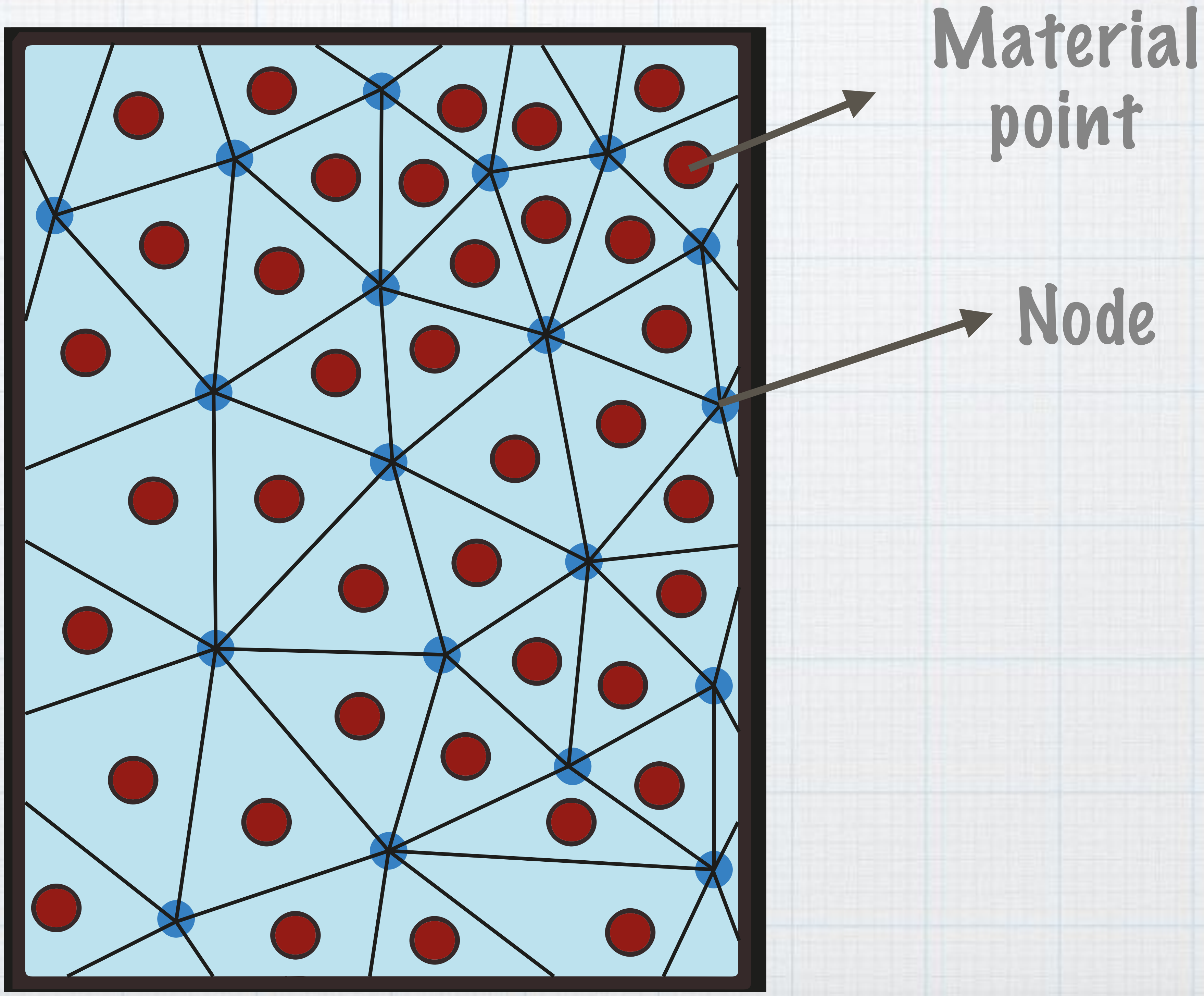
1. Spatial discretization: OTM

Optimal Transportation Meshfree

Optimal Transportation Meshfree (OTM) (Li, Habbal and Ortiz, 2010)

DAAS (Saucedo and Yu, 2012)



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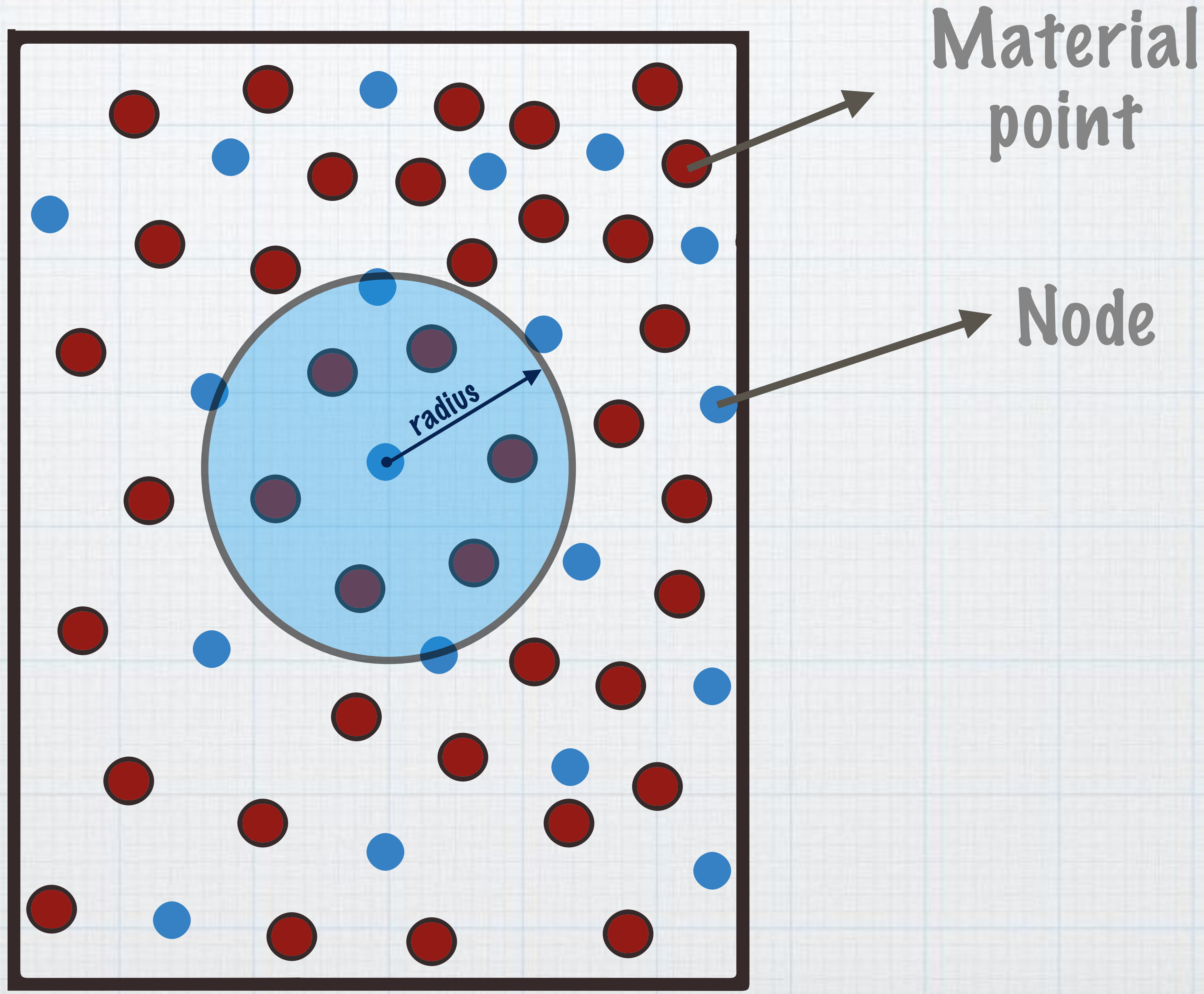


1. Spatial discretization: OTM

Local Max-Ent

M. Arroyo and M. Ortiz (2006)

$$\begin{aligned} f_\beta(\mathbf{x},\mathbf{p}) &= \beta H(\mathbf{x},\mathbf{p}) - H(\mathbf{p}) \\ \text{subject to} \quad p_a &\geq 0, \quad \text{, a=1,...,n} \\ \sum_{a=1} p_a &= 1 \\ \sum_{a=1} p_a \mathbf{x_a} &= \mathbf{x} \end{aligned}$$

the unique solution is:

$$p(\mathbf{x}) = \frac{\exp\left[-\beta|\mathbf{x}-\mathbf{x_a}|^{\mathbf{2}} + \lambda(\mathbf{x}-\mathbf{x_a})\right]}{Z(\mathbf{x},\lambda^*(\mathbf{x}))}$$

where

$$Z(\mathbf{x},\lambda) = \sum_{\mathbf{a}=\mathbf{a}}^{\mathbf{N}} \exp\left[-\beta|\mathbf{x}-\mathbf{x_a}|^{\mathbf{2}} + \lambda(\mathbf{x}-\mathbf{x_a})\right]$$

First derivatives expression is:

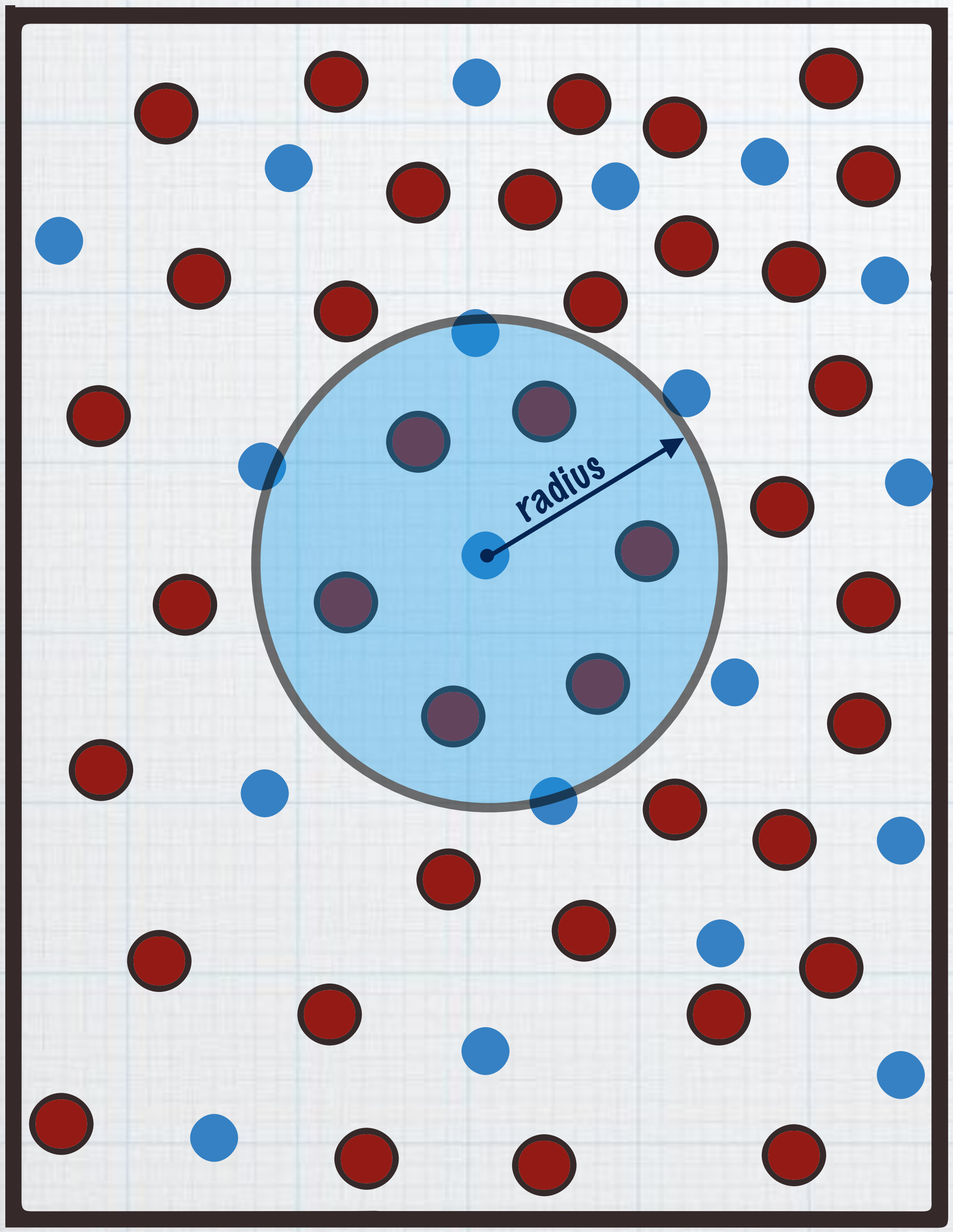
$$\nabla p_a^* = -p_a^*(\mathbf{J}^*)^{-1}(\mathbf{x}-\mathbf{x_a})$$

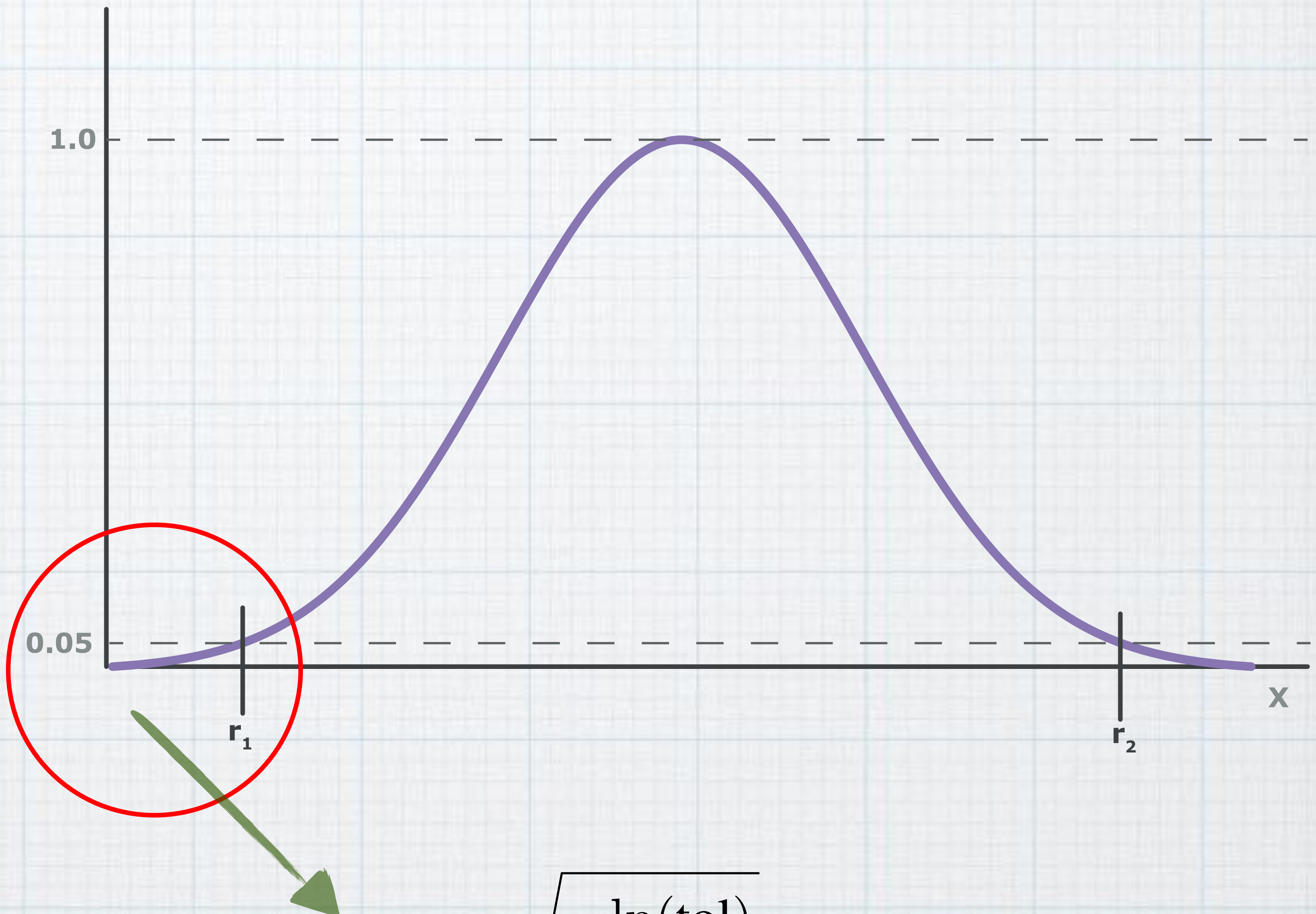
where

$$\begin{aligned} \mathbf{J}(\mathbf{x},\boldsymbol{\lambda},\beta) &= \frac{\partial \mathbf{r}}{\partial \boldsymbol{\lambda}} \\ \mathbf{r}(\mathbf{x},\boldsymbol{\lambda},\beta) &\equiv \partial_{\boldsymbol{\lambda}} \log Z(\mathbf{x},\boldsymbol{\lambda}) = \sum_a p_a(\mathbf{x},\boldsymbol{\lambda},\beta)(\mathbf{x}-\mathbf{x_a}) \end{aligned}$$

1. Spatial discretization: OTM

Optimal Transportation Meshfree

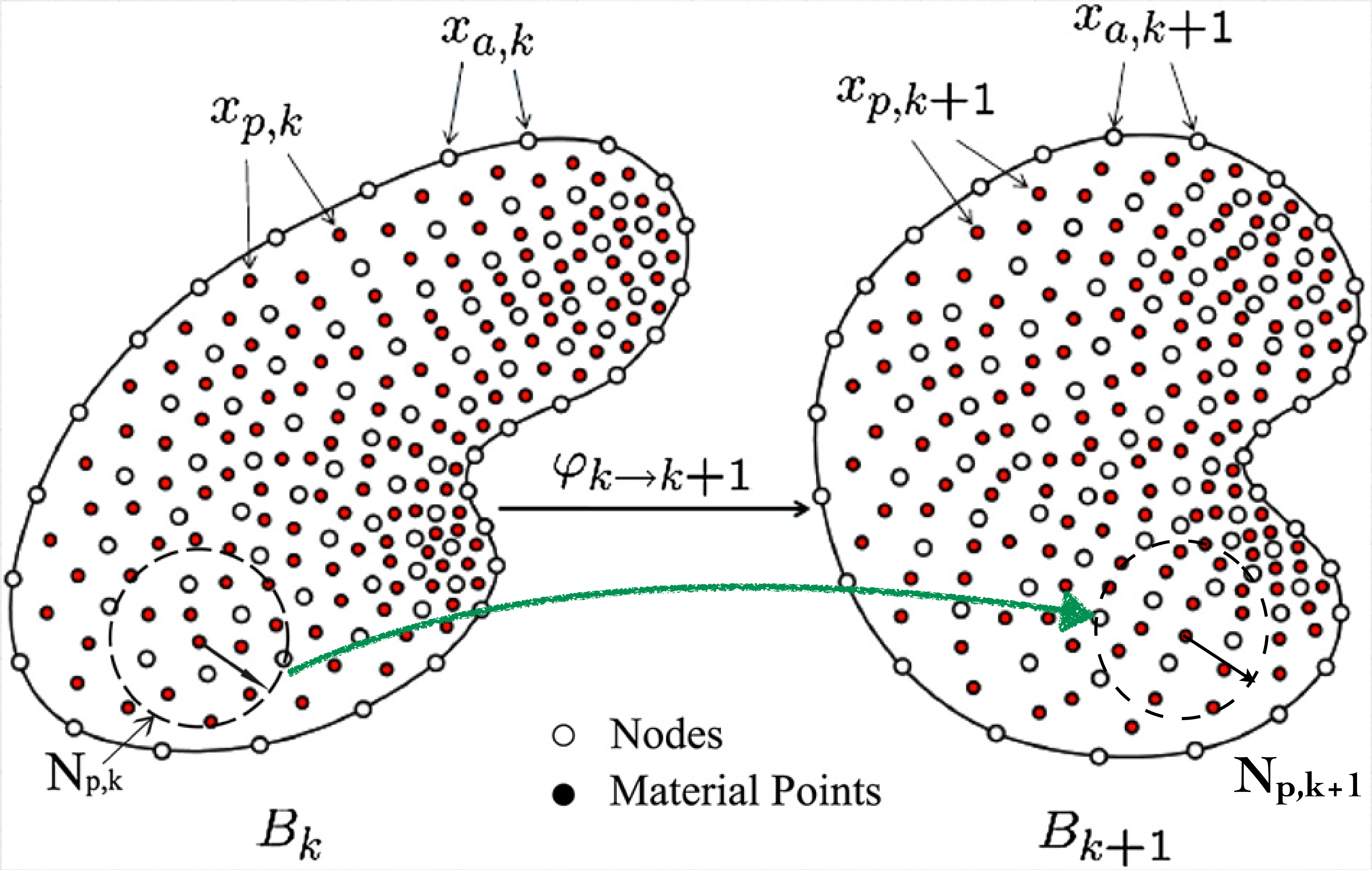




$$r_i = \sqrt{-\frac{\ln(\text{tol})}{\beta}}, \ i = 1 \text{ or } 2$$

1. Spatial discretization: OTM

Optimal Transportation Meshfree Li, Habbal and Ortiz (2010)



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2. Constitutive law

Hyperelastic

Neo-Hookean with compaction point   Ehlers and Eipper (2001)

$$\boldsymbol{\tau} = G(\boldsymbol{b} - \mathbf{I}) + \lambda\, n_0^2 \left(\frac{J}{n_0} - \frac{J}{J - 1 + n_0} \right) \mathbf{I}$$

$$\begin{aligned} \boldsymbol{c}^e \quad &= \quad 2 \left[G - \lambda n_0 J \frac{J-1}{J + n_0 - 1} \right] \mathbf{1} \\ &\quad + \lambda \left[n_0 J \frac{J^2 + (1 - n_0)(1 - 2J)}{(J + n_0 - 1)^2} \right] (\boldsymbol{I} \otimes \boldsymbol{I}) \end{aligned}$$

2. Constitutive law

Elasto-plastic

Drucker-Prager flow rule

Sanavia et al. (2002)

$$p_{lim} = \frac{3\alpha_Q K}{2G}\|\mathbf{s}^{trial}_{k+1}\| + \frac{\beta}{3\alpha_F}\left(\frac{\|\mathbf{s}^{trial}_{k+1}\|}{2G} H\sqrt{1+3\alpha_Q^2} + c_k\right)$$

Classical

$$\Phi^{cl} = \|\mathbf{s}^{trial}_{k+1}\| - 2G\Delta\gamma + 3\alpha_F[p^{trial}_{k+1} - 3K\alpha_Q\Delta\gamma] - \beta c_{k+1}$$

$$\begin{aligned}\boldsymbol{c}^{ep} = \quad & K\left[1 - \frac{9\alpha_Q\alpha_F K}{c_1}\right](\mathrm{I}\otimes\mathrm{I}) \\ & +2G\left[1 - \frac{2G\Delta\gamma}{\|\mathbf{s}^{trial}_{k+1}\|}\right](\mathbf{1} - \frac{1}{3}\mathrm{I}\otimes\mathrm{I}) \\ & -\frac{6\alpha_Q KG}{c_1}(\boldsymbol{I}\otimes\boldsymbol{n}^{tr}_{k+1}) - \frac{6\alpha_F KG}{c_1}(\boldsymbol{n}^{tr}_{k+1}\otimes\mathrm{I}) \\ & -4G^2\left[\frac{1}{c_1} - \frac{\Delta\gamma}{\|\mathbf{s}^{trial}_{k+1}\|}\right](\boldsymbol{n}^{tr}_{k+1}\otimes\boldsymbol{n}^{tr}_{k+1}),\end{aligned}$$

$$c_1 = 9\alpha_F\alpha_Q K + 2G + \beta H\sqrt{1+3\alpha_Q^2}].$$

Apex

$$\begin{aligned}\Phi^{ap} = & \frac{\beta}{3\alpha_F}\left[c_k + H\sqrt{\Delta\gamma_1^2 + 3\alpha_Q^2(\Delta\gamma_1+\Delta\gamma_2)^2}\right] \\ & -p^{trial}_{k+1} + 3K\alpha_Q(\Delta\gamma_1+\Delta\gamma_2)\end{aligned}$$

$$\boldsymbol{c}^{ep} = \quad Kc_2(\mathrm{I}\otimes\mathrm{I}) + \frac{Kc_2}{2\alpha_Q G\Delta\gamma_T}(\mathrm{I}\otimes\boldsymbol{s}^{tr}_{k+1}),$$

$$c_2 = \frac{\alpha_Q\beta H\Delta\gamma_T}{3\alpha_Q K\sqrt{\Delta\gamma_1^2 + 3\alpha_Q^2\Delta\gamma_T^2} + \alpha_Q\beta H\Delta\gamma_T},$$

$$\Delta\gamma_T = \Delta\gamma_1 + \Delta\gamma_2.$$

$$\boldsymbol{n}^{tr}_{k+1} = \frac{\boldsymbol{s}^{tr}_{k+1}}{\|\mathbf{s}^{tr}_{k+1}\|}$$

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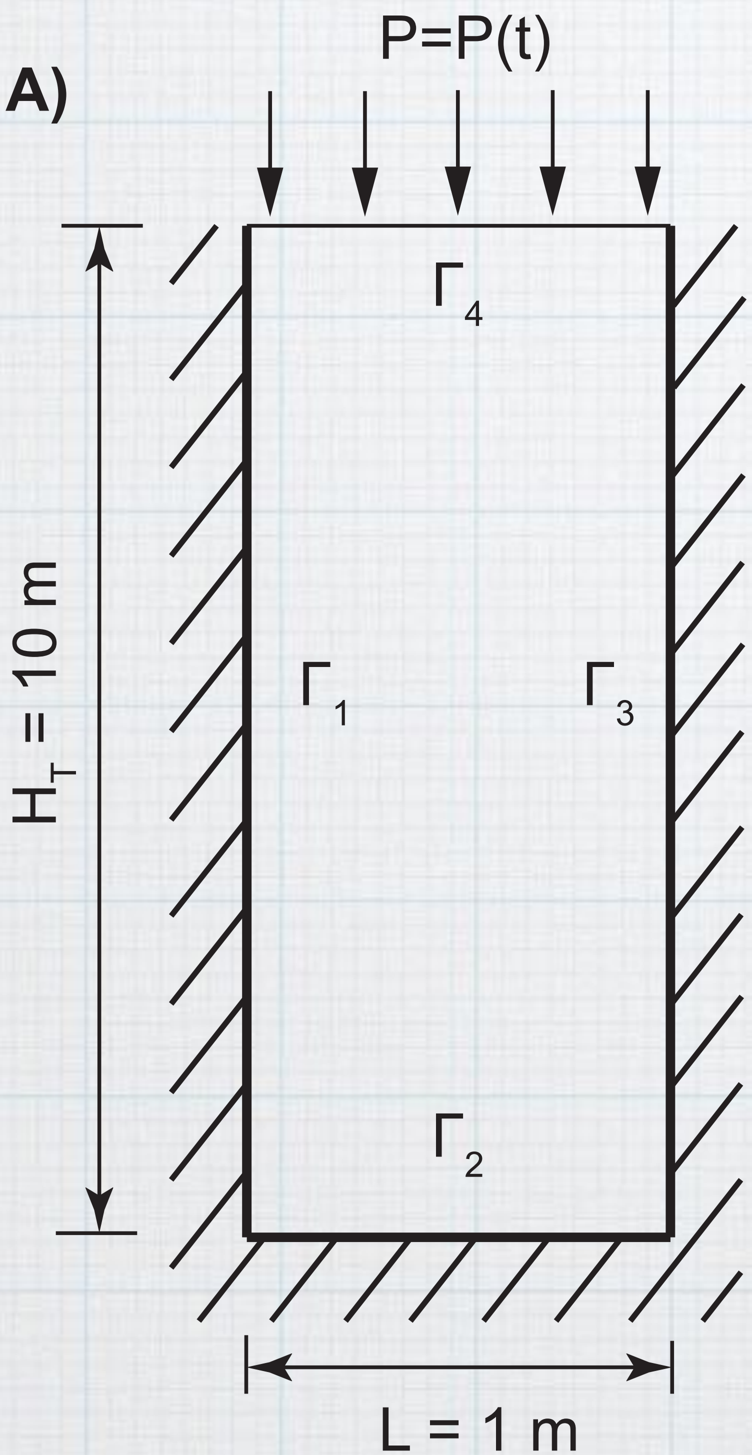
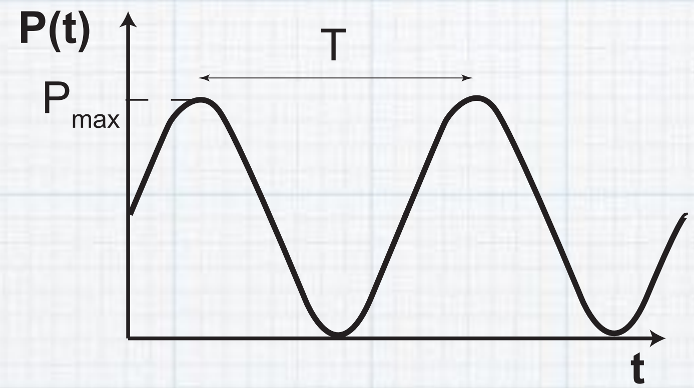
4. Benchmark examples

Consolidation

A)

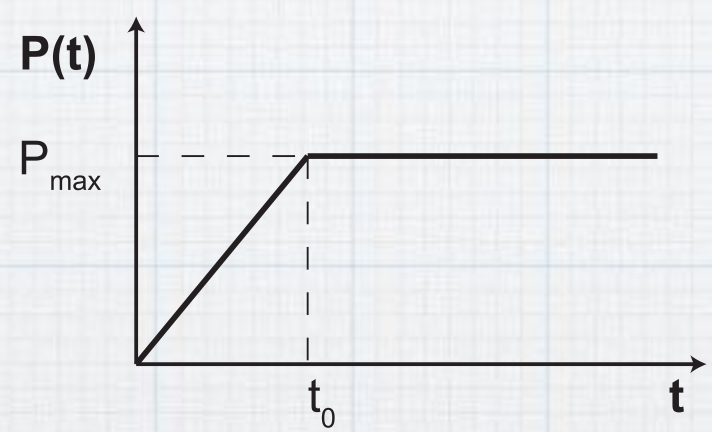


B) Dynamic consolidation



G [MPa]	312.5	K_w [MPa]	10^4
ν	0.2	K_s [MPa]	10^{34}
n	0.333	ρ_w [kg/m^3]	1000
V_c [m/s]	3205	ρ_s [kg/m^3]	3003

C) Large deformation consolidation



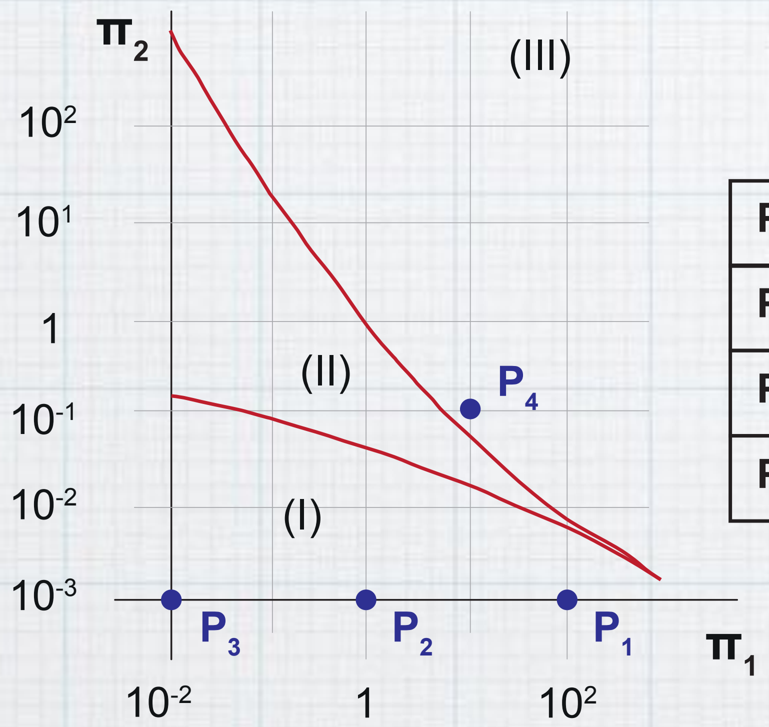
λ [MPa]	29	K_w [MPa]	$2.2 \cdot 10^4$
G [MPa]	7	K_s [MPa]	10^{34}
n	0.42	ρ_w [kg/m^3]	1000
k [m/s]	0.1	ρ_s [kg/m^3]	2700

Γ_1 : $u_x=0$, $w_x=0$
Γ_2 : $u_y=0$, $w_y=0$
Γ_3 : $u_x=0$, $w_x=0$
Γ_4 : free

4. Benchmark examples

Dynamic Consolidation

$$\Pi_1 = \frac{k\,V_c^2}{g\,\frac{\rho_f}{\rho}\,\omega\,H_T^2} = \frac{k\,\omega}{g\,\frac{\rho_f}{\rho}\,\Pi_2}, \quad \Pi_2 = \frac{\omega^2\,H_T^2}{V_c^2}$$



	π_1	π_2	ω [rad/s]	k [m/s]
P_1	10^2	10^{-3}	10.14	$3.22 \cdot 10^{-2}$
P_2	10^0	10^{-3}	10.14	$3.22 \cdot 10^{-4}$
P_3	10^{-2}	10^{-3}	10.14	$3.22 \cdot 10^{-6}$
P_4	10^1	10^{-1}	101.4	$3.22 \cdot 10^{-2}$

Zone (I) - Slow phenomena: ü and ẅ can be neglected

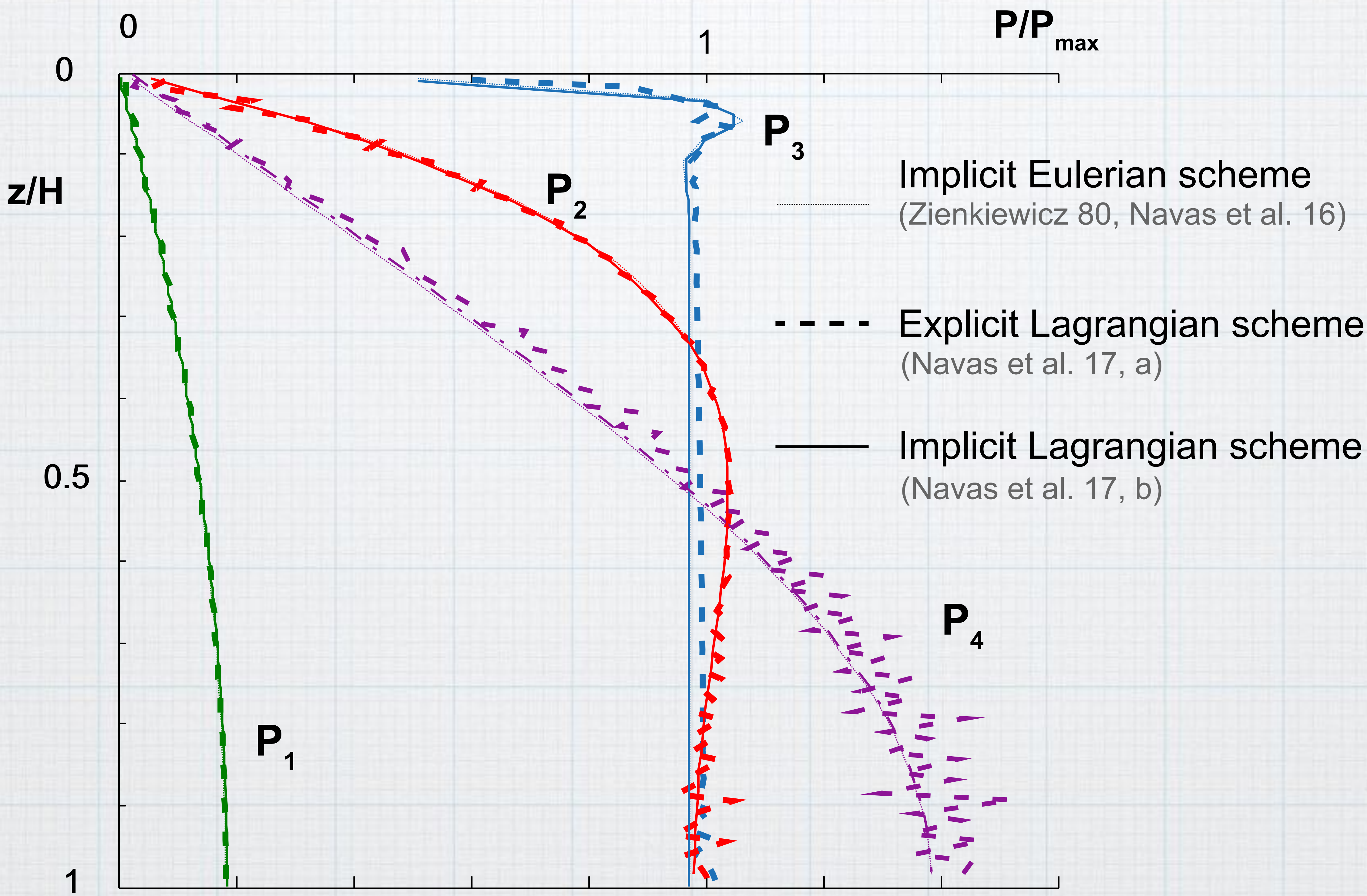
Zone (II) - Moderate speed: ẅ can be neglected

Zone (III) - Fast phenomena: only full Biot eq. valid

O.C. Zienkiewicz, C.T. Chang, and P. Bettes. Drained, undrained, consolidating and dynamic behaviour assumptions in soils. Geotechnique, 30(4):385-395,1980.

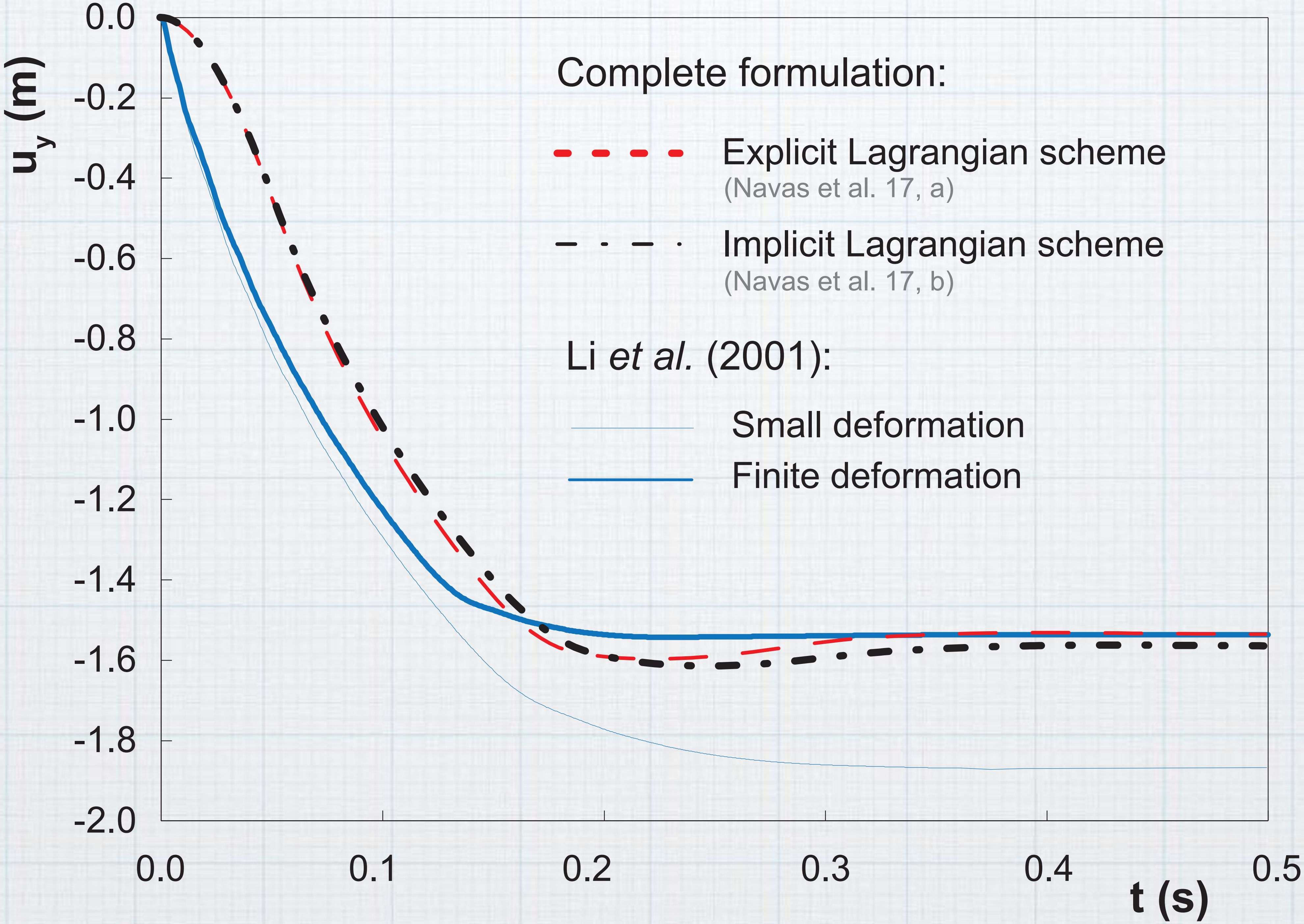
4. Benchmark examples

Dynamic Consolidation



4. Benchmark examples

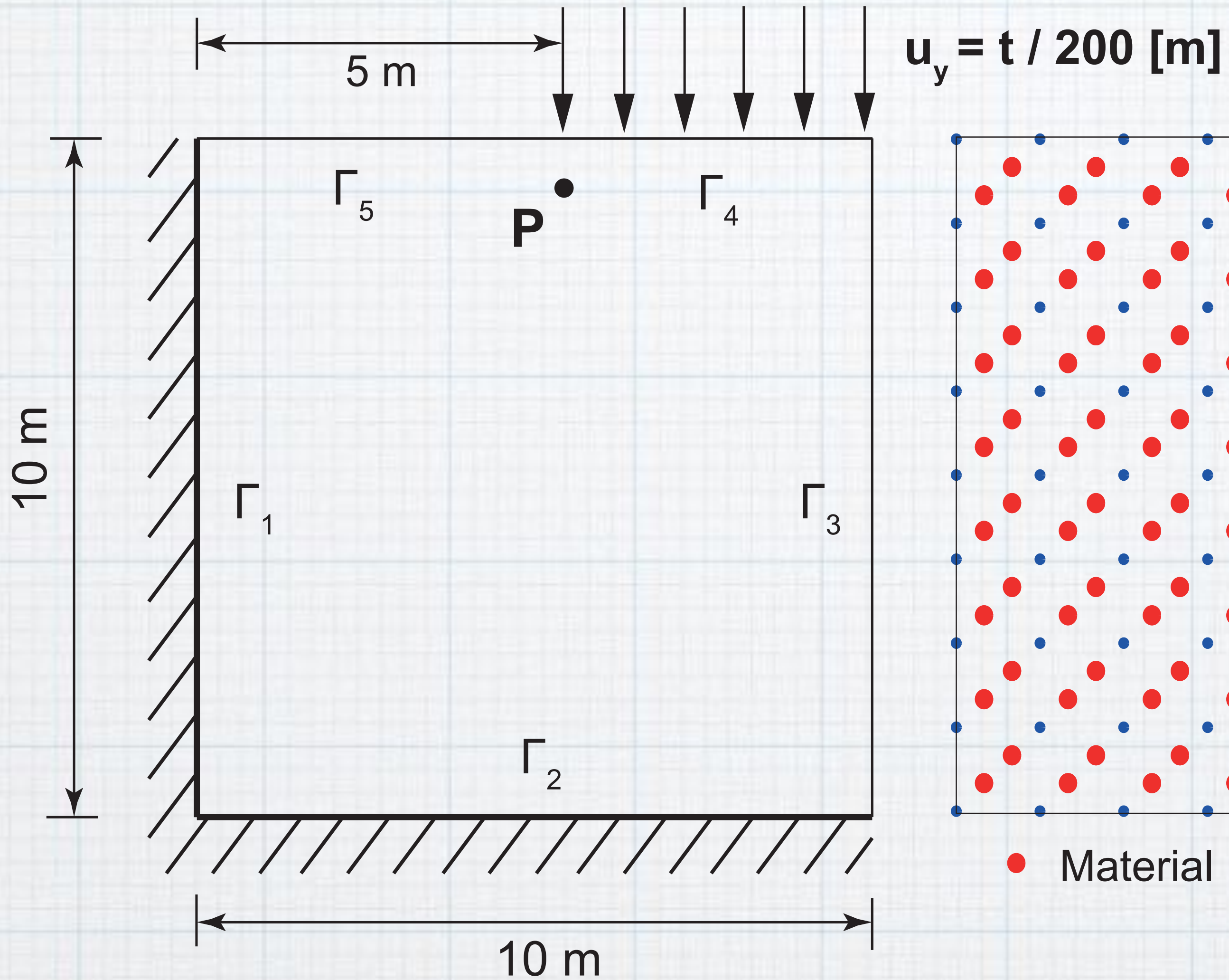
Large strain consolidation

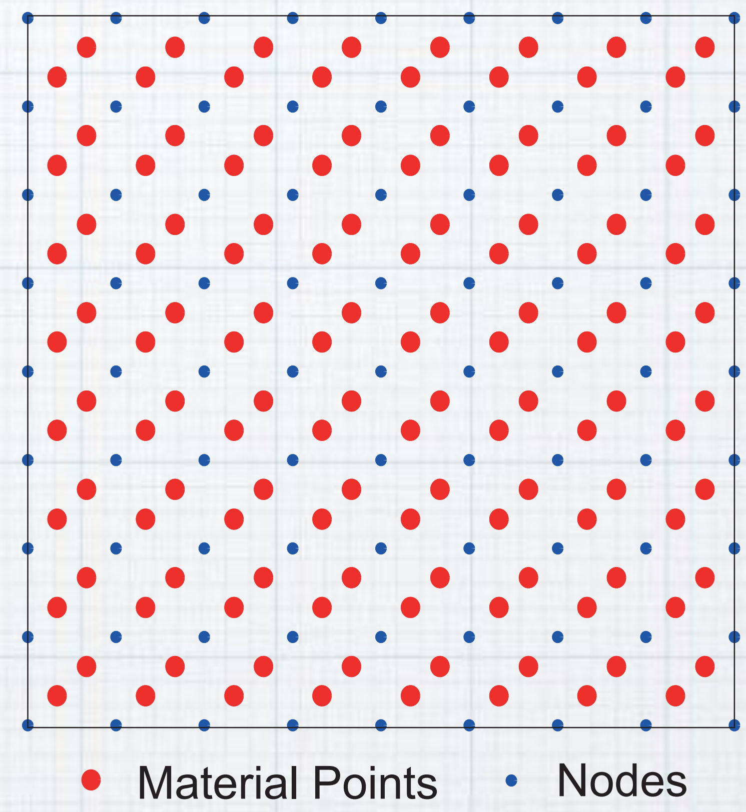


C. Li, R.I. Borja, and R.A. Regueiro. Dynamics of porous media at finite strain. Computer Methods in Applied Mechanics and Engineering, 193:3837-3870, 2004.

4. Benchmark examples

Rigid footing in a saturated square plate





$K = 8333$ kN/m^2

$G = 3486$ kN/m^2

$c_0 = 100$ kN/m^2

$H = -10$ kN/m^2

$\Phi = 20^o$

$\Psi = -10^o, 0^o, 10^o, 20^o$

$K_w = 50000$ kN/m^2

$k = 0.0001$ m/s

$n = 0.33$

$\rho_s = 2700$ kg/m^3

$\rho_w = 1000$ kg/m^3

Γ_1: $u_x=0$, $w_x=0$

Γ_2: $u_y=0$, $w_y=0$

Γ_3: $w_x=0$

Γ_4: $u_y=u_y(t)$, $w_y=0$

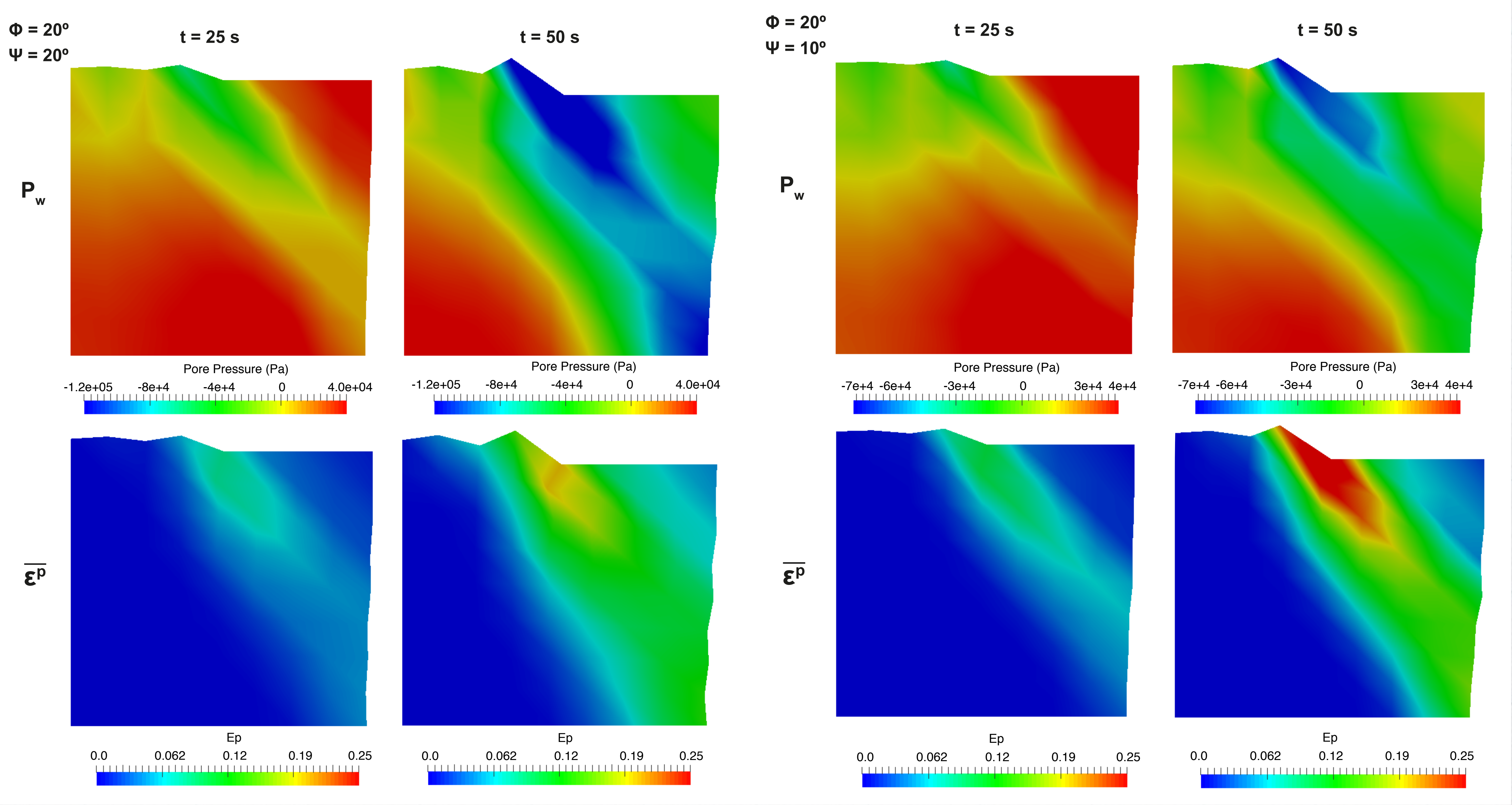
Γ_5: free

L. Sanavia, B.A. Schrefler, E. Stein, and P. Steinmann. Modelling of localisation at finite inelastic strain in fluid saturated porous media. Proc. In: Ehlers W (ed.), IUTAM Symposium on Theoretical and Numerical Methods in Continuum Mechanics of Porous Materials, Kluwer Academic Publishers, pages 239-244, 2001.

4. Benchmark examples

Rigid footing in a saturated square plate

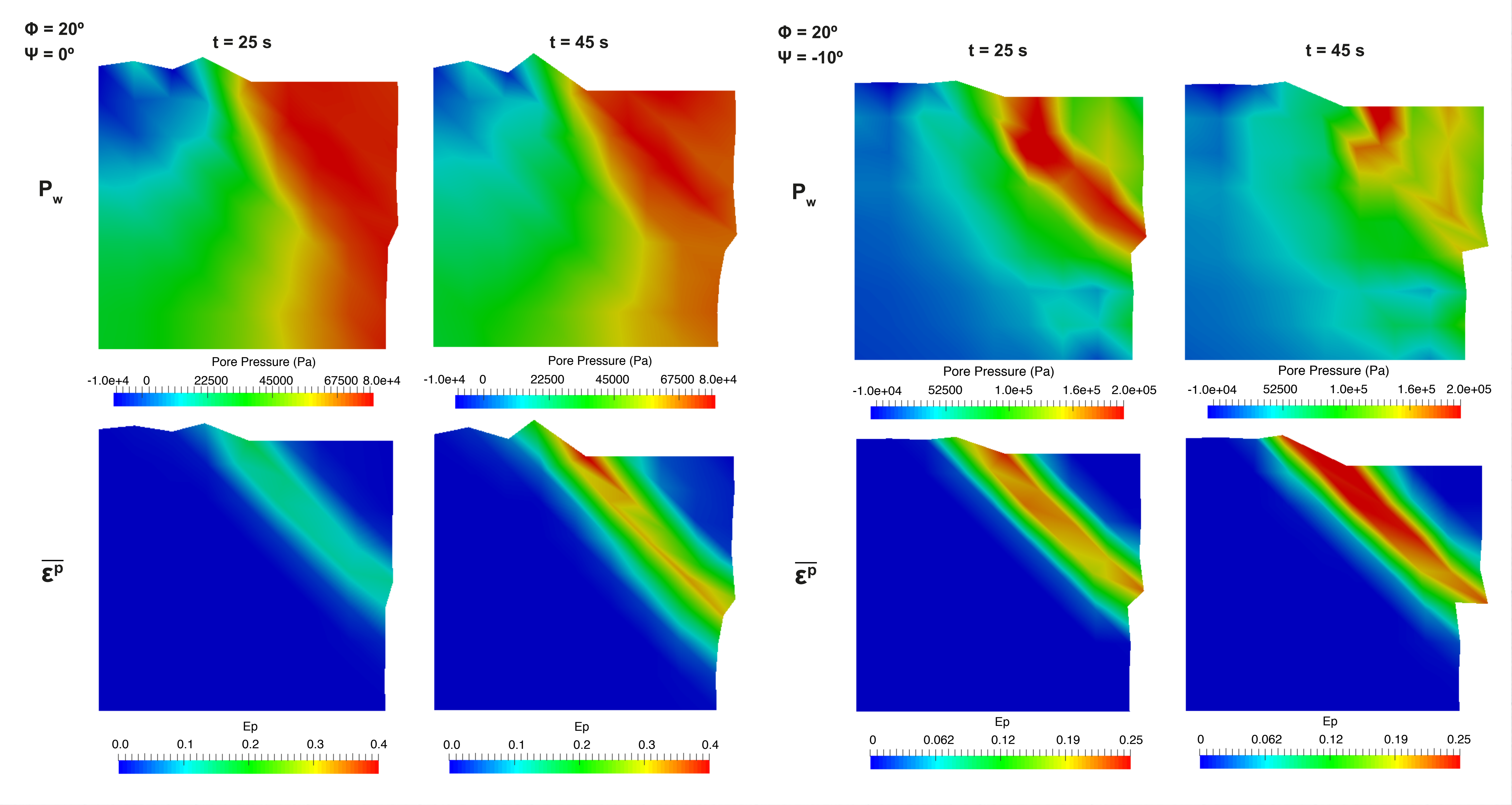
Explicit scheme: 2e-2 m/s



4. Benchmark examples

Rigid footing in a saturated square plate

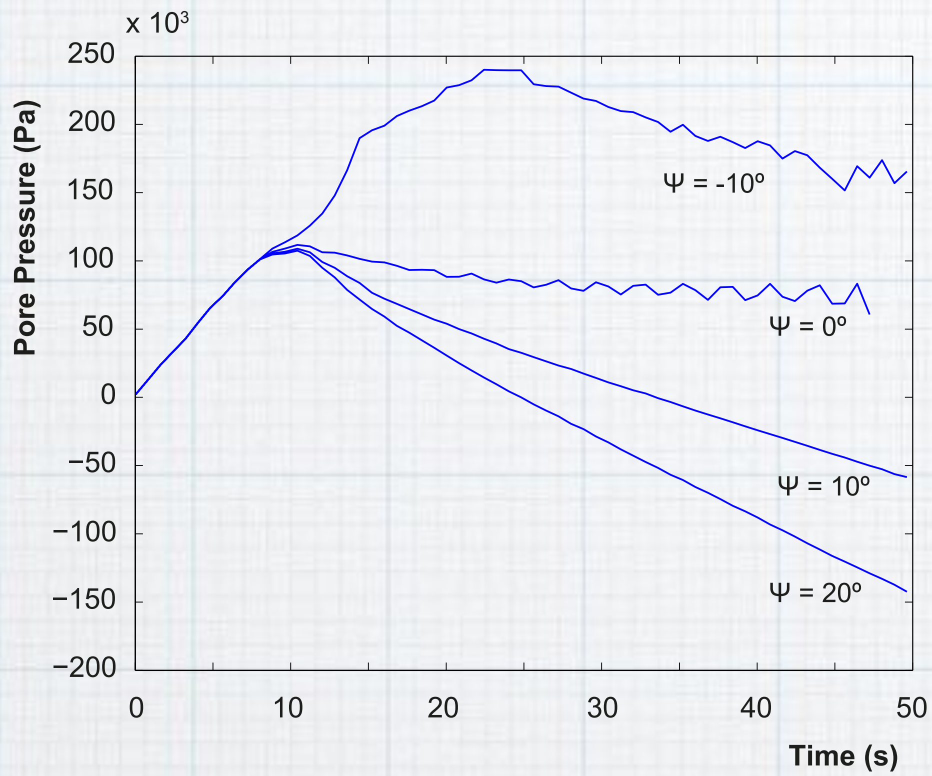
Explicit scheme: 2e-2 m/s

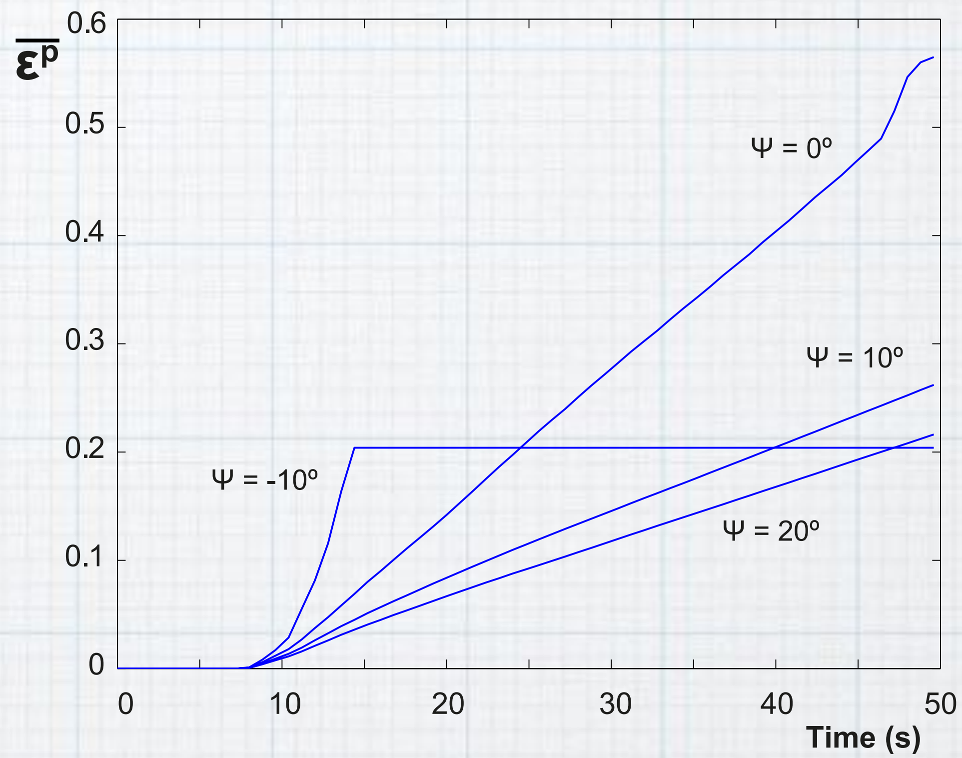


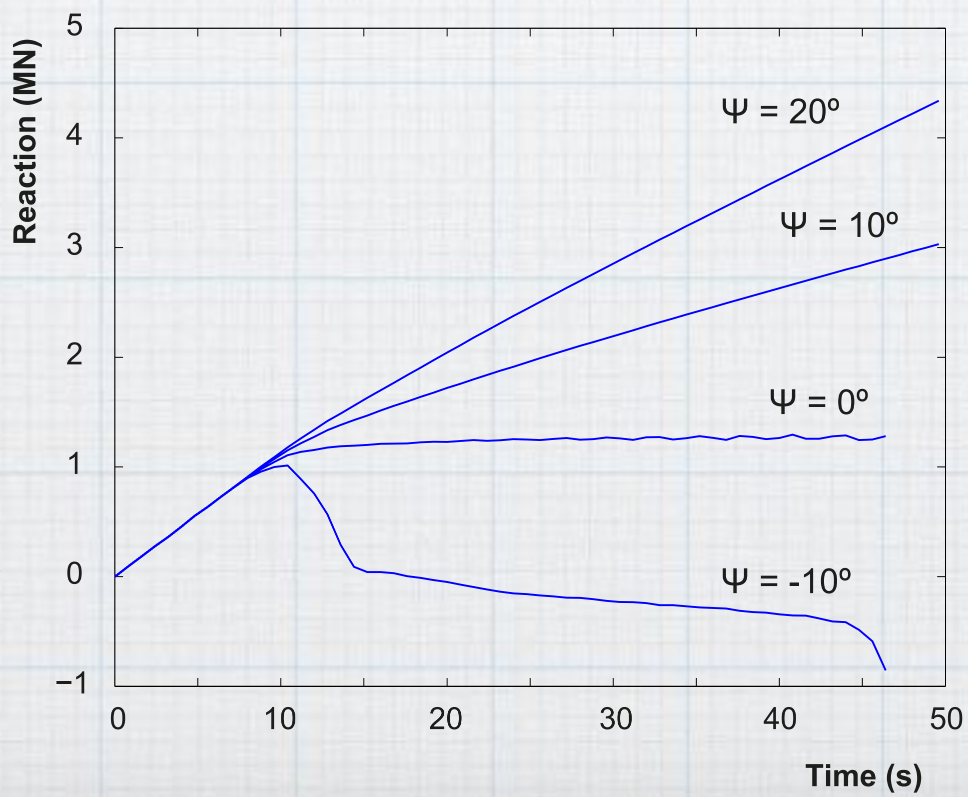
4. Benchmark examples

Rigid footing in a saturated square plate

Explicit scheme: 2e-2 m/s



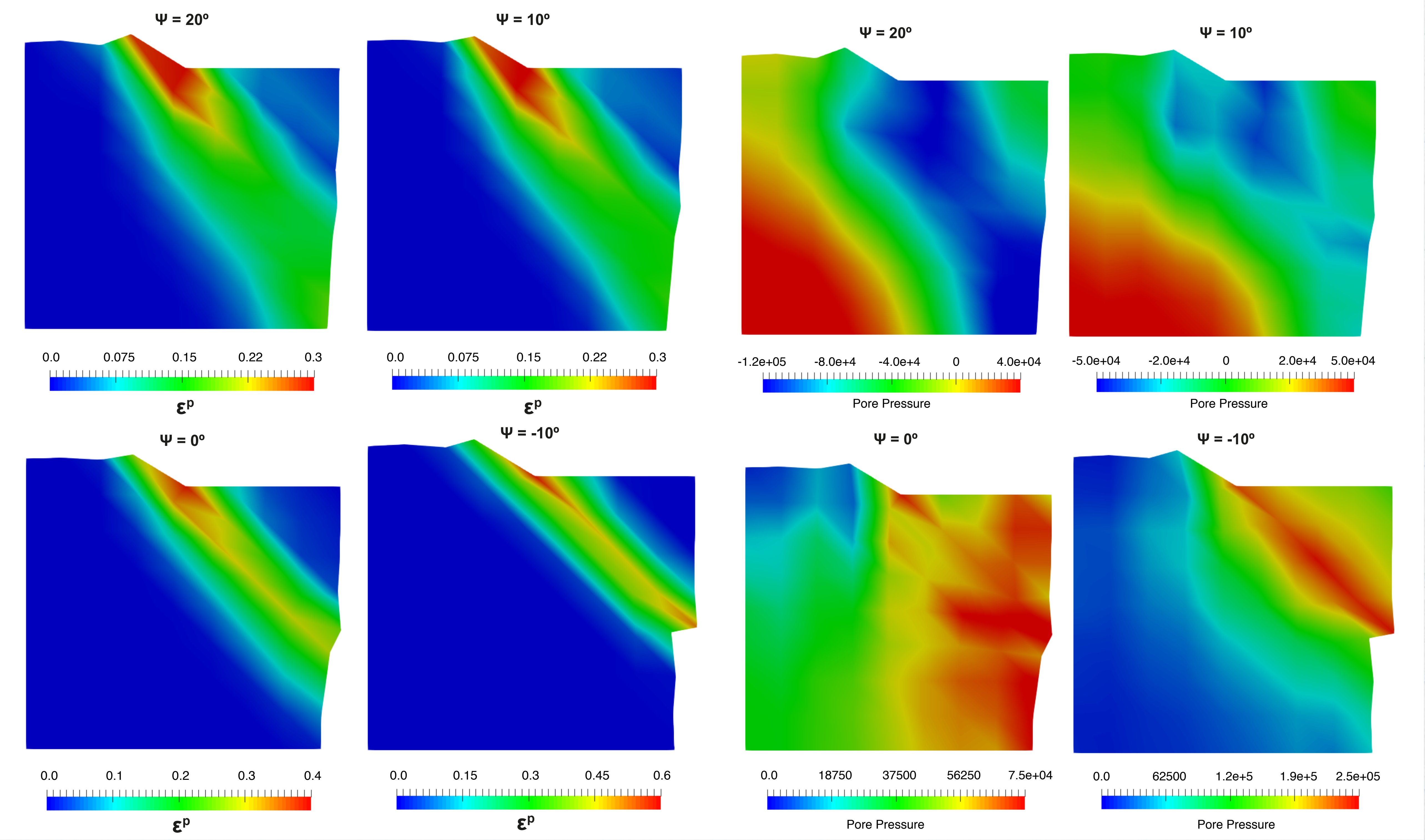




4. Benchmark examples

Rigid footing in a saturated square plate

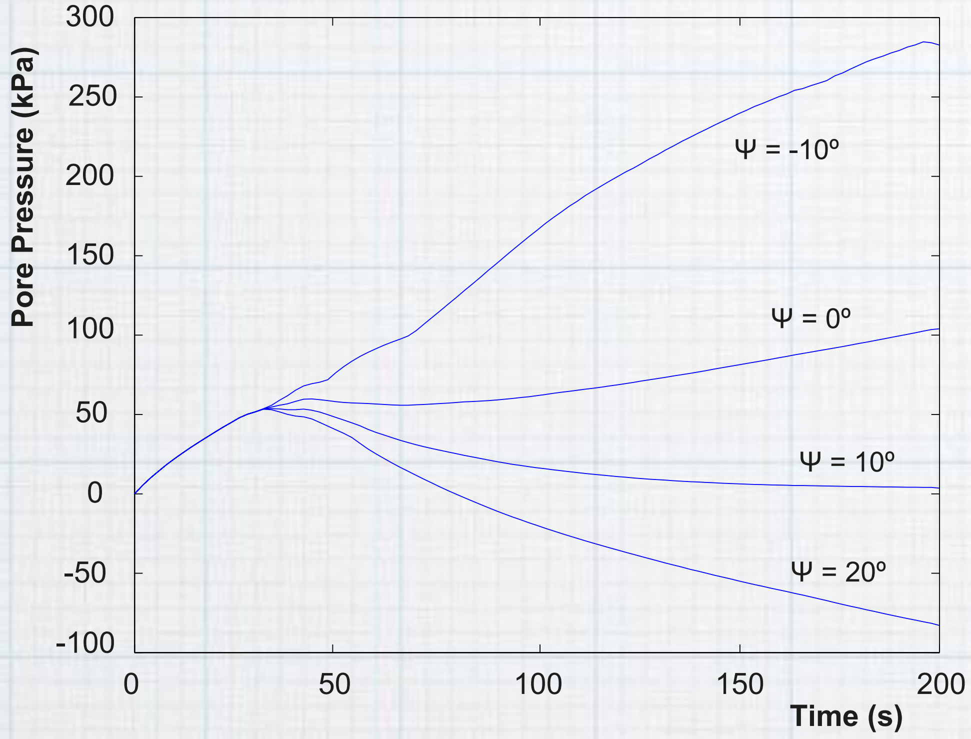
Implicit scheme: 5 mm/s

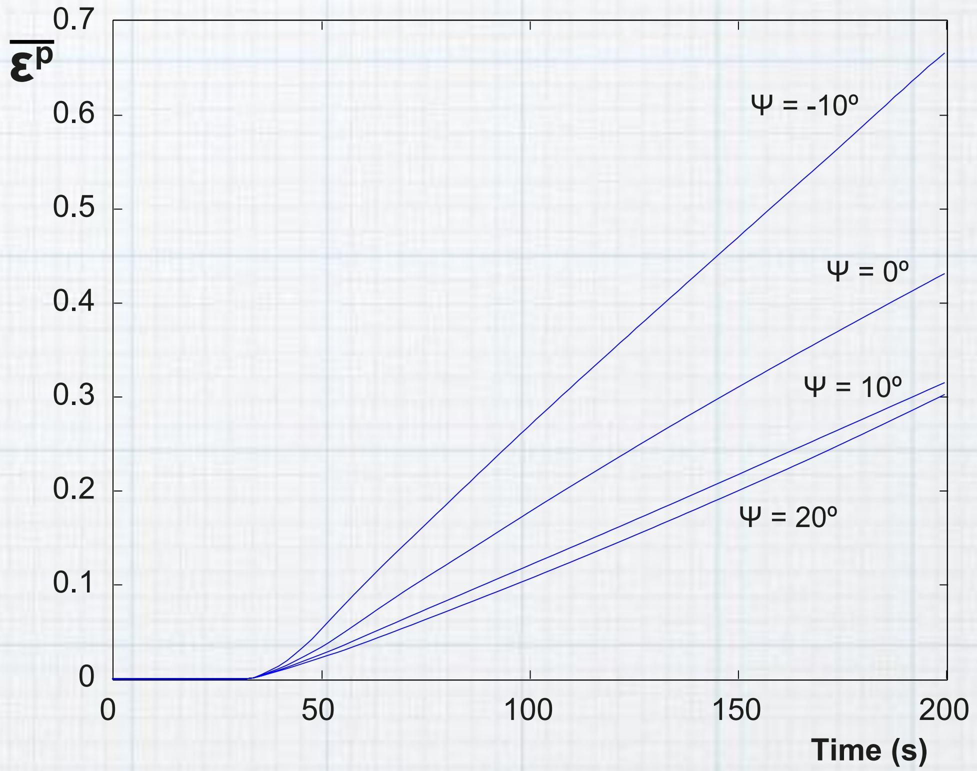


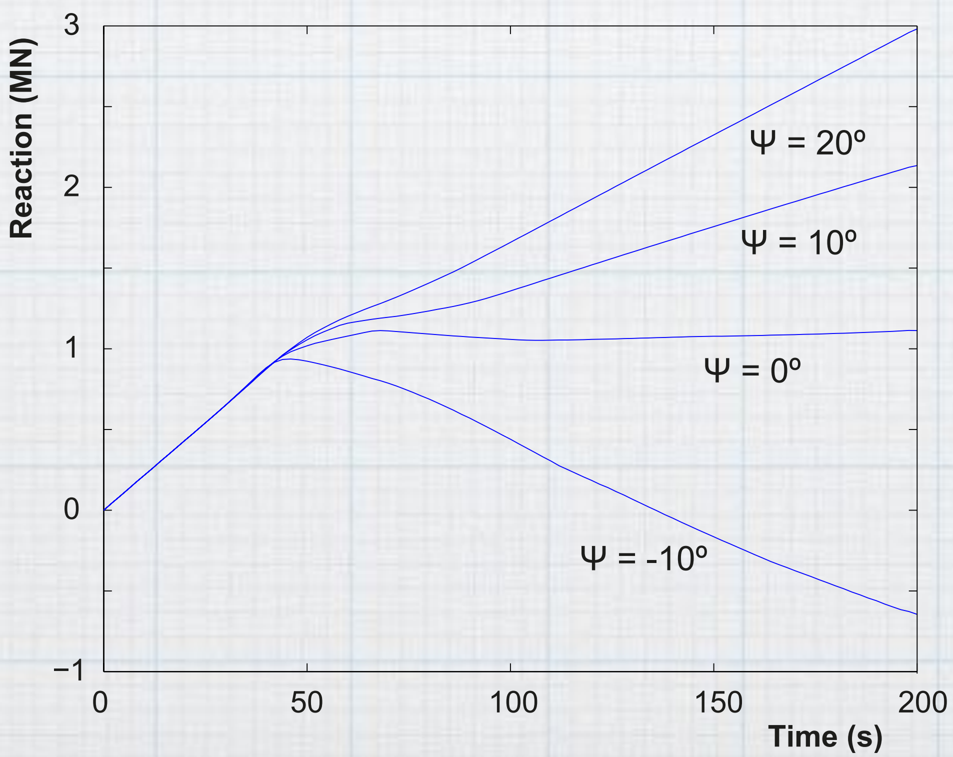
4. Benchmark examples

Rigid footing in a saturated square plate

Implicit scheme: 5 mm/s



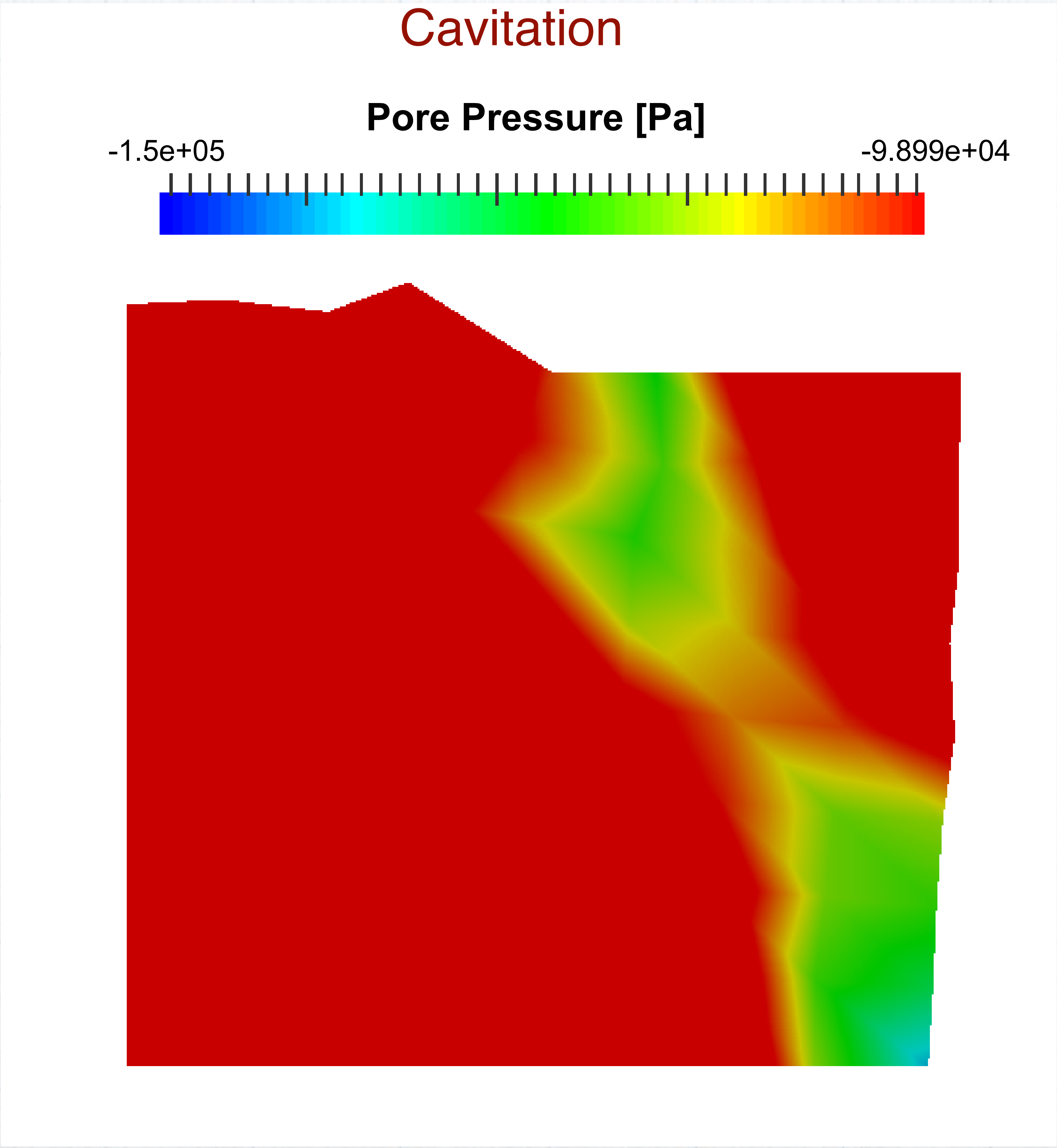




4. Benchmark examples

Rigid footing in a saturated square plate

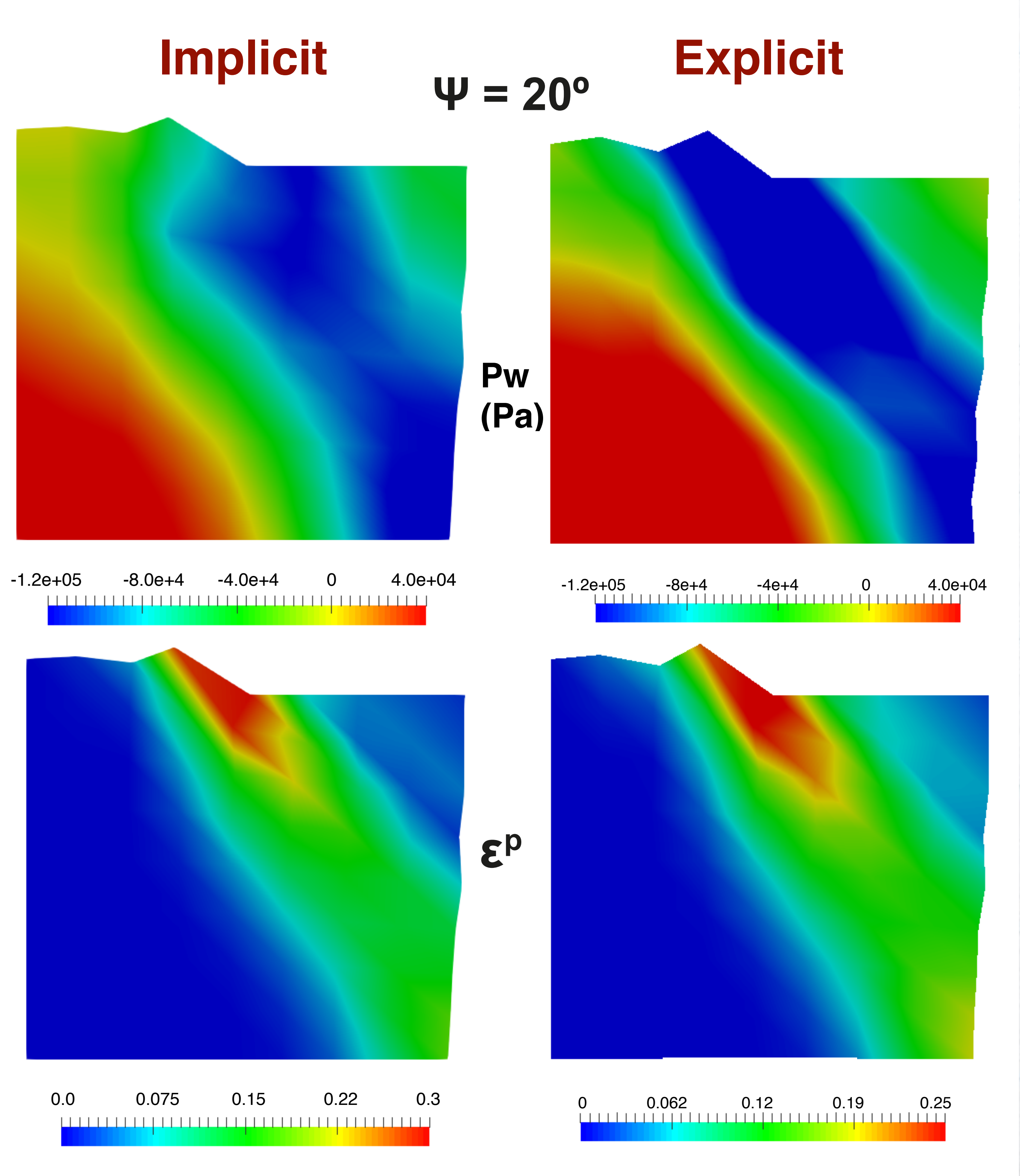
Implicit scheme: 5 mm/s



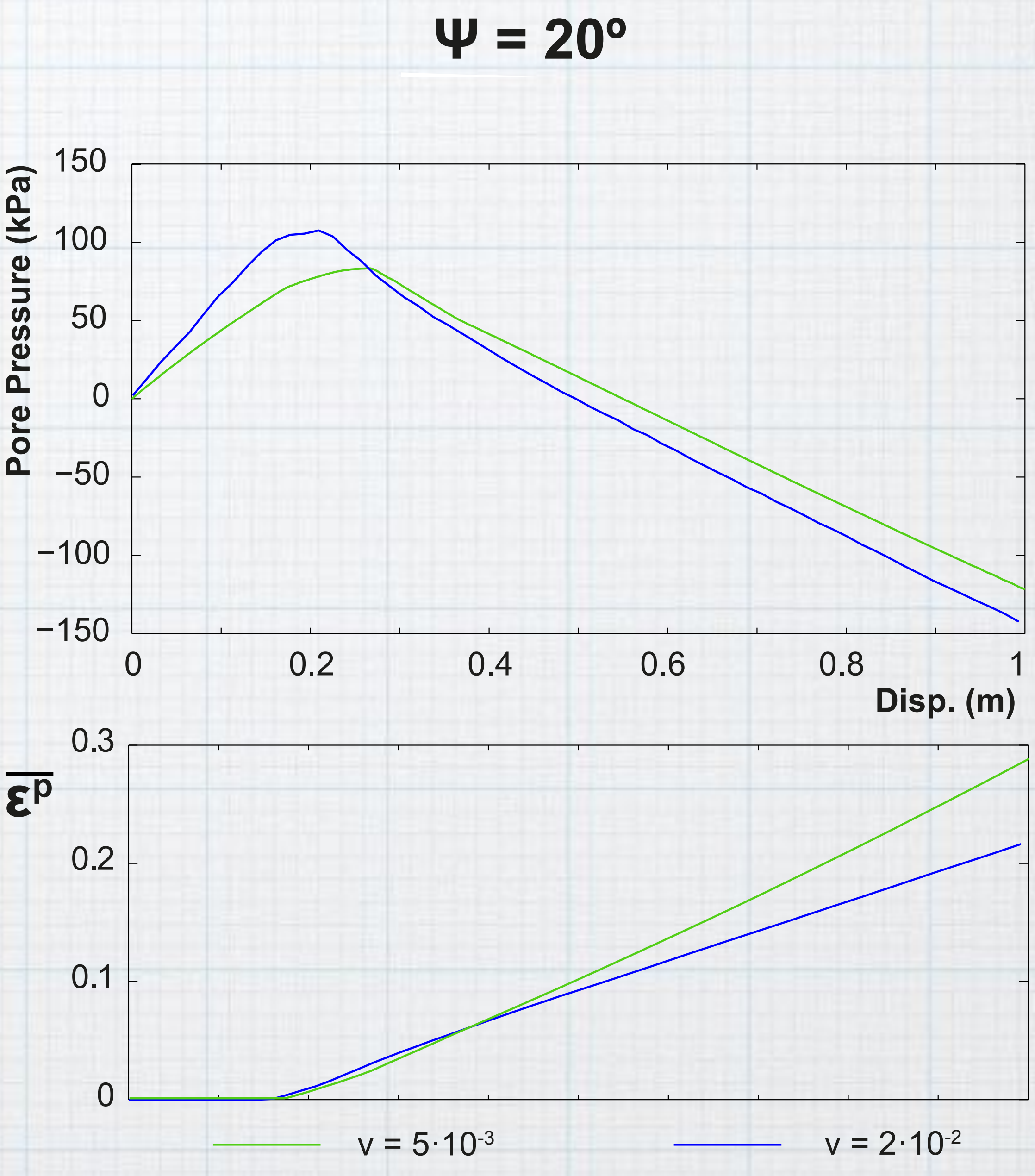
4. Benchmark examples

Rigid footing in a saturated square plate

Implicit vs Explicit scheme: 5 mm/s

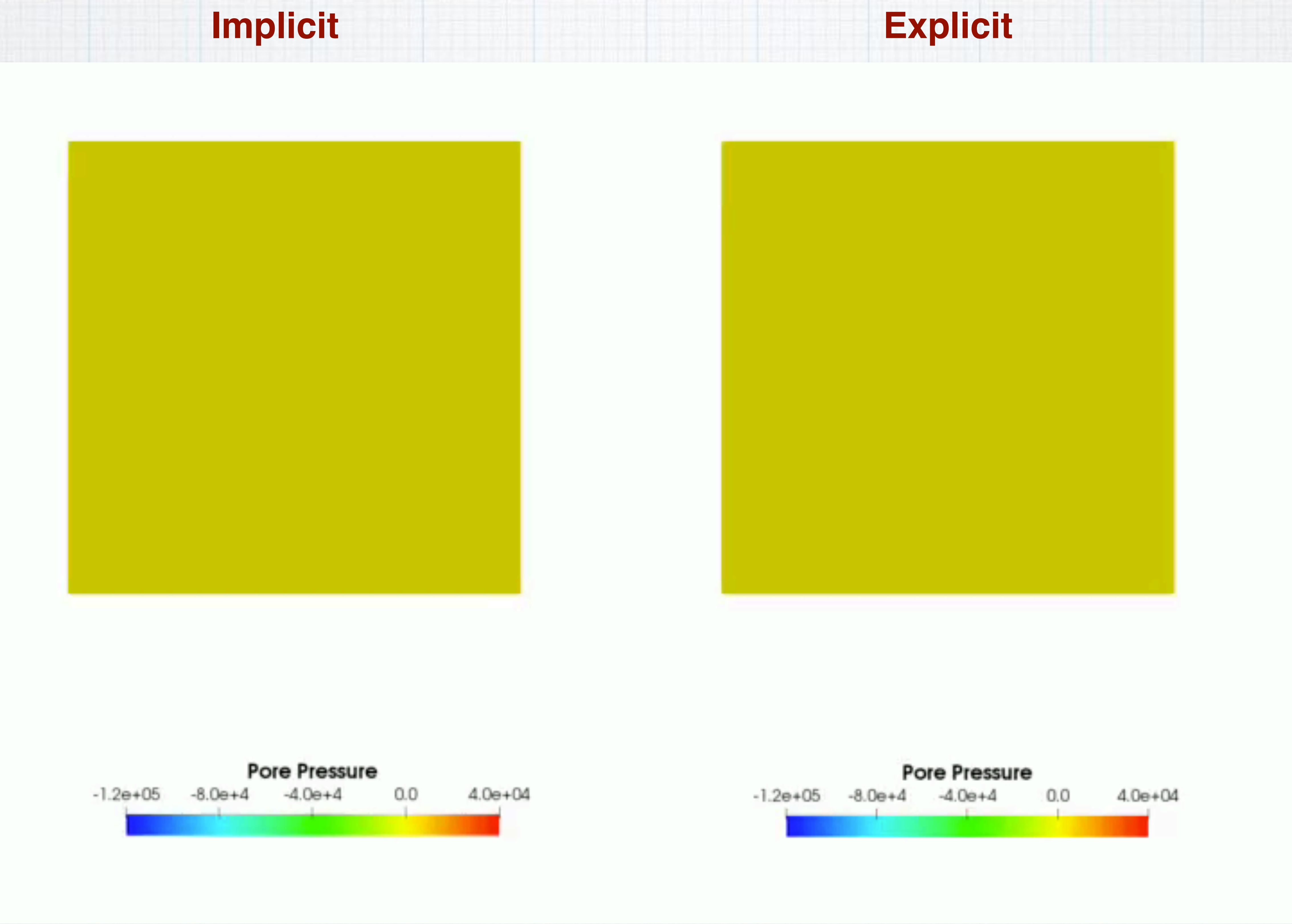


Comparison between velocities



4. Benchmark examples

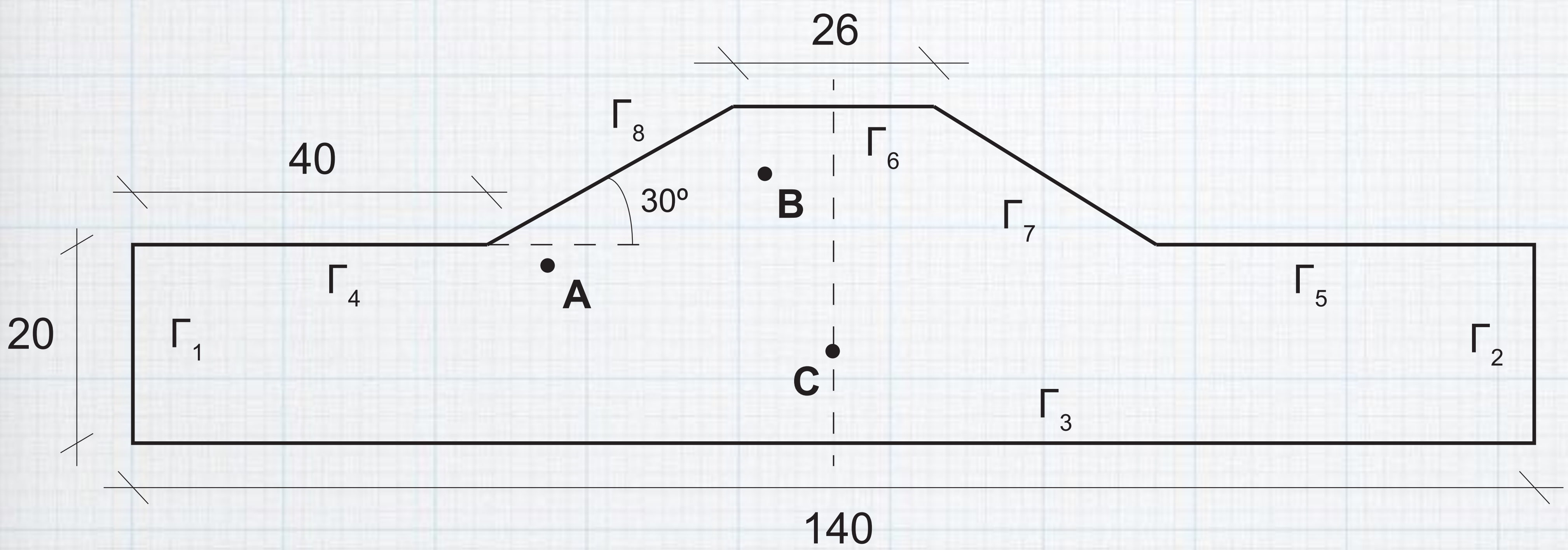
Rigid footing in a saturated square plate



Outline

1. Governing equations
2. Spatial discretization: OTM
3. Constitutive law
4. Benchmark examples
5. **Embankment application**
6. Conclusions and future work

5. Embankment application



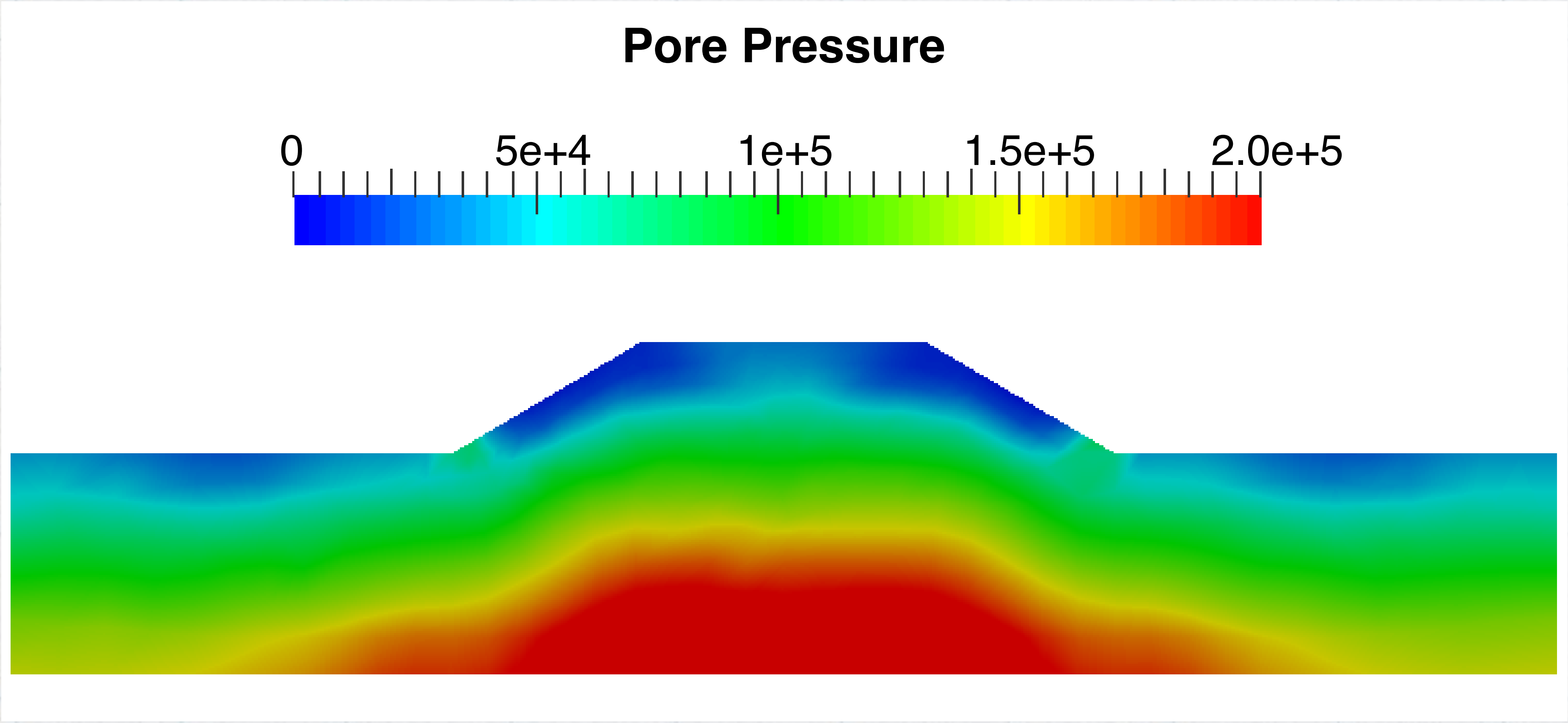
$$a = \frac{g}{2}\sin(2\pi f t)$$

Γ_1: $u_x=0$, $w_x=0$

Γ_2: $u_x=0$, $w_x=0$

Γ_3: $u_y=0$, $w_y=0$

$\Gamma_4, \Gamma_5, \Gamma_6, \Gamma_7, \Gamma_8$: free



$E = 50000$ kN/m^2

$G = 19230$ kN/m^2

$c_0 = 50$ kN/m^2

$H = -20$ kN/m^2

$\Phi = 15^o$

$\Psi = -3^o, 5^o$

$K_w = 10000$ kN/m^2

$k = 0.00001$ m/s

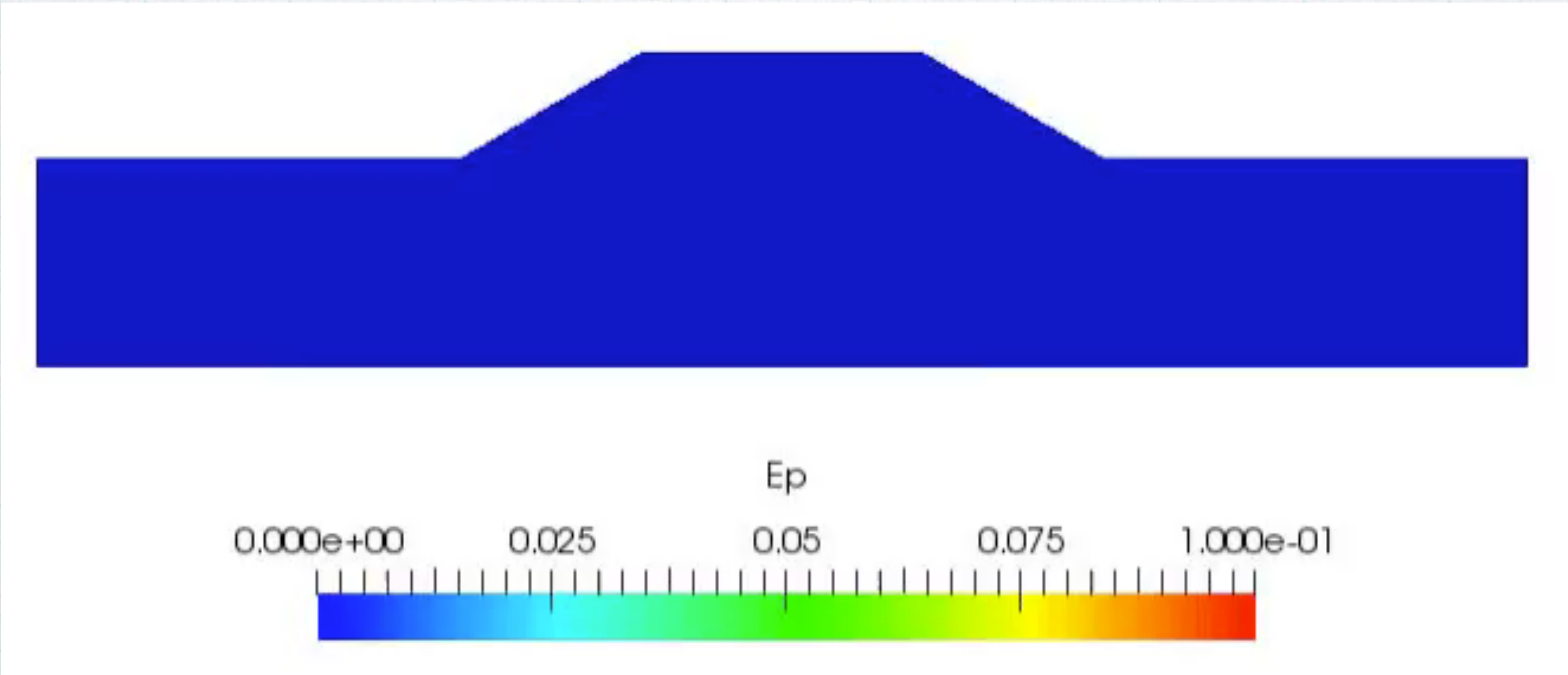
$n = 0.322$

$\rho_s = 2647$ kg/m^3

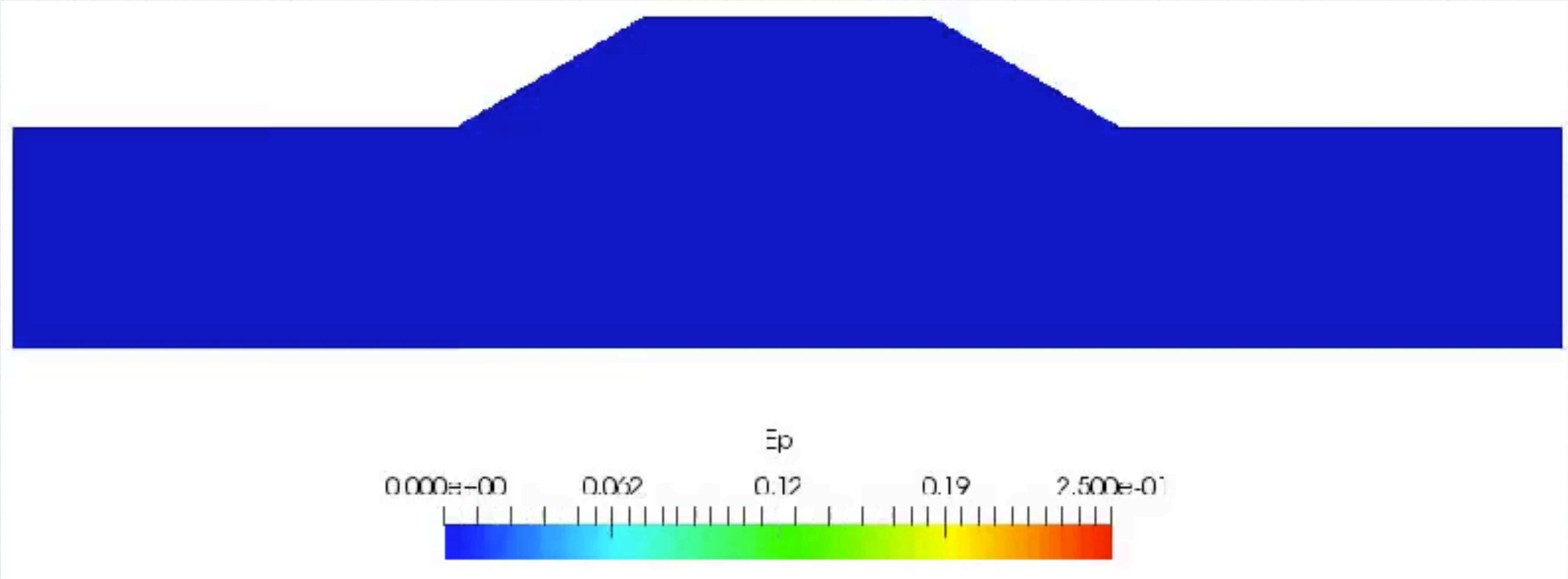
$\rho_w = 1000$ kg/m^3

5. Embankment application

Ψ = 5º



Ψ = -3º



Outline

1. Governing equations
2. Spatial discretization: OTM
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4. Benchmark examples
5. Embankment application
6. **Conclusions and future work**

6. Conclusiones and future work

Conclusions

- **Complete formulation** u-w
  - Implicit — Robustness
  - Explicit — Easyness
- **Dynamic consolidation**
  - Implicit — Stability
  - Explicit — Oscillations
- **Large deformation consolidation**
  - Implicit
  - Explicit
  - ✓ Dynamic effects
- **Plastic square loaded by a strip footing**
  - Implicit — Stability
  - Explicit — Oscillations
- **Embankment** — Implicit — Excellent performance

6. Conclusiones and future work

Future work

Employment within different computational methods

G-PFEM

MPM

Traditional techniques: FEM

Different constitutive models

Cam-Clay

Multi-phase governing equations

Unsaturated soils

Time integration schemes assessment

Explicit - Implicit

Publications

- Navas, P., Sanavia, L., López-Querol, S., Yu, R. (2017): Explicit meshfree solution for large deformation dynamic problems in saturated porous media. DOI: 10.1007/s11440-017-0612-7

- Navas, P., Sanavia, L., López-Querol, S., Yu, R. (2017): u-w formulation for dynamic problems in large deformation regime solved through an implicit meshfree scheme. Computational mechanics. DOI: 10.1007/s00466-017-1524-y

Local Max-Ent meshfree method applied to large deformation problems in saturated soils

P. Navas, L. Sanavia, S. López-Querol and **R.C. Yu**

Thanks!

28th ALERT Workshop Program

Aussois, 4th October 2017