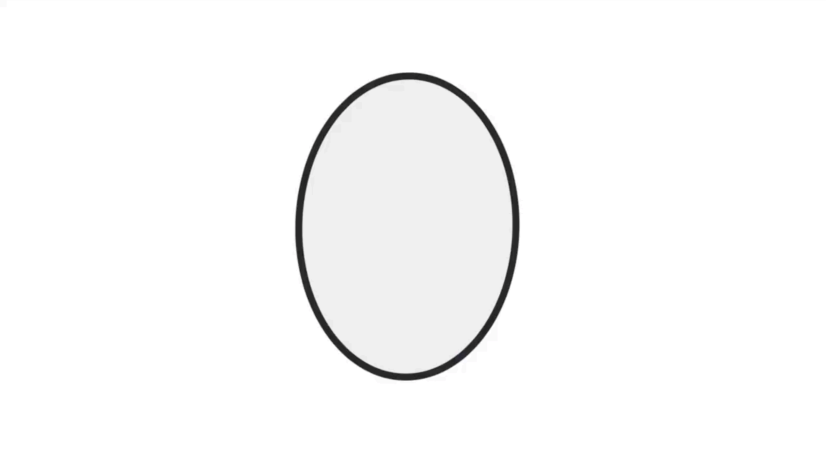
Thermo-hydro-mechanical couplings in geomaterials

Jean-Michel Pereira

ALERT Doctoral School, 2025, Aussois

Laboratoire Navier, École des ponts

Breaking eggs...



(https://www.youtube.com/@TheScienceClassroom)

(EPFL – Mathieu Nuth)

Applications:

- shallow energy geostructures
- slope stability, incl. permafrost
- energy production and storage
- nuclear waste disposal
- CO_2 geological storage
- …

(EPFL – Mathieu Nuth)

Physical processes:

- humidity effects
- thermal stress/strains
- thermal pressurisation
- phase changes
- ...

Outline

1. Starting from thermodynamics
2. Basics of constitutive modelling
   - Thermal problem
   - Hydraulic problem
   - Mechanical problem
3. THM couplings
   - Transport properties
   - Thermal expansion
   - Thermal consolidation
4. THM models
   - Unsaturated geomaterials
   - Thermoporoelastoplastic models
5. Application

Starting from thermodynamics

Pioneers


Pioneers

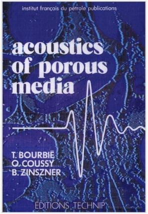
Prof. Karl von Terzaghi
(1883-1963)

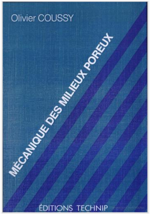
Prof. Maurice A. Biot
(1905-1985)

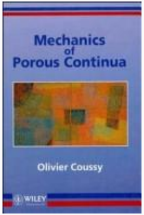
(1953 – 2010)

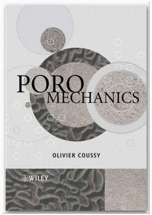
A poromechanics legacy

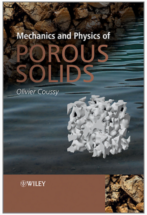
- 5 books
- c.a. 100 papers in scientific journals
- Laboratoire Navier (ENPC/UGE/CNRS)



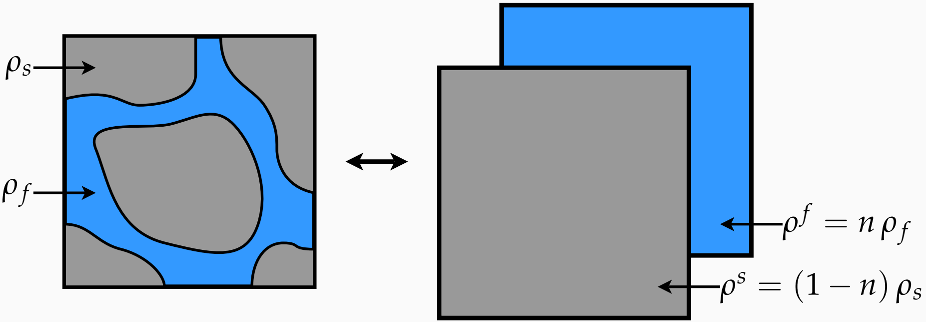








Porous media: a few definitions



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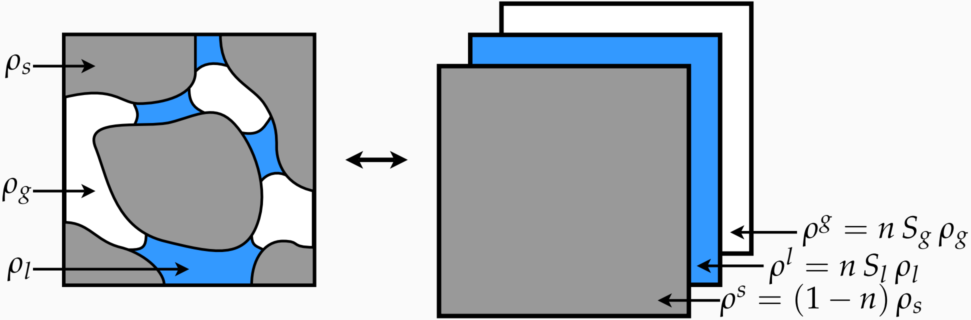

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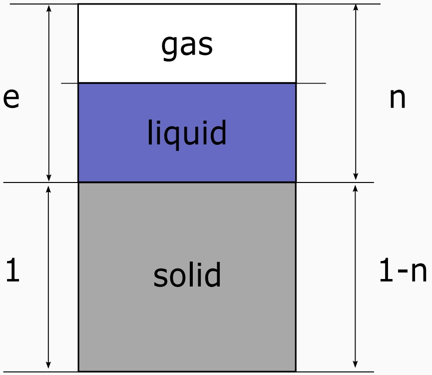

Porosity and degree of saturation:

$$n \text{ or } \phi = \frac{\text{pores vol.}}{\text{total vol.}}$$

$$S_l = \frac{\text{liquid vol.}}{\text{pores vol.}} = 1 - S_g$$

Thermodynamic/energetic approach: overview

- Define the system!
- Balance of energy at continuum scale[1] (Coussy 2004)
  - state equations (energy potential and state variables)
  - conjugate variables
- Identify an energy potential
  - deduce the constitutive relations
  - e.g. quadratic potential provides linear behaviour

[1] RVE, macroscale

Energetic approach – illustration

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Work input (reversible case)

Infinitesimal work to the initial system

$$\text{for a 1D spring:} \qquad dW = F\,\mathrm{d}x$$

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$$\begin{aligned} \text{for a 1D spring:} \quad dW &= F\,\mathrm{d}x \\ \text{for a non porous solid:} \quad dW &= -p\,\mathrm{d}V \end{aligned}$$

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\text{for a porous solid:} \quad & dW = -p\,\mathrm{d}V + p_l\,\mathrm{d}(n\,V)
\end{aligned}
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Infinitesimal strain work (solid skeleton) $\mathrm{d}W = \mathrm{d}w\,V_0$

$$\text{for a porous solid:} \quad dw = p\,\mathrm{d}\epsilon_v + p_l\,\mathrm{d}\phi$$

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Conjugate variables

Work input (extended to triaxial space)

$$dw = p\,d\epsilon_v + q\,d\epsilon_q + p_l\,d\phi$$

Strain-work conjugate variables

$$\begin{aligned} p &\longleftrightarrow \epsilon_v \\ q &\longleftrightarrow \epsilon_q \\ p_l &\longleftrightarrow \phi \end{aligned}$$

Conjugate variables – Terzaghi stress

Incompressibility of the solid grains ($K_s \gg K$) (Coussy 2004)

$$\mathrm{d}\epsilon_v = -\mathrm{d}\phi$$

Strain work input (Schofield and Wroth 1968)

$$\begin{aligned} dw &= (p - p_l)\,\mathrm{d}\epsilon_v + q\,\mathrm{d}\epsilon_q \\ &= p'\,\mathrm{d}\epsilon_v + q\,\mathrm{d}\epsilon_q \end{aligned}$$

Conjugate variables

$$p' = \frac{1}{3}(\sigma'_1 + 2\sigma'_3) \quad \longleftrightarrow \quad \epsilon_v = \epsilon_1 + 2\epsilon_3$$

$$q = \sigma_1 - \sigma_3 \quad \longleftrightarrow \quad \epsilon_q = \frac{2}{3}(\epsilon_1 - \epsilon_3)$$

Conjugate variables – tensorial form

Work input

$$\mathrm{d}w = \boldsymbol{\sigma} : \mathrm{d}\boldsymbol{\epsilon} + p_l \,\mathrm{d}\phi = \sigma_{ij} \,\mathrm{d}\epsilon_{ij} + p_l \,\mathrm{d}\phi$$

Strain-work conjugate variables

$$\boldsymbol{\sigma} \longleftrightarrow \boldsymbol{\epsilon}$$
$$p_l \longleftrightarrow \phi$$

Work input (incompressibility)

$$\mathrm{d}w = \boldsymbol{\sigma}' : \mathrm{d}\boldsymbol{\epsilon} = \sigma'_{ij} \,\mathrm{d}\epsilon_{ij}$$

Strain-work conjugate variables

$$\boldsymbol{\sigma}' \longleftrightarrow \boldsymbol{\epsilon}$$

How to get the constitutive laws?

Work input and dissipation – Clausius-Duhem inequality

Beyond reversibility?

Application of first and second laws of thermodynamics

$$\mathrm{d}D = \mathrm{d}w - \mathrm{d}F \geq 0$$

D: dissipation
F: free energy of the solid skeleton

State equations in reversible case

Clausius-Duhem inequality

$$\mathrm{d}D \quad = \quad \sigma_{ij}\,\mathrm{d}\epsilon_{ij} + p_l\,\mathrm{d}\phi - \,\mathrm{d}F \geq 0$$

Elasticity $\Leftrightarrow$ reversibility i.e. no dissipation ($\mathrm{d}D = 0$ and $\boldsymbol{\epsilon} = \boldsymbol{\epsilon}^e$ and $\mathrm{d}\phi = \mathrm{d}\phi^e$)

$$\mathrm{d}F \quad = \quad \sigma_{ij}\,\mathrm{d}\epsilon_{ij} + p_l\,\mathrm{d}\phi$$

Hence $F = F(\boldsymbol{\epsilon}, \phi)$ and the following state equations hold

$$\boldsymbol{\sigma} \quad = \quad \frac{\partial F}{\partial \boldsymbol{\epsilon}}$$

$$p_l \quad = \quad \frac{\partial F}{\partial \phi}$$

Linear poroelasticity

Energy potential F

Inspecting the state equations, it appears that a linear behaviour stems from a quadratic potential

→ From a stress- and pressure-free reference state

$$\begin{aligned} \boldsymbol{\sigma} &= \mathbb{D}\,\boldsymbol{\epsilon} - b\,p_l\,\boldsymbol{\delta} \\ \phi - \phi_0 &= -b\,\epsilon_v + \frac{p_l}{N} \end{aligned}$$

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$\mathbb{D}$: stiffness matrix

b: Biot coefficient, $b = 1 - \dfrac{K}{K_s}$

N: Biot modulus, $\dfrac{1}{N} = \dfrac{b - \phi_0}{K_s}$

Energy potential F

Inspecting the state equations, it appears that a linear behaviour stems from a quadratic potential

→ From a pre-stressed state

$$
\begin{aligned}
\boldsymbol{\sigma} - \boldsymbol{\sigma}_0 &= \mathbb{D}\,\boldsymbol{\epsilon} - b\,(p_l - p_{l,0})\,\boldsymbol{\delta} \\
\phi - \phi_0 &= -b\,\epsilon_v + \frac{p_l - p_{l,0}}{N}
\end{aligned}
$$

$\mathbb{D}$: stiffness matrix

b: Biot coefficient, $b = 1 - \dfrac{K}{K_s}$

N: Biot modulus, $\dfrac{1}{N} = \dfrac{b - \phi_0}{K_s}$

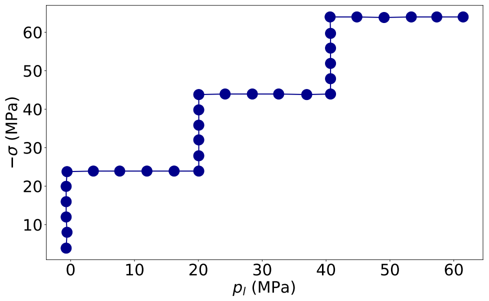
Nonlinear poroelasticity

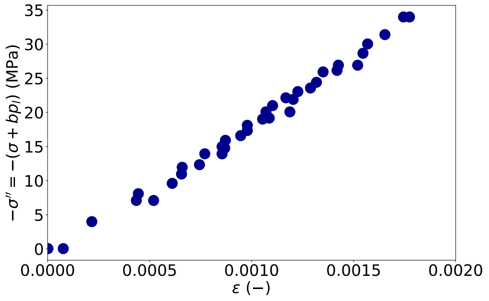
Incremental form of the constitutive equations

$$\begin{aligned} \mathrm{d}\boldsymbol{\sigma} &= \mathbb{D}(\boldsymbol{\sigma}, p_l)\,\mathrm{d}\boldsymbol{\epsilon} - b(\boldsymbol{\sigma}, p_l)\,\mathrm{d}p_l\,\boldsymbol{\delta} \\ \mathrm{d}\phi &= -b(\boldsymbol{\sigma}, p_l)\,\mathrm{d}\epsilon_v + \frac{\mathrm{d}p_l}{N(\boldsymbol{\sigma}, p_l)} \end{aligned}$$

Material parameters are tangent properties, and depend on material state

From $\mathrm{d}\sigma = K\,\mathrm{d}\epsilon_v + b\,\mathrm{d}p_l$, introduce the Biot stress: $\mathrm{d}\sigma'' = \mathrm{d}\sigma - b\,\mathrm{d}p_l$ so that $\mathrm{d}\sigma'' = K\,\mathrm{d}\epsilon_v$





Unjacketed test on a limestone ($K_s = 52.7$ GPa) (Coussy 2004)

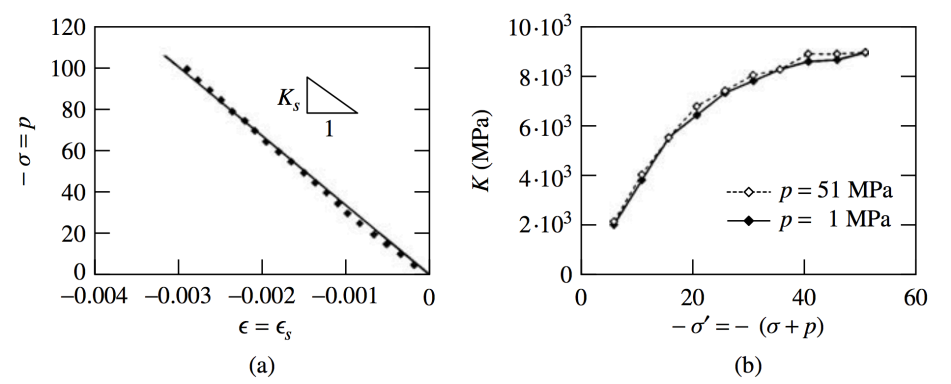
Stress variable(s) – poromechanical properties

Material	ϕ (%)	K (MPa $\times 10^3$)	b (–)	N (MPa $\times 10^3$)
Cement paste	40–63	15–2	0.07–0.37	1170–20
Mortar	27–40	15–3	0.04–0.35	2340–40
Bone	5	12	0.14	160
Granites	1–2	25–35	0.22–0.44	280–370
Marble	2	40	0.20	280
Sandstones	2–26	4.6–13	0.69–0.85	$\sim$17
Limestones	4–29	5–39	0.34–0.88	100–400

Order of magnitude of poroelastic properties for different materials (Coussy 2004)

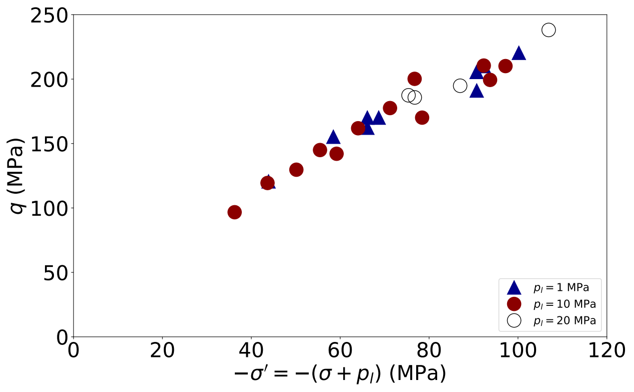
For soils: $b = 1 - \dfrac{K}{K_s} \approx 1$ and $N \to \infty$.

Biot (effective?) stress – case of non-linearity



Experimental confirmation of non-linear constitutive equations for a sandstone (after Bemer et al., 2001), cited by (Coussy 2004)

Biot (effective?) stress – yield function



Experimental validation of yield function in terms of Terzaghi stress for a limestone with $b = 0.63$ (after Vincké et al., 1998), cited by (Coussy 2010)

Back to Clausius-Duhem inequality, beyond reversibility?

$$\mathrm{d}D \quad = \quad \sigma_{ij}\,\mathrm{d}\epsilon_{ij} + p_l\,\mathrm{d}\phi - \mathrm{d}F \geq 0$$

Assuming that elasticity still gives

$$\mathrm{d}F \quad = \quad \sigma_{ij}\,\mathrm{d}\epsilon^e_{ij} + p_l\,\mathrm{d}\phi^e$$

But this time, $\mathrm{d}D \neq 0$ and $\boldsymbol{\epsilon} = \boldsymbol{\epsilon}^e + \boldsymbol{\epsilon}^p$ and $\mathrm{d}\phi = \mathrm{d}\phi^e + \mathrm{d}\phi^p$

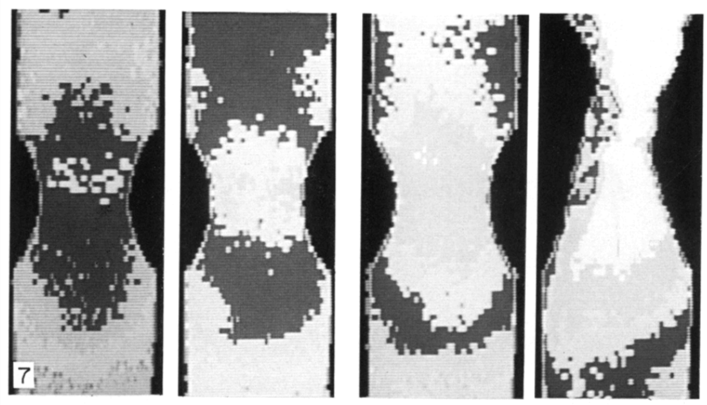
Dissipation

$$\mathrm{d}D \quad = \quad \sigma_{ij}\,\mathrm{d}\epsilon^p_{ij} + p_l\,\mathrm{d}\phi^p \geq 0$$

See also hyperplasticity theory (Houlsby and Puzrin 2006) to go further

Dissipation?

Dissipation?



Infrared thermovision of unstable failure in rock salt (Luong 1990)

Basics of constitutive modelling

"A model is a lie that helps you see the truth."

— Howard Skipper

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Definitions

Constitutive relations

- Mathematical relation between conjugate variables
- Introduce material parameters
- Allow closing the problem

Example in mechanics

Unknowns	Equations
$\boldsymbol{\sigma}$, 6	Equilibrium, 3
$\boldsymbol{u}$, 3	Compatibility, 6
$\boldsymbol{\epsilon}$, 6	Constitutive law, 6

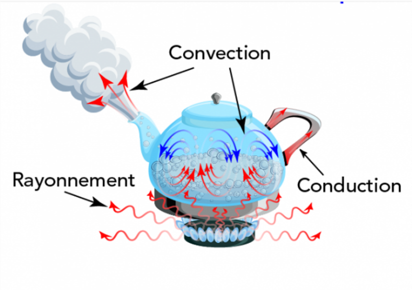
Examples of constitutive relations

Problem	Variables	Relation	Parameters
Thermal	$\boldsymbol{q}_t,\ T$	Fourier law	λ
Hydraulic	$\boldsymbol{q}_l,\ p_l$	Darcy law	κ
Mechanical	$\boldsymbol{\sigma},\ \boldsymbol{\epsilon}$	Hooke's law	$E,\ \nu$

Basics of constitutive modelling

Thermal problem

Heat transfer



(parlonssciences.ca)

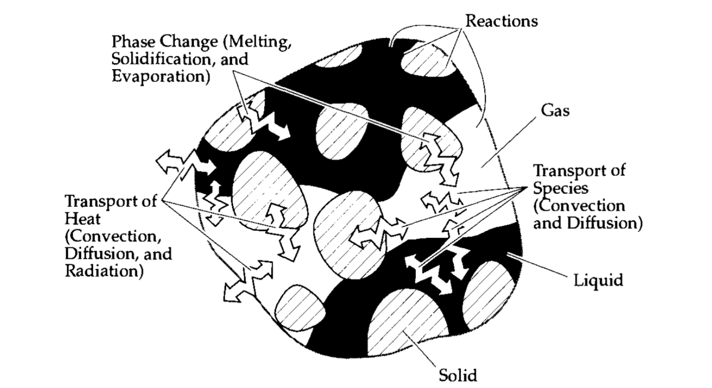
Definitions

Conduction Heat transfer through diffusion within the material (no mass transfer)

Convection Heat transfer through mass transfer

Radiation Heat transfer through electomagnetic waves (no mass support required)

Heat transfer in porous media



(Kaviany 1998)

Fourier law

Joseph Fourier
(1768–1830)

Isotropic material

$$q_{t,i} = -\lambda \frac{\partial T}{\partial x_i}$$

with λ: thermal conductivity (scalar) [W / K m]

Anisotropic material

$$q_{t,i} = -\lambda_{ij} \frac{\partial T}{\partial x_j}$$

with $\boldsymbol{\lambda}$: thermal conductivity tensor [W / K m]

Thermal diffusivity

Heat equation (energy balance equation & Fourier law)

$$\frac{\partial T}{\partial t} = \frac{\lambda}{\rho\, c}\Delta T + R$$

$D = \dfrac{\lambda}{\rho\, c}$: thermal diffusivity [$m^2/s$]
R: volumetric heat source

Phase change (e.g. water solidification)

Heat equation (energy balance equation & Fourier law)

$$\rho\, c\, \frac{\partial T}{\partial t} = \lambda\, \Delta T + L_f \frac{\rho_{ice}}{\rho_w} \frac{\partial \theta_{ice}}{\partial t} + R'$$

R': heat source

Need for a freezing curve $\theta_{ice} = \mathcal{F}(T)$

$$\left(\rho\, c + L_f'\right) \frac{\partial T}{\partial t} = \lambda\, \Delta T + R'$$

Basics of constitutive modelling

Hydraulic problem

Darcy law

Henri Darcy
(1803–1858)

$$\boldsymbol{q}_l = -\frac{\kappa}{\mu}\left(\mathbf{grad}\, p_l - \boldsymbol{\gamma}\right)$$

with

μ: dynamic viscosity of water (1 mPa.s at 20 °C)
κ: intrinsic permeability [m^2]

k : hydraulic conductivity [m/s]

Unsaturated case: $\kappa \leftarrow \kappa_{app} = \kappa\, \kappa_{rel}(S_l)$

Basics of constitutive modelling

Mechanical problem

Mechanical constitutive models

Modelling framework: define your needs

- Elasticity vs elastoplasticity
- Cyclic behaviour
- Time and rate effects (viscosity, creep...)
- Humidity effects (capillarity, adsorption)
- Temperature effects
- Damage

Some definitions

- Elasticity: reversibility (energetic point of view)
- Plasticity: irreversible deformation
- Failure ≠ plasticity

Reversible deformations

Reversible behaviour

- the stress-strain relation is unique
- no energy dissipation (no hysteresis cycle)
- no permanent deformation after a loading-unloading cycle

Irreversible behaviour

- the stress-strain relation is no more unique
- energy dissipation $\rightarrow \int \mathrm{d}F = \int \sigma_{ij}\,\mathrm{d}\epsilon_{ij} \geq 0$
- permanent deformation after a loading-unloading cycle $\rightarrow \boldsymbol{\epsilon}^p$: plastic strains

Elastic models

- Isotropic linear elasticity (two constant parameters):
  ex. : E & ν or $E(z)$ & ν or K & G

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- Non-linear elasticity: $E(\boldsymbol{\sigma})$

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- Non-linear elasticity: $E(\boldsymbol{\sigma})$
- Hyper-elasticity: $\mathrm{d}\boldsymbol{\sigma} = \dfrac{\partial F}{\partial \boldsymbol{\epsilon}}$
- Hypo-elasticity: $\mathrm{d}\boldsymbol{\sigma} = \mathbb{D}\,\mathrm{d}\boldsymbol{\epsilon}$ (does not ensure energy conservation)

Undrained elasticity

Why is this important?

- Short term behaviour of geotechnical structures (foundations, retaining walls, excavations…)
- Some finite element codes offer undrained analyses

How to get undrained elastic moduli from drained elastic moduli?

How to get undrained elastic moduli from drained elastic moduli?

Use the water bulk moduli: $K_w = 2.2$ GPa at 20 °C.

It can be shown that $K_u = K_d + \frac{K_w}{n}$ where n is the porosity.

Since water does not transmit shear stresses (good approximation), $G_u = G_d = G$.

Then, one uses the established relations between elastic moduli:

$$E_u = \frac{9K_u G}{3K_u + G} \qquad \nu_u = \frac{3K_u - 2G}{2(3K_u + G)}$$

If $K_u >> G$ (usually the case), then ?

Undrained elasticity

Soil incompressibility:

If $K_u >> G$ (usually the case), then $\nu_u \rightarrow 0.5$

$$K_u = \frac{E_u}{3(1 - 2\nu_u)} \qquad G_u = \frac{E_u}{2(1 + \nu_u)}$$

Do not use $\nu_u = 0.5$ in numerical tools, but $\nu = 0.49$ for instance (remember that $\nu = 0.5$ implies volumetric incompressibility, see K).

Elasto-plastic models: main ingredients

Most of elasto-plastic models rely on the strain partition

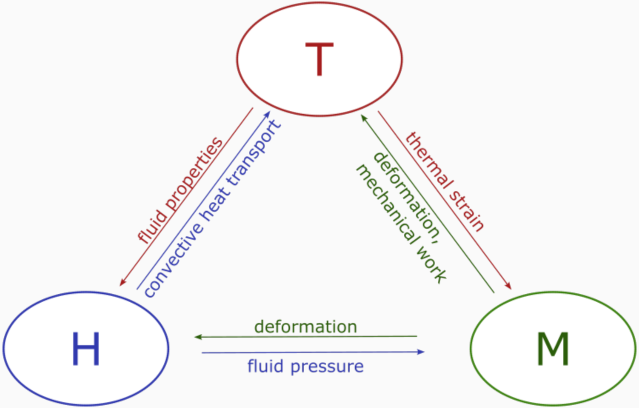
$$\boldsymbol{\epsilon} = \boldsymbol{\epsilon}^e + \boldsymbol{\epsilon}^p$$

What should we know, beyond elasticity?

1. WHEN?
   plastic criterion – yield surface f
2. HOW?
   increment of plastic strain tensor – flow rule
3. CONSEQUENCE ?
   evolution of the elastic domain – hardening law

THM couplings

About couplings



- **Direct couplings**
  between balance equations
- **Indirect couplings**
  affecting material parameters

About couplings – example

Direct coupling

Balance of momentum: $\operatorname{div}\boldsymbol{\sigma} + \rho\boldsymbol{b} = 0$ with $\boldsymbol{\sigma} = \boldsymbol{\sigma}' + p_l\,\mathbf{1}$

Indirect coupling

Darcy law: $\boldsymbol{q}_l = -\frac{\kappa}{\mu}\left(\mathbf{grad}\; p_l - \boldsymbol{\gamma}\right)$ with $\kappa = \kappa(\phi)$ (e.g. Kozeny-Carman model)

$\rightarrow$ depending on deformation

Question

- What about thermal expansion? Direct or indirect coupling?
- Other examples of indirect coupling?

THM couplings

Transport properties

Possible use of **apparent properties** (macro- or REV scale) obtained experimentally or through back analysis but need for state surfaces (porosity, water saturation, temperature...)

$$\text{e.g. } \pi = \pi(n, S_w, T \ldots)$$

Or, use **homogenisation** (upscaling) schemes; they readily account for couplings

Thermal properties and couplings

Volumetric heat capacity easy to estimate

$$C = (1 - n)\,\rho_s\,c_s + n\,S_w\,\rho_w\,c_w + n\,(1 - S_w)\,\rho_g\,c_g$$

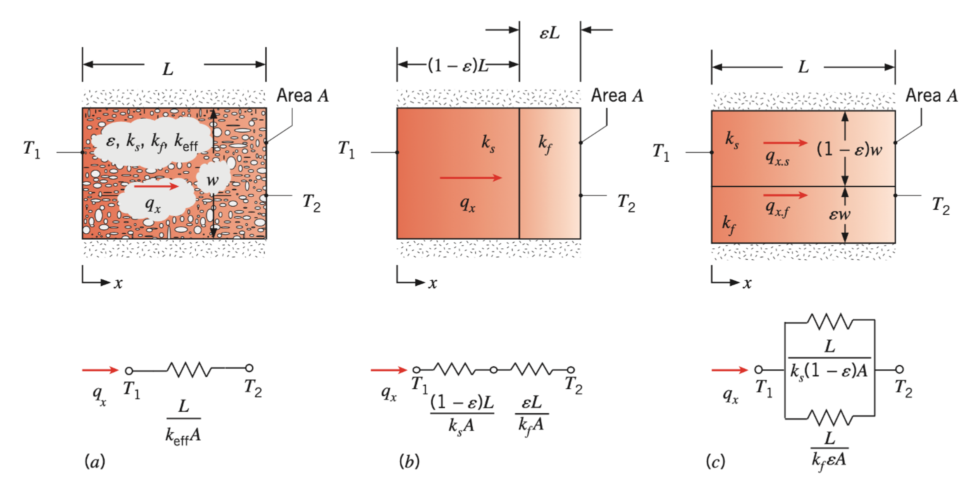
Thermal conductivity? Not so easy

Lazy guess

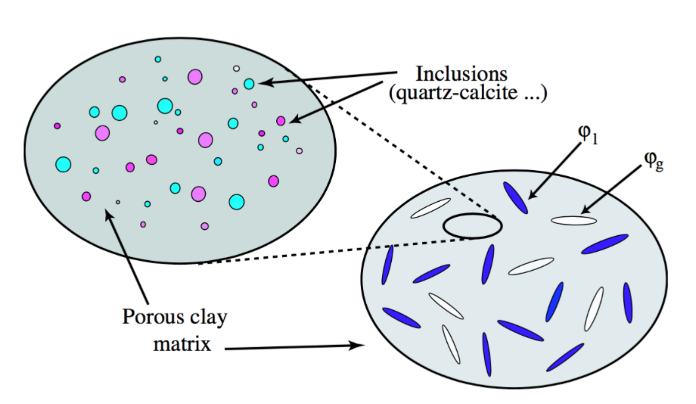
$$\lambda = (1 - n)\lambda_s + n\,S_w\,\lambda_w + n\,(1 - S_w)\,\lambda_g$$

Critical review for soils in (Dong, McCartney, and Lu 2015)

Microstructure must be accounted for (Bergman et al. 1996)



More sophisticated homogenisation schemes, e.g. on claystone (Gruescu et al. 2007)



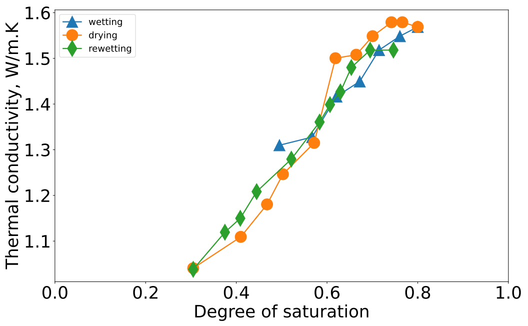
Thermal conductivity – unsaturated case

Thermal conductivity after (Johansen 1975)

$$\lambda_{eff} = \prod_{\alpha} \lambda_{\alpha}^{f_\alpha}$$

Unsaturated cases

$$\lambda(S_w) = (\lambda_{sat} - \lambda_{dry})\,\beta(S_w) + \lambda_{dry}$$



Thermal conductivity of Bapaume loess (Nguyen, Heindl, et al. 2017)

THM couplings

Thermal expansion

Thermal expansion – an introductory example

Triaxial sample, no stress, perfectly drained

Initial void ratio $e_0 = 1.0$

Soil thermal expansion $\alpha = 10^{-2}\ \mathrm{K}^{-1}$

Temperature increment $\Delta T = 10\ \mathrm{K}$

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Final stress? Final pore pressure?

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Soil thermal expansion $\alpha = 10^{-2}$ K^{-1}

Temperature increment $\Delta T = 10$ K

Final stress? Final pore pressure? $\boldsymbol{\sigma} = \mathbf{0}$ and $p_l = 0$

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Initial void ratio $e_0 = 1.0$

Soil thermal expansion $\alpha = 10^{-2}\ \mathrm{K}^{-1}$

Temperature increment $\Delta T = 10$ K

Final stress? Final pore pressure? $\boldsymbol{\sigma} = \mathbf{0}$ and $p_l = 0$

Final volumetric strain? Final void ratio?

Triaxial sample, no stress, perfectly drained

Initial void ratio $e_0 = 1.0$

Soil thermal expansion $\alpha = 10^{-2}$ K^{-1}

Temperature increment $\Delta T = 10$ K

Final stress? Final pore pressure? $\boldsymbol{\sigma} = \mathbf{0}$ and $p_l = 0$

Final volumetric strain? Final void ratio? $\epsilon_v = 0.1$ and $e = 1.0$

Thermal expansion – an introductory example

Triaxial sample, no stress, perfectly drained

Initial void ratio $e_0 = 1.0$

Soil thermal expansion $\alpha = 10^{-2}$ K^{-1}

Temperature increment $\Delta T = 10$ K

Final stress? Final pore pressure? $\boldsymbol{\sigma} = \mathbf{0}$ and $p_l = 0$

Final volumetric strain? Final void ratio? $\epsilon_v = 0.1$ and $e = 1.0$

Plaxis response?

Thermal expansion – an introductory example

Triaxial sample, no stress, perfectly drained

Initial void ratio $e_0 = 1.0$

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Final stress? Final pore pressure? $\boldsymbol{\sigma} = \mathbf{0}$ and $p_l = 0$

Final volumetric strain? Final void ratio? $\epsilon_v = 0.1$ and $e = 1.0$

Plaxis response? $\epsilon_v = 0.1$ and $e = 1.2$ Why?

Triaxial sample, no stress, perfectly drained

Initial void ratio $e_0 = 1.0$

Soil thermal expansion $\alpha = 10^{-2}$ K^{-1}

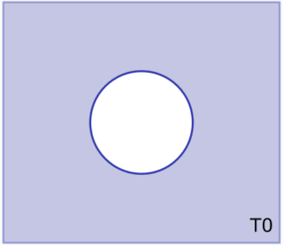
Temperature increment $\Delta T = 10$ K

Final stress? Final pore pressure? $\boldsymbol{\sigma} = \mathbf{0}$ and $p_l = 0$

Final volumetric strain? Final void ratio? $\epsilon_v = 0.1$ and $e = 1.0$

Plaxis response? $\epsilon_v = 0.1$ and $e = 1.2$ Why?

Probably use of $\Delta e = (1 + e_0) \times \epsilon_v = 0.2$. Why is this wrong?



Isotropic behaviour (Cheng 2016; Coussy 2004)

$$p - p_0 = K\,\epsilon_v - b\,(p_w - p_{w,0}) - 3\alpha\,K\,(T - T_0)$$

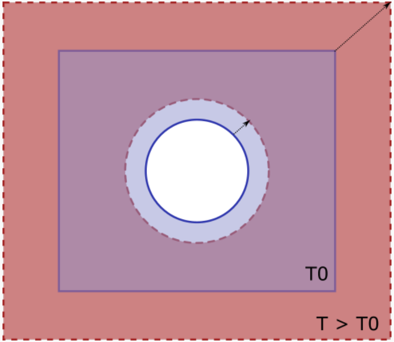
$$\phi - \phi_0 = b\,\epsilon_v + \frac{p_w - p_{w,0}}{N} - 3\alpha_\phi\,(T - T_0)$$

Relation with microscopic properties

$$\epsilon_v = (1 - \phi_0)\,\epsilon_s + \phi - \phi_0$$

$$b = 1 - \frac{K}{K_s}; \qquad \frac{1}{N} = \frac{b - \phi_0}{K_s}$$

$$\alpha = \alpha_s; \qquad \alpha_\phi = \alpha\,(b - \phi_0)$$



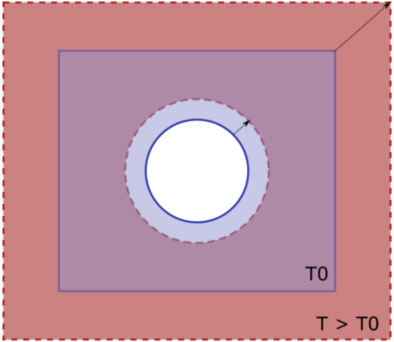
Temperature change assuming matrix incompressibility (drained and stress-free conditions)

$$\epsilon_v = (1-\phi_0)\,\epsilon_s + \phi - \phi_0 \neq \phi - \phi_0$$

$$b = 1 - \frac{K}{K_s} \approx 1; \qquad \frac{1}{N} = \frac{b-\phi_0}{K_s} \approx 0$$

$$\alpha = \alpha_s; \qquad \alpha_\phi = \alpha\,(b-\phi_0) \approx \alpha\,(1-\phi_0)$$

For homogeneous and isotropic solid, solid skeleton and porosity deform homothetically, so that...



Lagrangian porosity

$$\phi = \frac{V_v}{V_0} \neq \phi_0$$

Eulerian porosity

$$n = \frac{V_v}{V} = n_0$$

(Eulerian by nature) void ratio

$$e = \frac{V_v}{V_s} = e_0$$

...but this is not verified in all numerical codes...

So what?

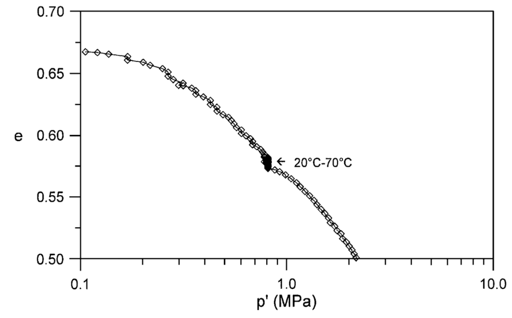
Even in isothermal conditions

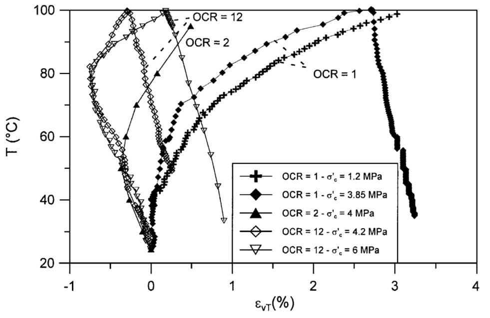
- Usually, no large difference in case of small deformation...
- But, **do use both** lagrangian and eulerian porosities!
  - Eulerian porosity for indirect couplings (updating permeability, thermal conductivity...)
  - Lagrangian porosity tracks deformation of the porous network and should be used to solve the mass balance equation (in a conservative manner)
  - See (Melot et al. 2020) for a study on bitumen, using BIL FEM code (P. Dangla)

THM couplings

Thermal consolidation

Experimental observations (Baldi et al. 1991; Sultan, Delage, and Cui 2002)





THM models

THM models

Unsaturated geomaterials

Context

Simply (?) wet sand (Sculpture of Sagrada Familia) (photo by SetosPuppy / CC BY-SA)

Context

(courtesy: E. Alonso)

Stress state variables

- Extension of Terzaghi's effective stress
  $\boldsymbol{\sigma}' = \boldsymbol{\sigma} - p_g \mathbf{1} + \chi\, s \mathbf{1}$ (Bishop 1959)
- Two state variables approaches
  - simple (measurable) variables
    $\boldsymbol{\sigma} - p_g \mathbf{1},\ \boldsymbol{\sigma} - p_l \mathbf{1},\ s$ (Coleman 1962; Fredlund and Morgenstern 1977)
  - use of an "effective" stress
    $\boldsymbol{\sigma} + \pi \mathbf{1},\ s$

Stress state variables

- Extension of Terzaghi's effective stress
- Two state variables approaches
  - simple (measurable) variables
  - use of an "effective" stress

Three classes of models (Gens 1995)

$$\begin{cases} \boldsymbol{\Sigma}_1 = \boldsymbol{\sigma} - p_g \mathbf{1} + \mu_1(s, S_l)\mathbf{1} \\ \boldsymbol{\Sigma}_2 = \mu_2(s, S_l)\mathbf{1} \end{cases}$$

Stress state variables

- Extension of Terzaghi's effective stress
- Two state variables approaches
  - simple (measurable) variables
  - use of an "effective" stress

Three classes of models (Gens 1995)

$$\begin{cases} \boldsymbol{\Sigma}_1 & = & \boldsymbol{\sigma} - p_g \mathbf{1} + \mu_1(s, S_l)\mathbf{1} \\ \boldsymbol{\Sigma}_2 & = & s\mathbf{1} \end{cases}$$

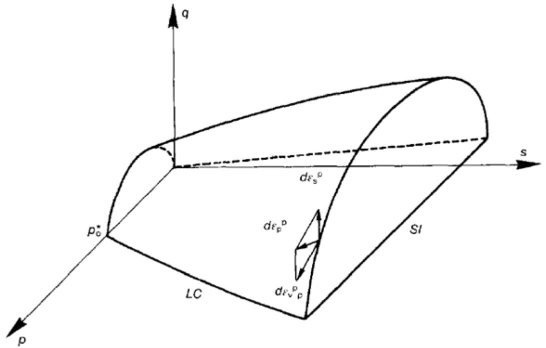
Classe I $\mu_1 = 0$ (Alonso, Gens, and Josa 1990)...

Classe II $\mu_1 = \mu(s)$ (Abou-Bekr 1995; Loret and Khalili 2000)...

Classe III $\mu_1 = \mu(s, S_l)$ (Dangla 2001; Wheeler, Sharma, and Buisson 2003)...

Barcelona Basic Model (BBM)

- First elastoplastic model for unsaturated soils (Alonso, Gens, and Josa 1990)
- Based on modified cam-clay



Going further

Accounting for water adsorption effects, osmotic effects...

See for instance:

- On drying induced shrinkage of cement pastes: (Rahoui 2018; Rahoui et al. 2021)
- Recent works by Prof. Ning Lu, e.g. (Wang et al. 2022)

THM models

Thermoporoelastoplastic models

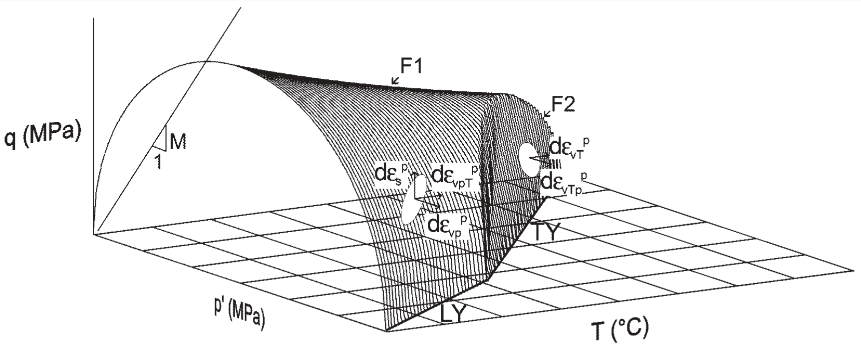
What we know

- Little effect of temperature on elastic properties
- Same for failure properties (friction angle and cohesion little affected)
- Yield stress is temperature dependent (cf. thermal consolidation)

See (Abuel-Naga et al. 2009; Cui, Sultan, and Delage 2000; Laloui and Cekerevac 2003) for some founding models

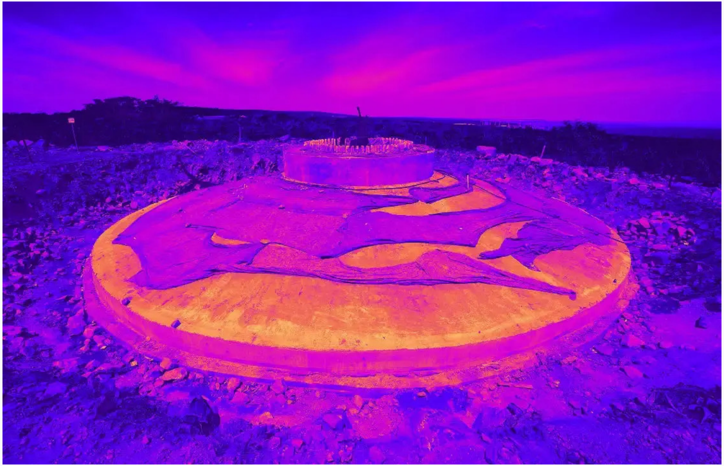
Temperature dependent yield surface

Thermal elastic strain ($d\epsilon_v^e = d\epsilon_{v,M}^e + \alpha_T \, dT$) and temperature dependent yield surface $p_{c0,T} = p_{c0,T}(T)$



Yield surface in (p', q, T) space (Cui, Sultan, and Delage 2000)

Is this relevant for energy geostructures?



Infrared thermography of a shallow foundation during cement hydration
(`https://www.nxfem.com/`)

Application

Piles, diaphragm walls, tunnel support...

What we know?

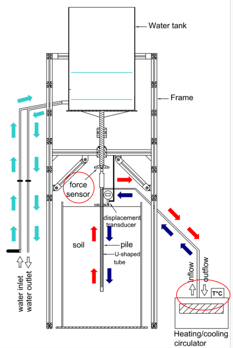
- Shear strength mostly temperature-independent (Yavari et al. 2016)
- Thermal consolidation in normally consolidated clays: might not be relevant
- Creep? Temperature enhanced

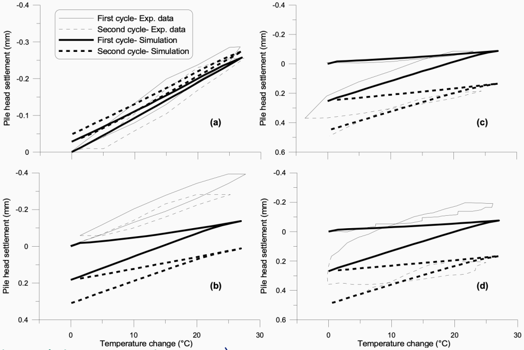
Mainly cyclic and long term effects on vertically (and laterally) loaded piles

Can we keep it simple?

Can we keep it simple?

Use of a **”decoupled” strategy** (Yavari et al. 2014) to model in situ and small scale (1*g*) lab piles: imposed volumetric strain and perfect plasticity





(Kalantidou et al. 2012)

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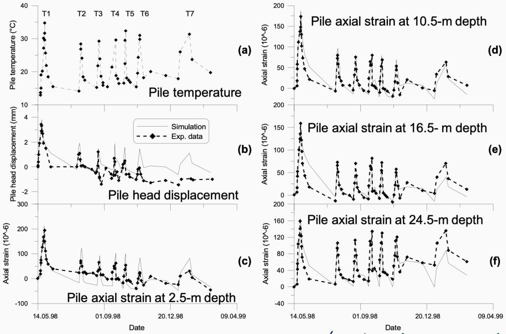
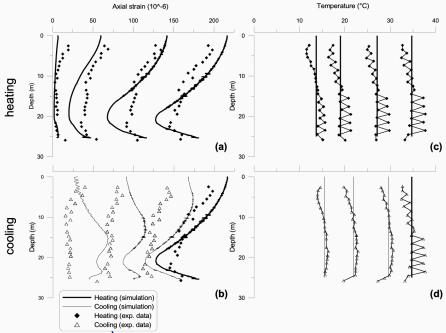
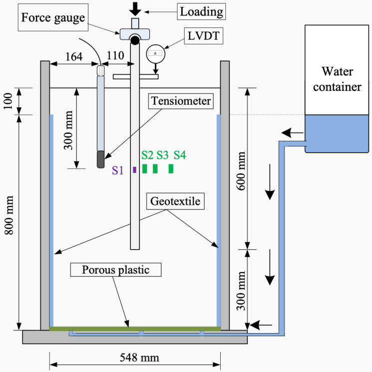




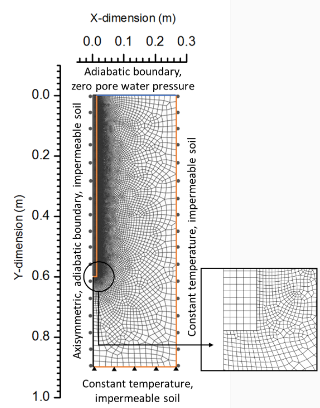
(Laloui, Moreni, and Vulliet 2003)

Essential role played by lateral stress variation on mobilisable shaft friction

More detailed analysis (THM coupled) using rather simple constitutive model (MCC) (Nguyen, Wu, et al. 2020)

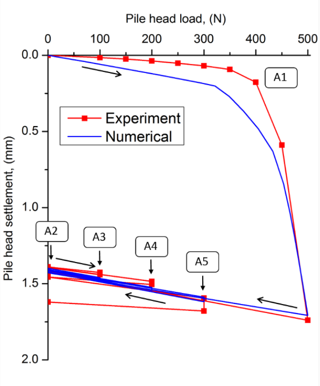


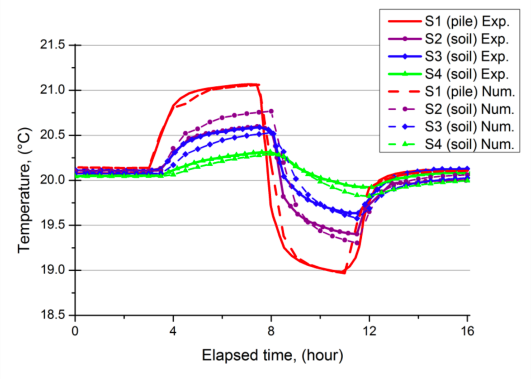
S1: Temperature transducer inside the pile
S2÷4: Temperature transducer distributed in soil



Refined THM analysis

More detailed analysis (THM coupled) using rather simple constitutive model (MCC) (Nguyen, Wu, et al. 2020)





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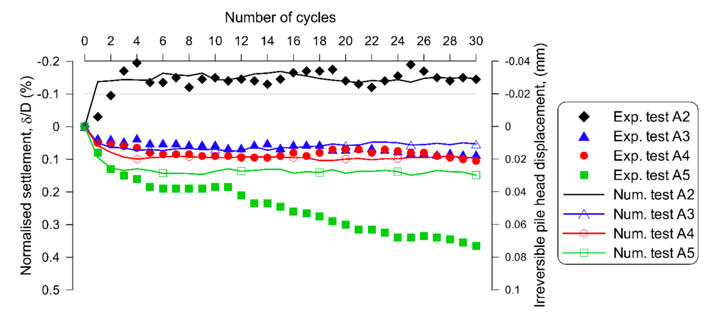

Influence of shaft strength mobilisation (Bourne-Webb and Bodas Freitas 2020; Pasten and Santamarina 2014)

"All models are wrong but some are useful."

— George Box

Thanks for your attention – Questions?

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