



# 25<sup>th</sup> ALERT Workshop

## Session II: Railway Geomechanics

Aussois – September 30<sup>th</sup> 2014



## A simplified procedure for settlement analysis of ballasted tracks

Gabriele Della Vecchia, Federico Pisanò, Andrea Galli, Claudio di Prisco

in collaboration with



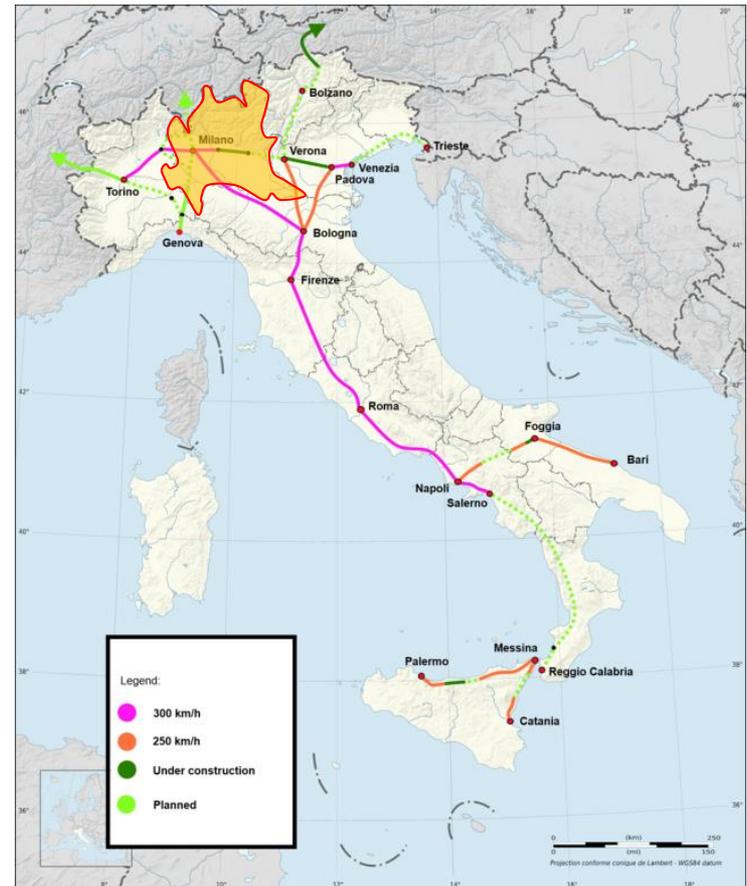
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# Some introductory remarks

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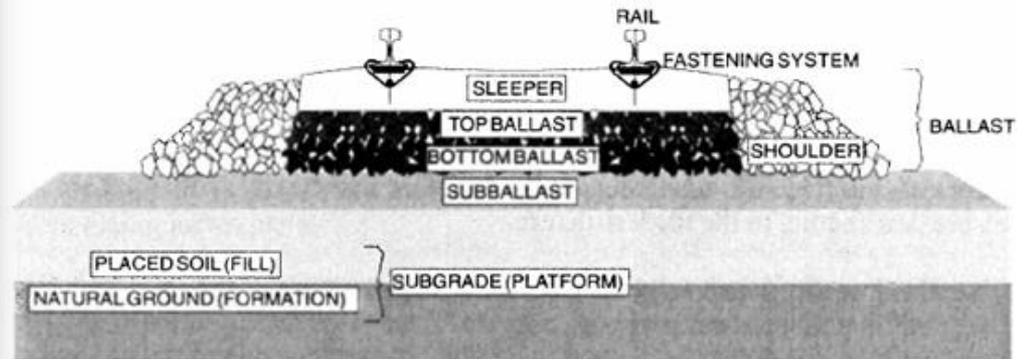
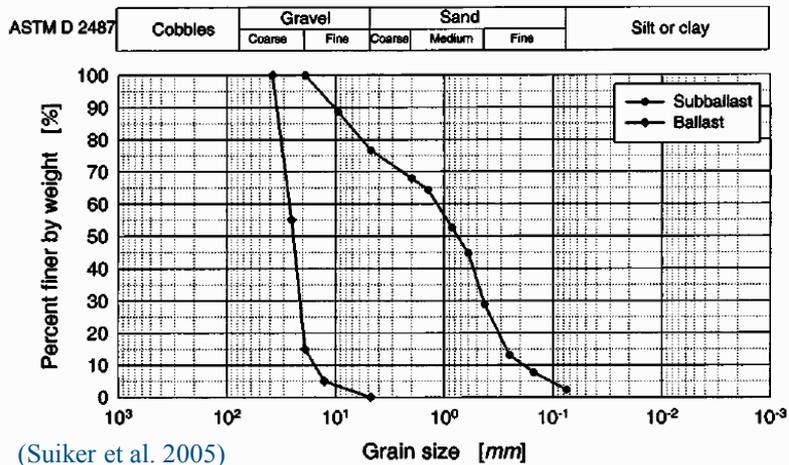
20255 km of railway lines in Italy

Strategic lifeline both at national ad regional level



- High technical/commercial **competition** with airlines, highways, and among railways operators
- Increasing **speed and weight** of trains
- Need for **cheaper** and more efficient railway tracks
- Need for **simpler** desing approaches
- **Maintenance** costs are often the most important part of railways management



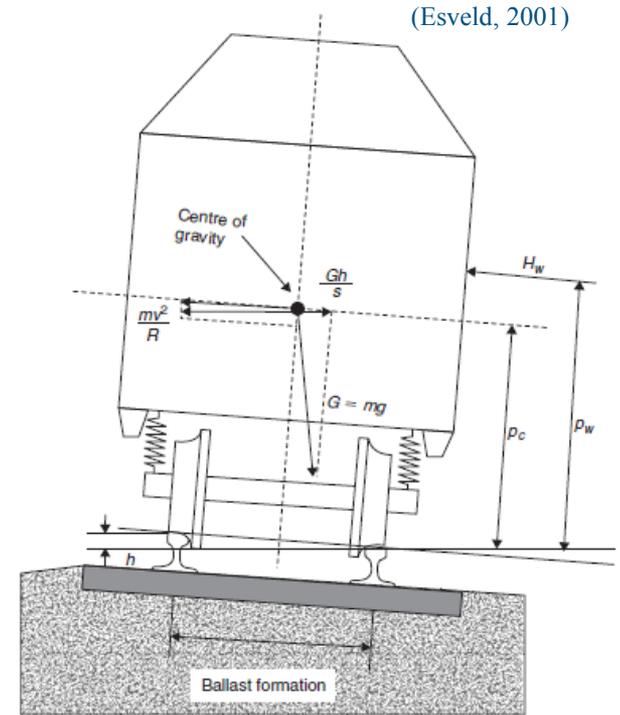
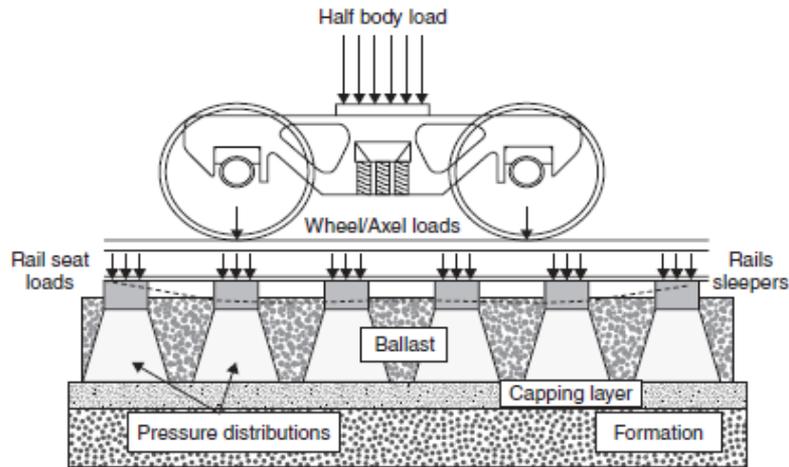


(Selig & Waters, 2004)

Layered strip foundation, subject to complex dynamic loads.

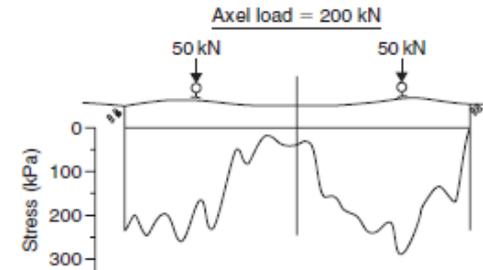
Cyclic settlement accumulation, due to permanent sliding between grains, ratcheting phenomena and grain breakage





Layered strip foundation, subject to complex dynamic loads.

Cyclic settlement accumulation, due to permanent sliding between grains, ratcheting phenomena and grain breakage



Differential settlement



(Sucker, 2002)

Local “buckling” of the track



(Indraratna et al., 2011)

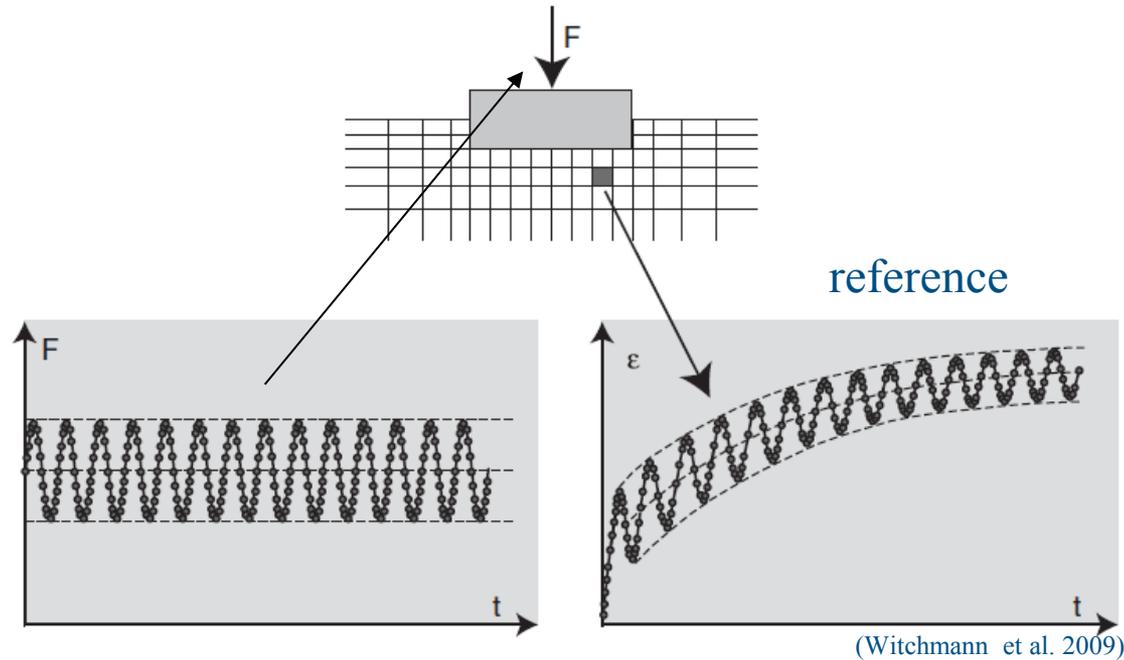


Insufficient drainage,  
pumping of fines



# Settlement computation: rigorous approach

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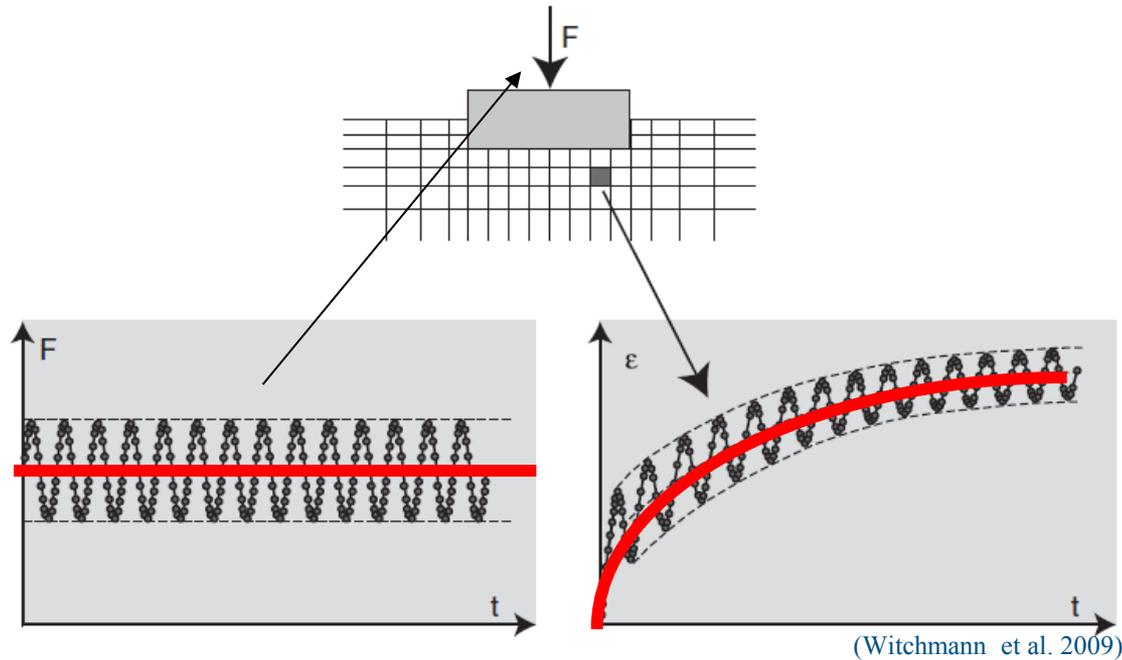


FE analyses by employing advanced constitutive models specifically conceived for cyclic loads

High computational effort; difficult parameter calibration

# Settlement computation: simplified approach

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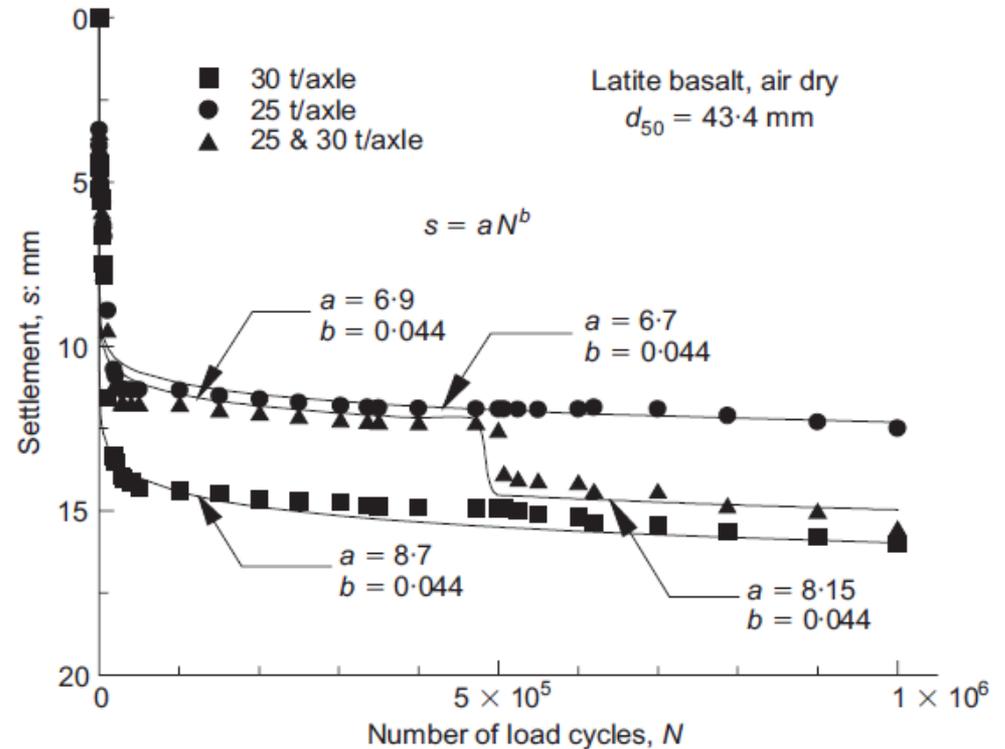
Simplified constitutive models, describing the accumulation of plastic strain

→ Viscous equivalent approach

# Settlement computation: simplified approach – global vs local

**GLOBAL** (Indraratna et al. 2000; Indraratna et al 2003): direct evaluation of the settlement as a function of number of cycles  $N$

$$s_N = a \cdot (1 + k \cdot \log N)$$
$$s_N = a \cdot N^b$$

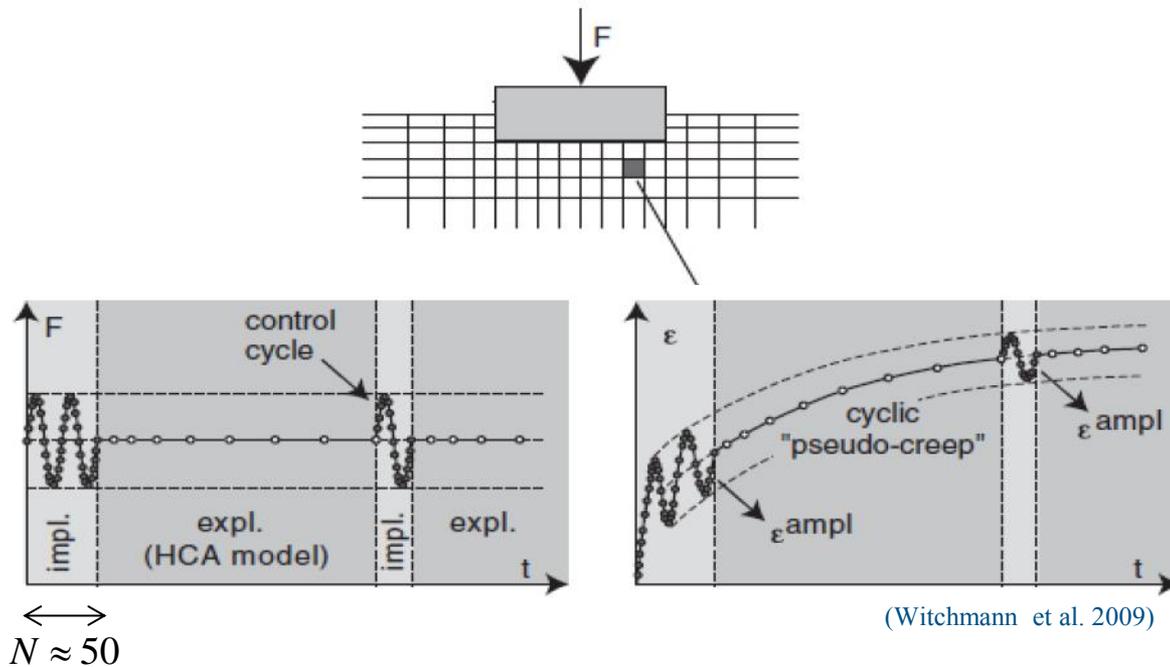


parameters do not have actually any physical meaning and depend both on structural mechanical properties and geometry and on the applied load

# Settlement computation: simplified approach – global vs local

## LOCAL

- FEM analyses on BVP
- Local constitutive relationship between strain and cycle number
- High number of parameters, sophisticated models
- Combination of “implicit” and “explicit” methods



# Settlement computation: simplified approach – global vs local

## LOCAL

HCA - “High Cyclic Accumulation Model” (Niemunis e Triantafyllidis, 2005)

$$\dot{\epsilon}_{acc} = f_{ampl} \cdot \dot{f}_N \cdot f_e \cdot f_p \cdot f_Y \cdot f_\pi$$

effect of cycle amplitude  $f_{ampl} = \min \left\{ \left( \frac{\epsilon^{ampl}}{\epsilon_{ref}^{ampl}} \right)^2 ; 100 \right\}$  effect of confining pressure  $f_p = e^{-C_p \left( \frac{p}{p_{pref}} - 1 \right)}$

effect of cycle number  $\dot{f}_N = \frac{C_{N1} C_{N2}}{1 + C_{N2} N} + C_{N1} C_{N3}$  effect of void index  $f_e = \frac{(C_e - e)^2}{1 + e} \frac{1 + e_{ref}}{(C_e - e_{ref})^2}$

$$C_{N1} = 0,0002 \exp(-0,65 d_{50}) \exp(0,91 U_c);$$

$$C_{N2} = 0,95 \exp(0,33 d_{50}) \exp(-0,90 U_c);$$

$$C_{N3} = 0,00003 \exp(-0,69 d_{50}) \exp(0,26 U_c);$$

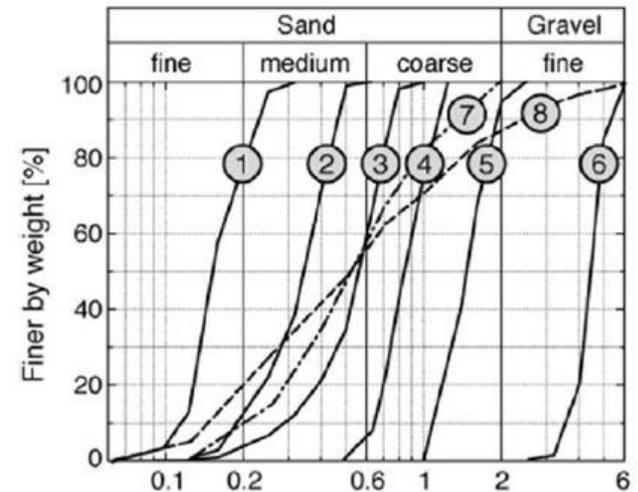
effect of stress level  $f_Y = e^{(C_Y Y^{av})}$

$$C_Y = 2,6;$$

$$Y_{av} = \frac{Y - 9}{Y_c - 9};$$

$$Y_c = \frac{9 - \sin^2 \phi_c}{1 - \sin^2 \phi_c};$$

$$Y = \frac{27(3 + \eta)}{(3 + 2n)(3 - n)};$$



(Triantafyllidis et al. 2009)

# Settlement computation: simplified approach – global vs local

## LOCAL

Tseng and Lytton (1989)

$$\varepsilon_p(N) = \left(\frac{\varepsilon_0}{\varepsilon_r}\right) e^{-\left(\frac{\rho}{N}\right)^\beta} \varepsilon_v$$

$$\log\left(\frac{\varepsilon_0}{\varepsilon_r}\right) = -1,69867 + 0,09121w_c - 0,11921\sigma_d + 0,91219 \log(E_{SG});$$

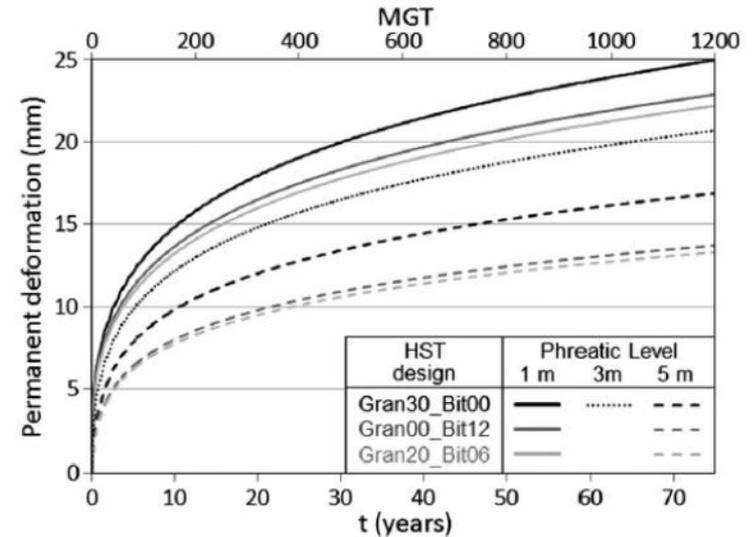
$$\log(\beta) = -0,9730 - 0,0000278 w_c^2 \sigma_d + 0,017165 \sigma_d - 0,00000338 w_c^2 \sigma_\theta;$$

$$\log(\rho) = 11,009 + 0,0000681 w_c^2 \sigma_d - 0,40260 \sigma_d + 0,00000545 w_c^2 \sigma_\theta;$$

$\sigma_d$  = deviatoric stress

$\sigma_\theta$  = isotropic pressure

$w_c$  = water content



(Tseng and Lytton, 1989)

# Settlement computation: simplified approach – global vs local

## LOCAL

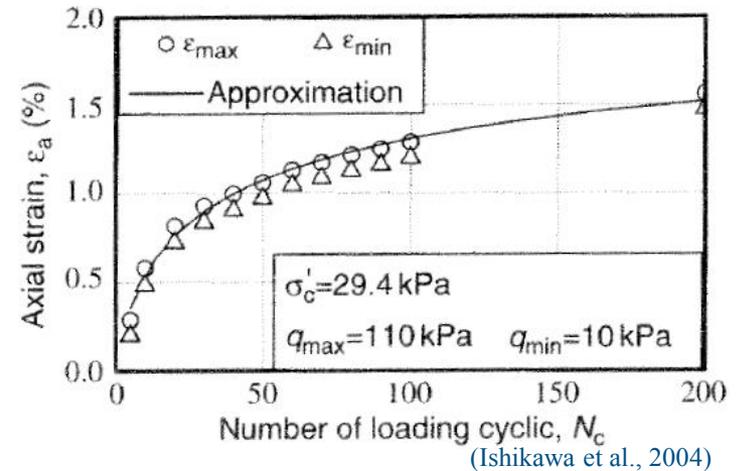
### Cumulative Damage Model (Ishikawa et al., 2004)

$$(\varepsilon_a)_{max} = \left( \frac{SR_d}{a_1 \cdot (1 - a_2 \cdot SR_s^{a_3}) \cdot N_c^{a_4}} \right)^{a_5 \cdot N_c^{a_6}}$$

$\sigma_d$  = deviatoric stress at the  $N_c^{th}$  cycle  
 $\sigma_s$  = initial deviatoric stress  
 $\sigma_m$  = mean principal stress  
 $a_1.. a_6$  = model parameters

$$SR_s = \frac{\sigma_s}{(2\sigma_m)}$$

$$SR_d = \frac{\sigma_d}{(2\sigma_m)}$$



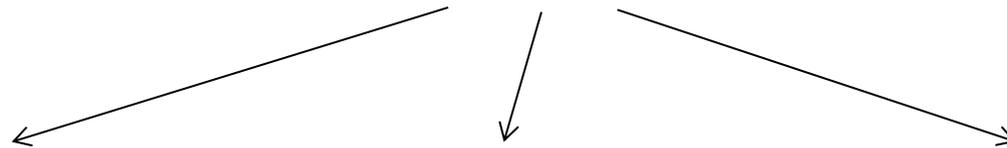
### Liu and Carter (2004)

$$\varepsilon_p = a \left( \frac{\Delta q}{q_f - \Delta q} \right) (\ln N)^b$$

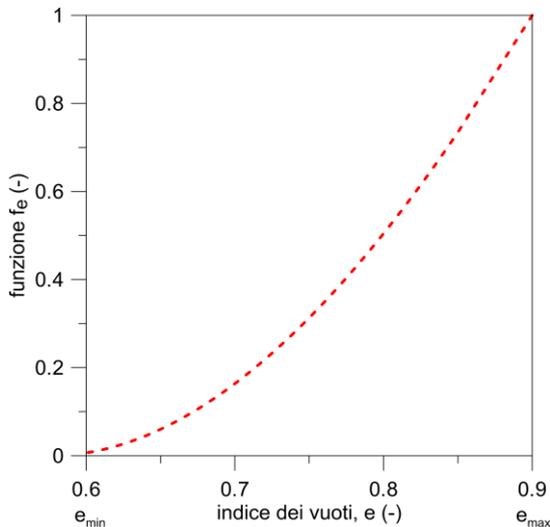
“distance” from failure  
 ↗

# Settlement computation: New simplified local model

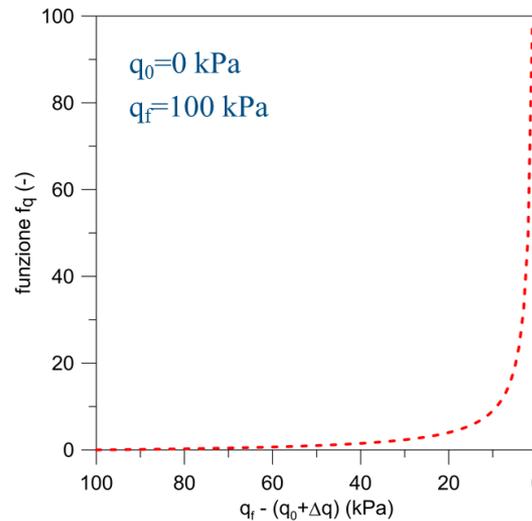
$$\dot{\epsilon}_{acc} = a \cdot f_e \cdot f_q \cdot f_N$$



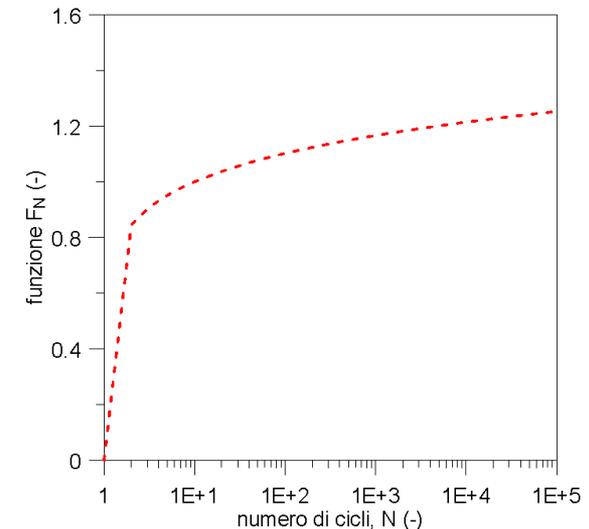
$$f_e = \left( \frac{(C_e - e)^2}{1 + e} \frac{1 + e_{ref}}{(C_e - e_{ref})^2} \right)$$



$$f_q = \left( \frac{q_0 + \Delta q}{q_f - (q_0 + \Delta q)} \right)^c$$



$$f_N = \frac{b(\log(N + 1))^{b-1}}{1 + N}$$



a, b, c = model parameters to be calibrated

$$F_N = \int f_N dN = (\log(N + 1))^b$$

# Settlement computation: New simplified local model

$$f_e = \left( \frac{(C_e - e)^2}{1 + e} \frac{1 + e_{ref}}{(C_e - e_{ref})^2} \right)$$

$$\begin{cases} \dot{\varepsilon}_v = \dot{\varepsilon}_1 + 2 \dot{\varepsilon}_3 \\ \dot{\varepsilon}_d = \frac{2}{3} (\dot{\varepsilon}_1 - \dot{\varepsilon}_3) \end{cases}$$

update of void index  $\varepsilon_v = -\frac{\Delta e}{1 + e_0}$

$$\dot{\varepsilon}_1 = \dot{\varepsilon}_{acc}$$

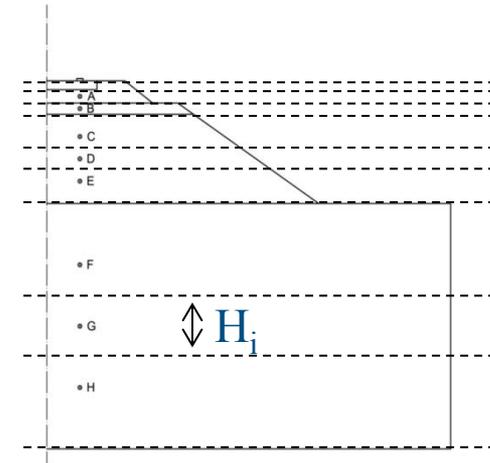
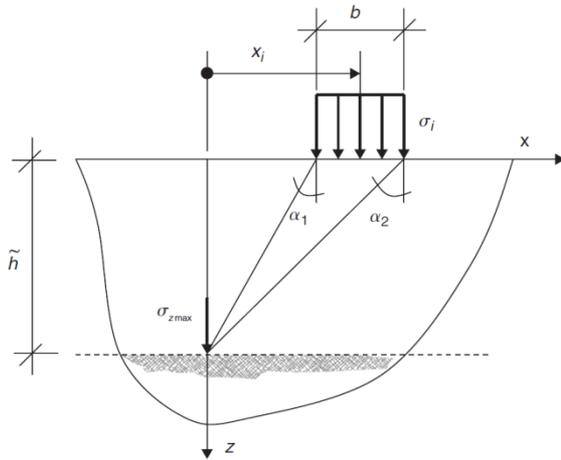
MMC flow rule  $\frac{\dot{\varepsilon}_v^{acc}}{\dot{\varepsilon}_d^{acc}} = \frac{M_C^2 - \eta^2}{2\eta} = d$

$$\dot{\varepsilon}_3 = -\sqrt{\frac{\dot{\varepsilon}_{acc}^2}{2 + \left(\frac{2 + \frac{2}{3}d}{1 - \frac{2}{3}d}\right)^2}}$$

Elastic analysis of the geostatic state of stress and of the stress increment due to trains



Evaluation of the cyclic strain increment in a given set of points along depth



$$\varepsilon_{acc,i}(N + \Delta N) = \varepsilon_{acc,i}(N) + \dot{\varepsilon}_{acc,i}(N) \cdot \Delta N$$

□

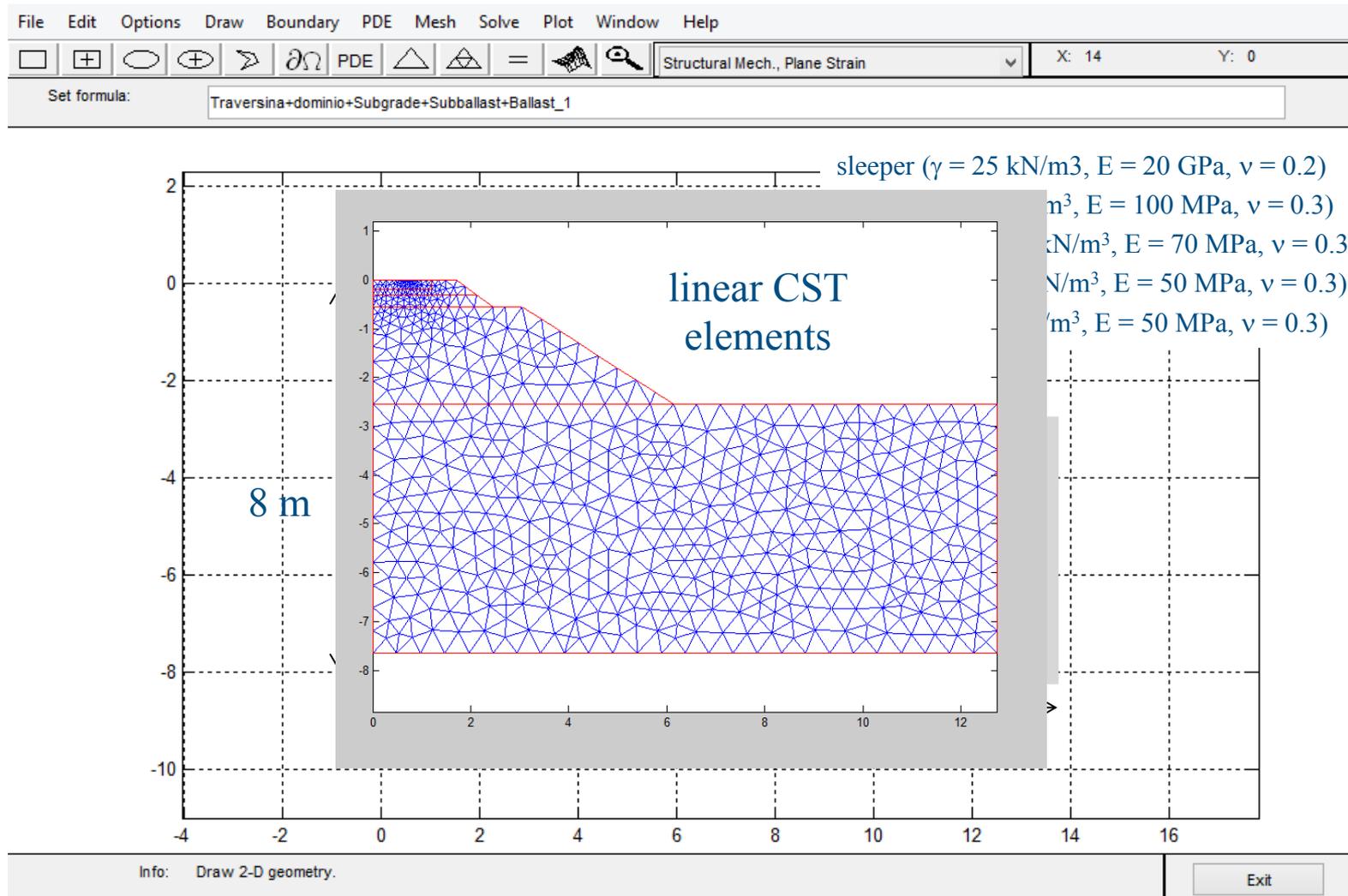


Integration of strain along depth

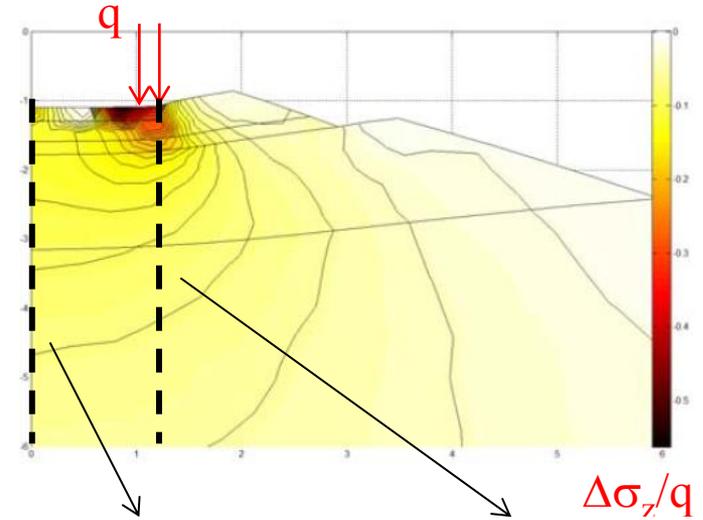
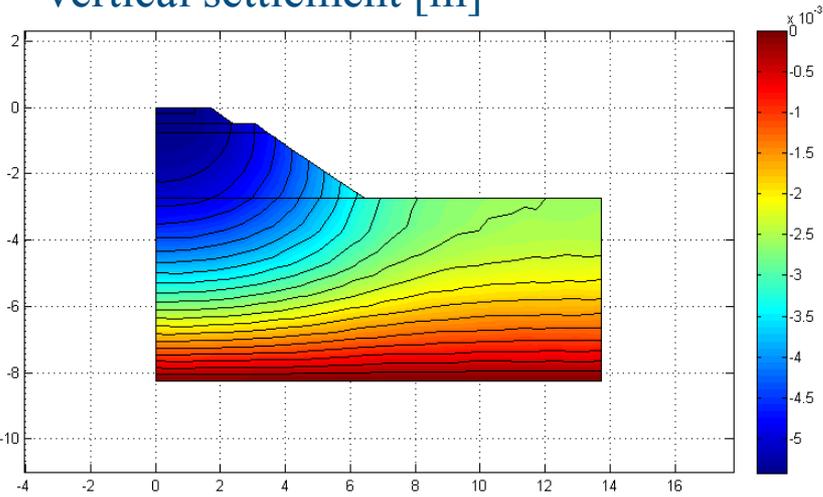
$$s_i(N) = \varepsilon_{acc,i}(N) H_i$$

$$s(N) = \sum_{i=1}^m s_i(N)$$

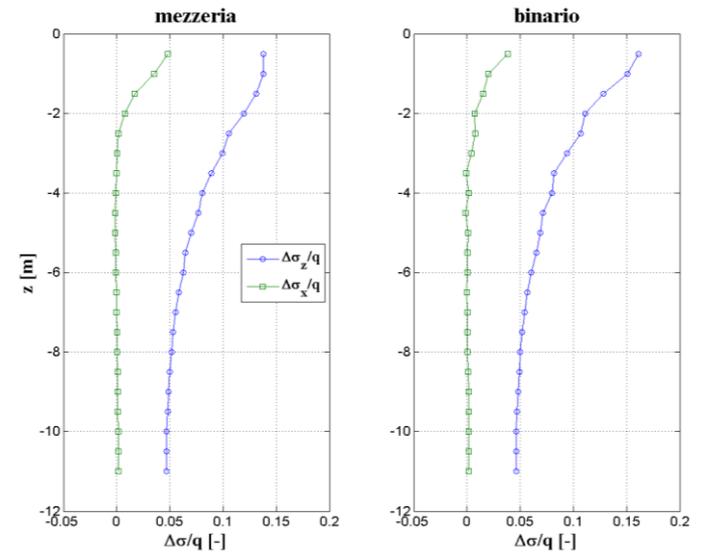
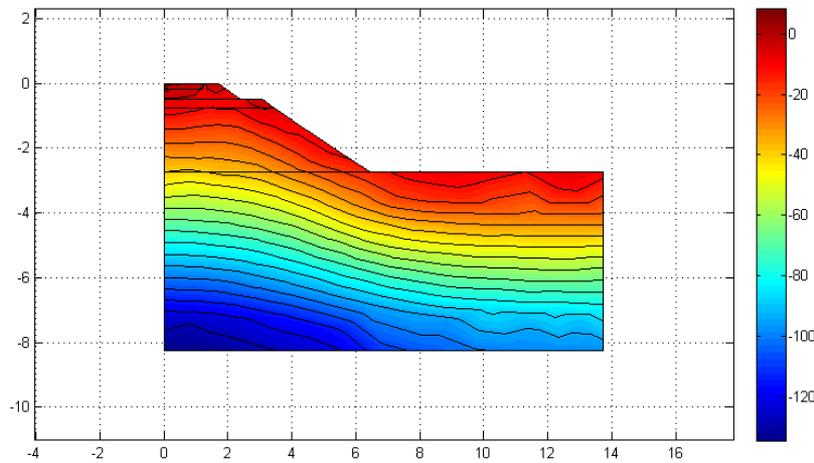
## Elastic solution by Matlab PDE tool (plane strain analysis, linear elasticity, static loads)



vertical settlement [m]



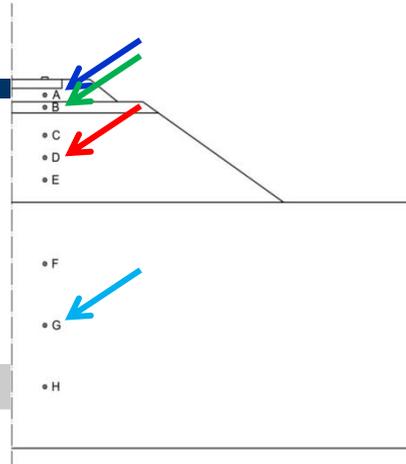
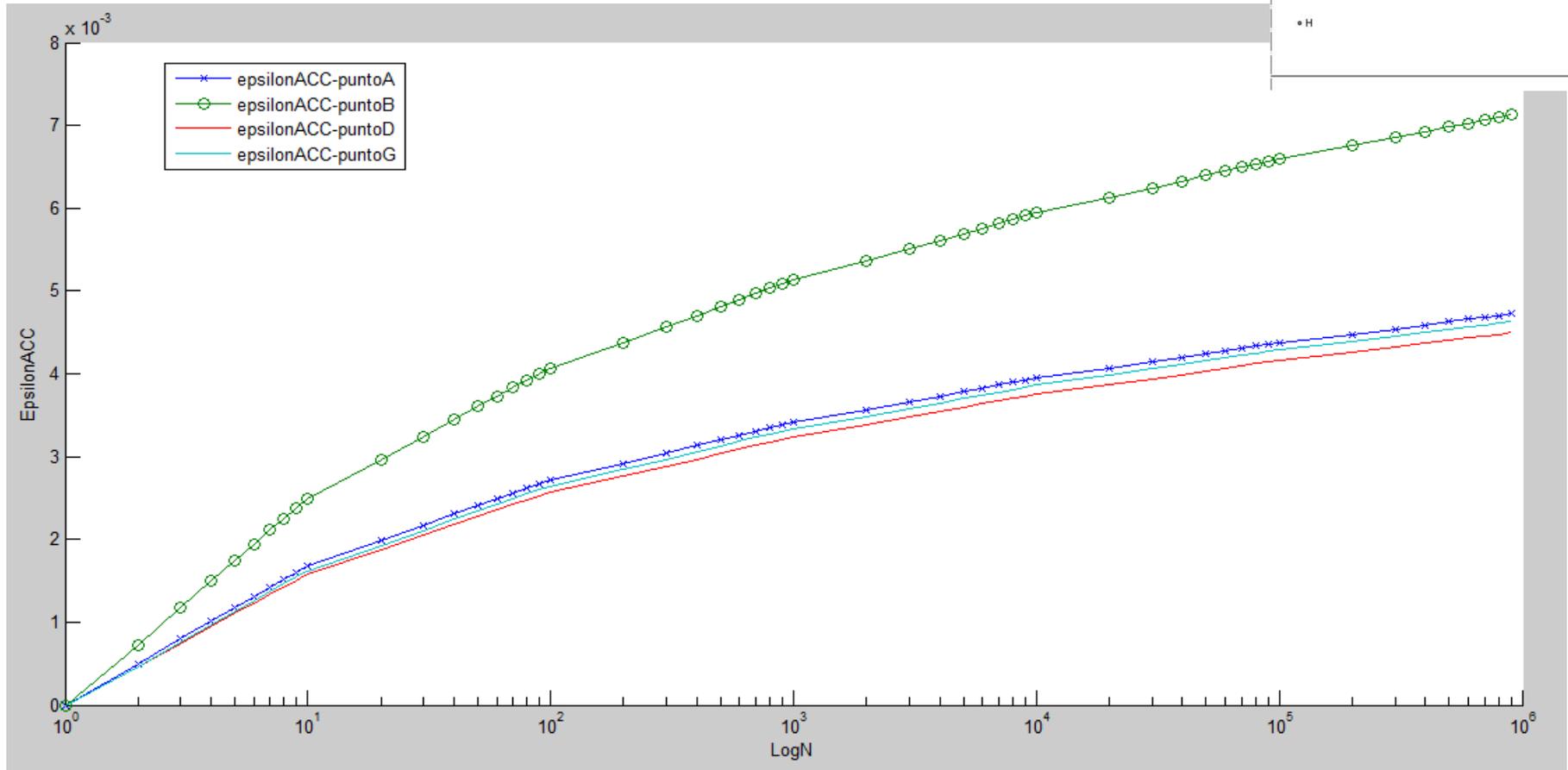
vertical stress [m]



# Example of application

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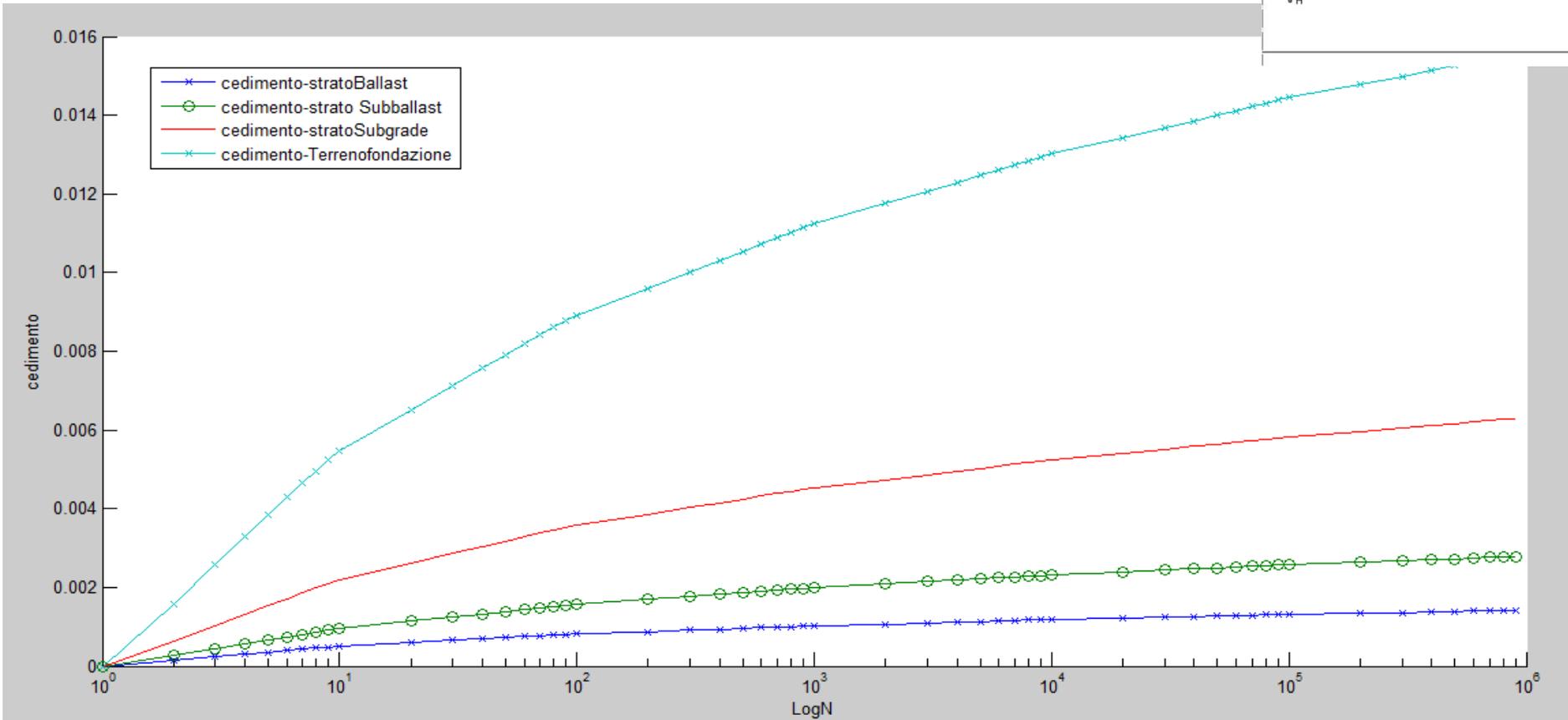
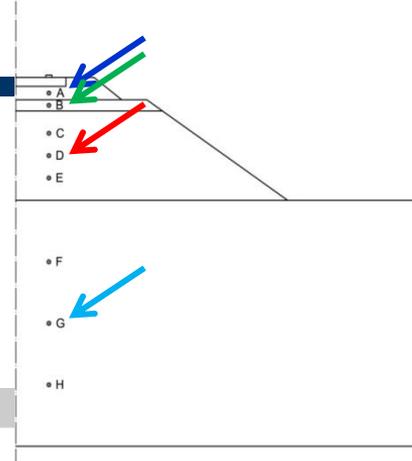
Cumulated vertical strain  $\epsilon_{acc}$



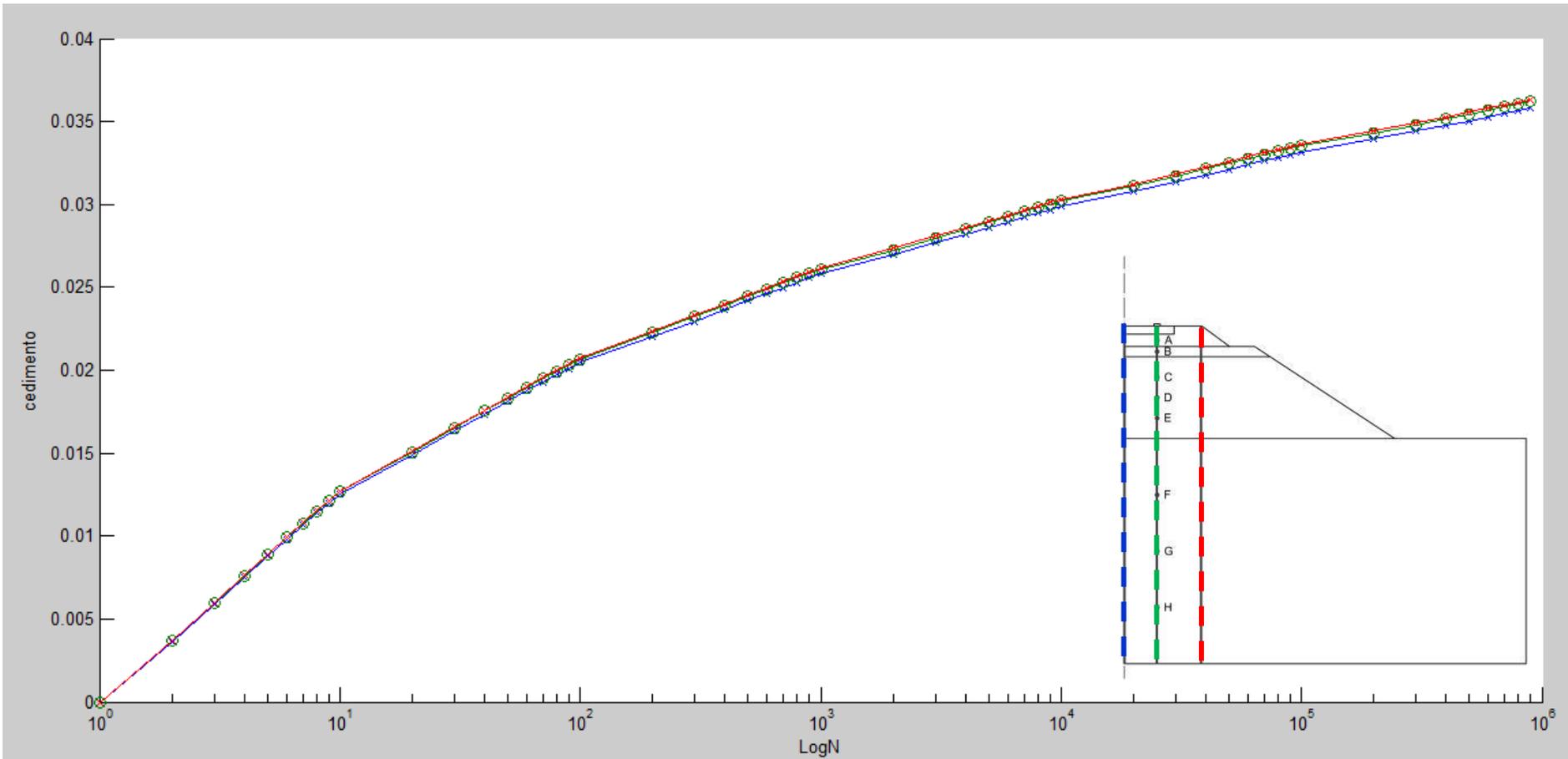
# Example of application

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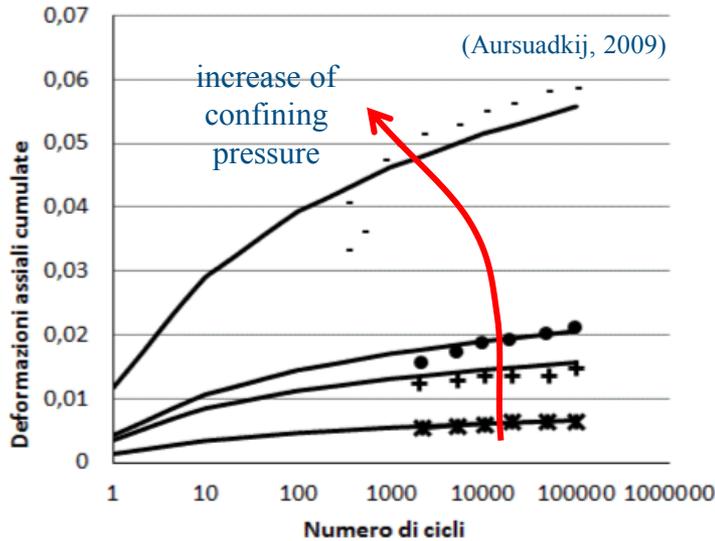
Cumulated vertical settlement of each layer



Total cumulated vertical settlement  
(and differential settlement along the sleeper)



## Cyclic triaxial tests on ballast

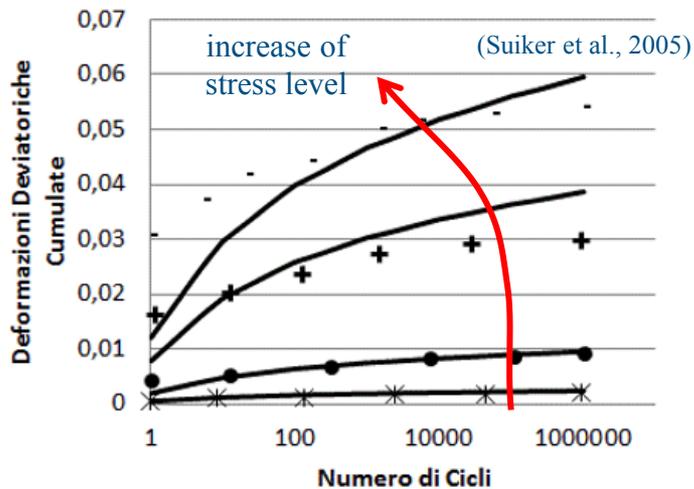


- aur 1
- aur 2
- aur 3
- aur 4
- \* sp1
- sp2
- + sp3
- sp4

|         |        |       |                           |
|---------|--------|-------|---------------------------|
| a=0,01  | b=0,14 | c=0,3 | (p <sub>c</sub> = 10 kPa) |
| a=0,035 | b=0,14 | c=0,3 | (p <sub>c</sub> = 30 kPa) |
| a=0,11  | b=0,14 | c=0,3 | (p <sub>c</sub> = 60 kPa) |
| a=0,1   | b=0,14 | c=0,3 | (p <sub>c</sub> = 60 kPa) |

parameter “a” is mainly affected by the confining pressure

parameter “b” and “c” appear to be independent of the confining pressure



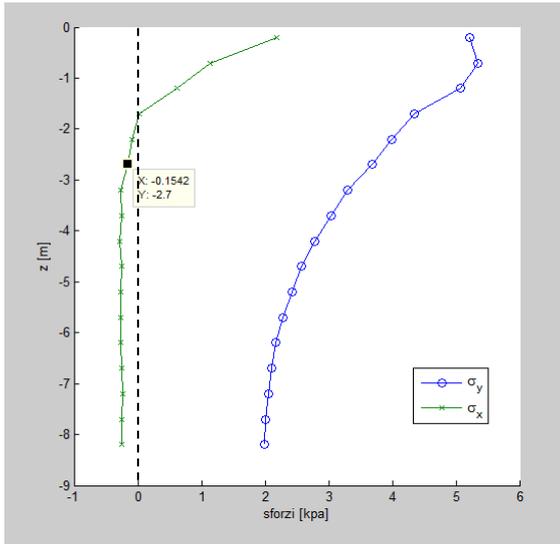
- b1
- b2
- b3
- b4
- \* sp1
- sp2
- + sp3
- sp4

|         |        |     |                             |
|---------|--------|-----|-----------------------------|
| a=0,046 | b=0,14 | c=1 | (p <sub>c</sub> = 68,9 kPa) |
|---------|--------|-----|-----------------------------|

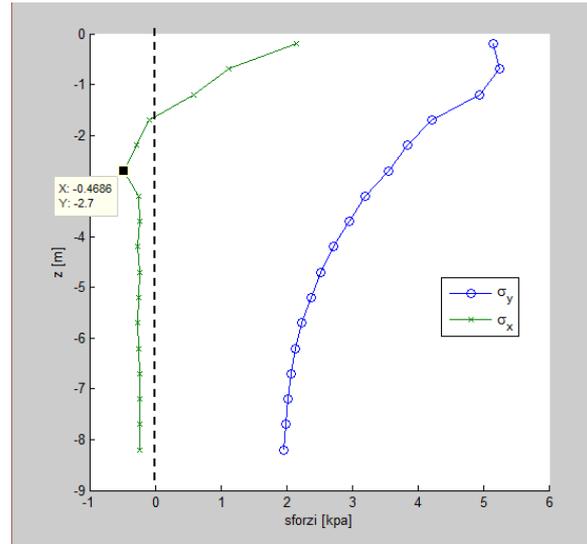
Actually “a” and “c” seem to depend on a combination of confining pressure and cyclic stress level.

Parameter “b” is apparently uniquely dependent on the type of material

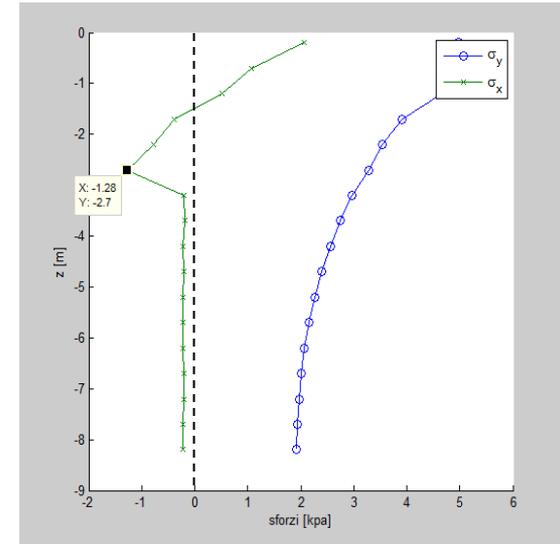
## Vertical and horizontal stress increments



$E_{sub} = 100 \text{ MPa}$

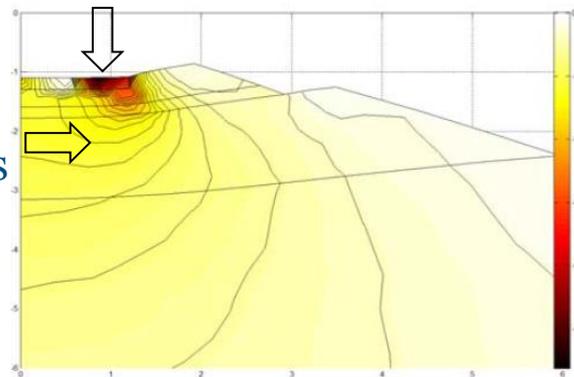


$E_{sub} = 80 \text{ MPa}$



$E_{sub} = 50 \text{ MPa}$

horizontal  
tensile stress





work in progress....