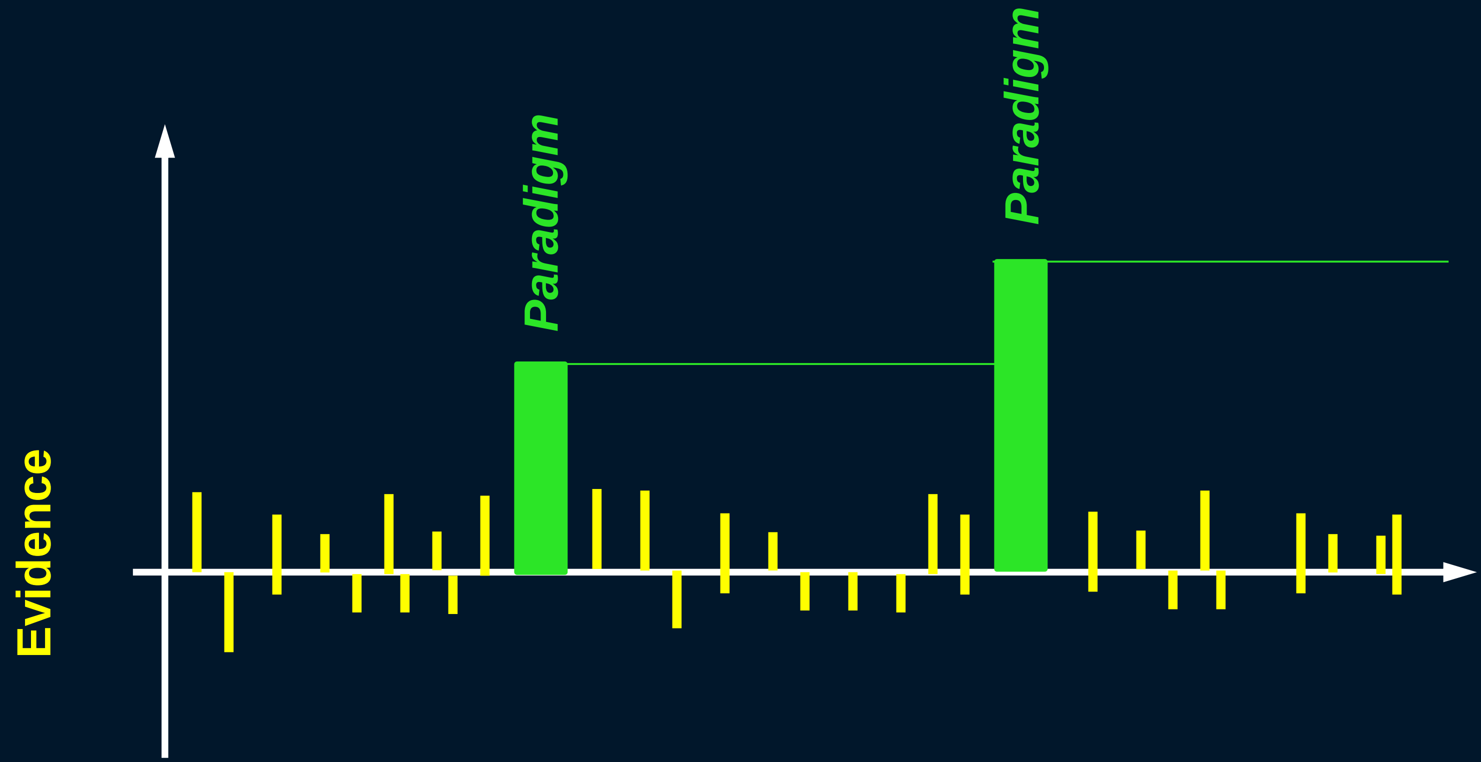
CRITICAL STATE:

MISLEADING ELEGANCE?

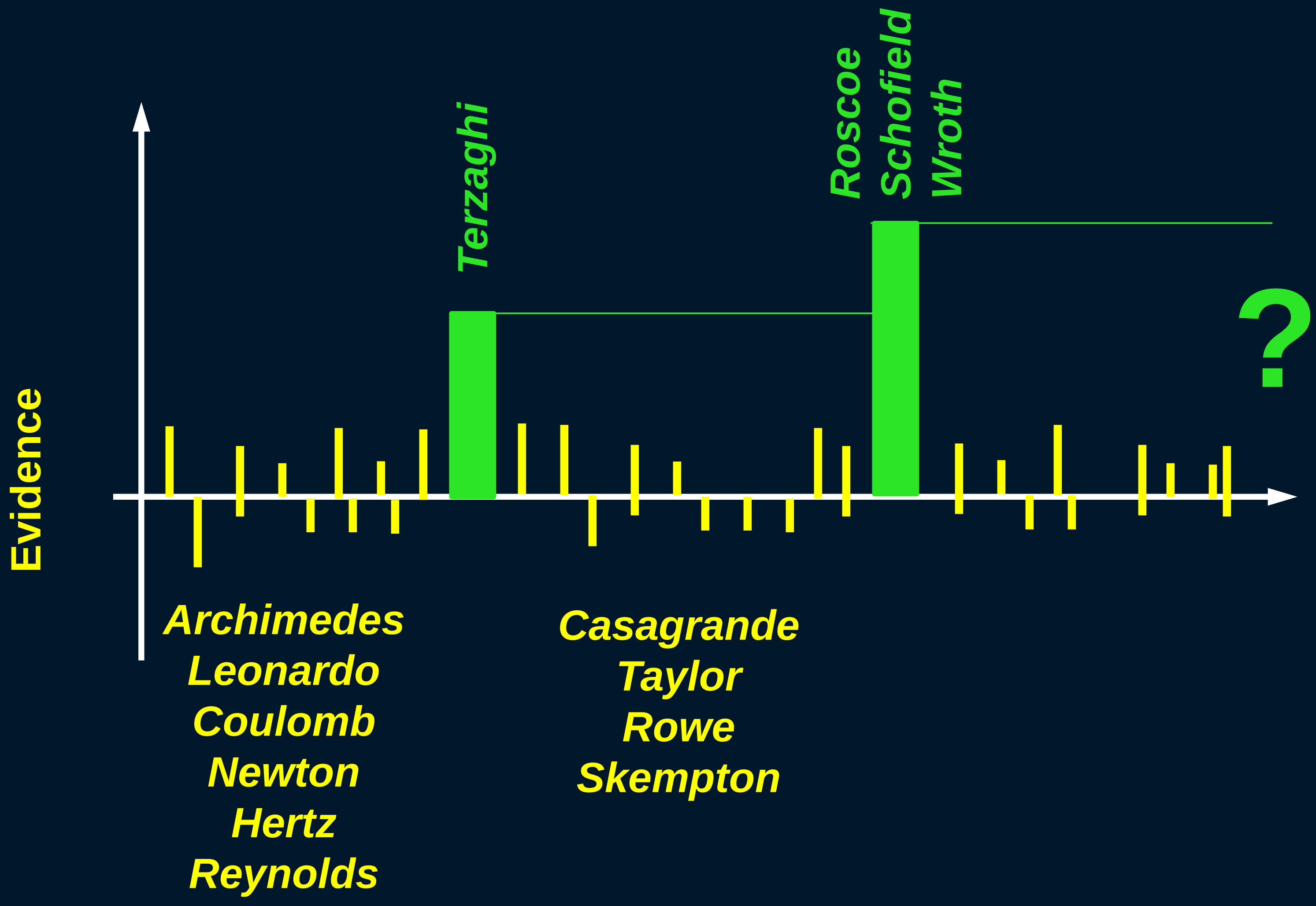
J. Carlos Santamarina
KAUST

Scientific Revolutions: Kuhn's "Paradigms"

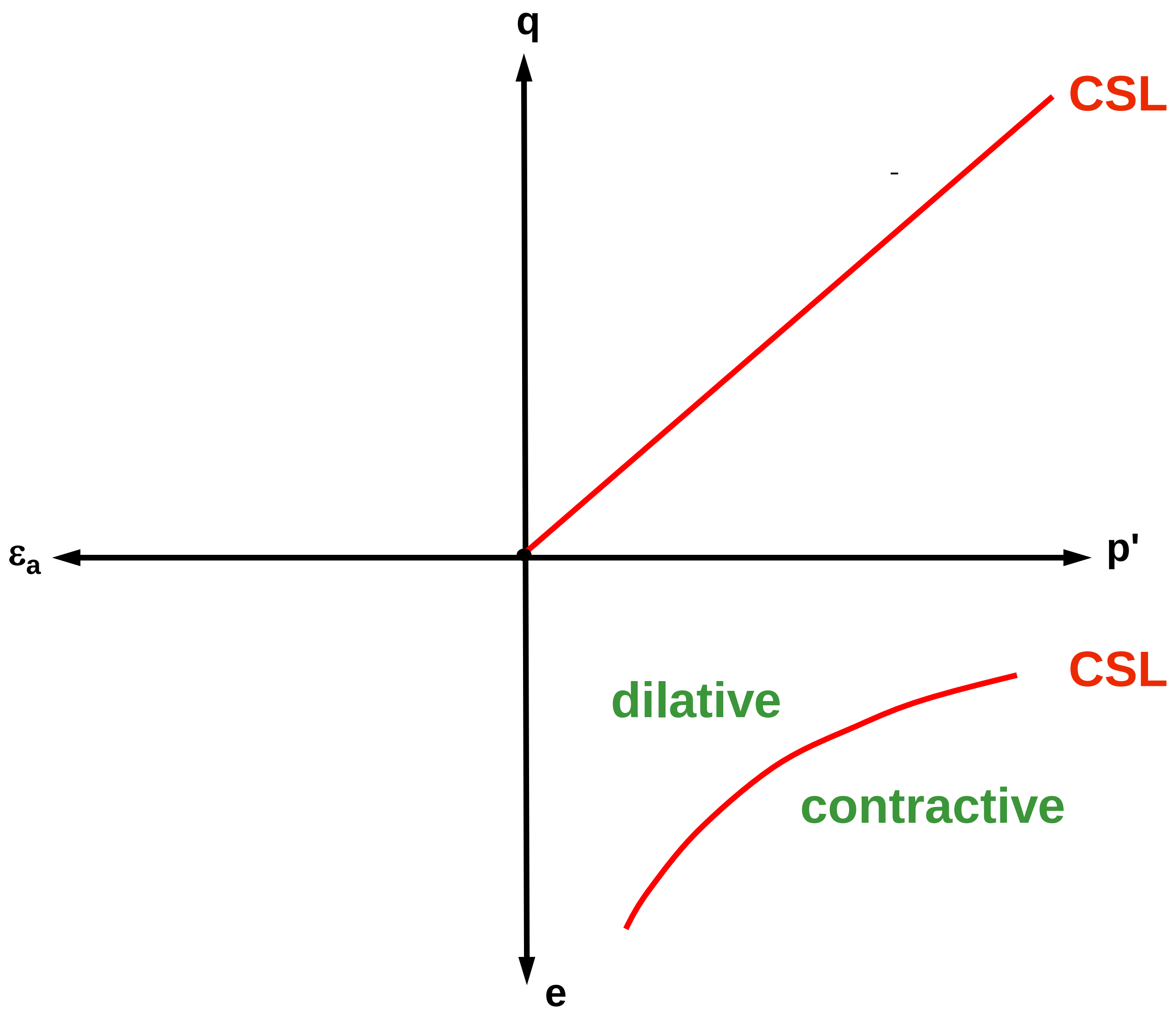


Paradigms:	*Copernicus*	*heliocentric*
	Newton	*optics mechanics calculus*
	Maxwell	*electromagnetism*
	Einstein	*relativity*

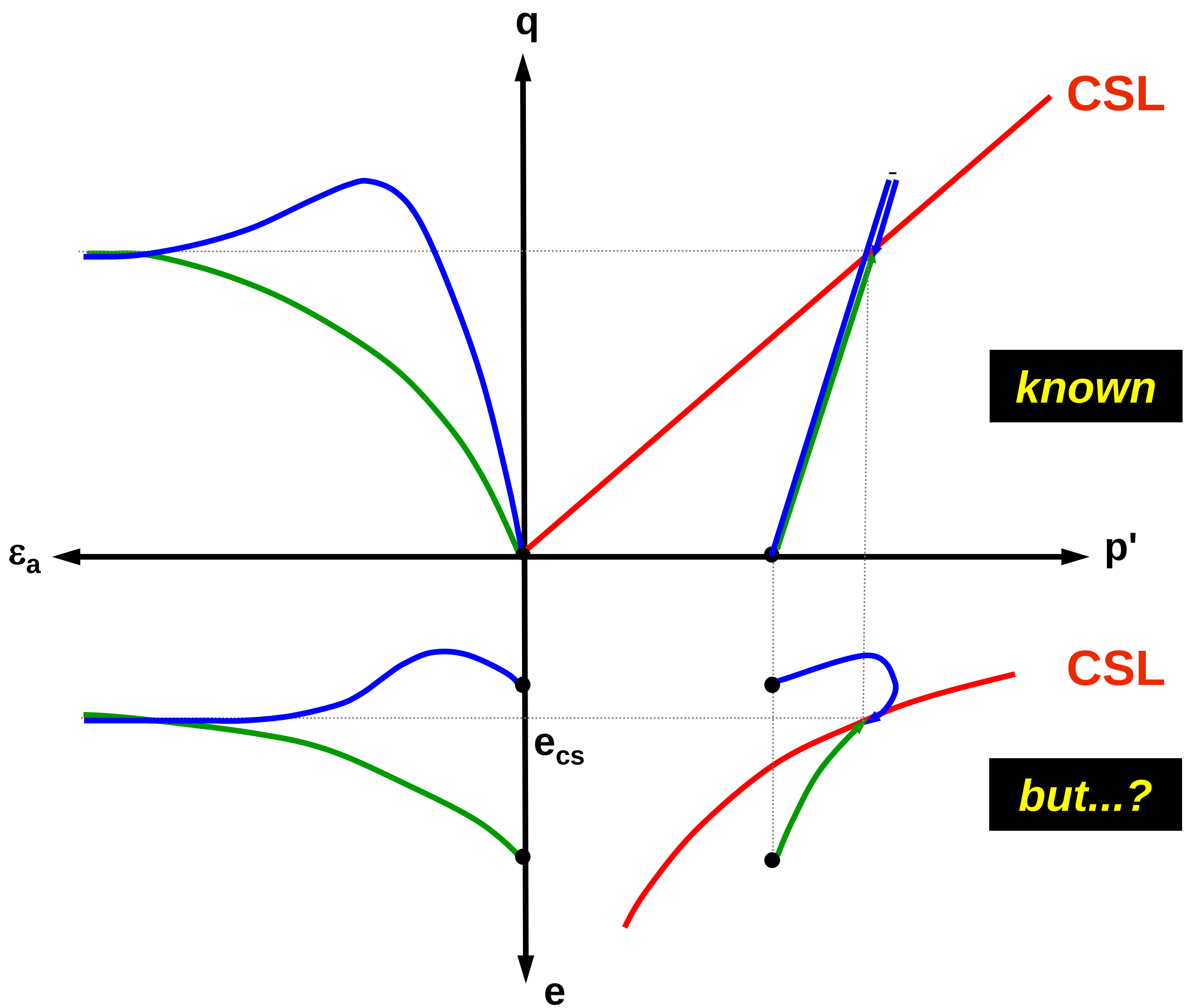
Scientific Revolutions: Geotech



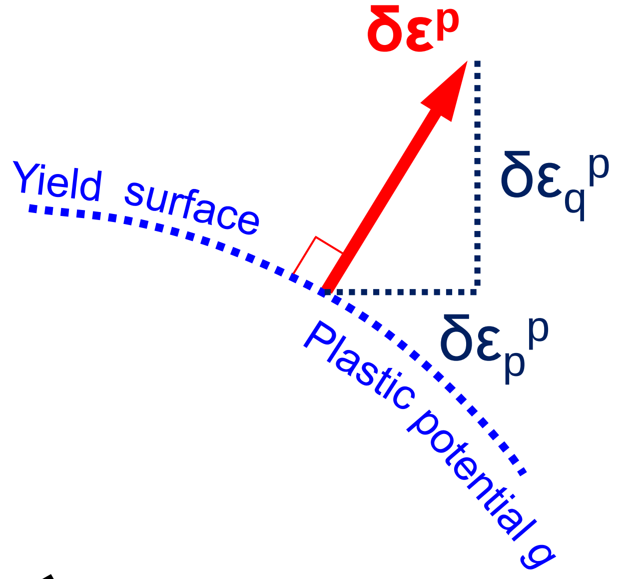
4D ... Projections: e-p'-q-ε_a

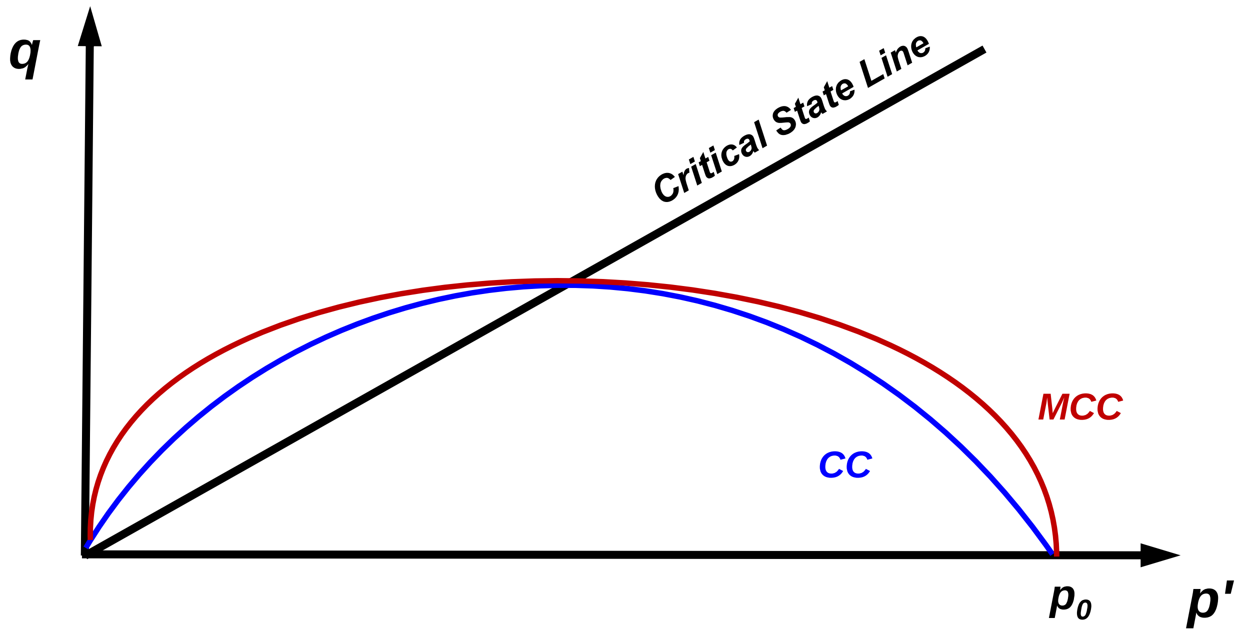


Drained Loading: u=constant



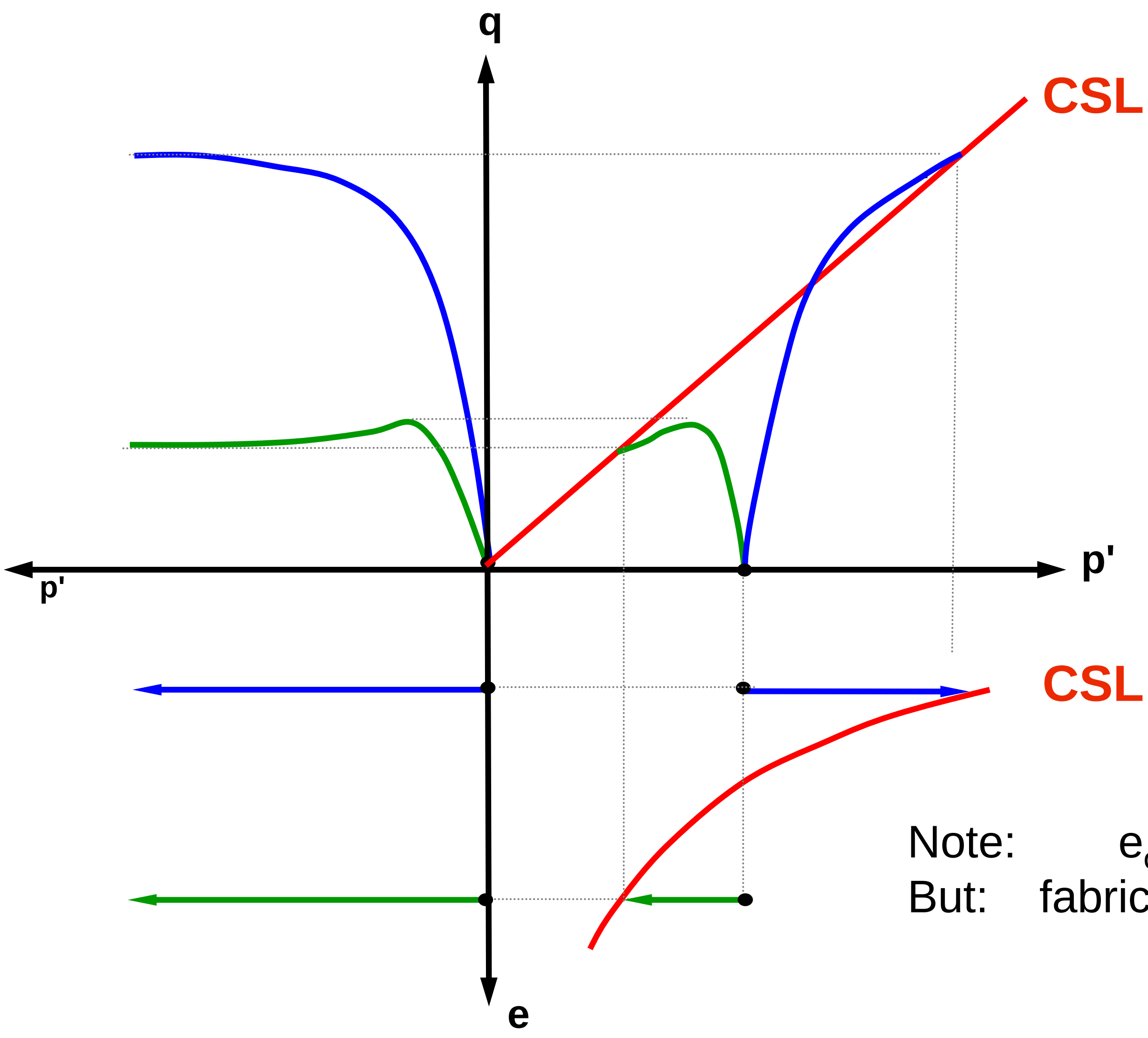
Critical State





Simple, few parameters, robust ➔ links p'-q-e-ε … within Okham's framwork

Undrained Loading: e=constant



Note: $e_o = e_{cs}$

But: $fabric_o \neq fabric_{cs}$

Allow us to change a state variable (p or e) without changing the other

Critical State:

Soil deforms at

constant stress (normal and shear)

constant void ratio

constant velocity

statistically steady state

... particle orientation

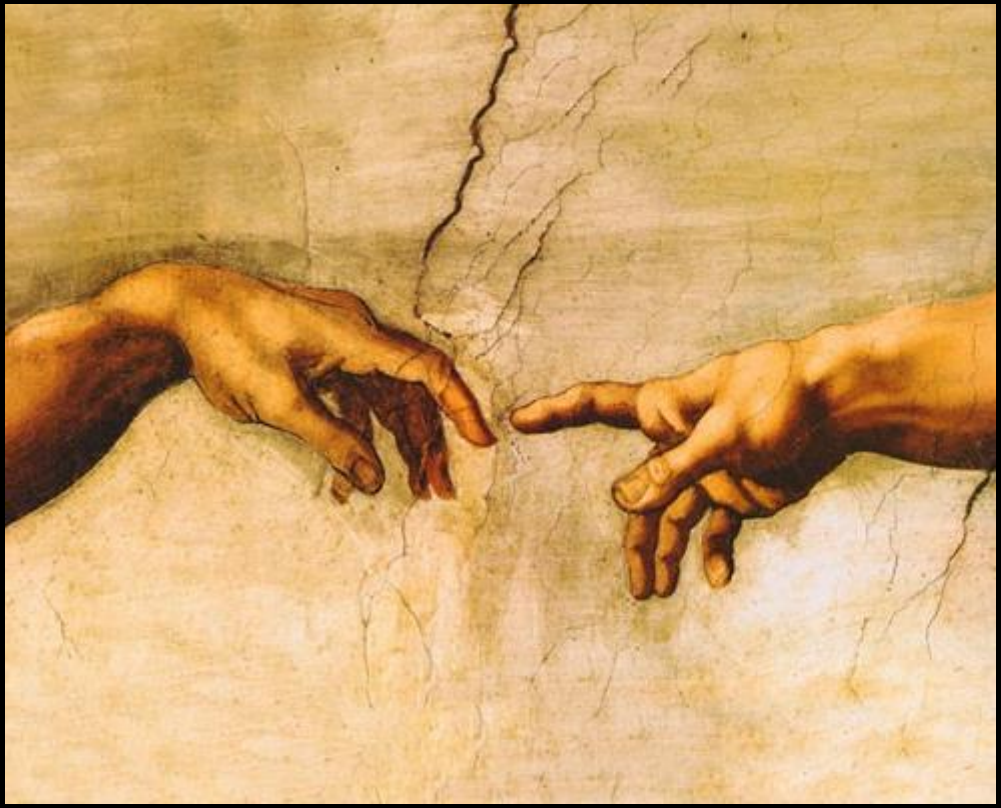
... particle breakage

Roscoe et al 1958

Poulos 1981
Steady State Line

memoryless characteristic fabric

Terminal/asymptotic states: robust design… essence of engineering philosophy

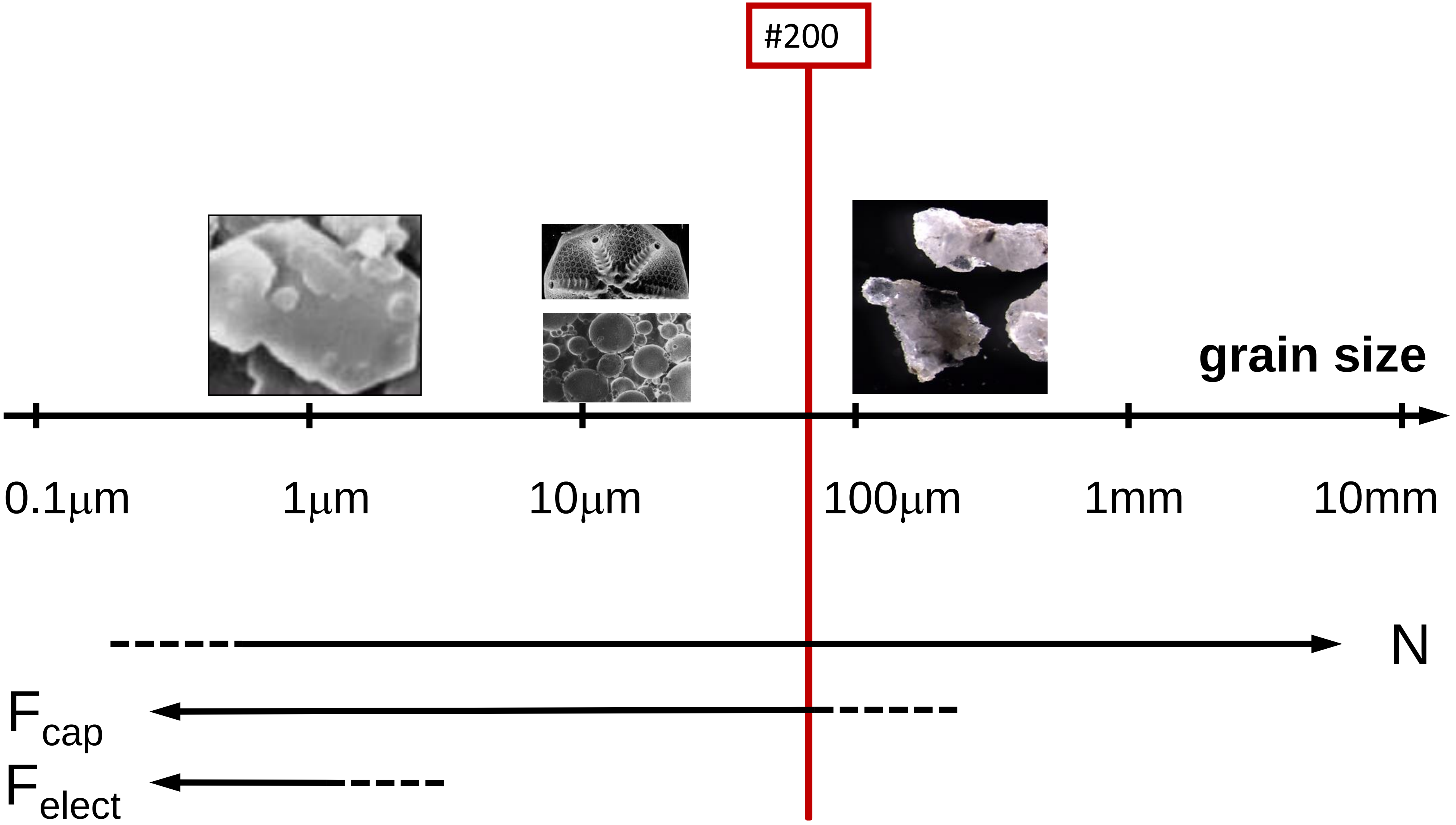


Michelangelo

Genesis

in the beginning...

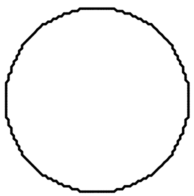
Genesis ➔ Size, Shape & Forces

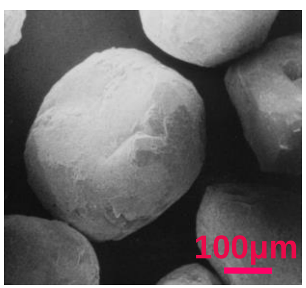


Modifications to CS if F_{cap} or F_{elect} change

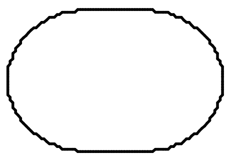
Genesis: Particle Shape

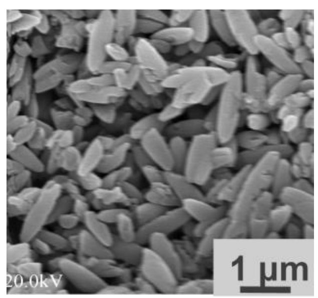
size d



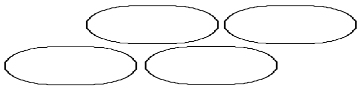


eccentricity

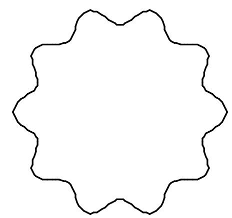


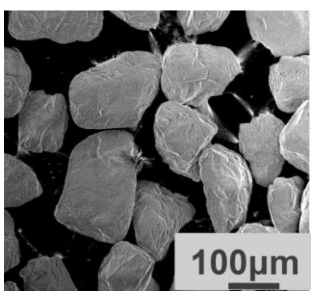


alignment

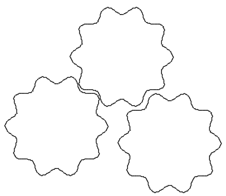


angularity

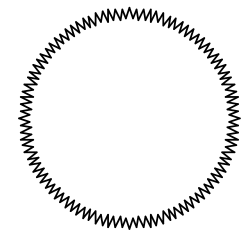


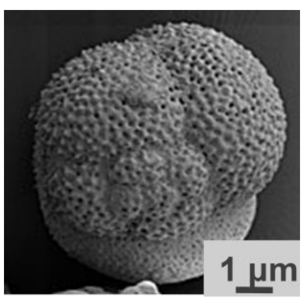


interlocking

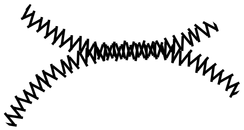


roughness



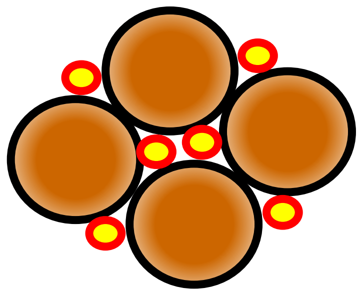


surface μ

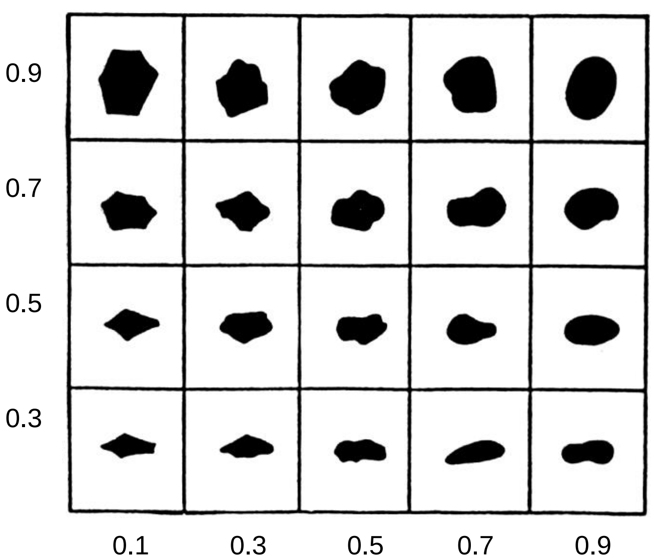


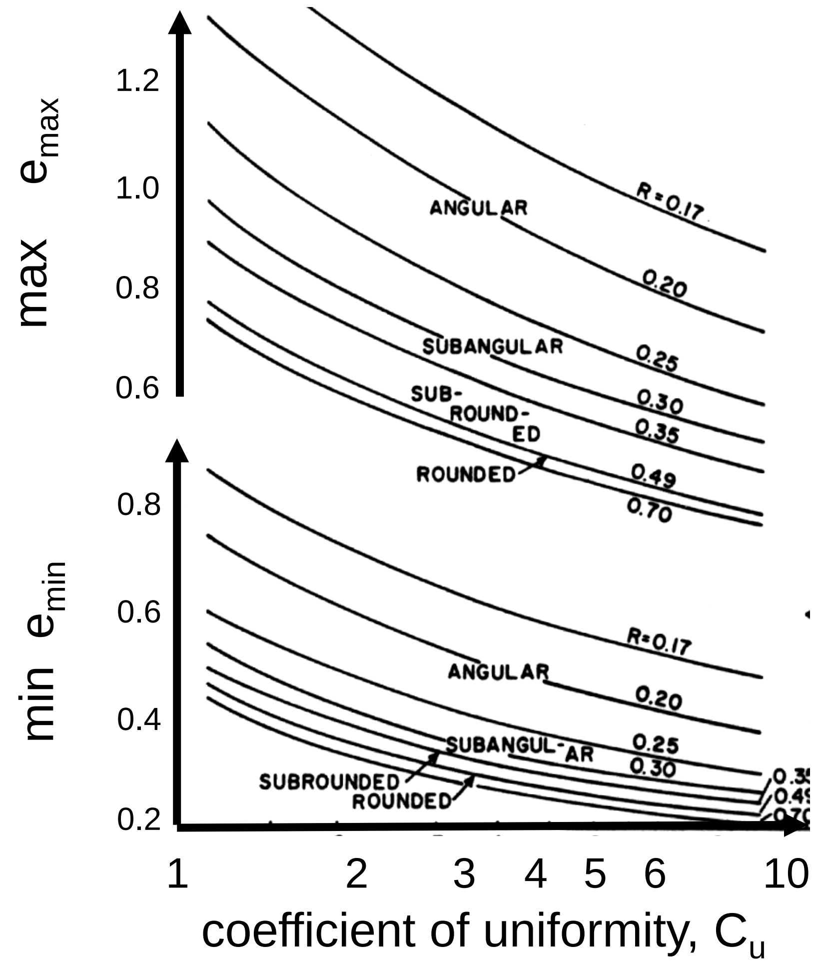
Packing *Coarse* Grained Soils:
Shape + Relative Size

C_u



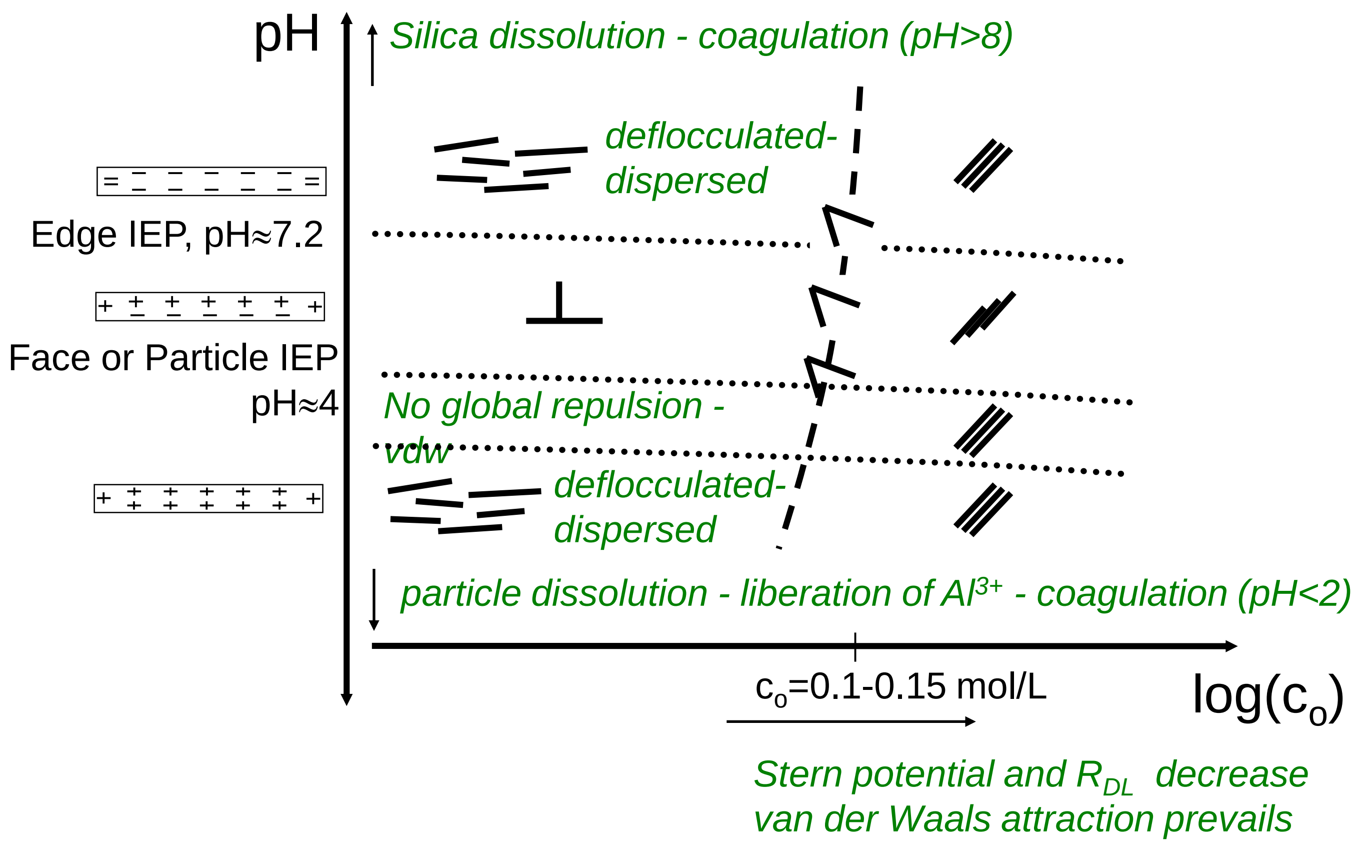
(Krumbein and Sloss, 1963)





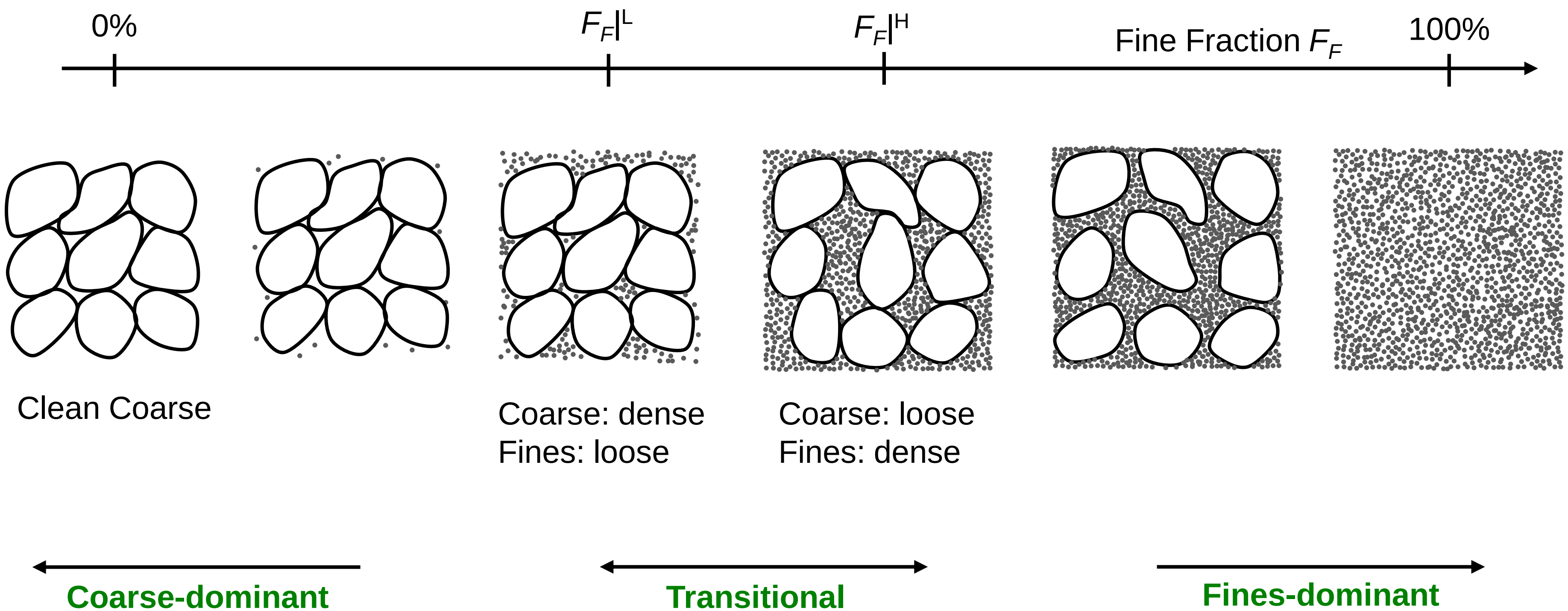
(Youd, 1973; see also Maeda, 2001)

Packing *Fine* Grained Soils:
Mineral & Fluid (pH c_{ion})



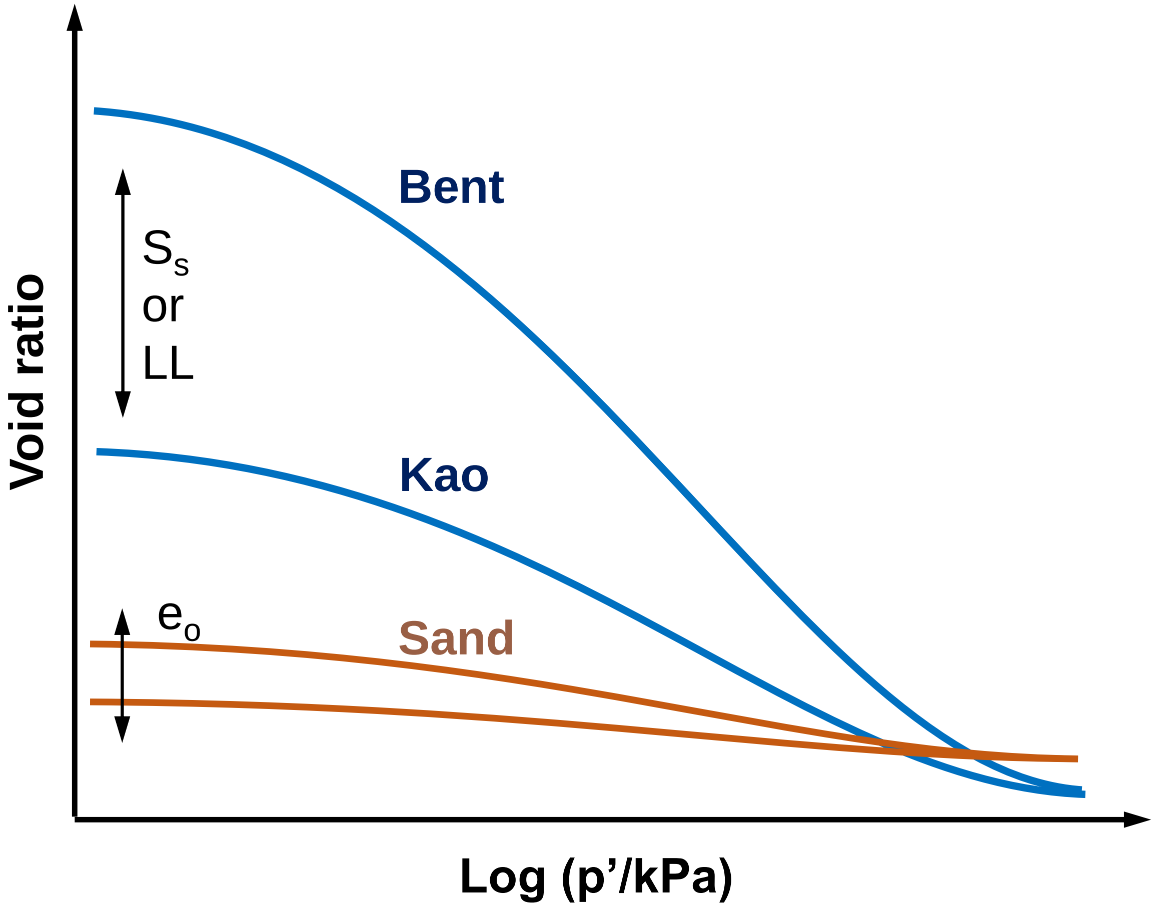
Packing *Mixed* Soils

$$F_F = \frac{M_{sand}}{M_{total}} = \frac{e_G}{1+e_G+e_s}$$



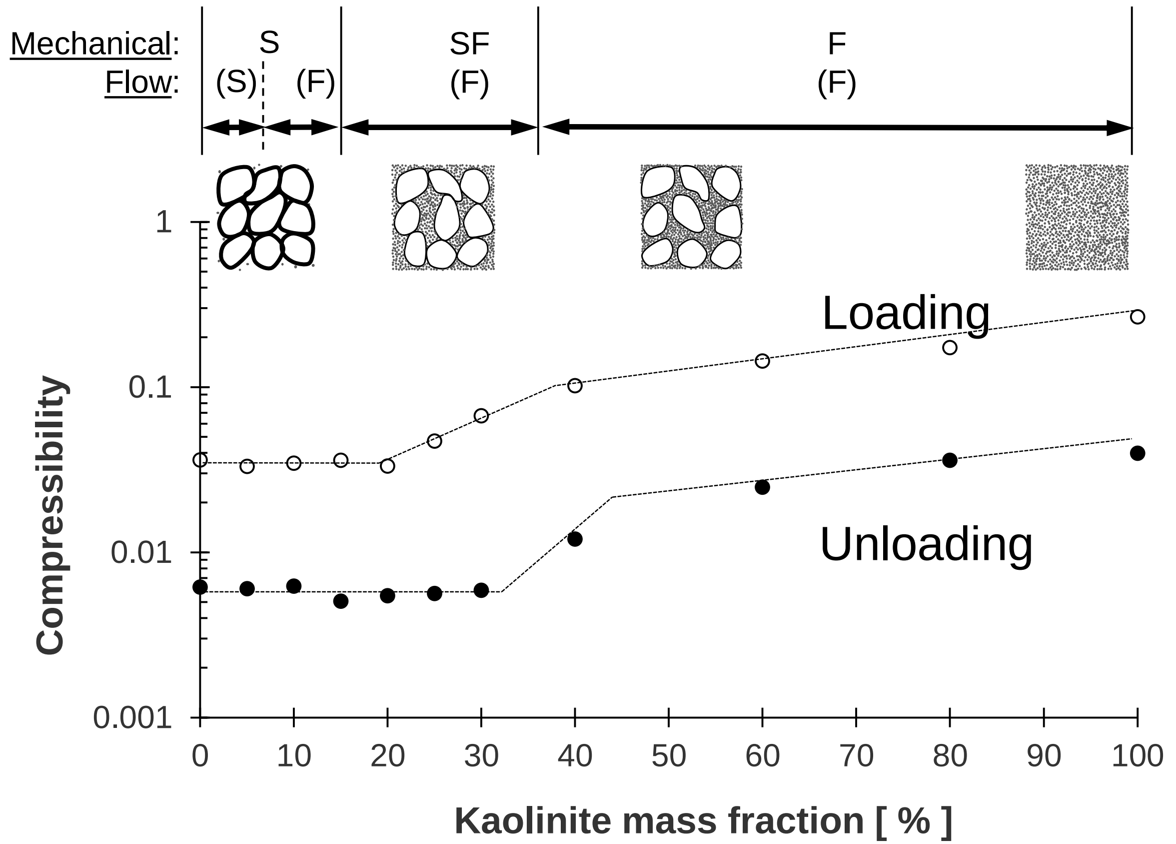
Compression

Compression: Fines and Coarse



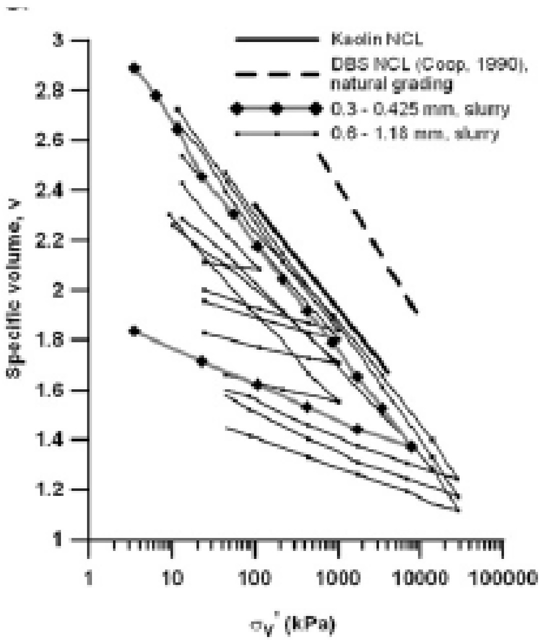
Sands (and most silts) do <u>not</u> have an inherent virgin consolidation line

Transition: From Clay-to-Sand Control

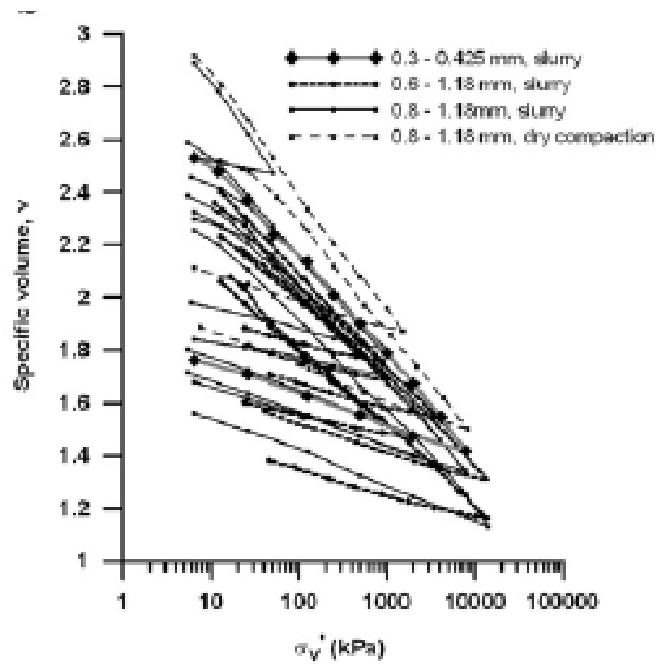


Transitional Behavior: Non-unique NCL & CSL

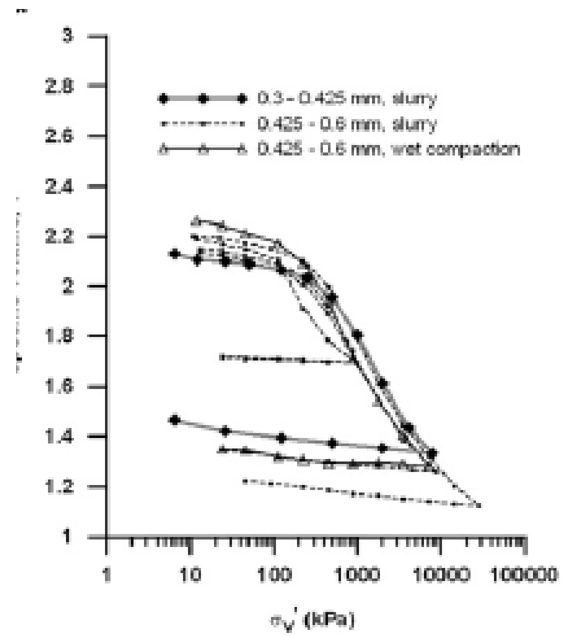
5% sand



10% sand



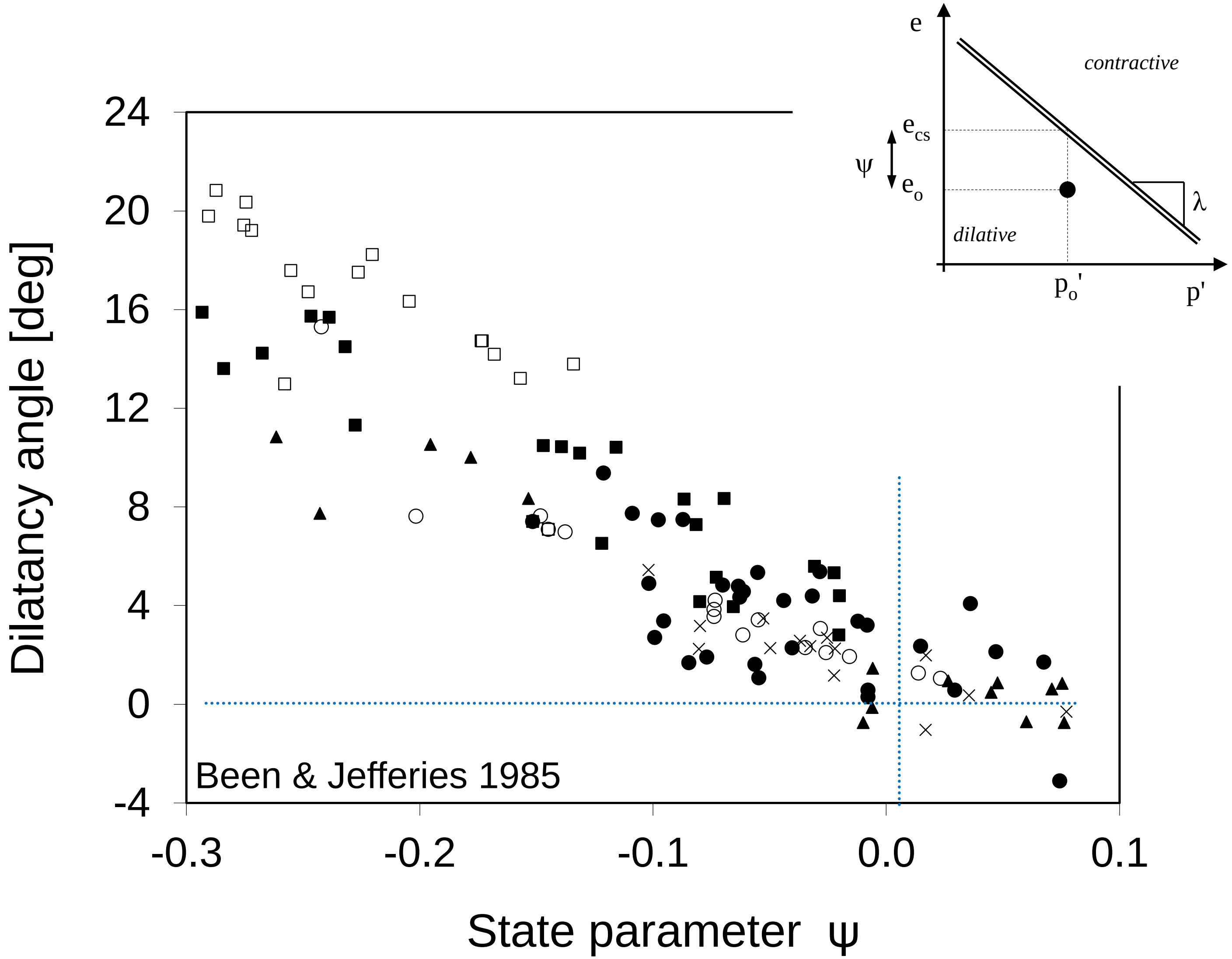
70% sand



Dogs Bay carbonate sand & kaolin
Shipton & Coop 2012

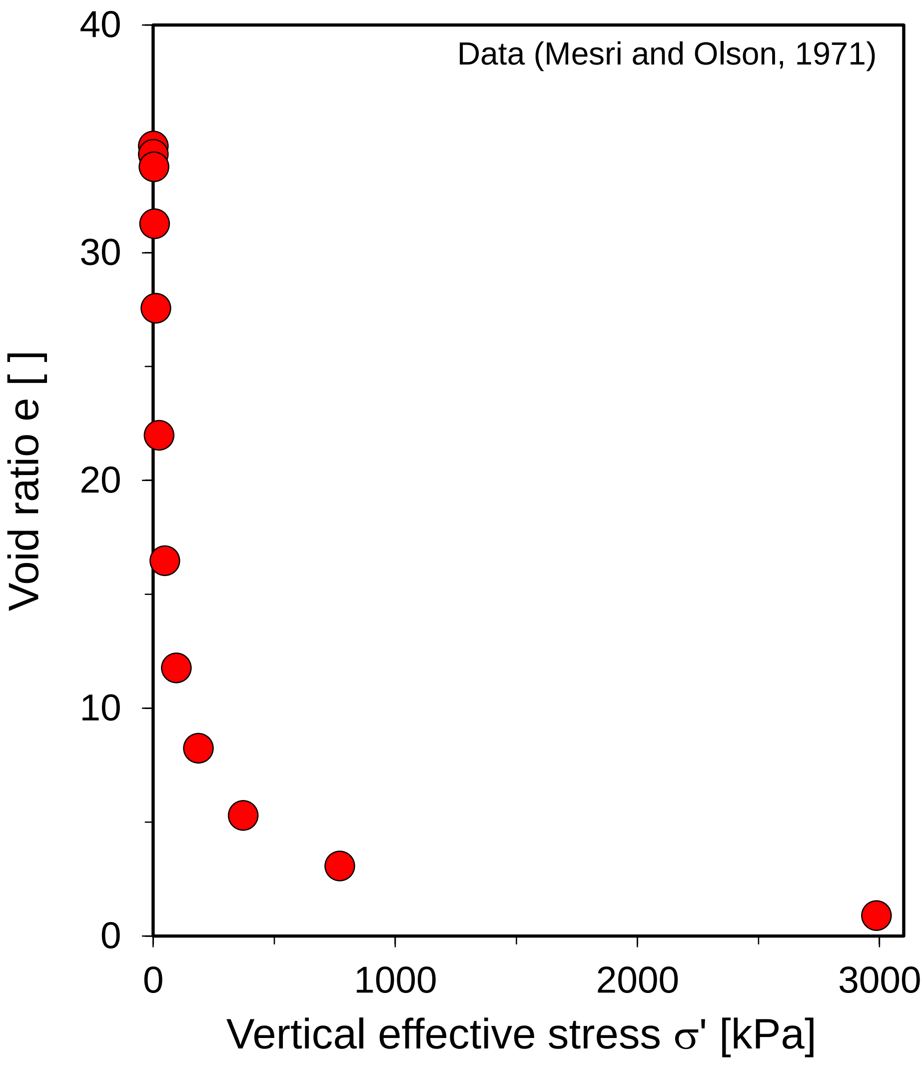
Non-unique NCL: mixed grading, mixed mineralogy, mixed particle types

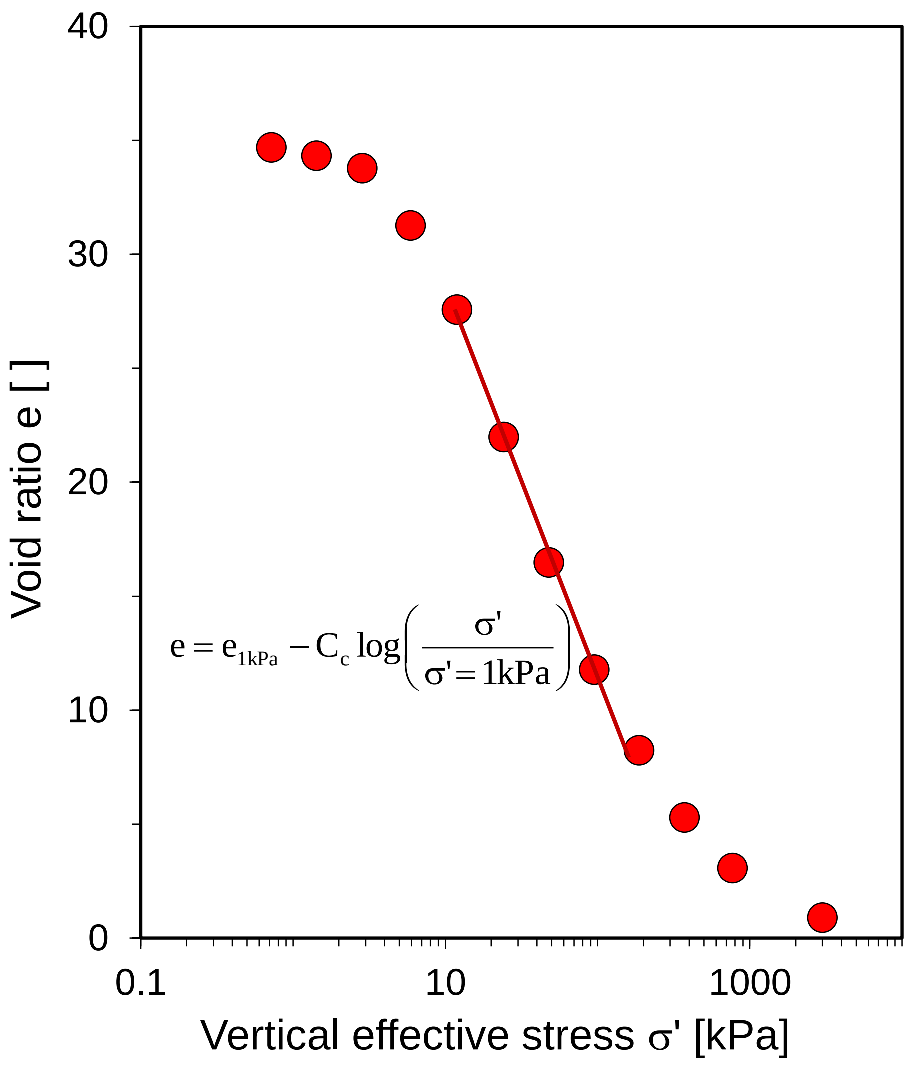
Sands: State Parameter



Sands do not have an intrinsic consolidation line ➔ Nor Sand

Compression



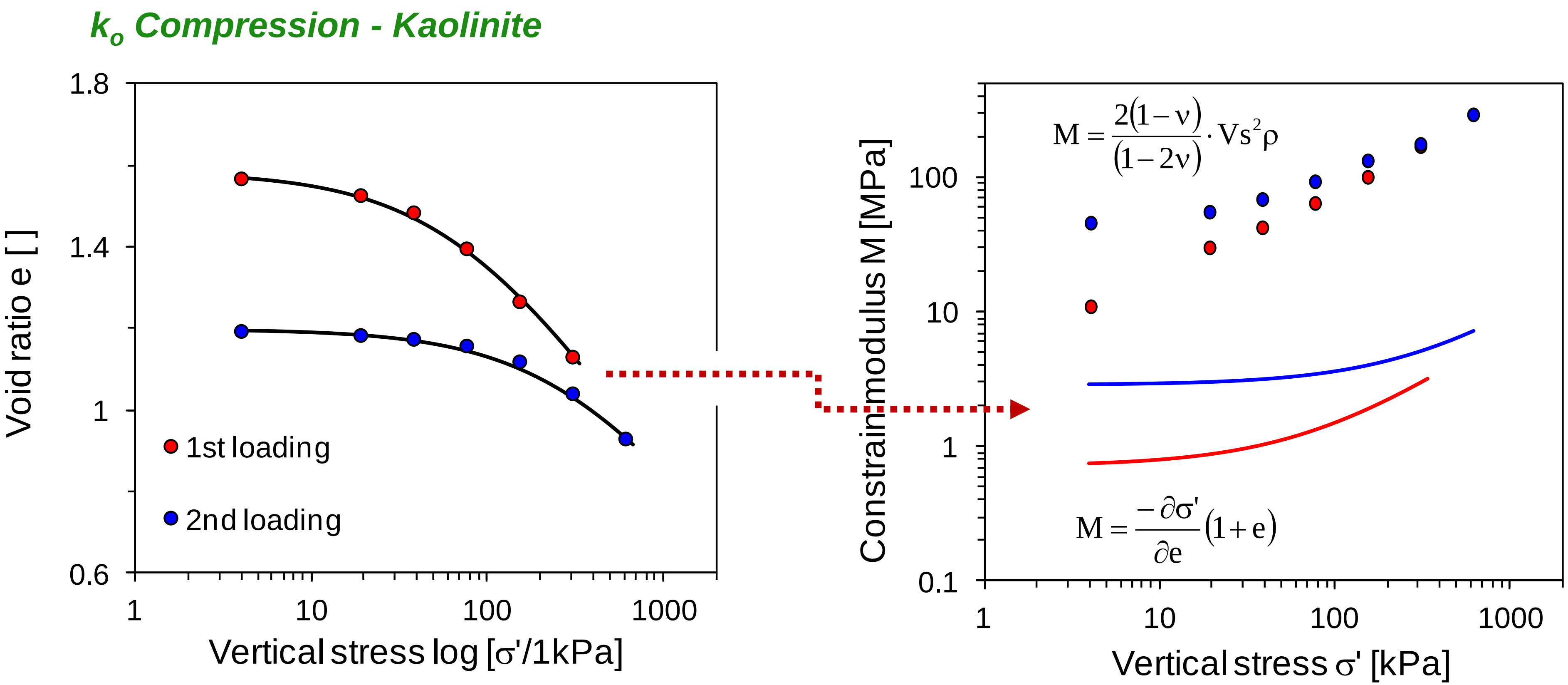


Why so much emphasis on yield stress?

Compression

Family	Model	Equation	References
Semi-log	*Classical*	$e = e_{ref} - C_c \log\left(\frac{\sigma'}{\sigma'_{ref}}\right)$	Terzaghi & Peck (1948) Schofield & Wroth (1968)
Semi-log	*Cubic (3rd order)*	$e = e_{ref} - \alpha \cdot \log\left(\frac{\sigma'}{\sigma'_{ref}}\right) + \beta \cdot \left[\log\left(\frac{\sigma'}{\sigma'_{ref}}\right)\right]^3$	Burland (1990)
Semi-log	*Modified*	$e = e_c - C_c \log\left(\frac{1kPa}{\sigma' + \sigma'_L} + \frac{1kPa}{\sigma'_H}\right)^{-1}$	
Power	*From gas to soil*	$e = e_H + (e_L - e_H)\left(\frac{\sigma' + \sigma'_c}{\sigma'_c}\right)^{-\beta}$	Hansen (1969) Butterfield (1979) Juárez-Badillo (1981) Houlsby & Wroth, (1991) Pestana & Whittle (1995)
Exponent.	*Gompertz function*	$e = e_H + (e_L - e_H) \cdot \exp^{-(\sigma'/\sigma'_c)^{\beta}}$	Gregory et al. (2006) Cargill (1984 – β=1)
Hyperbol	*Hyperbolic function (classical hyp: β=1)*	$e = e_L - (e_L - e_H)\frac{1}{1 + (\sigma'_c/\sigma')^{\beta}}$	
Arctang.	*S-shaped function*	$e = e_L + \frac{2}{\pi}(e_L - e_H)\arctan\left[-\left(\frac{\sigma'}{\sigma'_c}\right)^{\beta}\right]$	

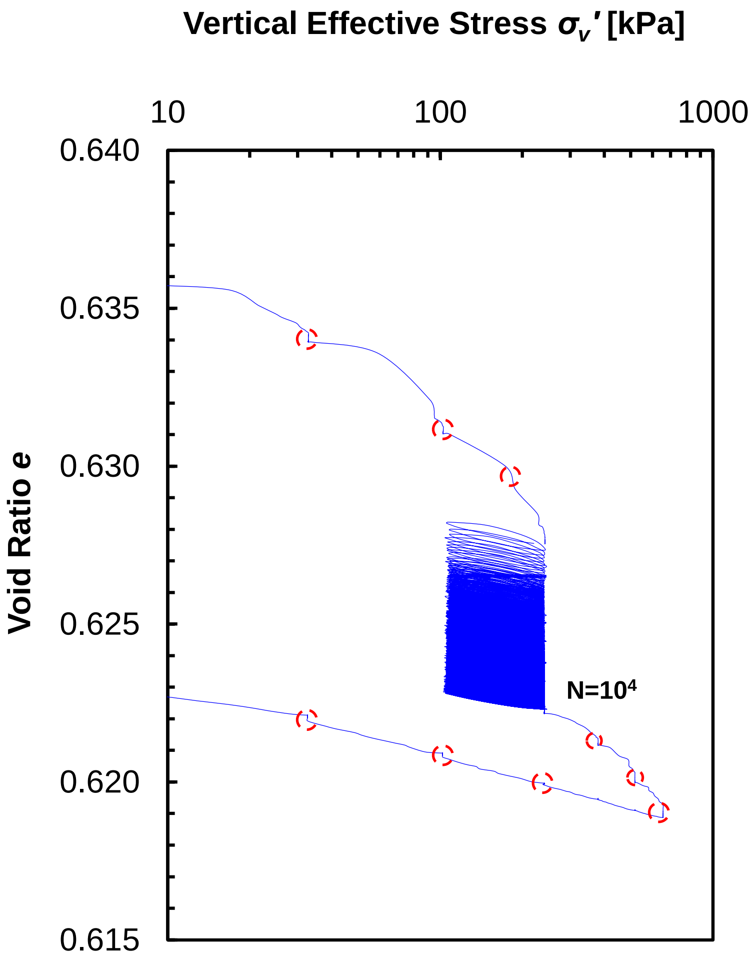
Stiffness: Small Strain vs. Tangent

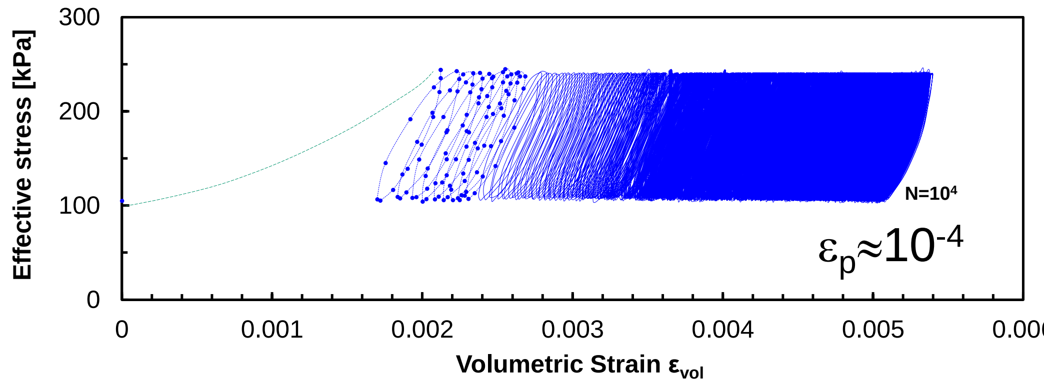


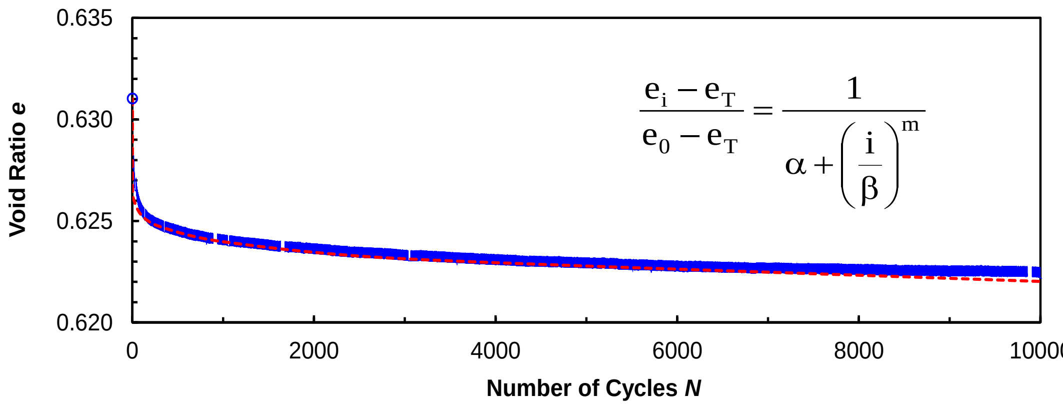
Elastic modulus ≠ Tangent to compression line … and it is non-linear with p'

Elastic Within Yield Surface?

Repetitive k_o-Loading







$$\frac{e_i - e_T}{e_0 - e_T} = \frac{1}{\alpha + \left(\frac{i}{\beta}\right)^m}$$

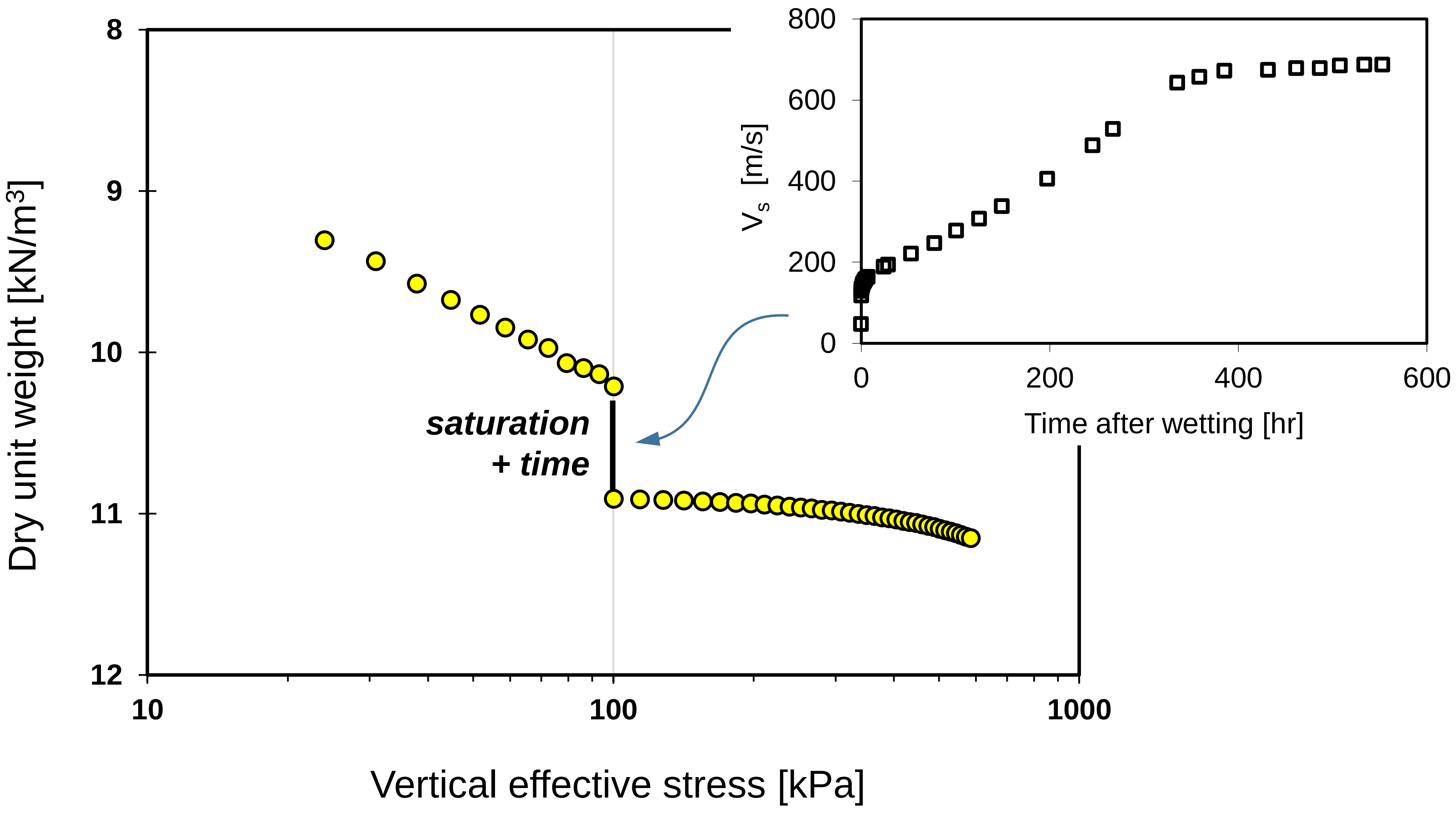
Why "yield-or-not-yield" ? *remains hysteretic* *$e_T = f(e_o)$ ➔ Memory !*

Diagenesis

Structure, cementation

residual soils, dissolution/precipitation

Diagenesis

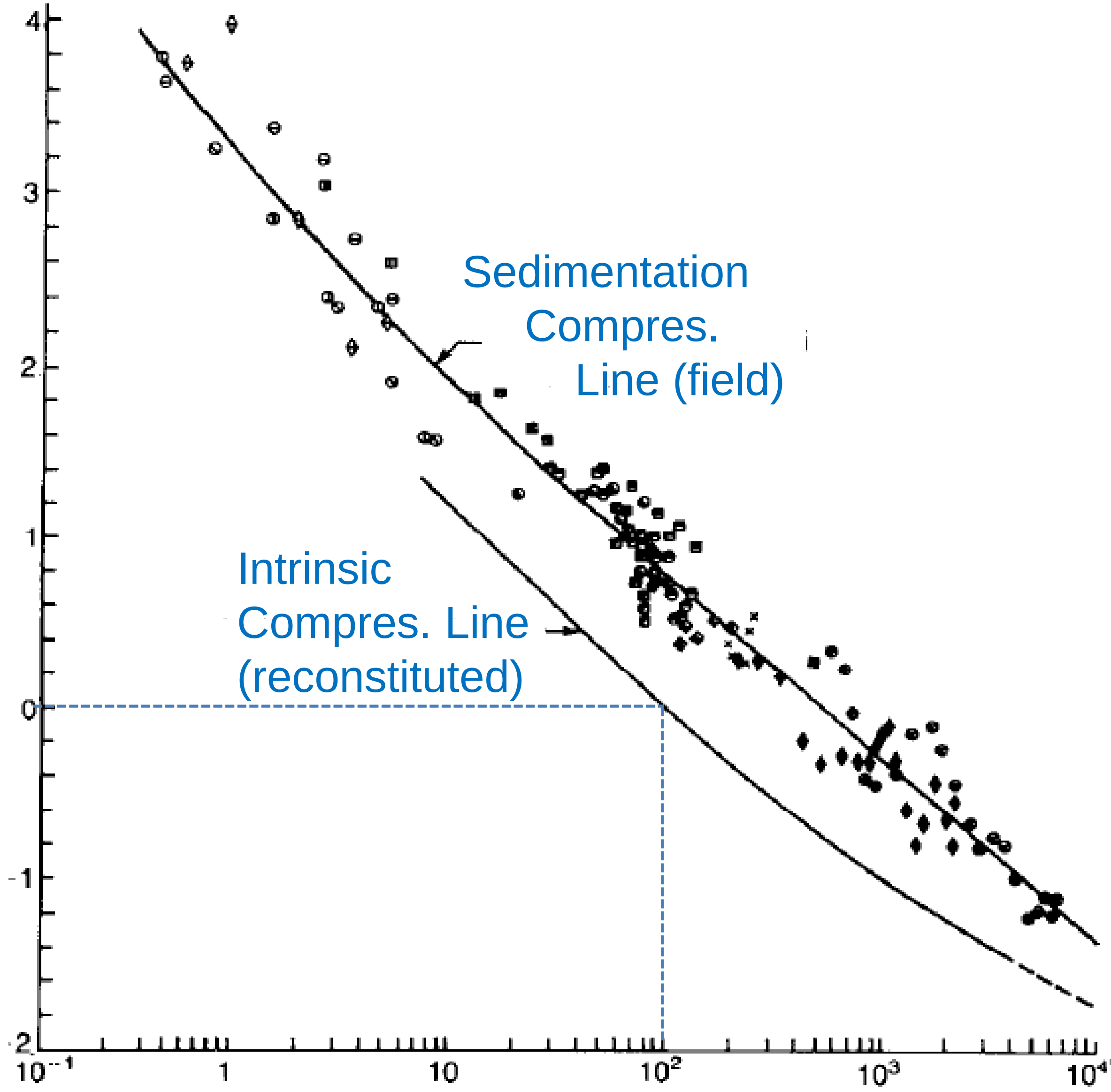


8
9
10
11
12
Dry unit weight [kN/m3]
10
100
1000
Vertical effective stress [kPa]
saturation
+ time
800
600
400
200
0
Vs [m/s]
0
200
400
600
Time after wetting [hr]

Locked-in Porosity

Void Index

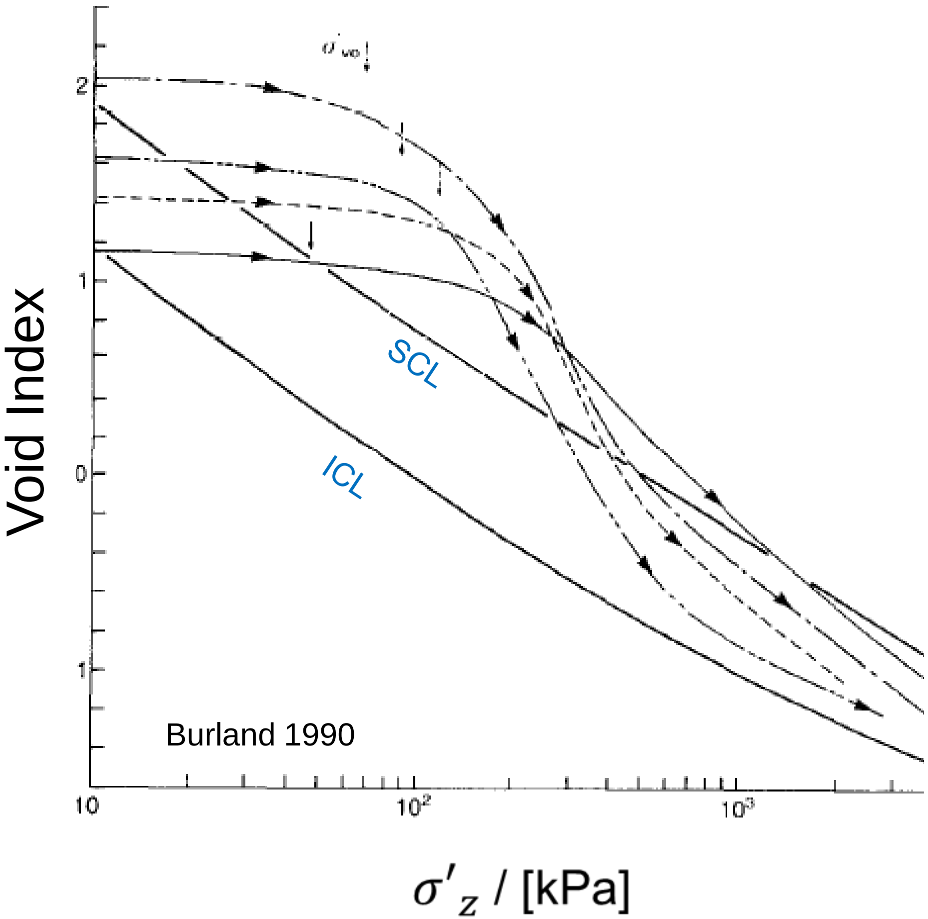
$$Iv = \frac{e - e^r_{100}}{e^r_{100} - e^r_{1000}}$$

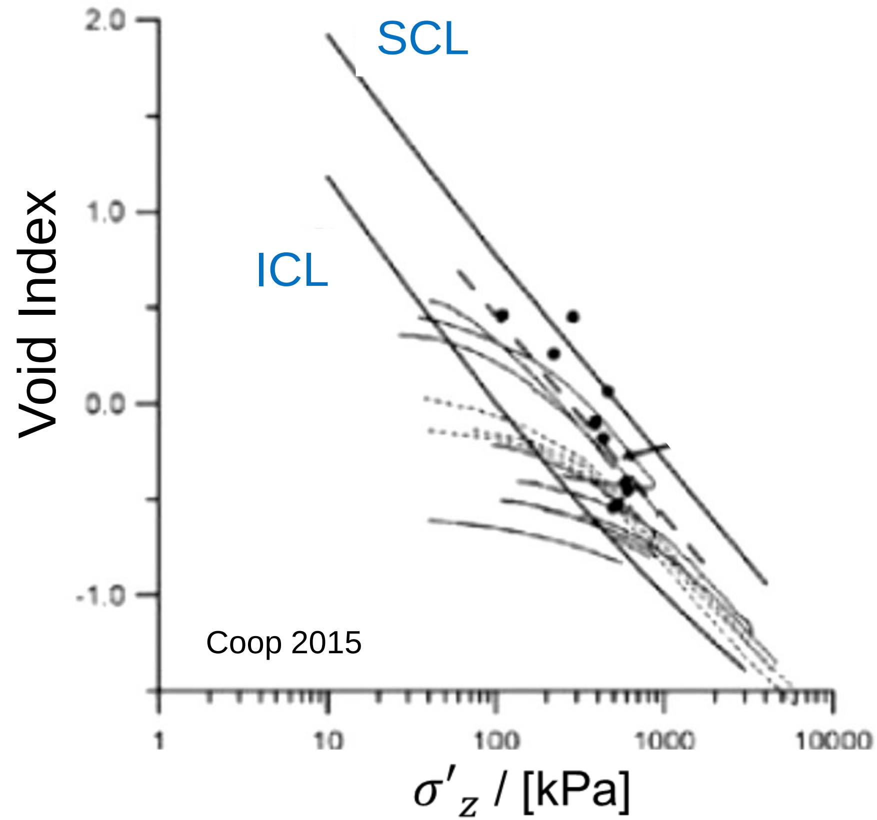


σ'_z / [kPa]

Burland 1990

Locked-in Porosity



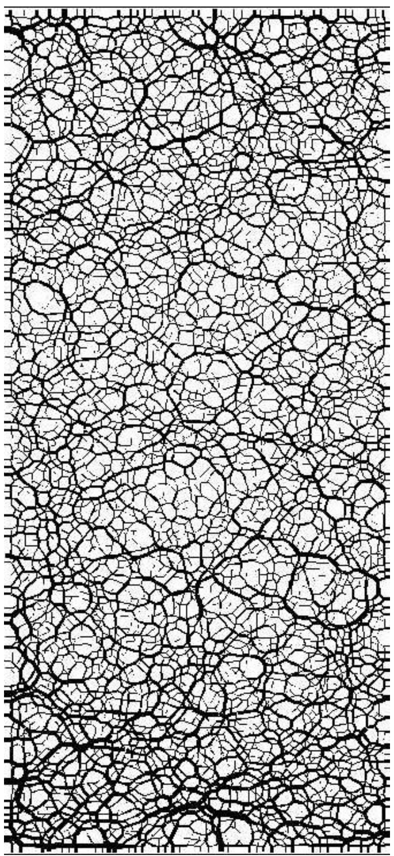
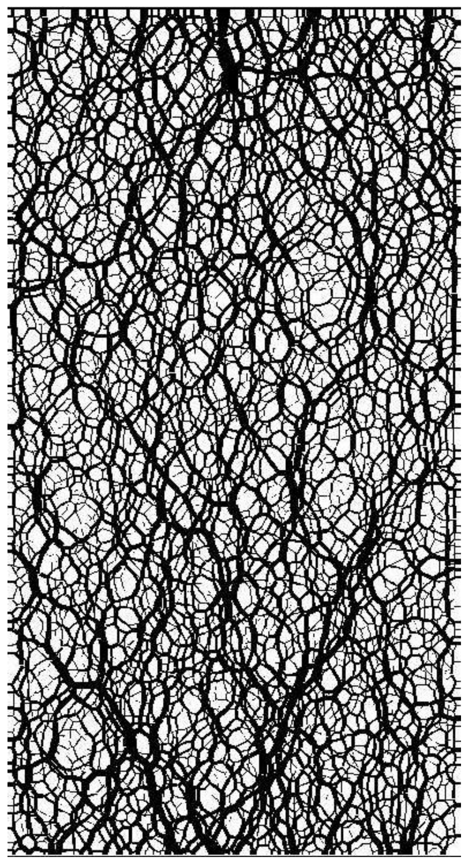
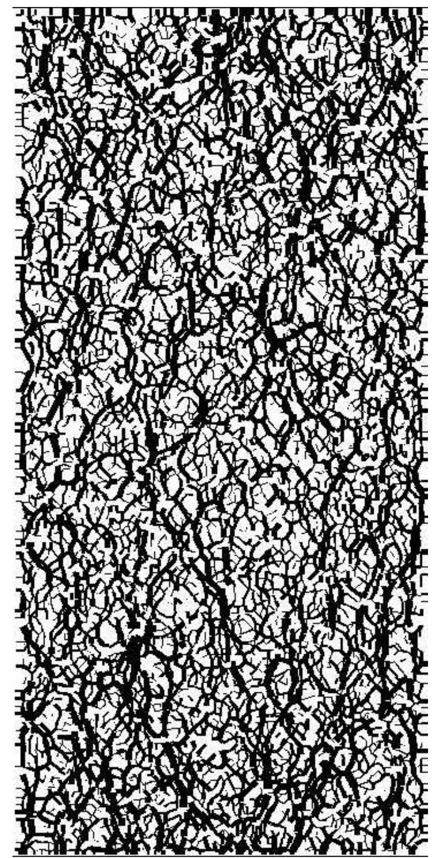
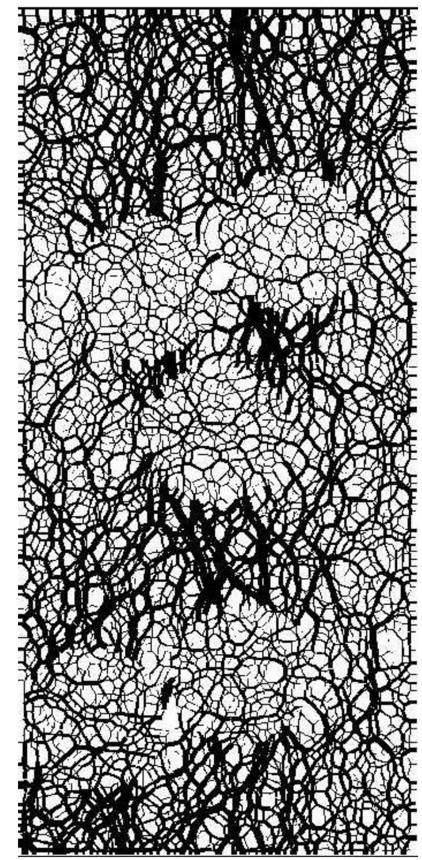


Coop et al. Cotecchia & Chandler (SEM fabric) Leroueil Gens

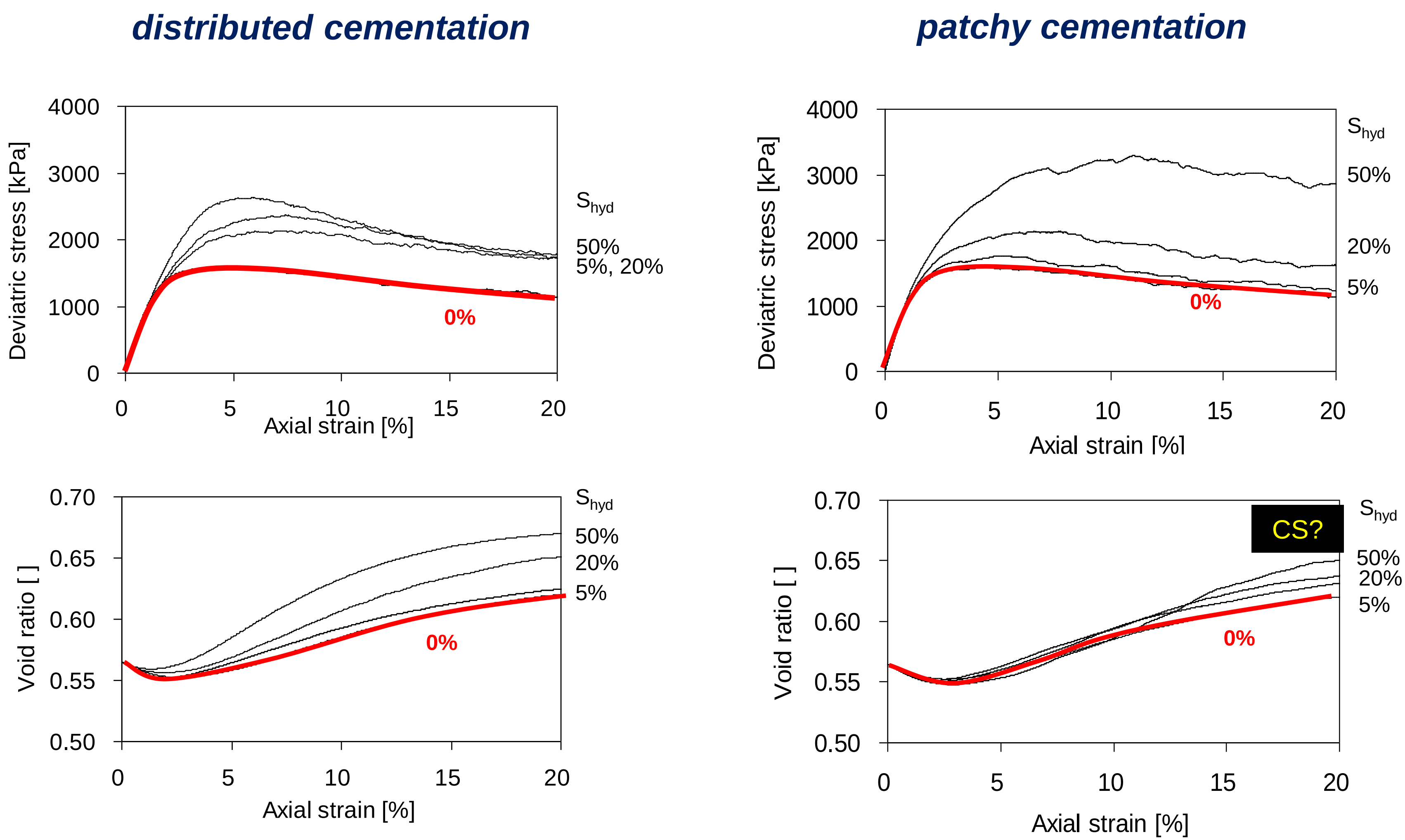
Compression: long memory...

D/L_{corr}~1

Response to Deviatoric Loading

Isotropic load	Under deviatoric load (σ_o=1MPa, σ_d=1.2MPa)		
	Cement-free	Distributed cement	Patchy cementation
			

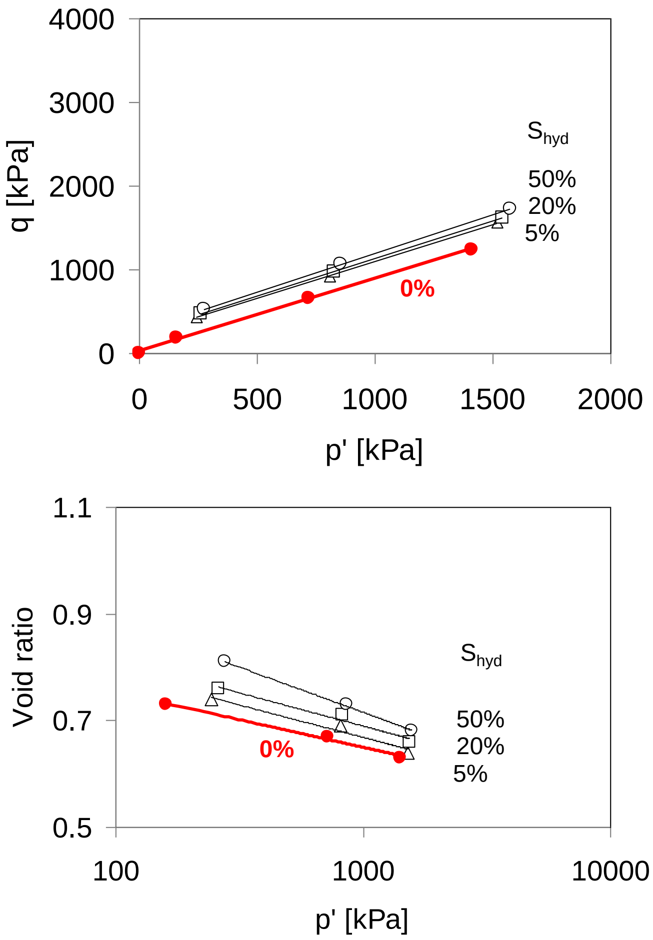
Critical State - large strain (3D)



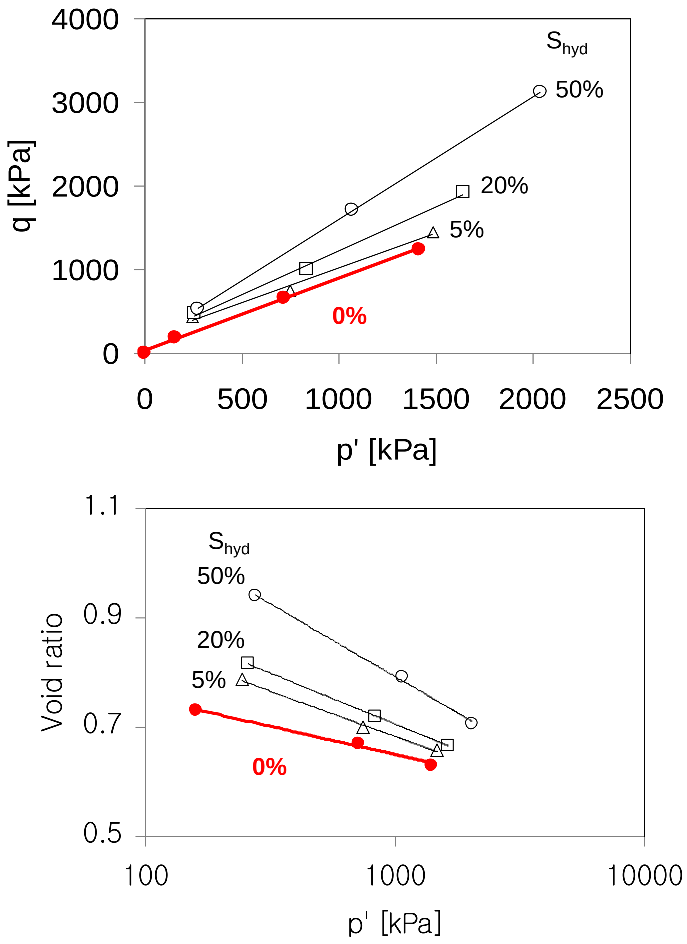
Increased stiffness, strength, dilation - Cement amount and pore habit

Critical State - large strain (3D)

distributed cementation



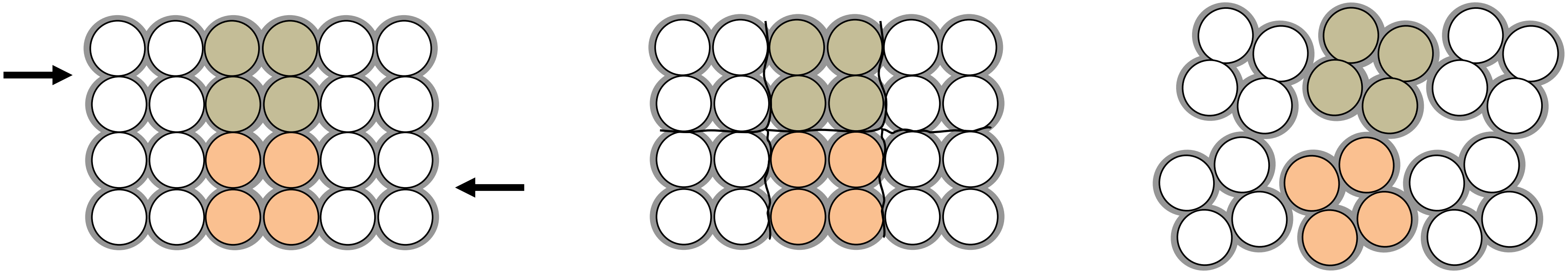
patchy cementation



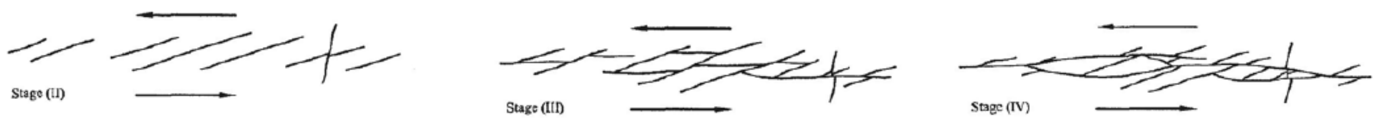
Cementation alters CS (e-p and q-p)

Destructuration: Dilative Blocky Structure

initial *early damage* *intermediate*

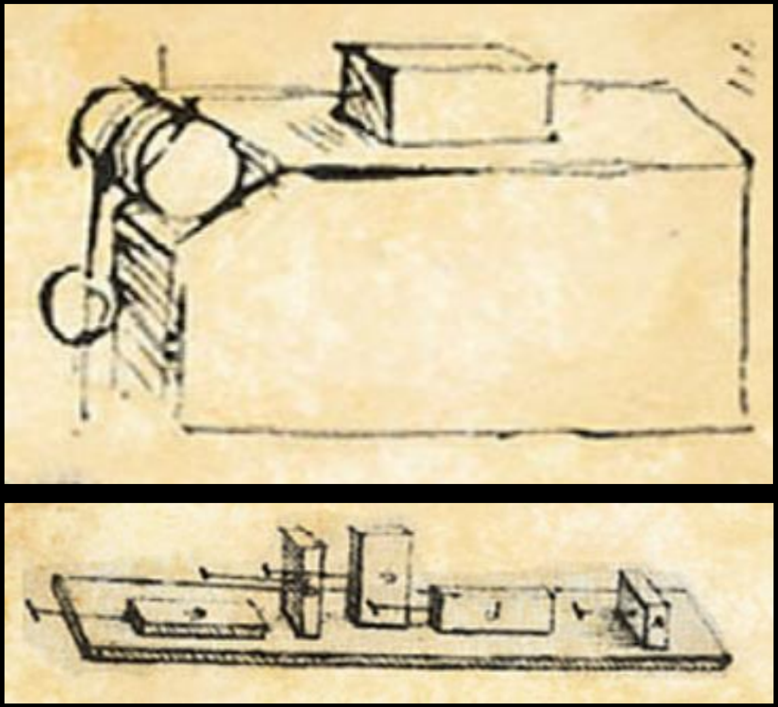


Skempton 1966



Burland's intrinsic properties (remolded clays at w>LL): relevance to field?

Position of CSL $f(e_o)$ ➔ CSL has long memory

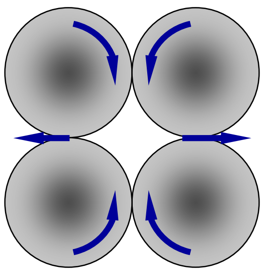


Leonardo (1452-1519)

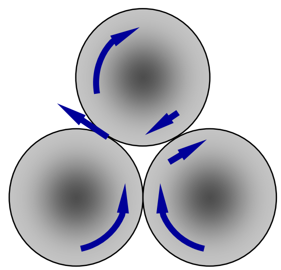
Shear

CS = statistically steady state
inherent and stress induced fabric

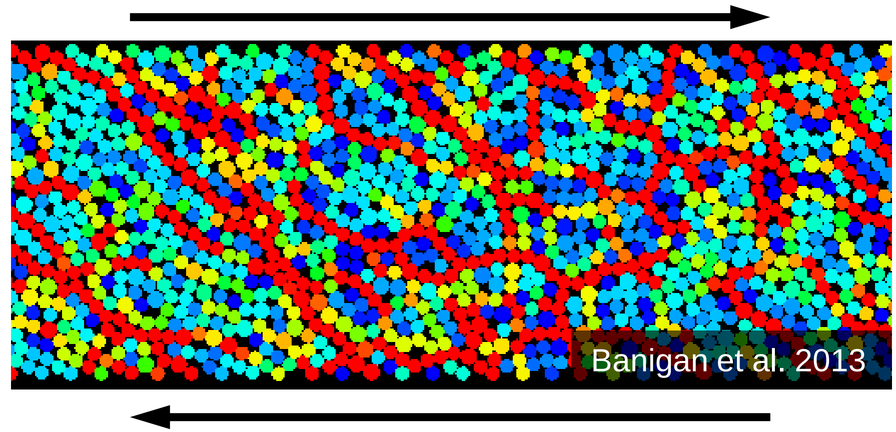
Critical State: “Statistically Steady” State



free
(high e)



frustrated
(low e)



frustration ➔ dilate or slip

slender columns buckle

local dilation - cn↓

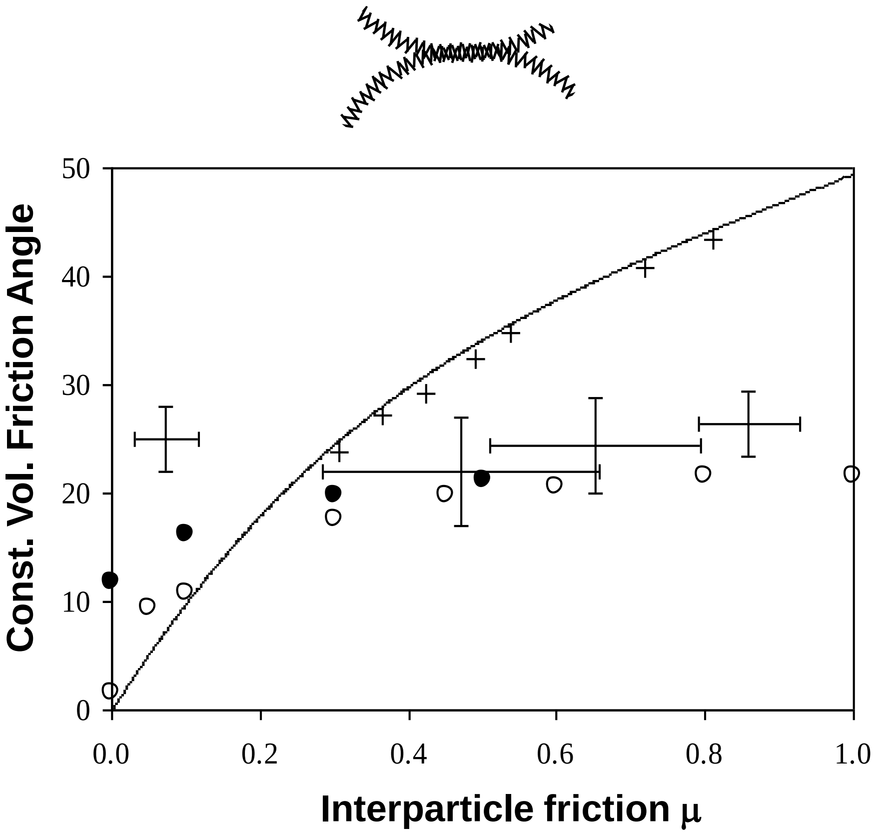
local contraction - cn↑

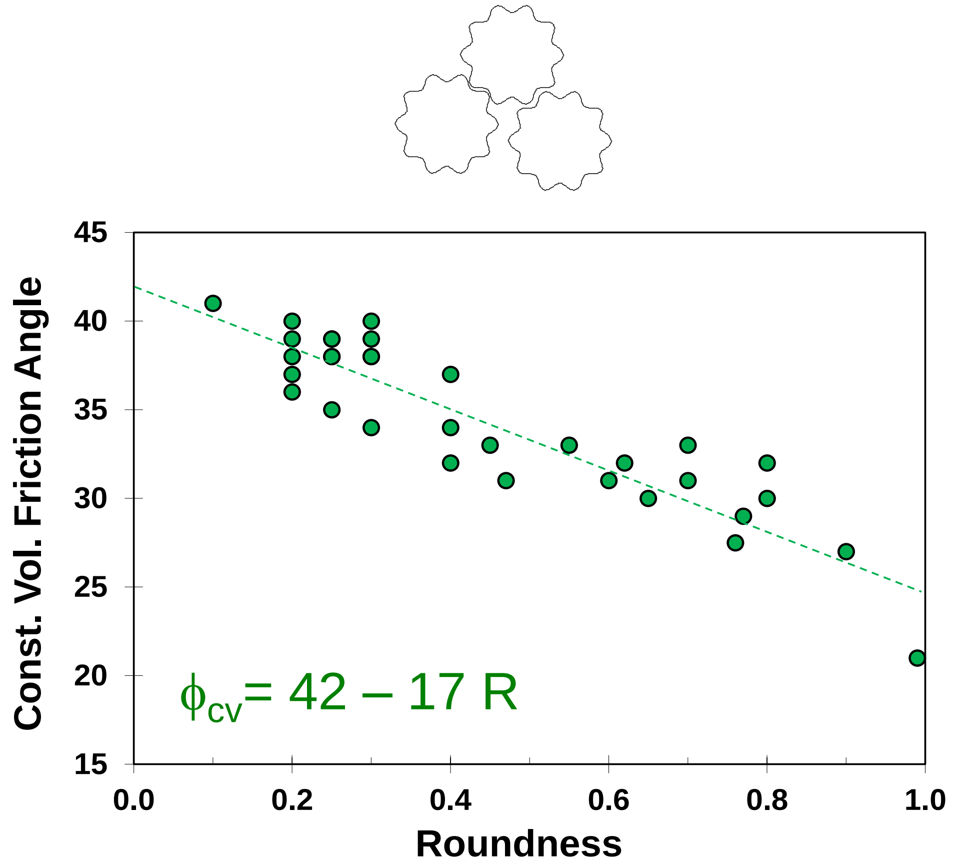
fabric_{cs} anisotropy_{cs} cn_{cs}

DEM: Rothenburg & Kruyt (2004)

Mobility ➔ various CS   *strain to critical state*   *ϕ_{cv}= f(shape)*

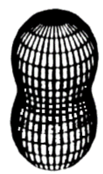
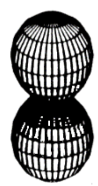
CS Constant Volume Friction





(—) theoretical: Horne (1969)
(-.-) experiments: Skinner (1969), (+) Rowe (1969)
(○) DEM 3D Thornton (2000),
(●) DEM 2D Kruyt & Rothenberg (2006)

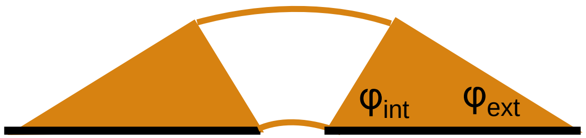
Shear strength = allowable anisotropy

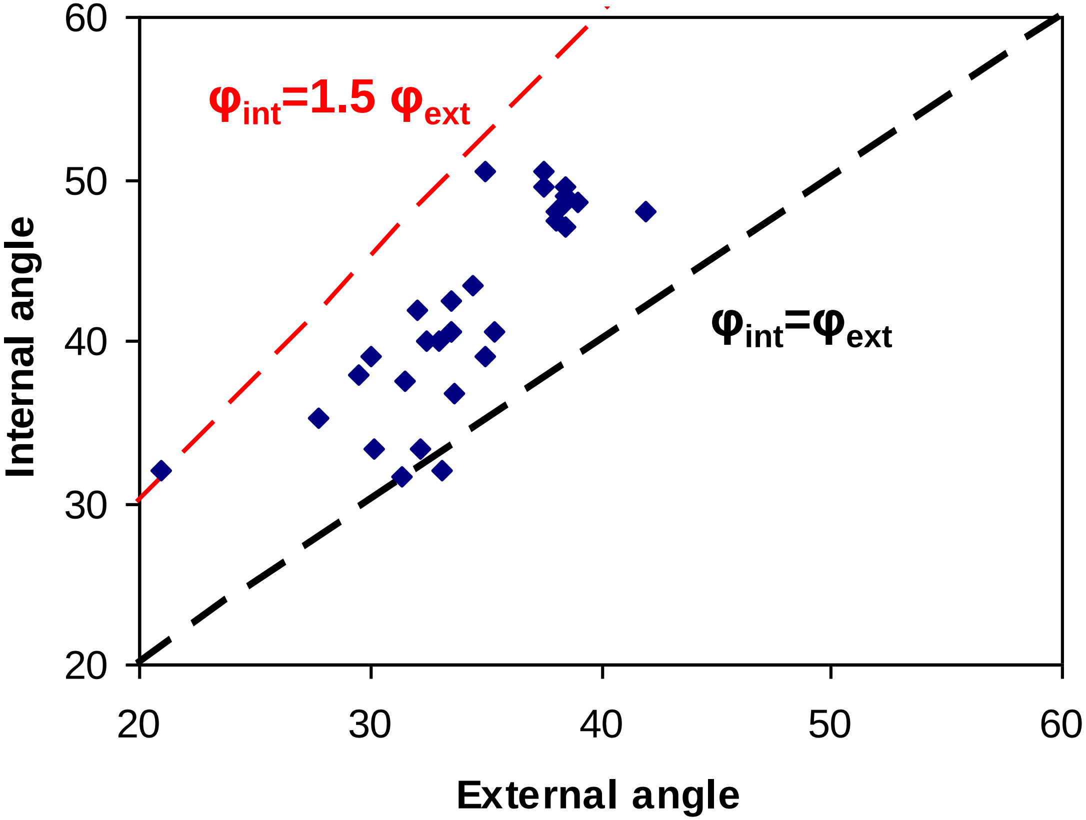
	Isotropic confinement	At peak dev. load	
		AC (b=0)	AE (b=1)
Contact normals			
N(θ)			
T(θ) (magnified x5)			

Chantawarangul and Rothenburg, 1993

$$\sin\phi_{mob} = \frac{a_c + a_n + a_t}{2}$$

Change in Coordination?



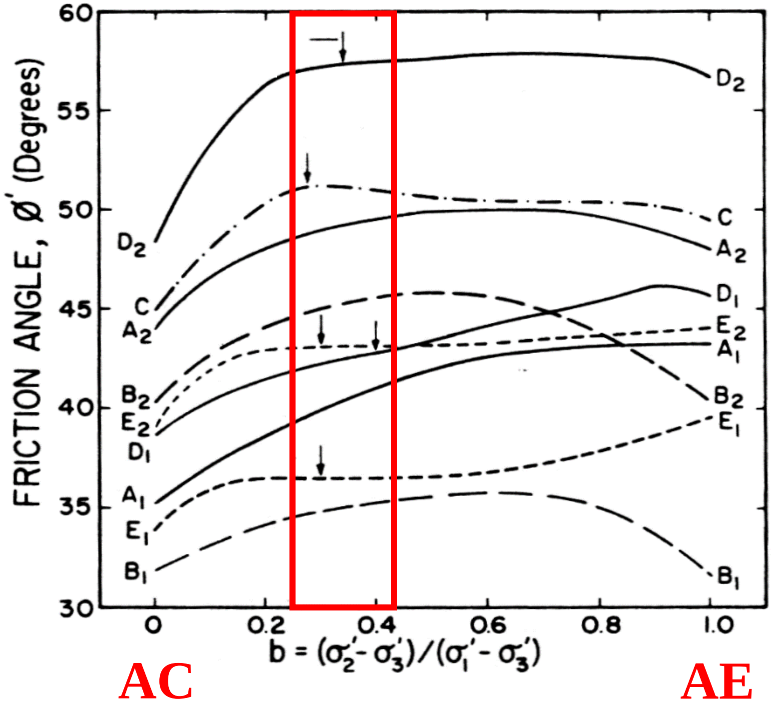


Narsilio, Dodds, Fugle, Trott, Kim, Yun

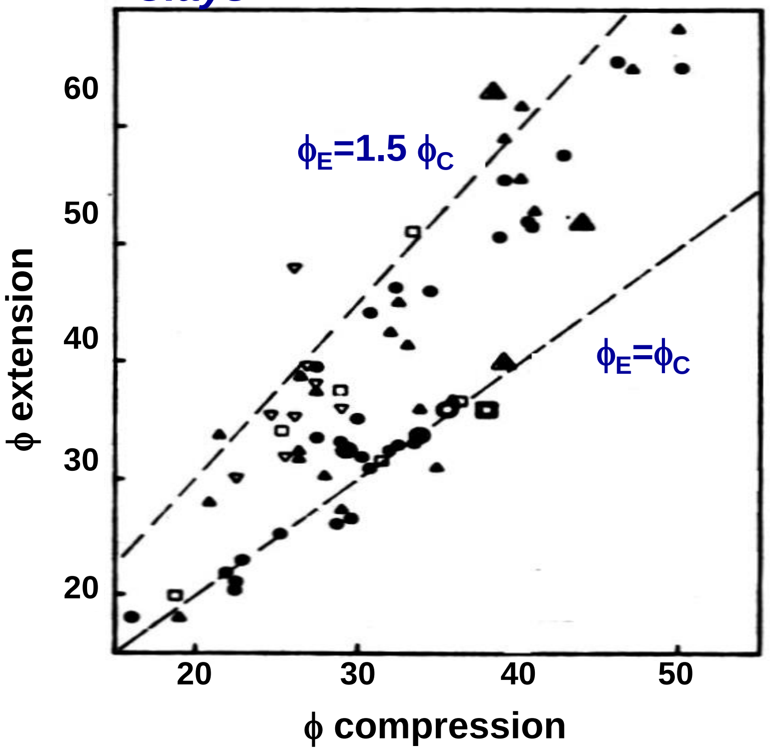
Frictional strength anisotropy

$\phi_E = 1.0$ to $1.5\ \phi_C$

Sands



Clays



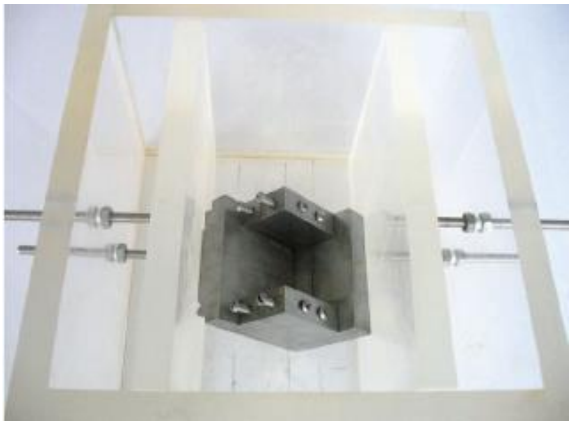
compiled by Ladd et al. 1977

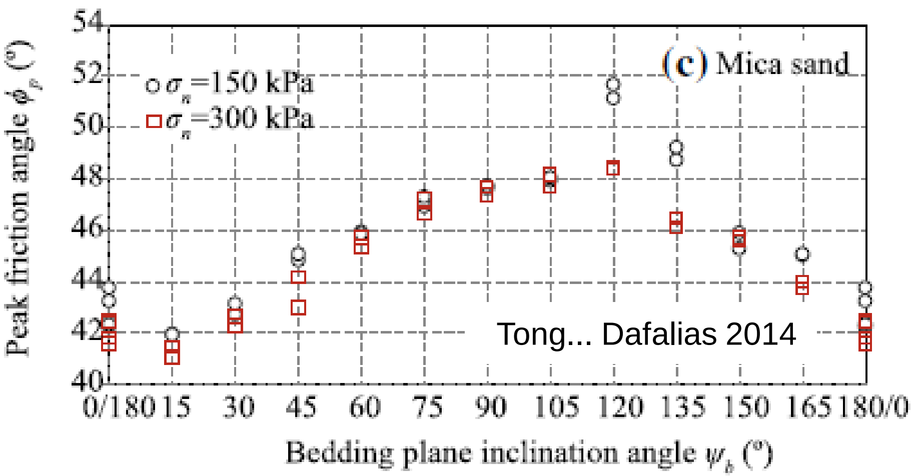
(see Lade and Duncan 1975)

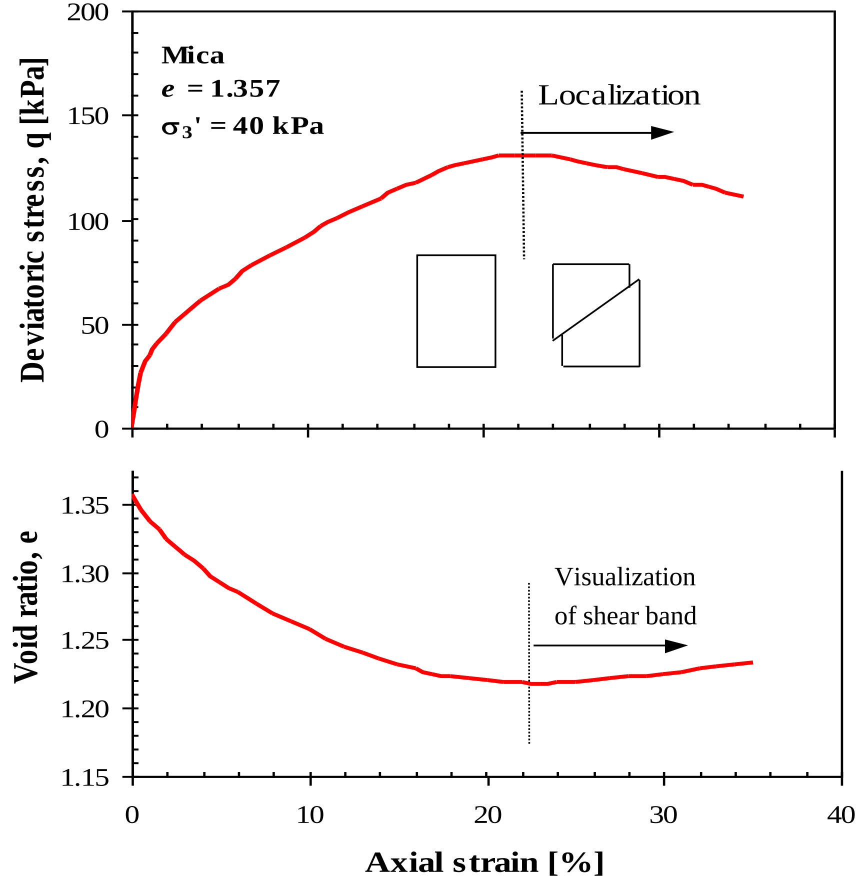
compiled by Mayne and Holtz (1985)

Inherent anisotropy

Oda et al. 1985
Rothenburg and Bathurst 1992/3
Aloufi, M. and Santamarina, J.C. 1995
Lade and Kirkgard (2000)
JCS & Cho 2003







Shear strength = f(α, ecc)

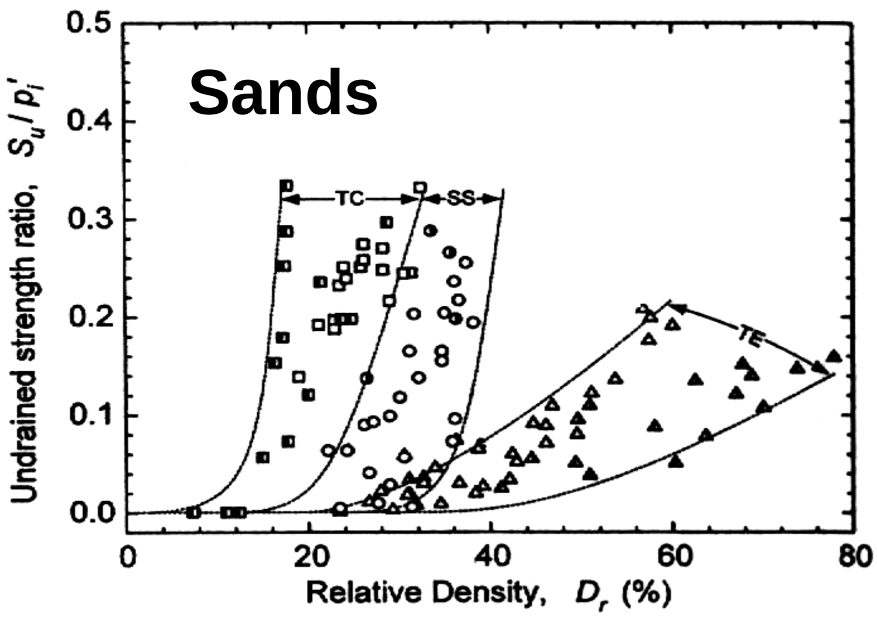
Eccentric → Localization

L/w ~1.1 is enough

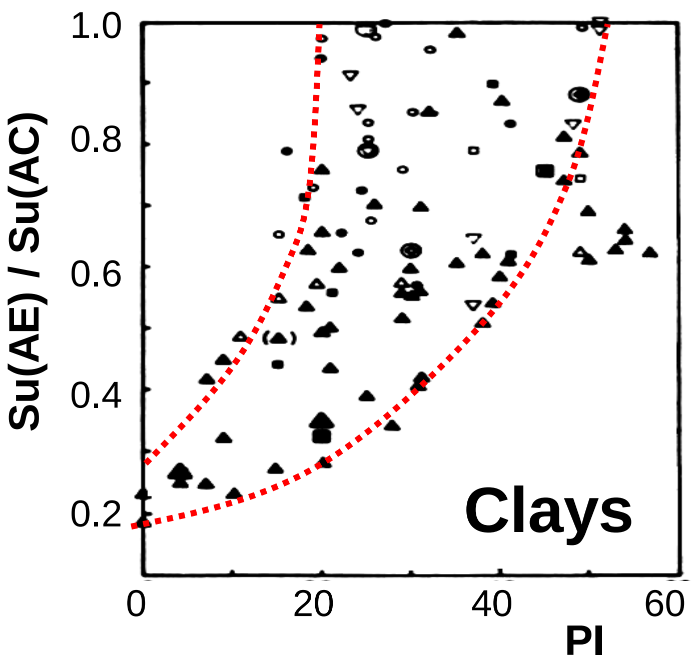
Undrained strength anisotropy (Ladd 1967)

Controlled by the generation of pore pressure

- ***chain buckling and skeletal stiffness***
- ***spatial variability of e***
- ***threshold strain***



Yoshimine et al 1999

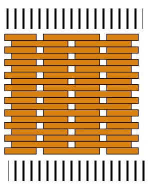


Mayne and Holtz 1985

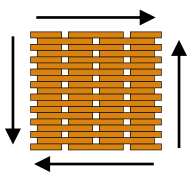
Undrained strength anisotropy

Controlled by the generation of pore pressure

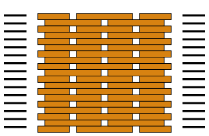
- ***chain buckling and skeletal stiffness***
- ***spatial variability of e***
- ***threshold strain***
- ***fabric anisotropy***



AC: b=0 α=0

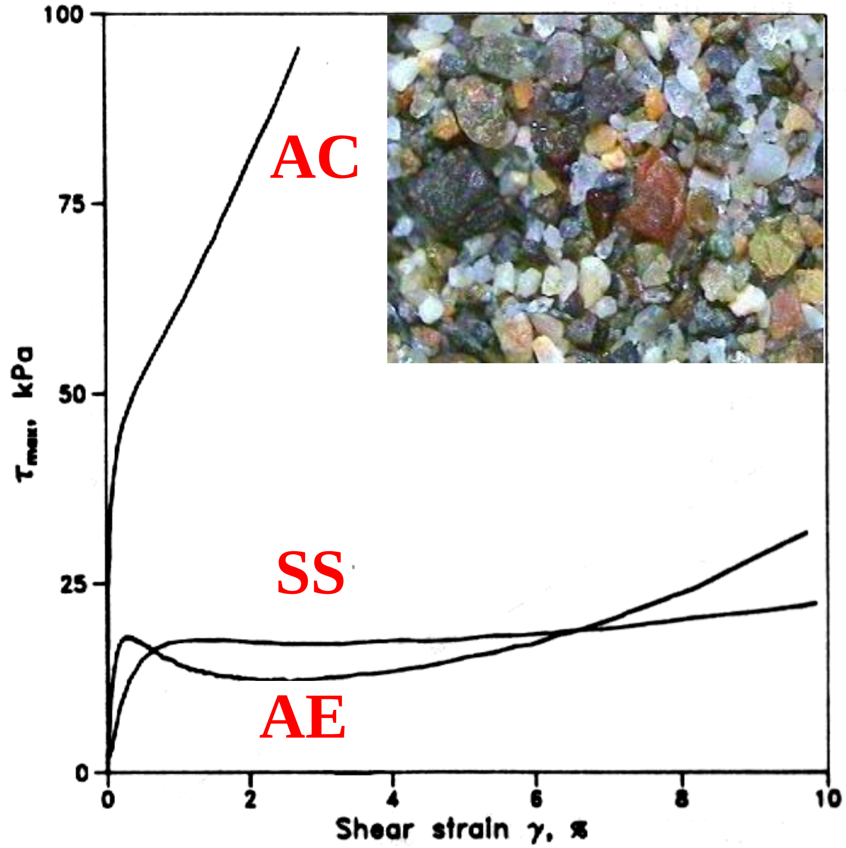


SS: b>0 α>0



AE: b=1 α=90

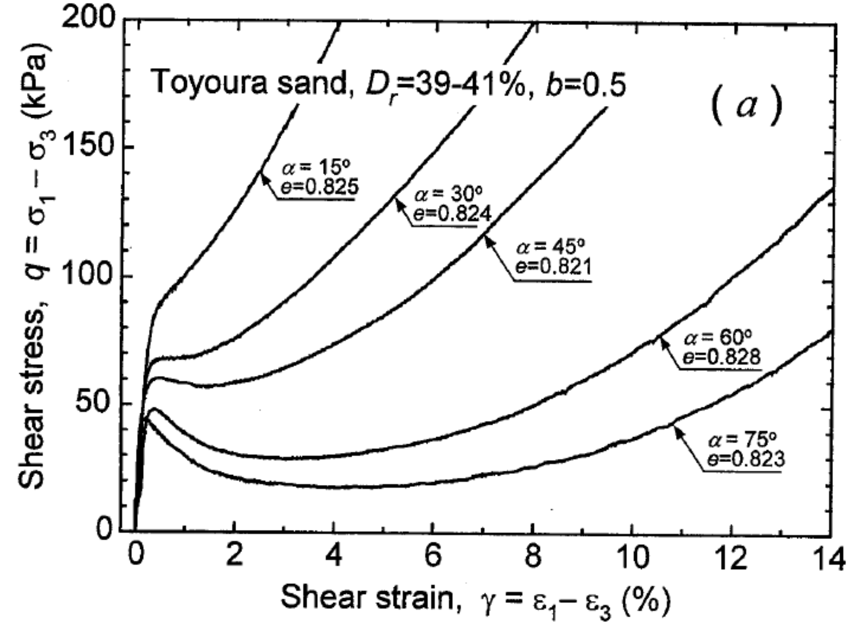
Fraser River Sand - Vaid & Sivathayalan 1996

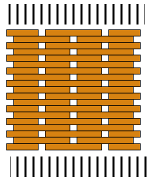


higher compressibility within bedding plane

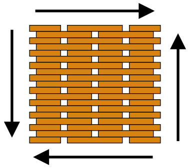
Undrained strength

$b = (\sigma_2 - \sigma_3) / (\sigma_1 - \sigma_3)$
α: angle between particle normal and σ_1

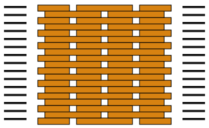




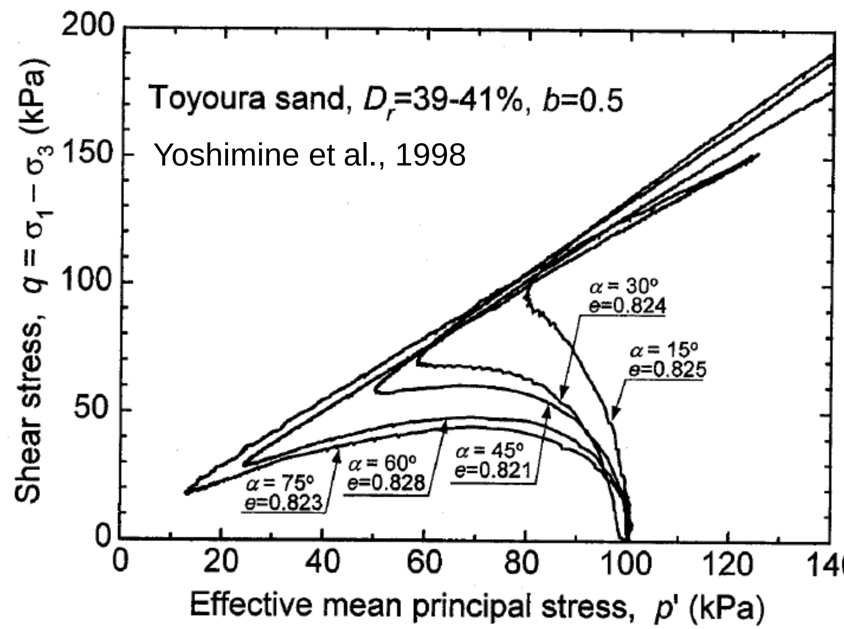
AC: b=0 α=0



SS: b>0 α>0



AE: b=1 α=90



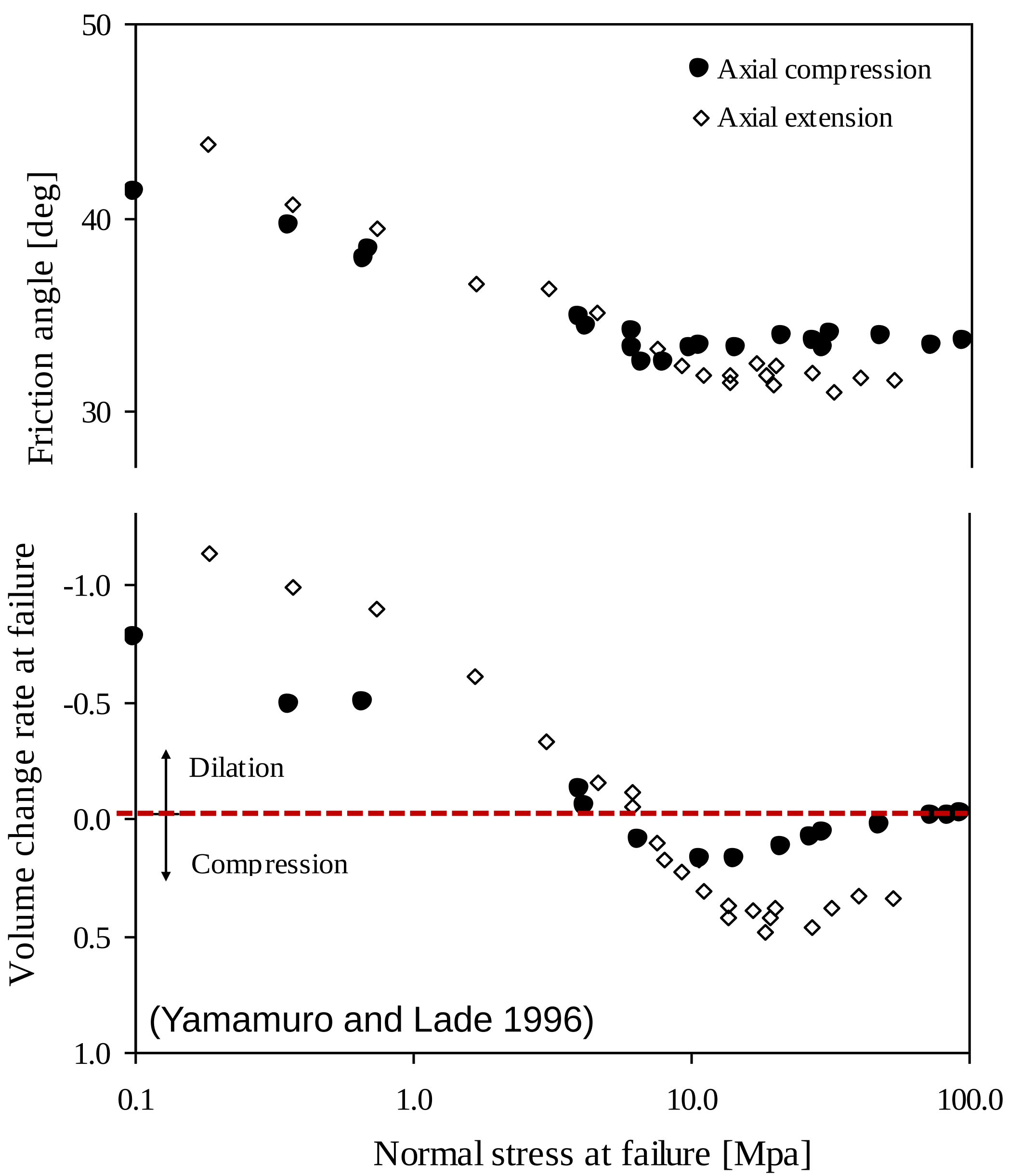
initial fabric affects behavior

Crushing

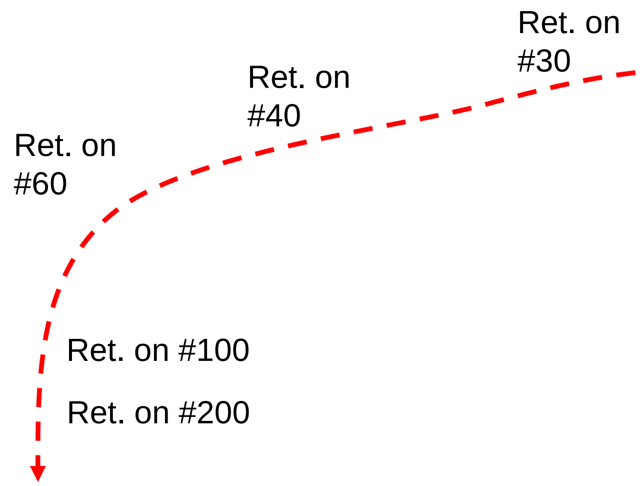
dilate or slide .... or BREAK

Grain Crushing

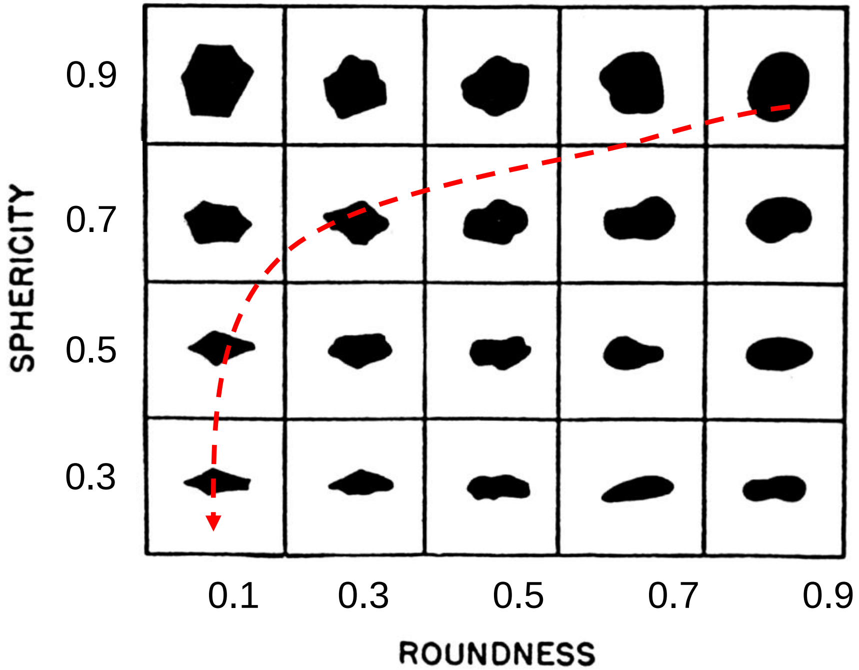
Dilation and ϕ_{peak}



Grain Crushing: Size and Shape



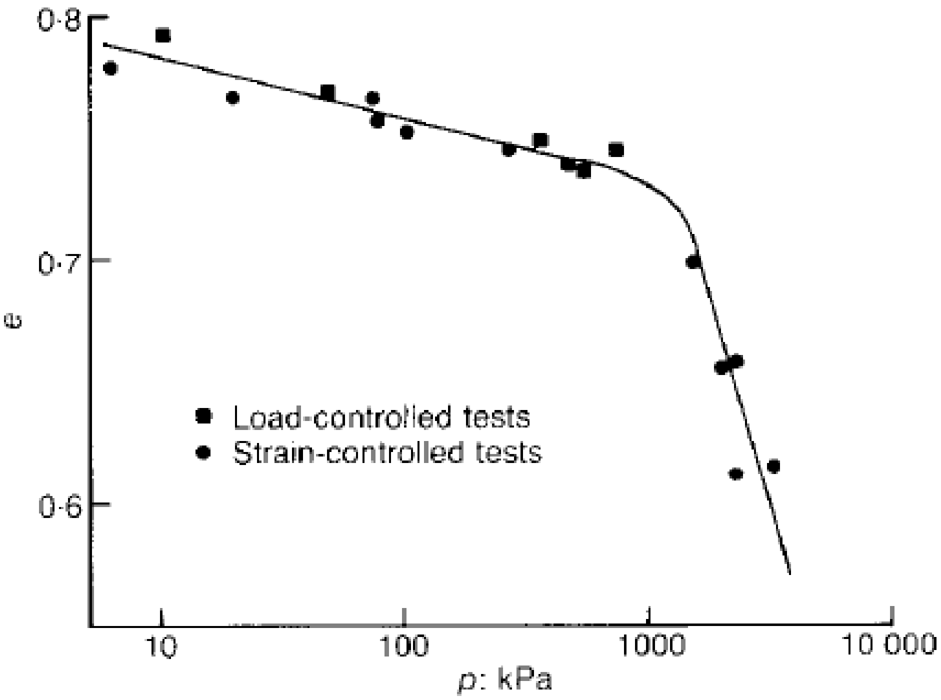
Krumbein and Sloss (1963)



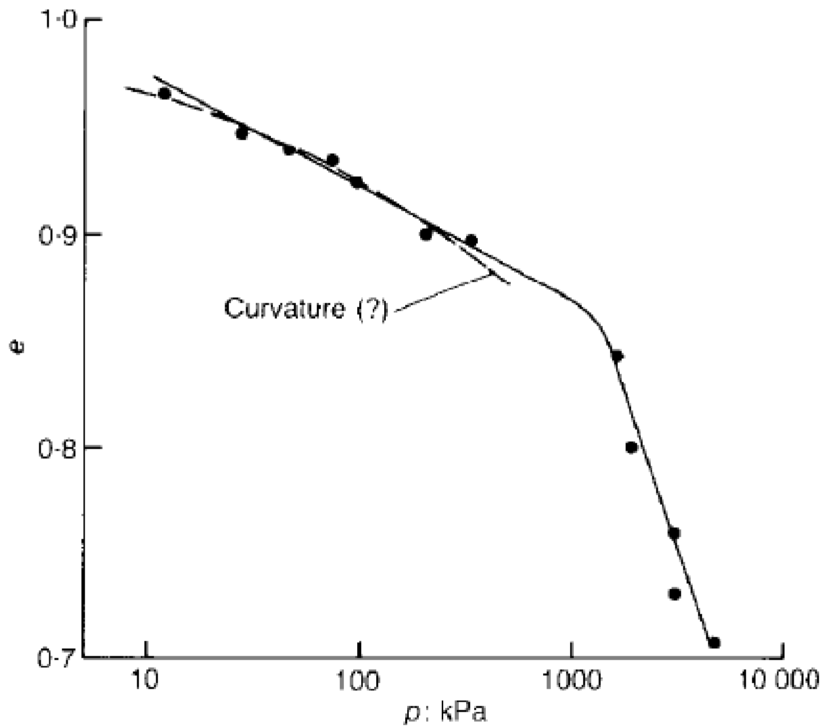
Change gradation, coordination number, shape ➔ change in critical state

Crushing ➔ CSL

Erksand sand



Leighton Buzzard sand



Been et al., 1991

Linear CSL in e-log(p’): subrounded sans at low stress p’<500 kPa

Strain to critical state

is CS a terminal state?

Threshold Strains

Contact sliding – Coarse soils

$$\frac{\delta^*}{d} = \frac{3}{4}\mu(2-\nu_g)\frac{\sigma'}{G_g}\frac{d}{d_c} \qquad \frac{d_c}{d} = \sqrt[3]{\frac{3(1-\nu_g)}{2}\frac{\sigma'}{G_g}}$$

$$\gamma_t = 1.26\mu\left(\frac{\sigma'}{G_g}\right)^{2/3}$$

Contact loss – Fine grained soils

$$\gamma_t = \frac{1\overset{o}{A}}{40\overset{o}{A} + t}$$

Strain level for constant volume shear

$$\gamma_t \approx 100\%$$

Strain level for particle alignment

$$\gamma_t > 100\%$$

Strain level for particle segregation

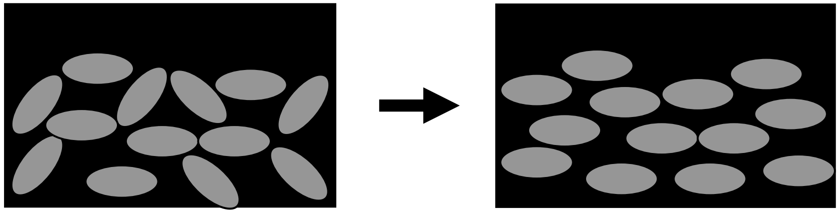
$$\gamma_t >> 100\%$$

If CS is not a terminal state... then, what is the strain level of interest?

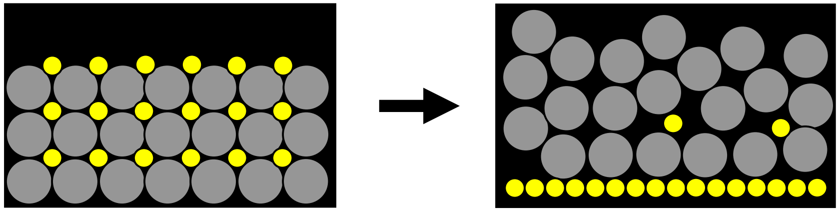
$\phi_r < \phi_{cv}$

Residual Friction Angle

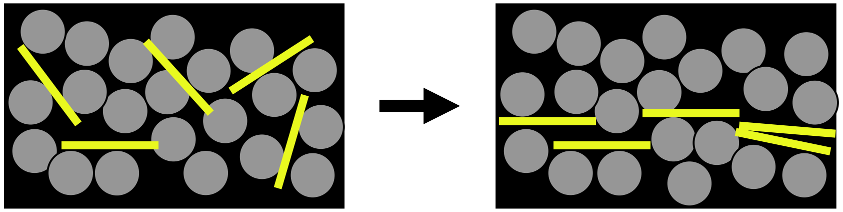
particle alignment



size segregation



shape segregation



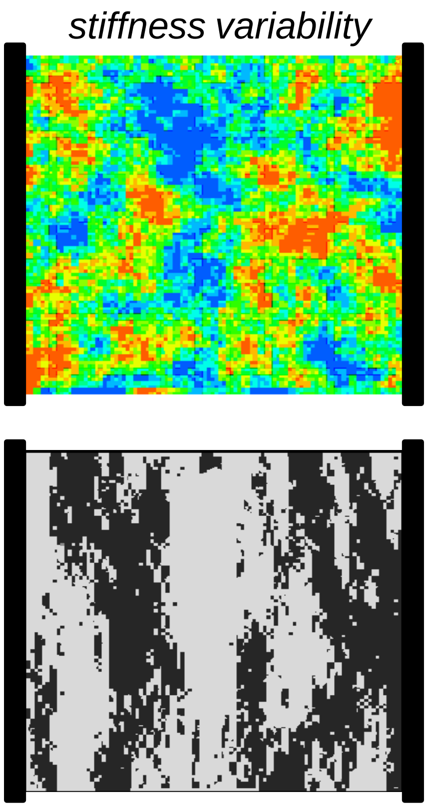
bond breakage ....destructuration

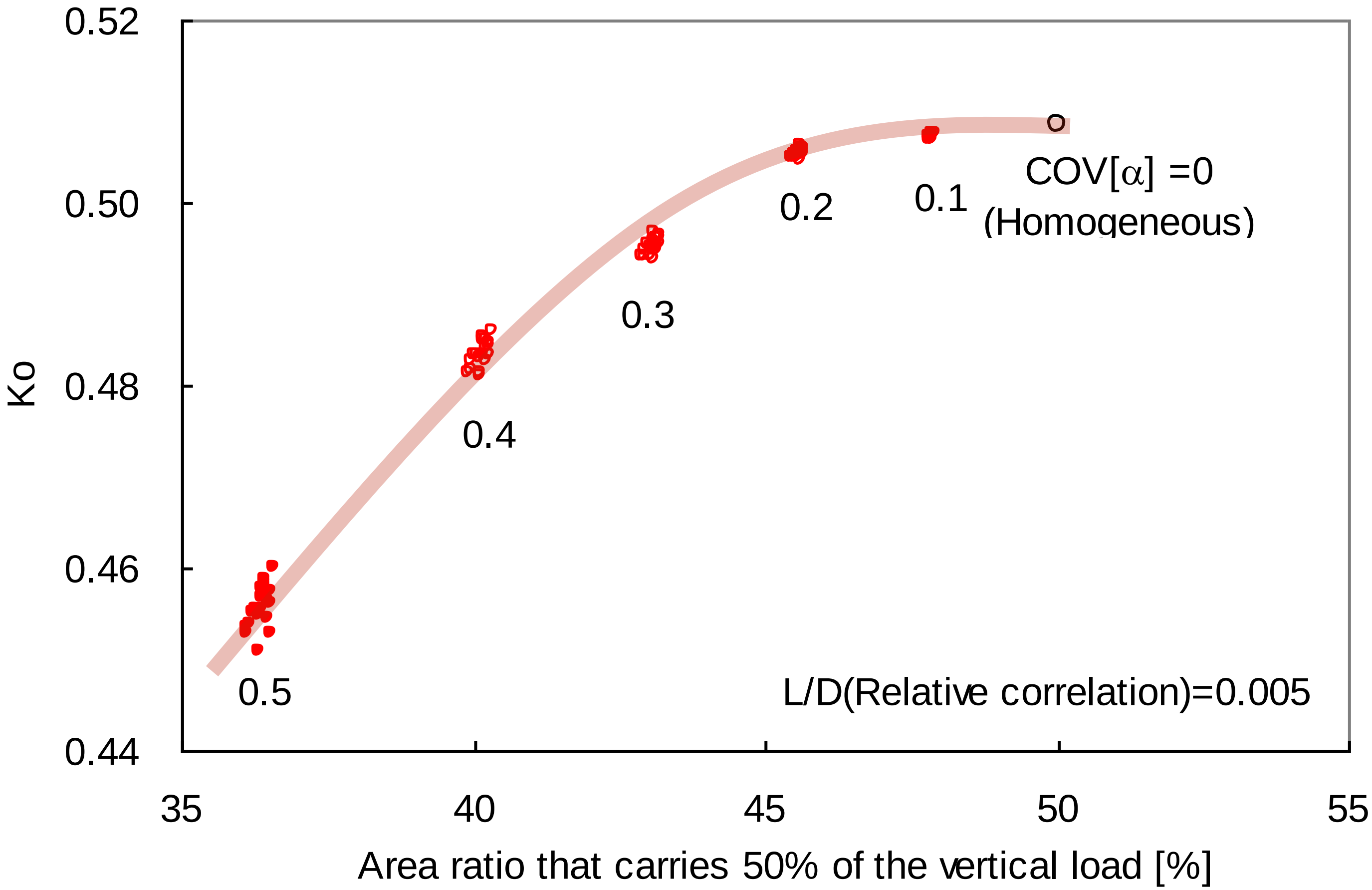
Mobility & large strain !!

Spatial Variability

heterogeneity

Compression (k_o)

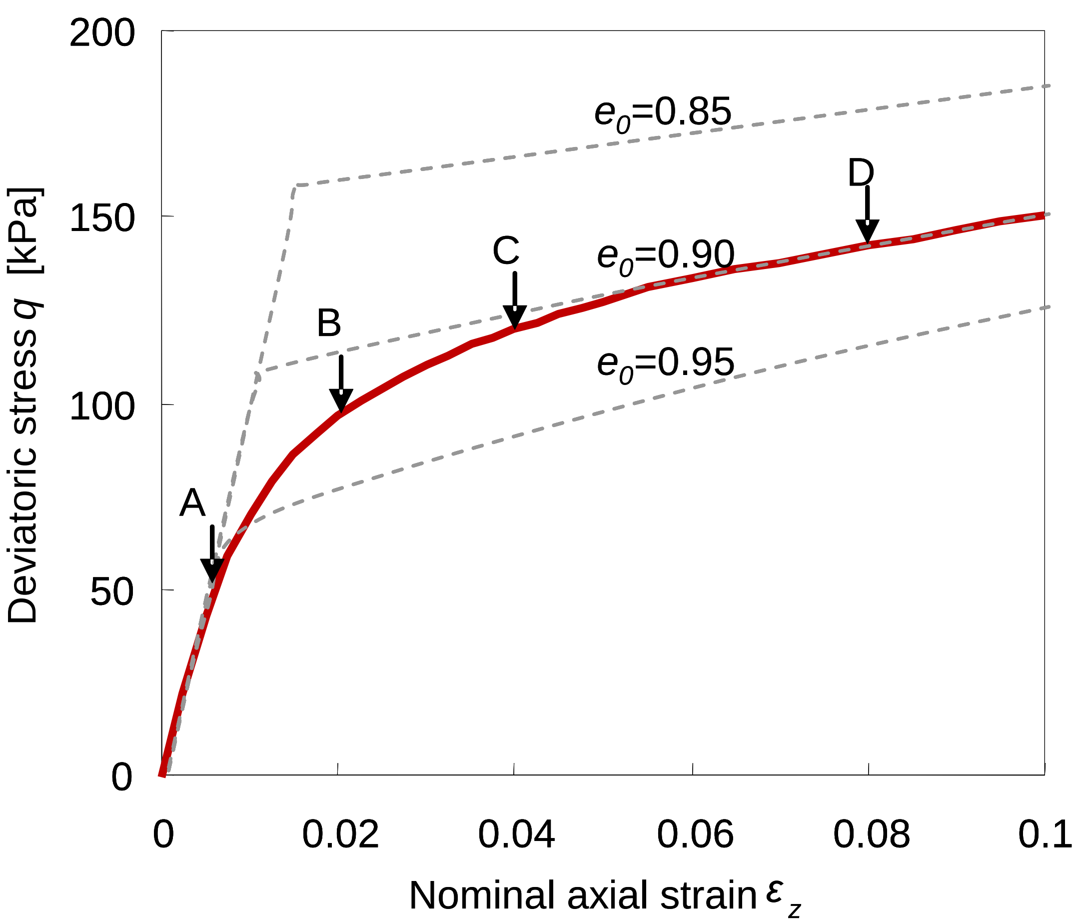


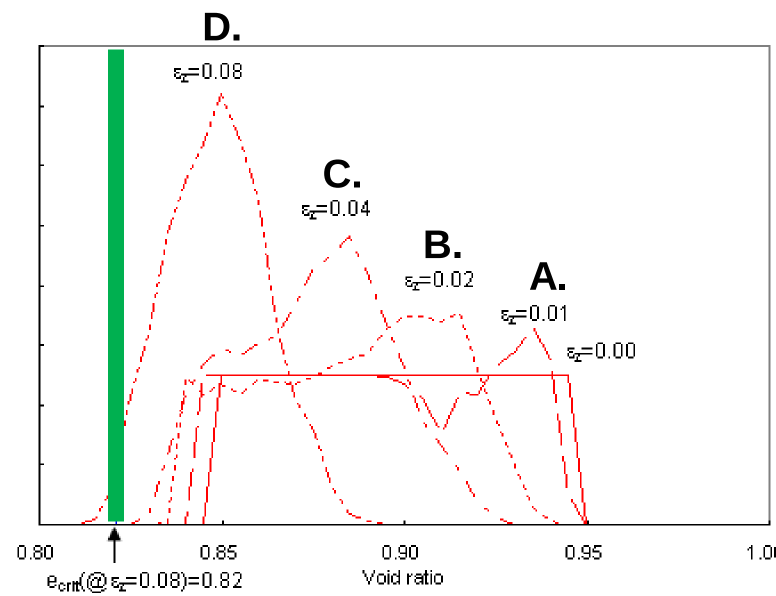


variability ↑ *stress focusing* ↑ k_o ↓

D/L_{corr}~1

Contractive - *Drained*

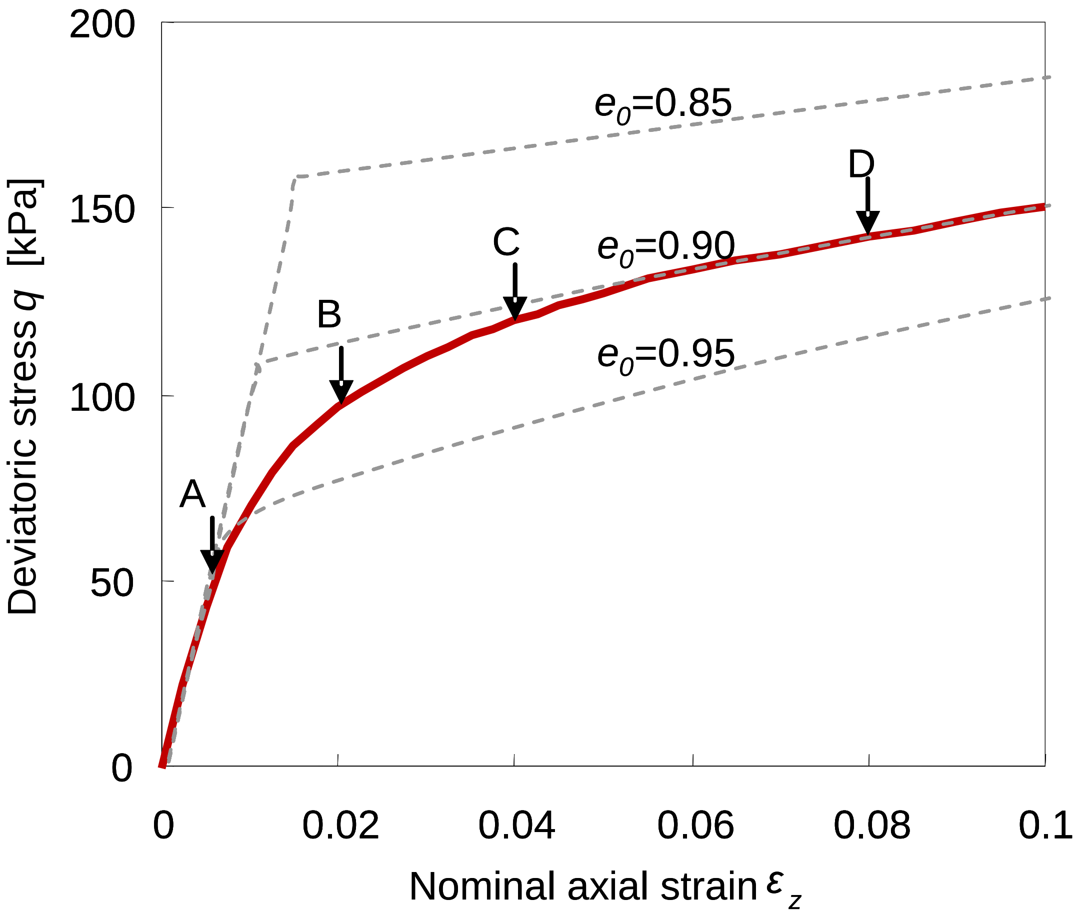




contractive & drained shear: homogenization towards critical state

D/L_{corr}~1

Contractive - *Drained*

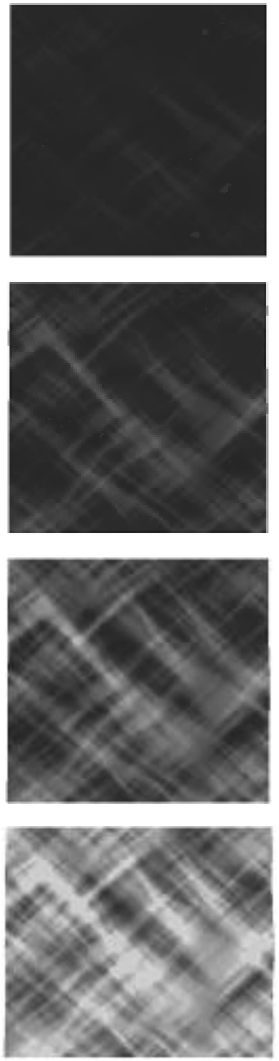


A.

B.

C.

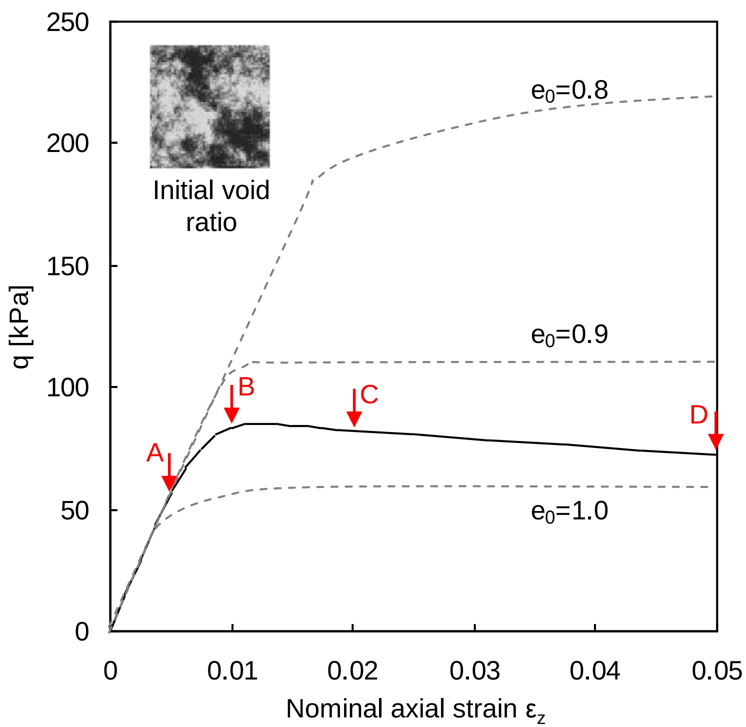
D.



contractive & drained shear: homogenization towards critical state

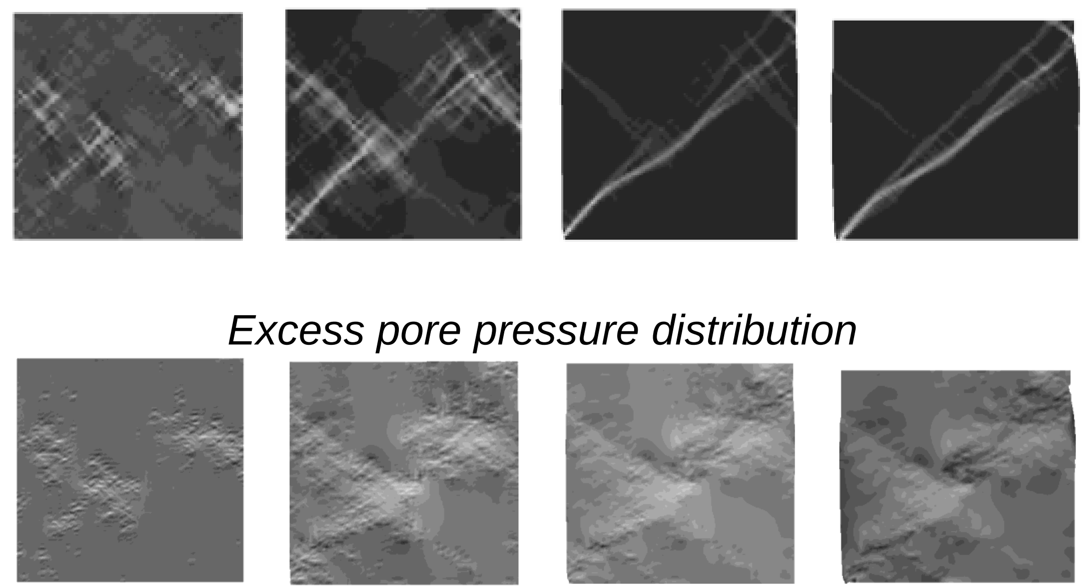
Contractive - *Undrained*

locally undrained



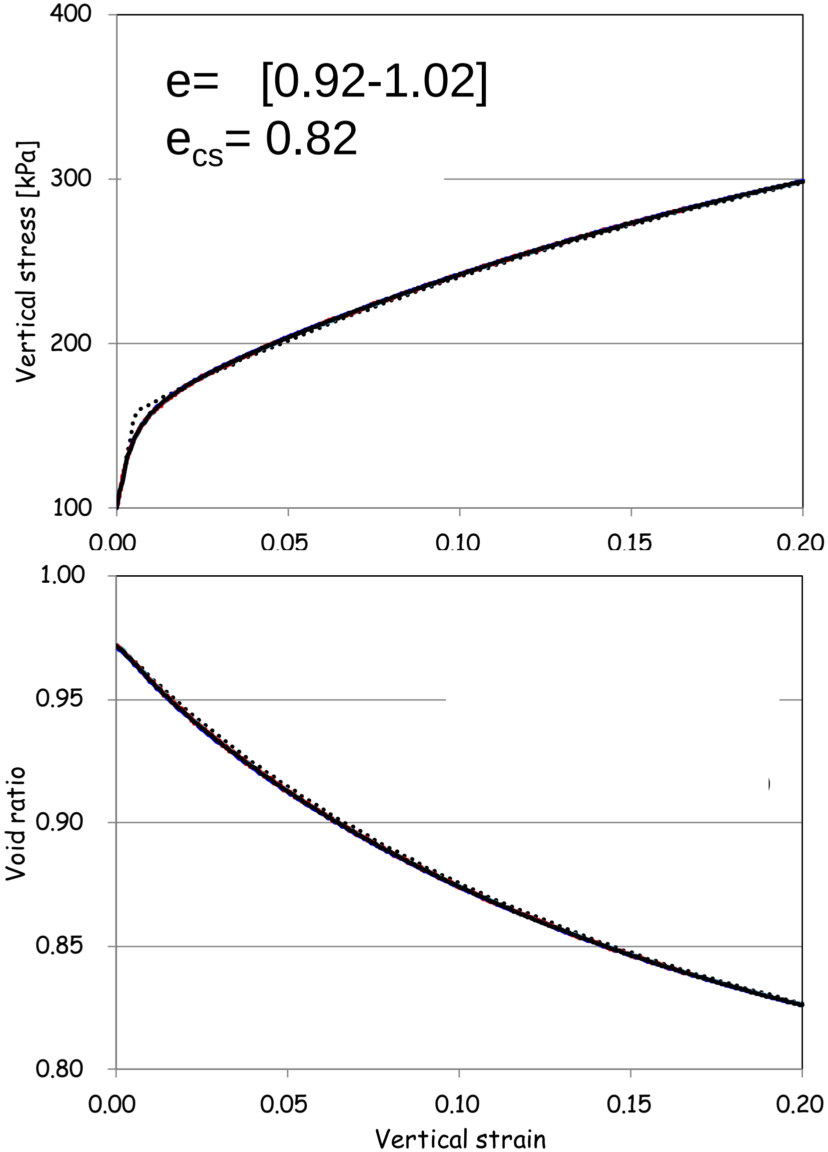
Shear strain distribution

Excess pore pressure distribution

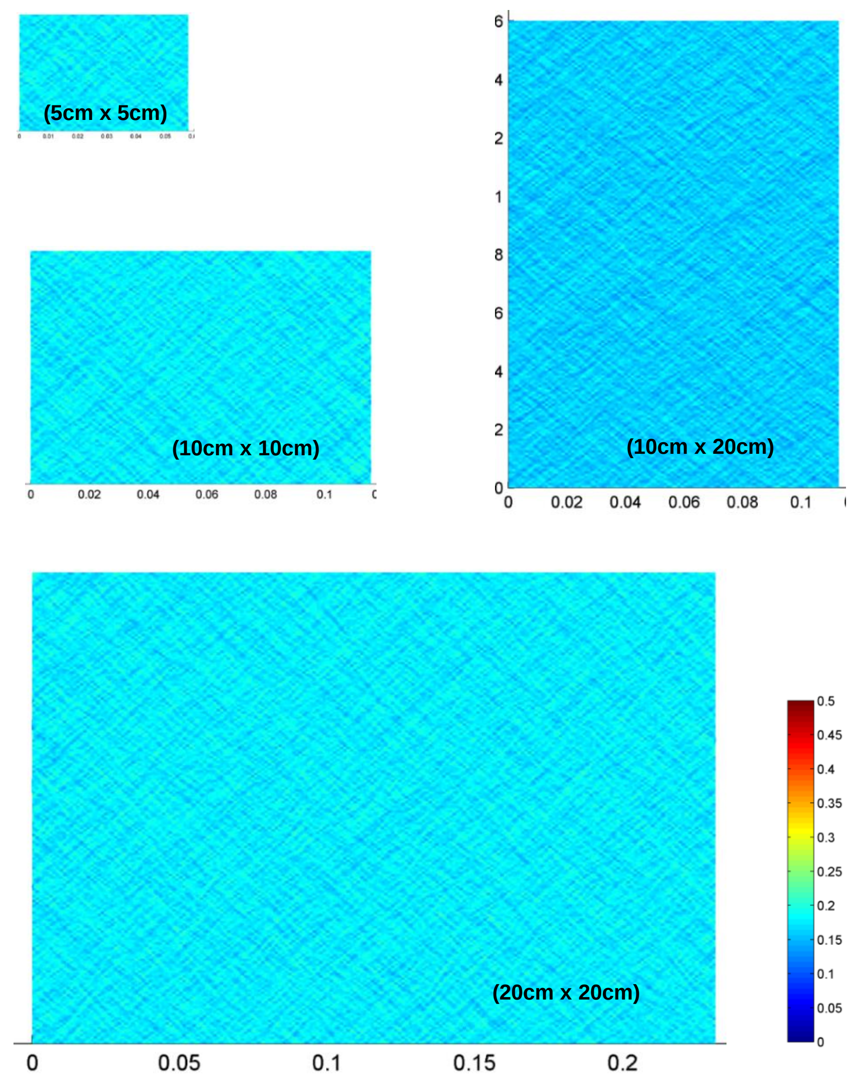


A. ε_z=0.005 B. ε_z=0.010 C. ε_z=0.020 D. ε_z=0.050

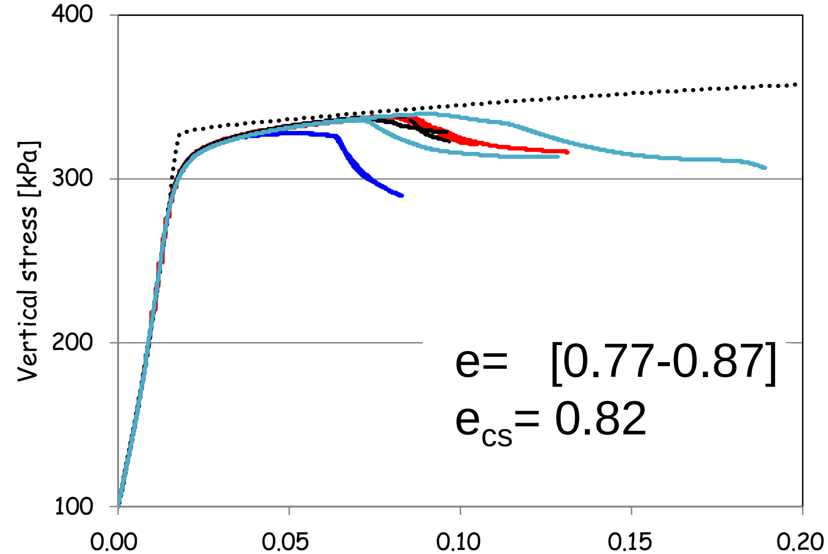
Specimen Size Effects: Contractive

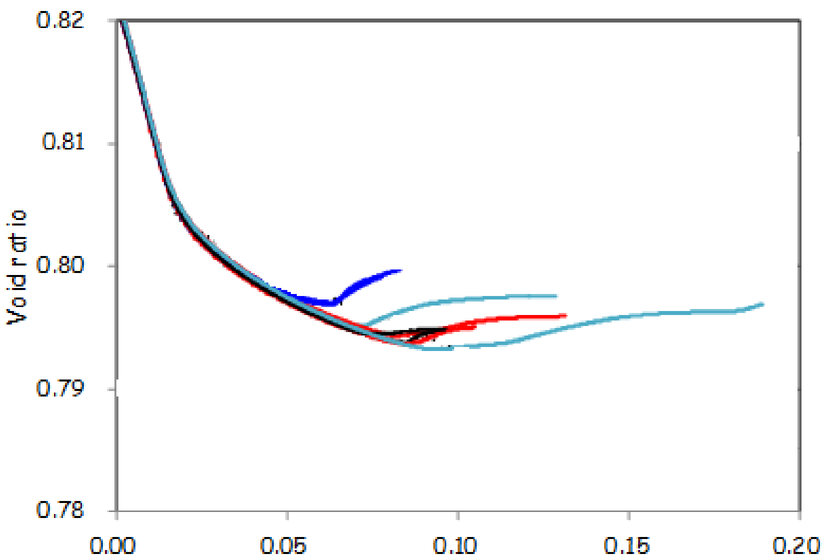


δ-fields @ ε_z=20%

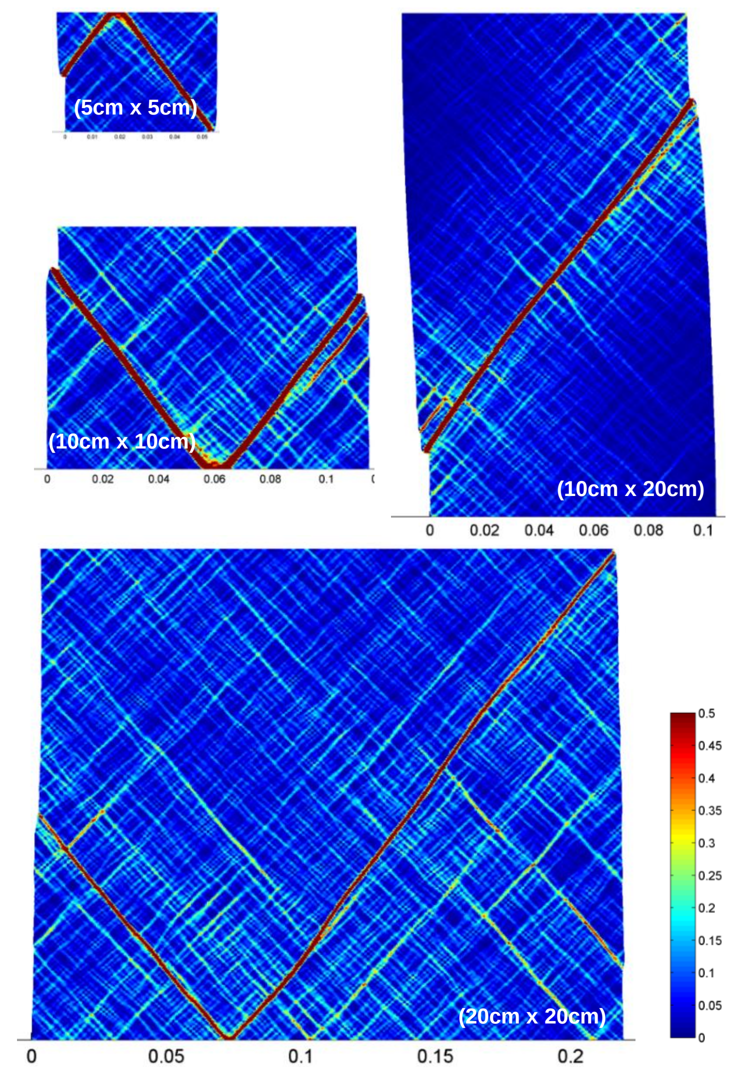


Specimen Size Effects

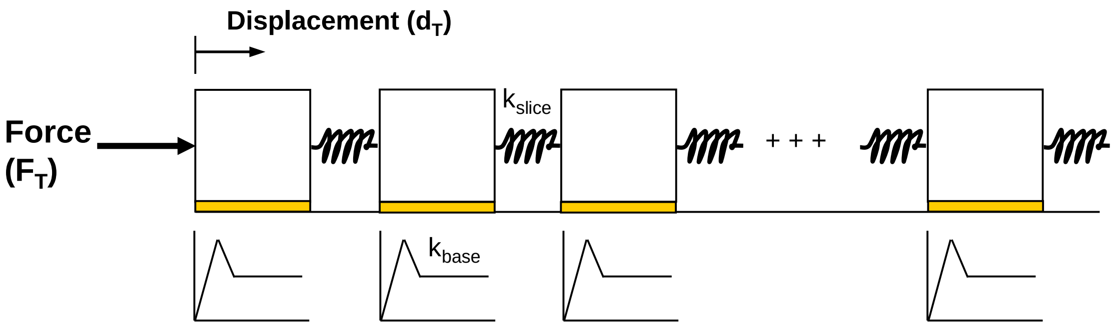


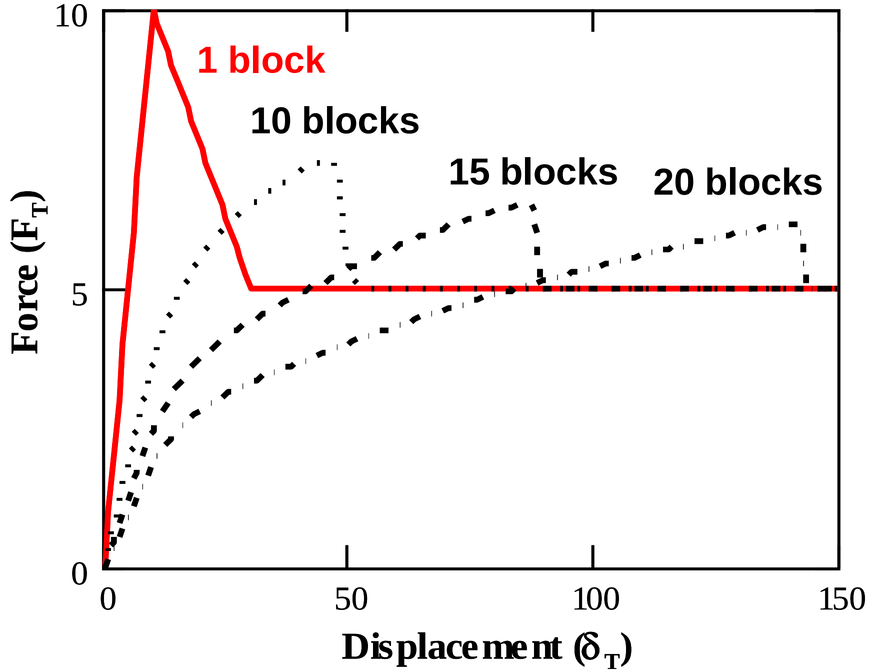


δ-fields @ end of test



Specimen-size dependent p-q-e

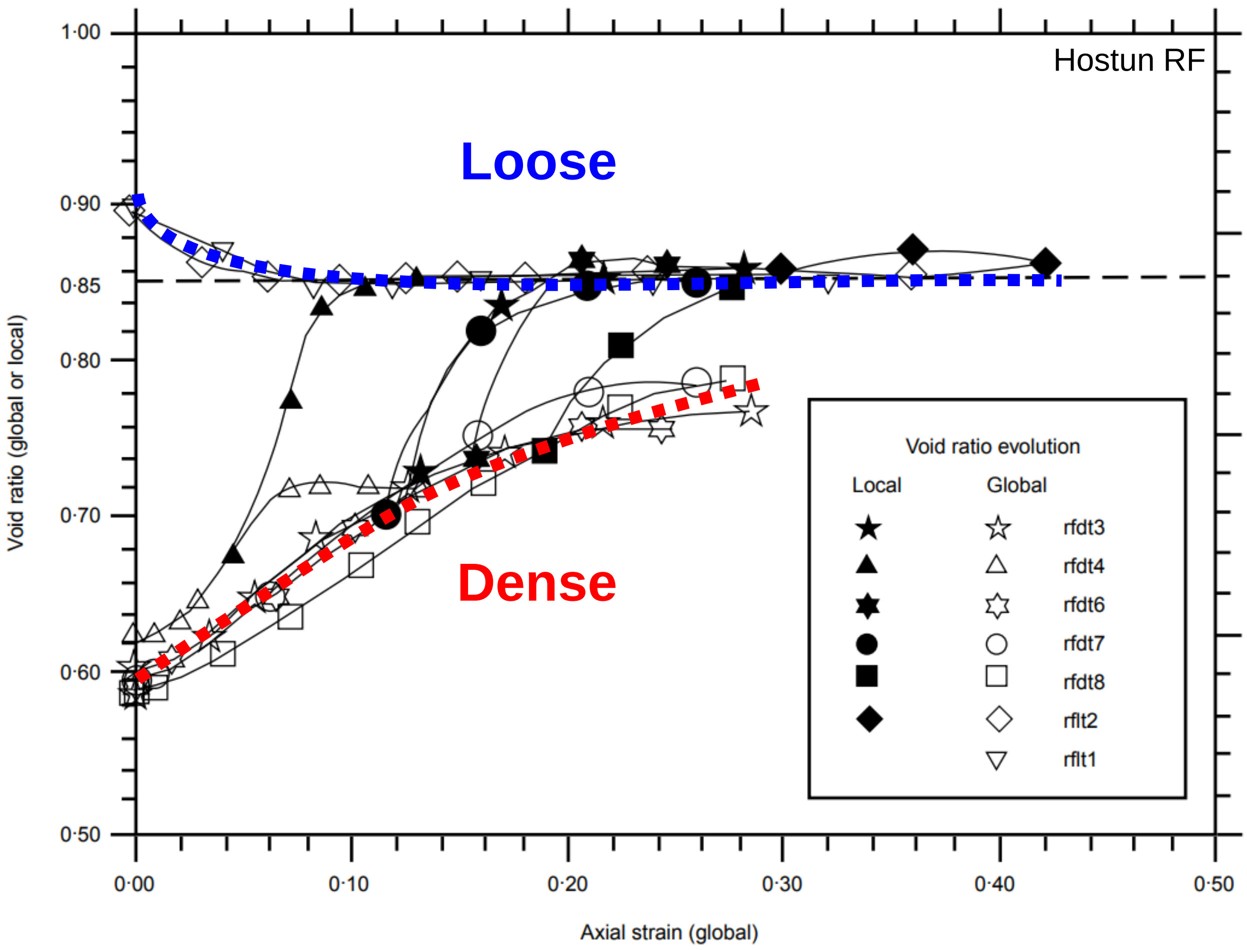




Localization ➔ stiffness, ψ, ϕ_{peak}, e_{cs} .... specimen-size dependent

Localizations

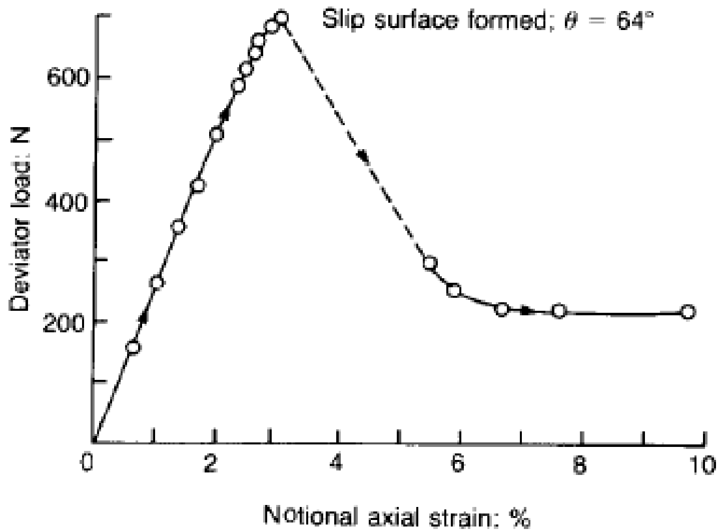
Experimental Limitations

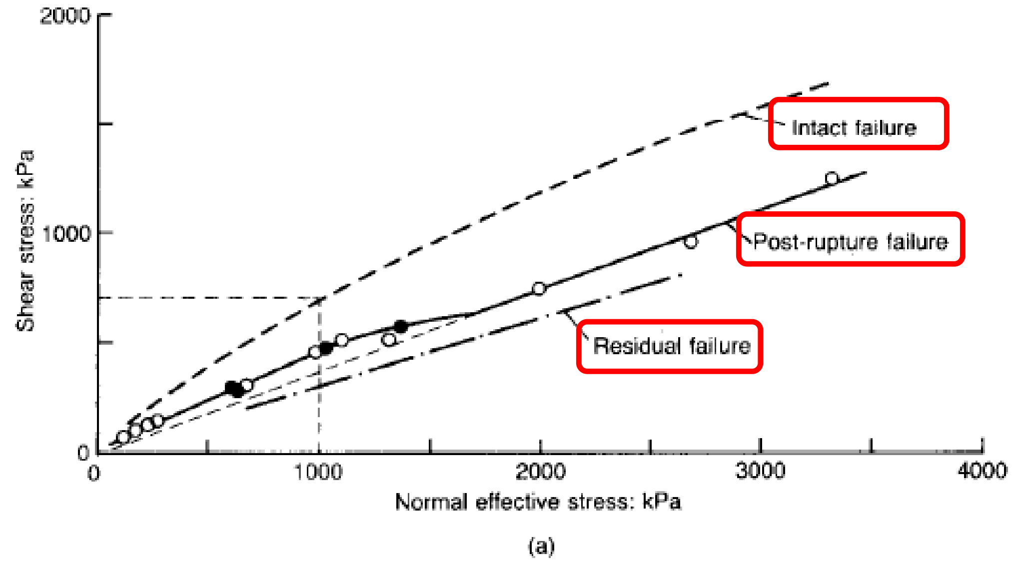


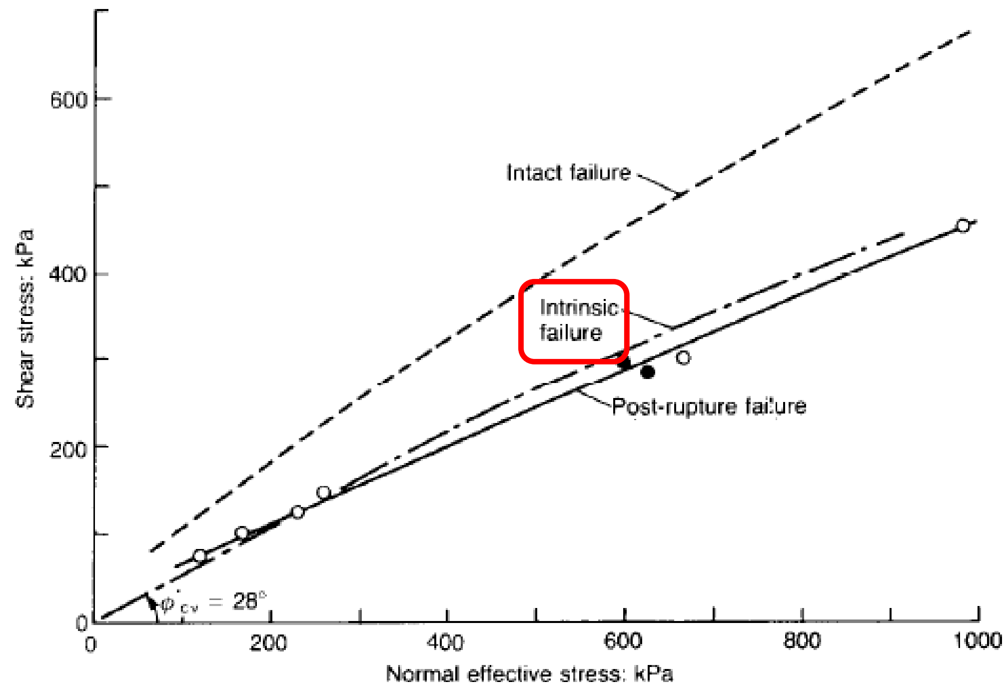
Desrues et al. 1991/6
Desrues & Viggiani 2004
Oda & Kazama 1998

Non CS-fabric within shear band

Localization







Burland 1990

Localization

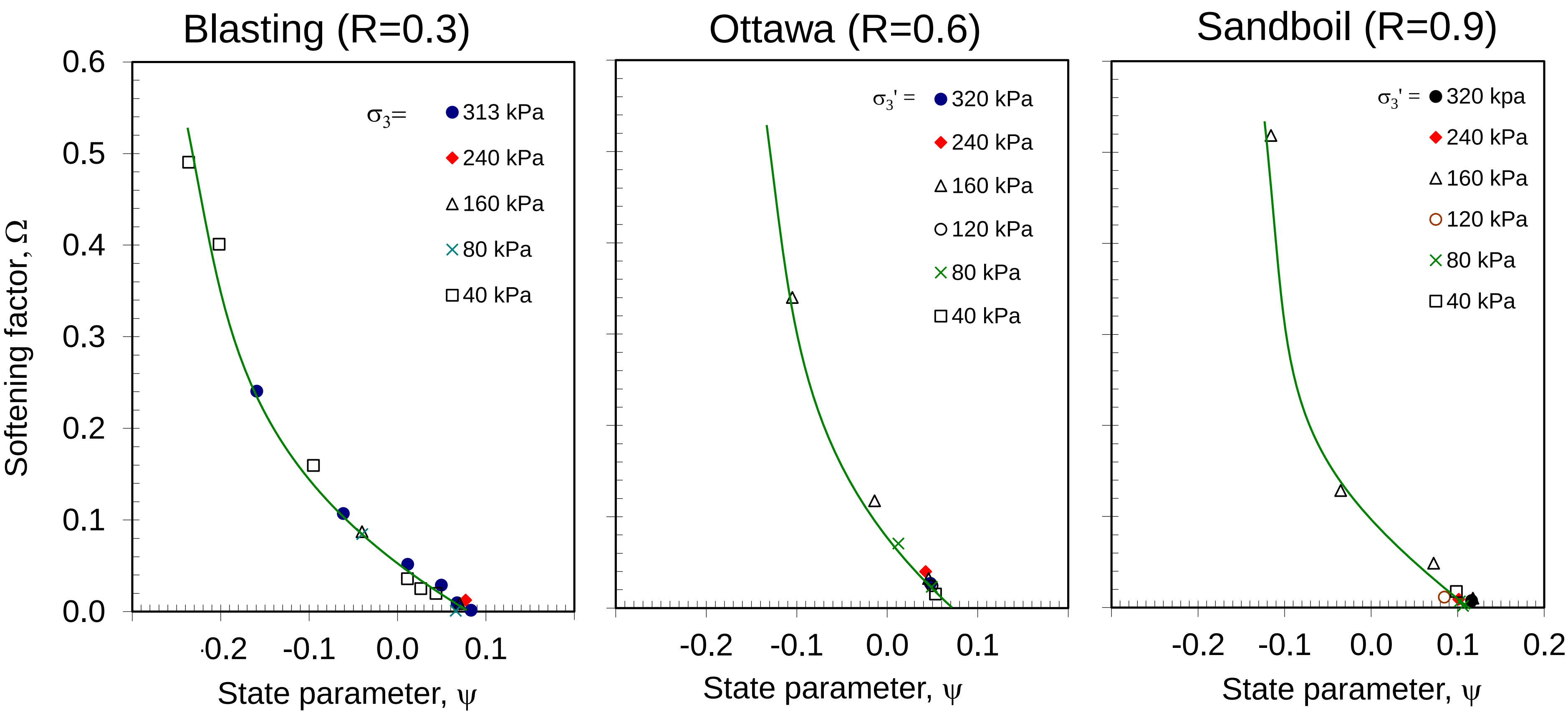
- Dilative drained
- Dilative undrained due to cavitation
- Contractive undrained *(locally undrained)*
- Eccentric particles - residual strength
- Cemented material due to breakage of cementation
- Unsaturated soils due to breakage of meniscus failure
- Non-homogeneous specimen in drained and undrained
- Non-uniform grain crushing or void collapse

omnipresent !!

State parameter - Softening - CSL

softening factor

$$\Omega = \frac{(\sigma'_1 - \sigma'_3)_{max}}{(\sigma'_1 - \sigma'_3)_{CS}} - 1$$

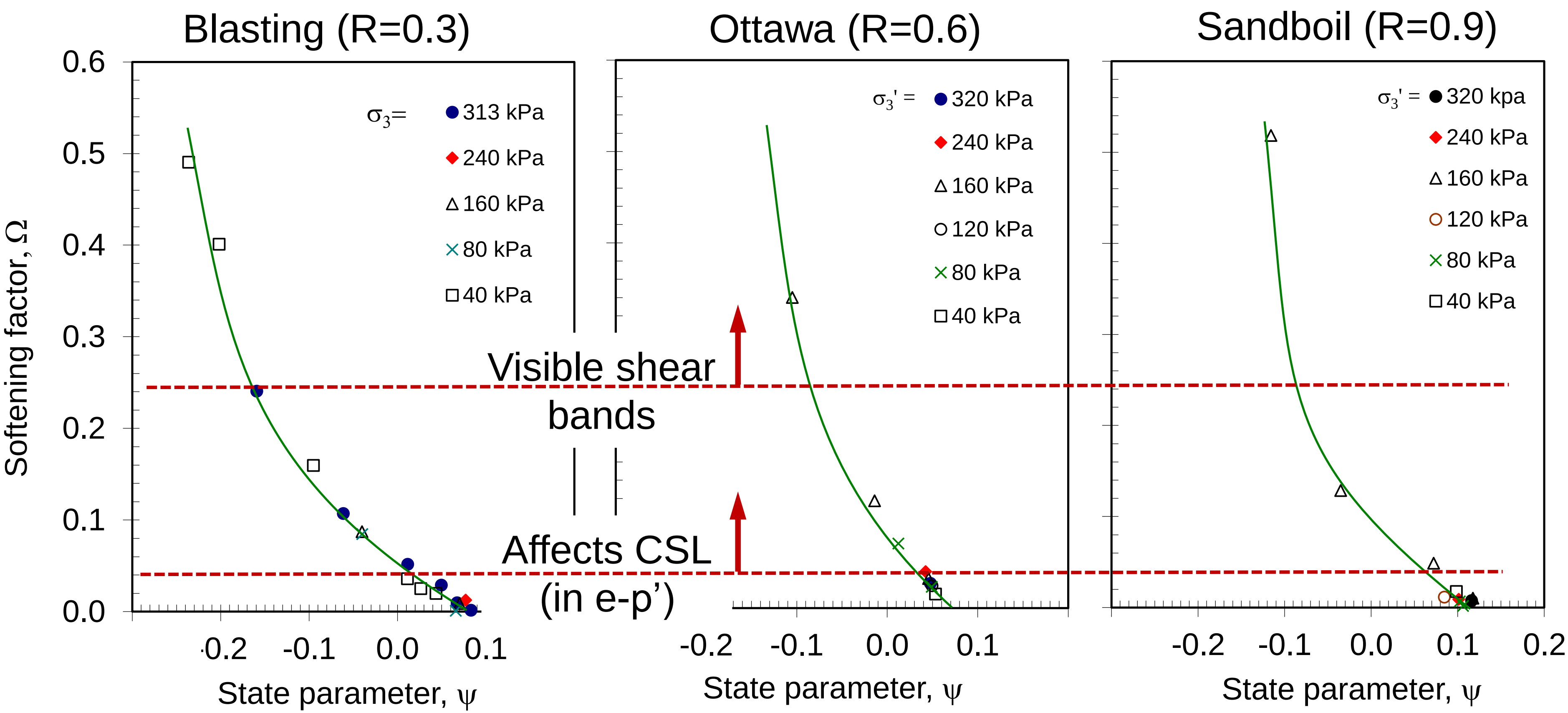


CS requires proper test design .... But relevance to field?

State parameter - Softening - CSL

softening factor

$$\Omega = \frac{(\sigma'_1 - \sigma'_3)_{max}}{(\sigma'_1 - \sigma'_3)_{CS}} - 1$$



CS: soil-intrinsic .... or a procedural definition (test-dependent)?

Closing Thoughts

CS: Education and Practice *(Limited survey - 2017)*

Education

ASIA:	Less than 1% of universities teach CSSM as a formal course A section: <10% (more in the 1980's)
EUROPE:	<50% in UK, Spain, Portugal, Italy, Greece ..... Much less in central & Nordic countries
AMERICA:	some introduction only (2 books in USA have a chapter ~10% universities)

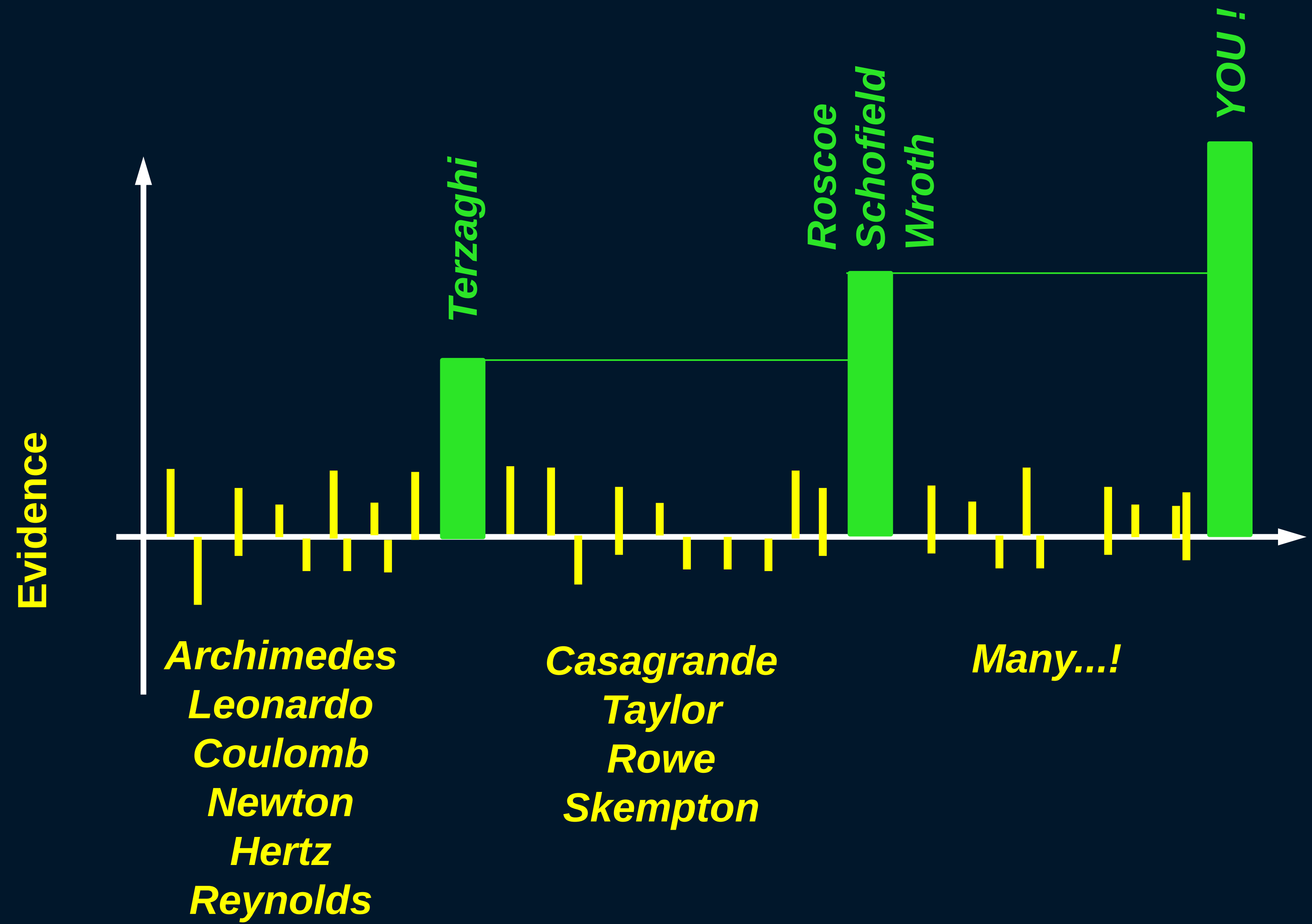
Engineering *jobs analyzed using critical-state (or related models)*

YES:	complex jobs by large companies
NO:	small companies and routine jobs some if built-in commercial software but without understanding

(Sources: C. Ng, GC Cho, HS Shin, A. Gens, M. Pantazidou R. Bachus, PW Mayne, A. Welker's

why is adoption so low?

Scientific Revolutions: Geotech



Sources not listed in figures: contributions by

GC Cho
HK Kim
HS Shin
JR Valdes
GA Narsilio
JW Jung
JH Park
SH Chong
A Palomino

Visit egel.kaust.edu.sa for publications

Thank you