ALERT Geomaterials Workshop 2017

ADVANCED NUMERICAL MODELLING OF GEOMATERIALS

Grupo de Modelos Matemáticos en Ingeniería Civil – M2i

POLITÉCNICA

Coupling solid and fluid phases in fast landslide propagation

Manuel Pastor, **Ángel Yagüe**, Miguel Martín Stickle, Diego Manzanal, Pablo Mira, Saeid Mousavvi, Chuan Lin, José Antonio Fernández Merodo and Caitlin Chalk

Alert Geomaterials

Contents

- Introduction
- Mathematical model
- Numerical models: SPH
- Examples and applications
- Conclusions

Contents

- Introduction ⇐
- Mathematical model
- Numerical models: SPH
- Examples and applications
- Conclusions

General Flow definition

- Spatially continuous movement of rock, debris or earth down a slope.

- Triggered from the failure of the materials which make up the hill slope and are driven by the force of gravity.

- Ussually extremely rapid.

- The distribution of velocities in the displacing mass seems like a viscous liquid.

Types of flows

Granular avalanches

Flow of fragmented rock

•One phase (solid)

Debris flows, lahars

Flow of partially or
fully saturated debris.

•Two phases (solid-water)

•Three phases (solid-water-air)

Mudflows

Flow of saturated plastic debris

•Two phases (solid-water)

Different types might be classify depending on the content of solid, liquid and/or air

Debris Flows

Prof.Sucheng Zhang

(Chengdu's Institute for Natural Hazards in Mountain Areas, CAS)

Important tasks to be model

- Triggering or initiation of landslide.
- Propagation of the mixture down the slope.
- Landslide mitigation.

Types of models needed in each task

- Mathematical models capable of reproducing interaction between possible phases.

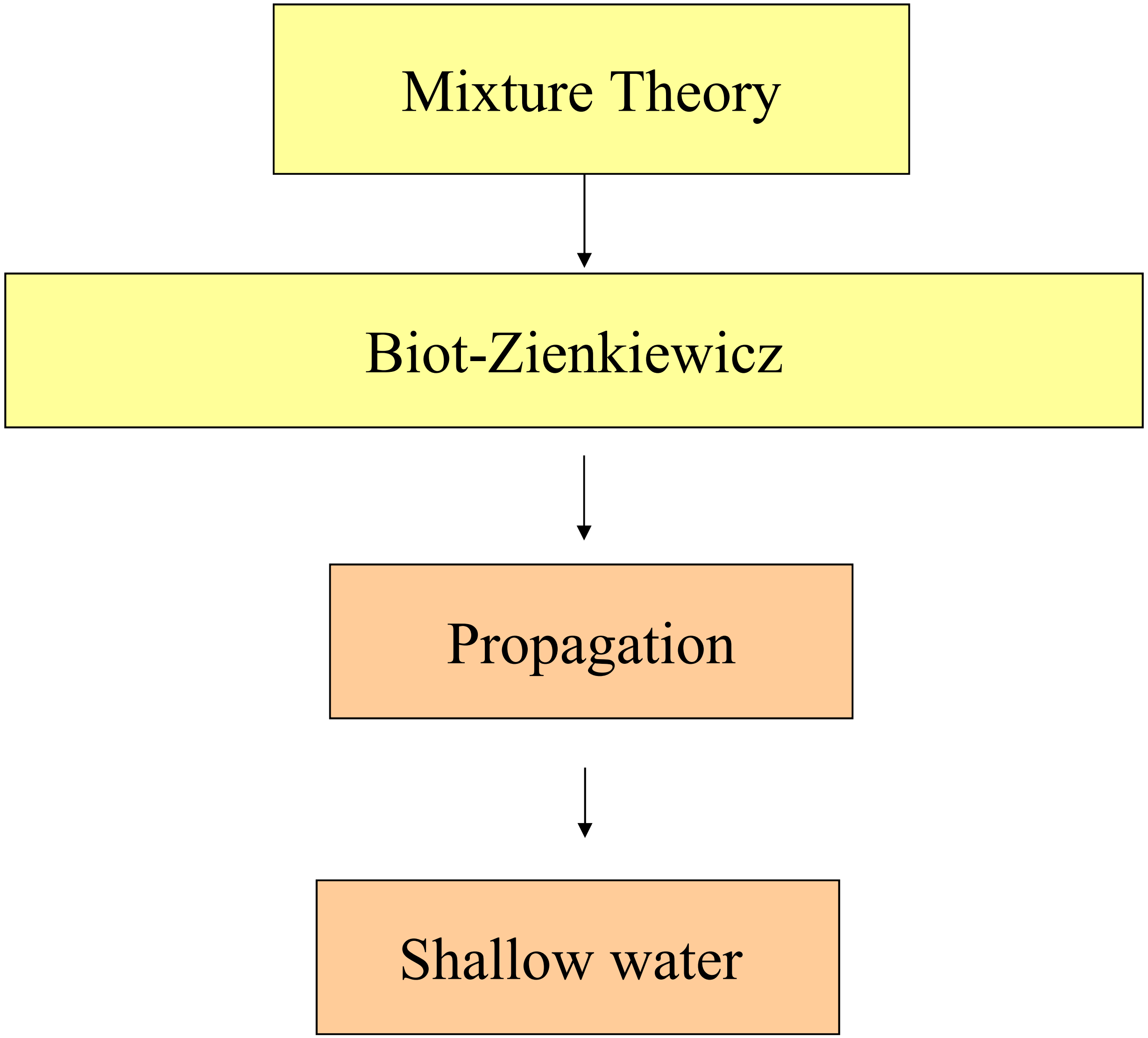
- Constitutive (initiation) or Rheological (propagation) models.

- Numerical models to obtain accurate, robust, and efficient solutions of real cases.

Contents

- Introduction
- Mathematical model
- Numerical models: SPH
- Examples and applications
- Conclusions

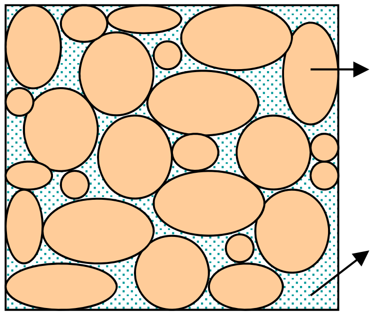
Outline of the Mathematical model



From Mixture Theory to Biot-Zienkiewicz eq.

Continuum description of the flow slide

REV(V)



$$\left.\begin{array}{l} volume\ \ fraction\ \ of\ \ solid\ \ n^s = \dfrac{V_s}{V} \\ volume\ \ fraction\ \ of\ \ fluid\ \ n^w = \dfrac{V_w}{V} \end{array}\right\} \begin{array}{c} n^s + n^w = 1 \\ (fully\ \ saturated) \end{array}$$

Average of real (intrinsic) quantities over REV
via multiplication by volumen fraction

The General Model

Variables

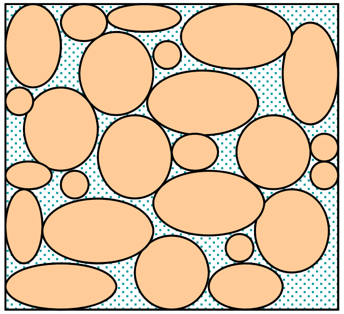
- n porosity
- σ_s, p_w stresses
- v_s, v_w velocities
- d_s rate of deformation
- ~~ρ_s~~, ρ_w densities

Equations

- Balance of mass (soil,water)
- Balance of momentum (soil,water)
- Constitutive or rheological (skeleton, soil, water)
- Relations velocities – rate of deformations

Vs-Vw-Pw: 6 equations - 6 unknonws

$n \;\; v_s \;\; v_w \, \sigma' \, p_w \, d_s$



Balance of mass

$$\frac{n}{K_w}\frac{d^{(w)}p_w}{dt} + \mathrm{div}\, w + \mathrm{div}\, v_s = 0$$

$$\frac{d^{(s)}n}{dt} = (1-n)\,\mathrm{div}\, v_s$$

Balance of momentum

$$(1-n)\rho_s \frac{d^{(s)}v_s}{dt} = \mathrm{div}\,\sigma' - (1-n)\,\mathrm{grad}\, p_w + (1-n)\rho_s b + R$$

$$n\rho_w \frac{d^{(w)}v_w}{dt} = -n\,\mathrm{grad}\, p_w + n\rho_w b - R$$

Constitutive $\frac{d^{(s)}}{dt}\sigma' = D^{ep}.d_s$

Kinematic $d_s = \mathrm{grad}_{sym} v_s$

Depth Integrated Model for Two phases (I)

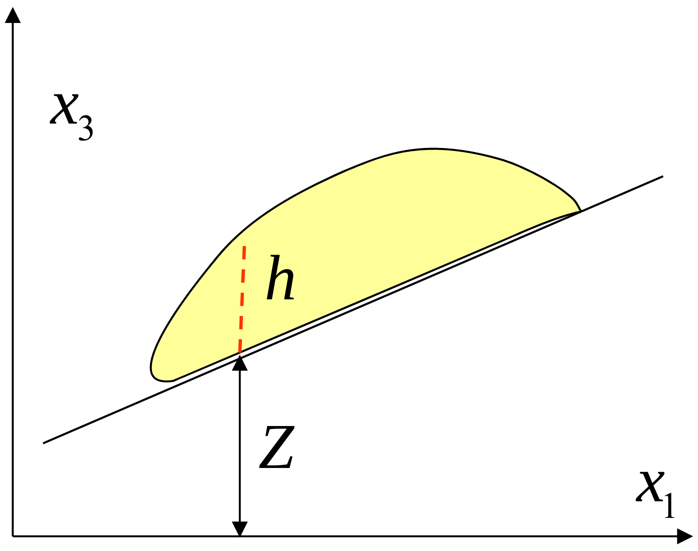
- As we are considering shallow landslides, we do not solve directly the equations to model propagation. Instead we integrate the equations in depth.

- We directly integrate from the bottom of the slope to the free surface of the displacing mass, taking in to account the Leibnitz rule and adding the boundary conditions at free surface and the bottom

$$\int_a^b \frac{\partial}{\partial s} F(r,s)\,dr = \frac{\partial}{\partial s}\int_a^b F(r,s)\,dr - F(b,s)\frac{\partial b}{\partial s} + F(a,s)\frac{\partial a}{\partial s}$$

- This technique eliminates:
    - The vertical variable
    - The free surface condition of the displacing mass (as this condition is directly considered within the depth integrated equations)

Depth Integrated Model for Two phases (II)



Unknowns:

$$\bar{v}_w\ \bar{v}_s\ \bar{n}\ h$$

$$h_s = \left(1-\bar{n}\right)h$$

$$h_w = \bar{n}h$$

$$\frac{\bar{d}^{(s)}}{dt}\left(\left(1-\bar{n}\right)h\right)+\left(1-\bar{n}\right)h\,\mathrm{div}\,\bar{\boldsymbol{v}}_s = \left(1-\bar{n}\right)e_R$$

$$\frac{\bar{d}^{(w)}}{dt}\left(\bar{n}\,h\right)+\bar{n}\,h\,\mathrm{div}\,\bar{\boldsymbol{v}}_w = \bar{n}\,e_R$$

$$\rho_s h\left(1-\bar{n}\right)\frac{\bar{d}^{(s)}\bar{\boldsymbol{v}}_s}{dt} = \mathrm{div}\left(h\,\bar{\boldsymbol{\sigma}}'\right)-\left(1-n\right)\mathrm{grad}\left(h\,\bar{p}_w\right)+\boldsymbol{\tau}_b^{(s)}$$

$$+\left(1-\bar{n}\right)h\,\bar{\boldsymbol{R}}_s+\left(1-\bar{n}\right)\rho_s\boldsymbol{b}h-\left(1-\bar{n}\right)\rho_s\bar{\boldsymbol{v}}_s e_R$$

$$\rho_w h\,\bar{n}\,\frac{\bar{d}^{(w)}\bar{\boldsymbol{v}}_w}{dt} = -n\,\mathrm{grad}\left(h\,\bar{p}_w\right)+\boldsymbol{\tau}_b^{(w)}+\bar{n}h\,\bar{\boldsymbol{R}}_w+\bar{n}\rho_w\boldsymbol{b}h-\bar{n}\,\rho_w\bar{\boldsymbol{v}}_w e_R$$

Contents

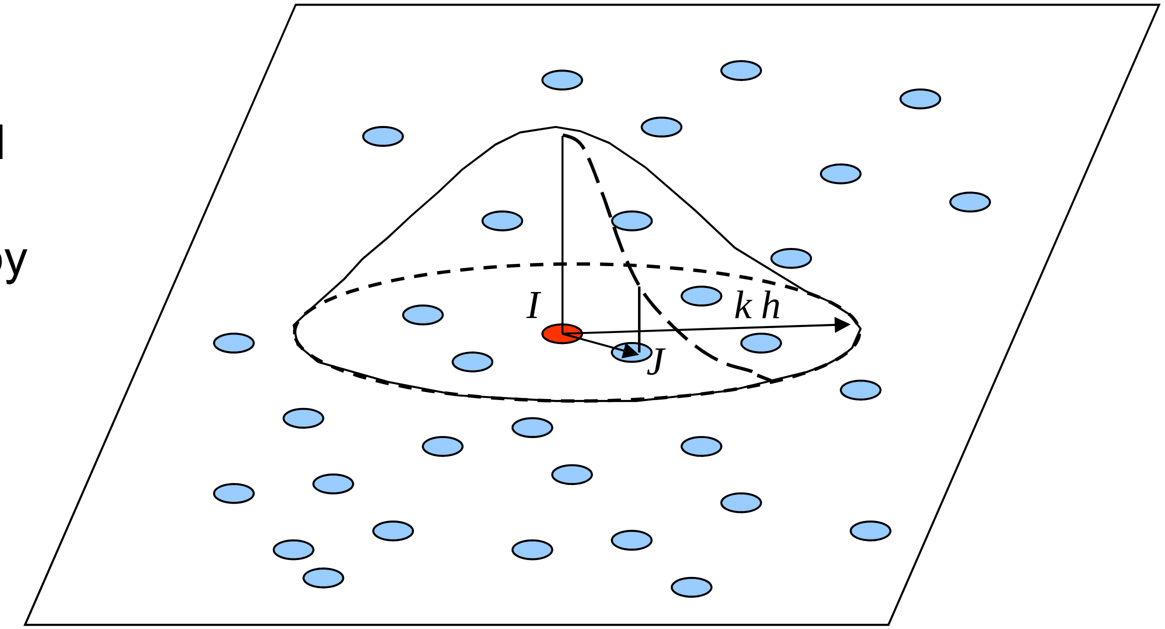
- Introduction
- Mathematical model
- Numerical models: SPH ⇦
- Examples and applications
- Conclusions

Smoothed Particle Hydrodynamics (I)

The approximation of a given function is written as:

$$< \phi(x) > \approx \int_{\Omega} \phi(x') W(x' - x, h) dx'$$

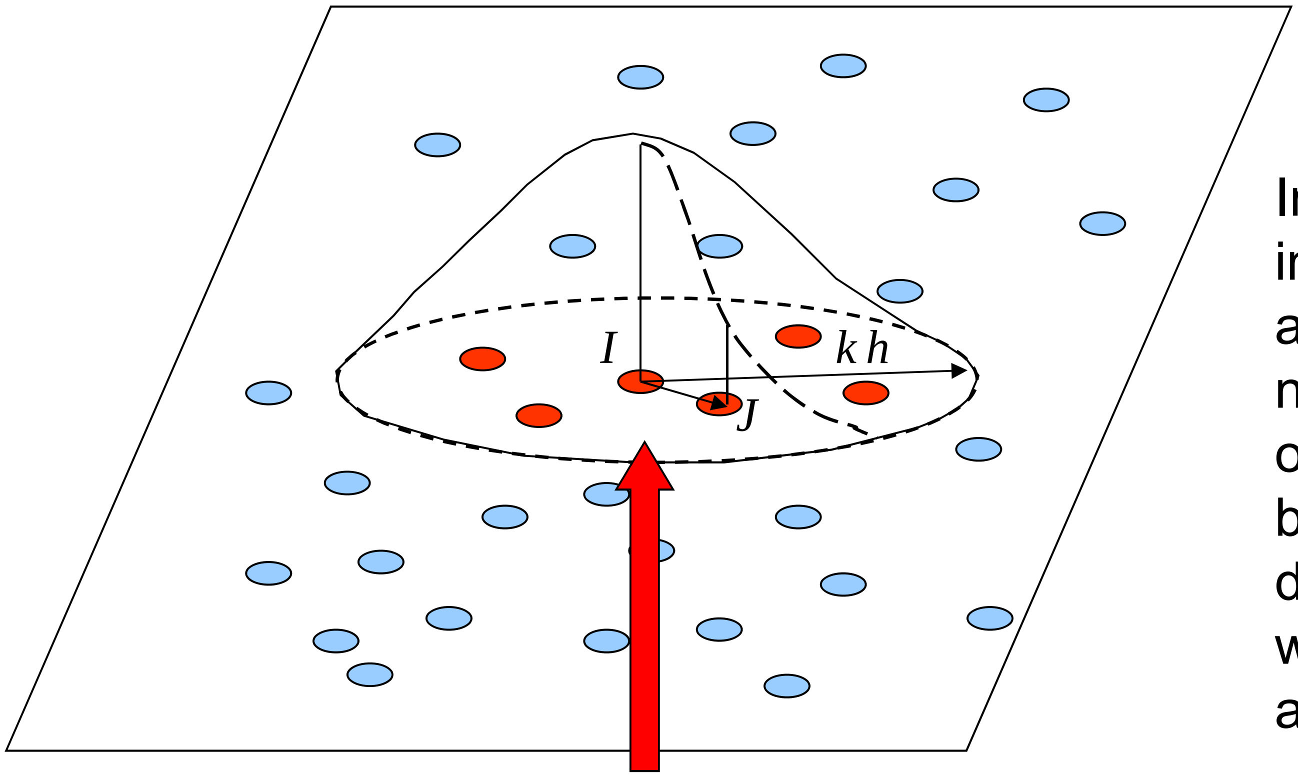
The SPH method is based on approximating functions and differential operators such as gradient or divergence by integral approximations defined in terms of a kernel



Where $W(x' - x, h)$ is referred to as the kernel of the linear functional, h being a parameter describing its decay (smoothing length)

Smoothed Particle Hydrodynamics (II)

$$\phi_I \approx \langle \phi(x_I) \rangle_h \approx \sum_{J=1}^{N} \phi(x_J) W(x_J - x_I, h) \omega_J$$



Summation extended to nodes within *kh*

In a <u>second step</u>, these integral representations are approximated numerically by a class of numerical integration based on a set of discrete point or nodes, without having to define any “element”

Contents

- Introduction
- Mathematical model
- Numerical models: SPH
- Examples and applications ⇦
- Conclusions

Examples and Applications

- The purpose of this section is to illustrate the performance of the proposed two-phase debris flow SPH model.

- It is important to know that no analytical solution has been proposed so far for this problem. The tests we present aiming to:

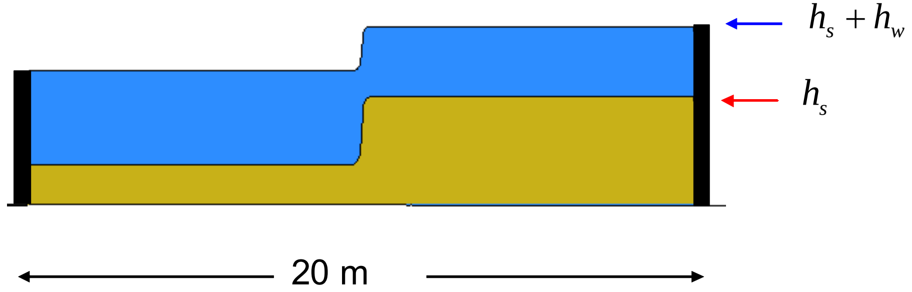
  ⇨ Illustrate the mathematical features of the model

  ⇨ Simple tests showing the main features of the two phase model

Shocks and expansion waves (I)

- Consists of a domain enclosed by two walls located at x = -10 m and x = 10 m filled with two masses having different heights and porosities.

- Debris flows having two phases with important relative mobility present a rich structure of shocks and rarefaction waves, which has to be properly modelled.



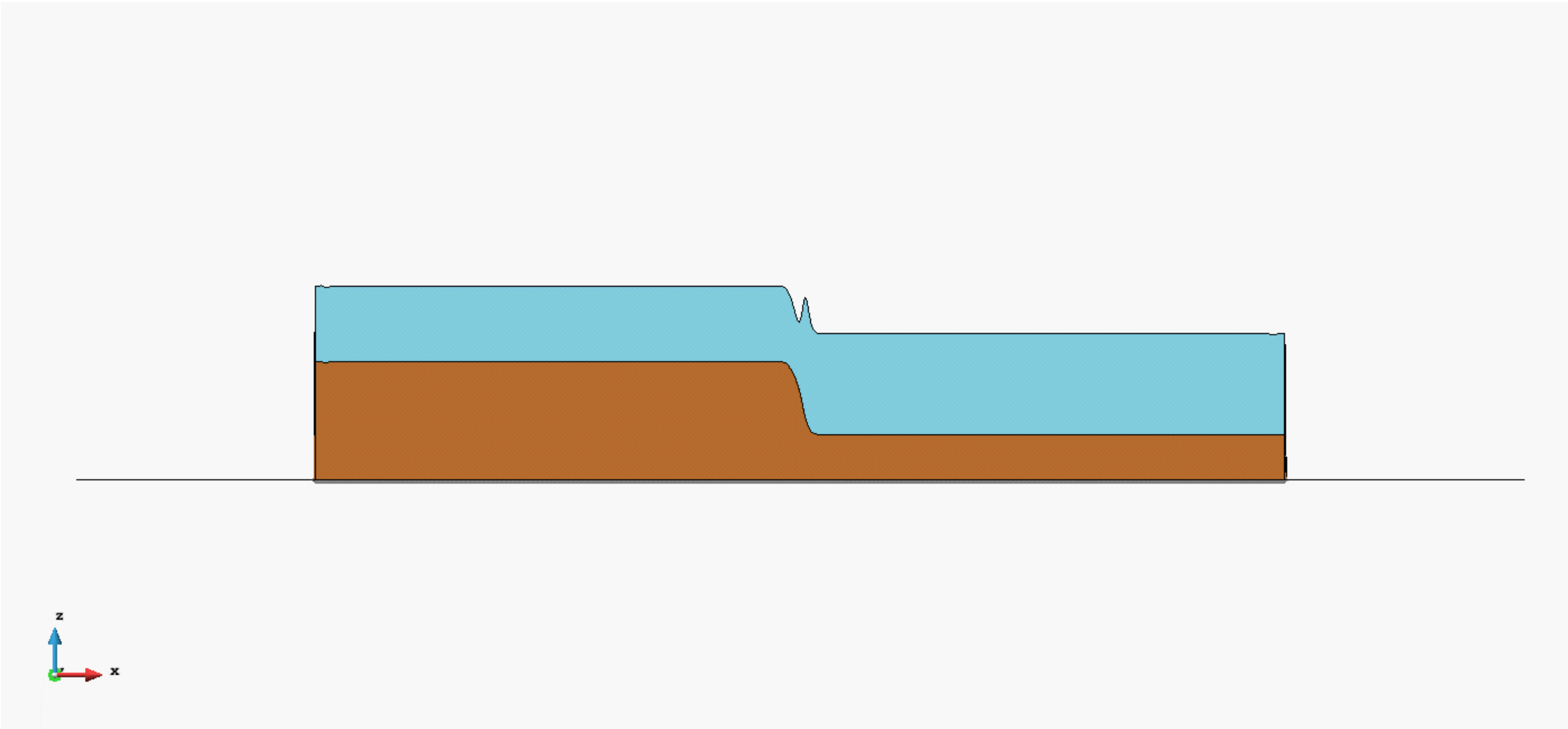
$$\rho_s = 2000\,Kg/m^3$$
$$\rho_w = 1000\,Kg/m^3$$
$$n_L = 0.69$$
$$n_R = 0.39$$
$$\tan\phi = 0.0$$

Interaction law (Anderson)

$$R = -nR_w = (1-n)R_s$$
$$R = C_d\left(v_w - v_s\right)$$
$$C_d = \frac{n\left(1-n\right)}{V_T n^m}\left(\rho_s - \rho_w\right)g$$

$$V_T = 1.0e-3$$
$$m = 1$$

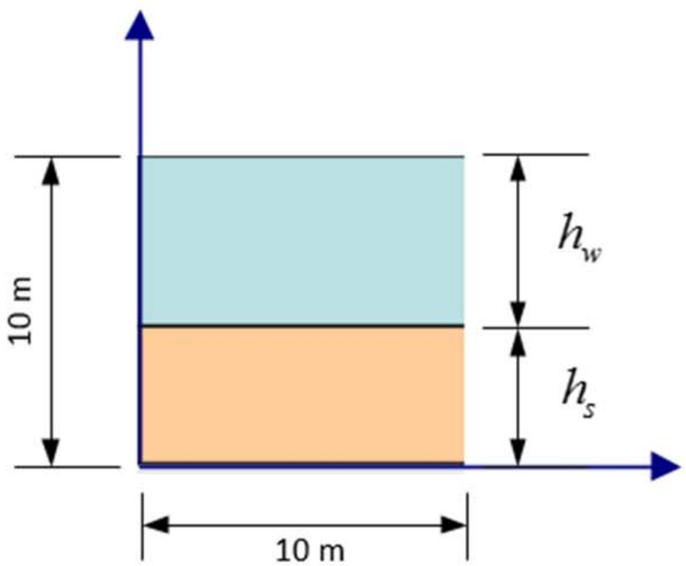
Shocks and expansion waves (II)



Breaking of a dam over a dry, horizontal basal surface (I)

- The second example we have selected is much more relevant, as it can provide some insight on the propagation of two-phase debris flows

- We have assumed that the granular soil has a friction angle of 45º, the basal angle of friction being the same. The drag coefficients are the same used in the previous example.



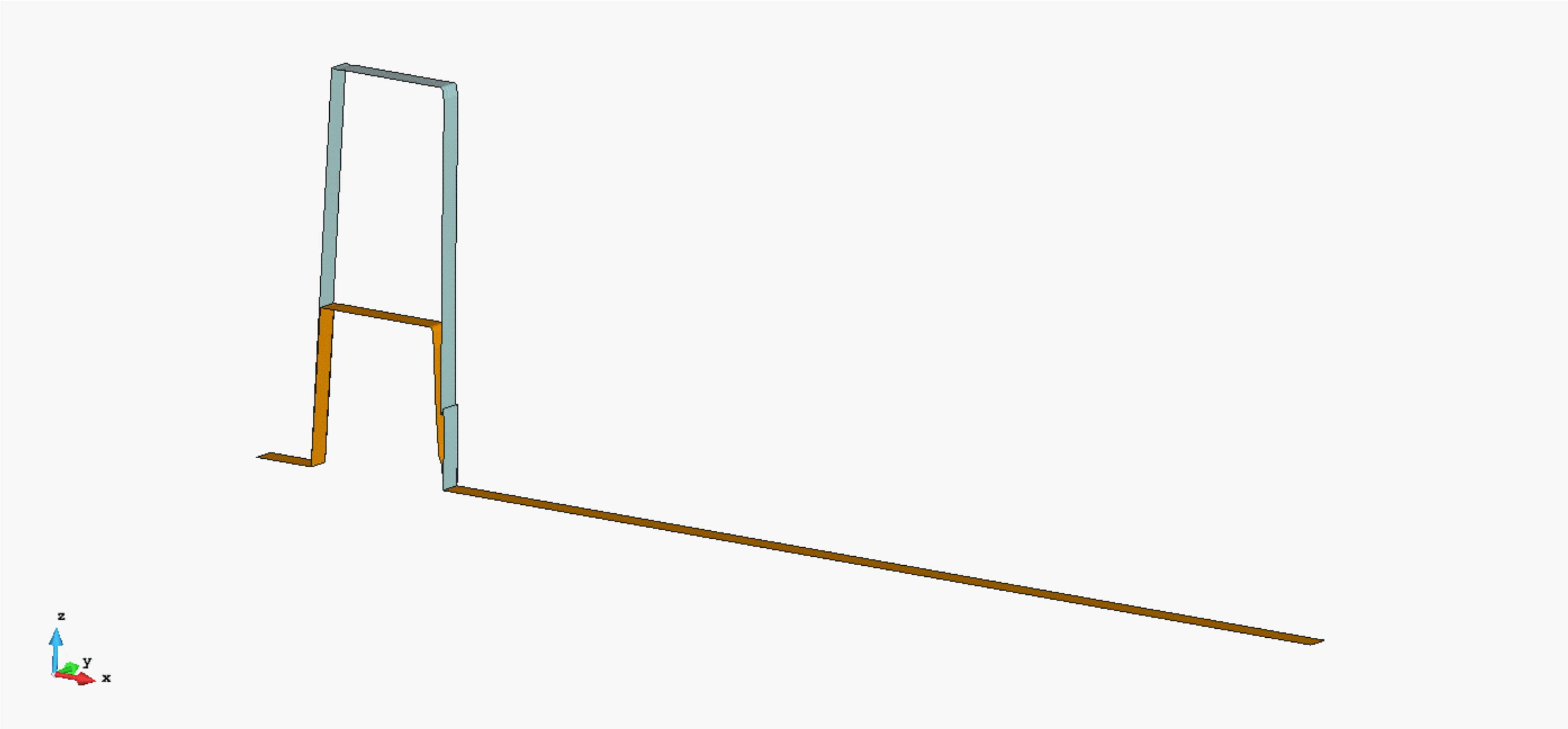
$$\rho_s = 2000\,Kg/m^3$$
$$\rho_w = 1000\,Kg/m^3$$
$$n = 0.6$$
$$\tan\phi = 1.0$$

Interaction law (Anderson)

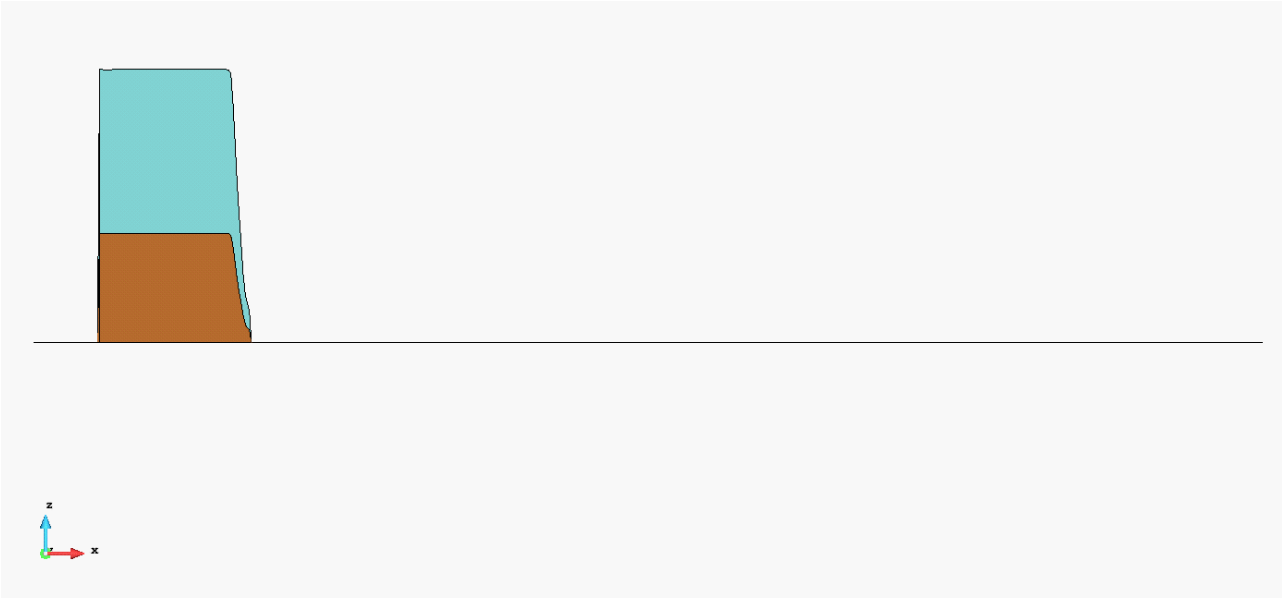
$$R = -nR_w = (1-n)R_s$$
$$R = C_d\left(v_w - v_s\right)$$
$$C_d = \frac{n(1-n)}{V_T n^m}(\rho_s - \rho_w)g$$

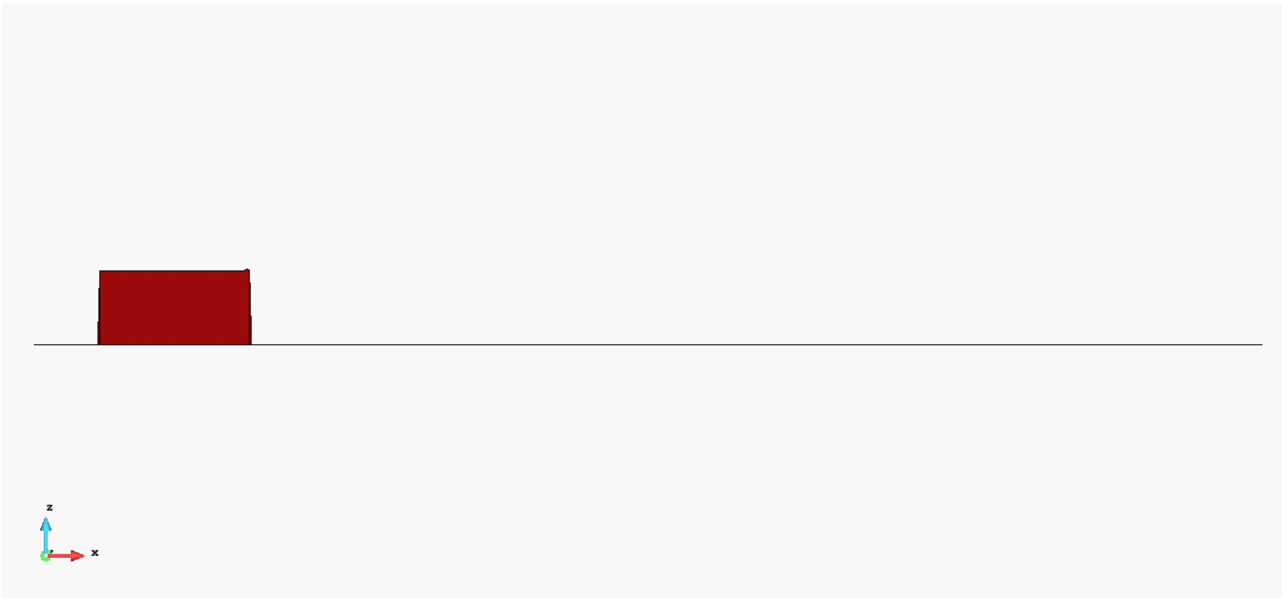
$$V_T = 1.0\,e-3$$
$$m = 1$$

Breaking of a dam over a dry, horizontal basal surface (II)

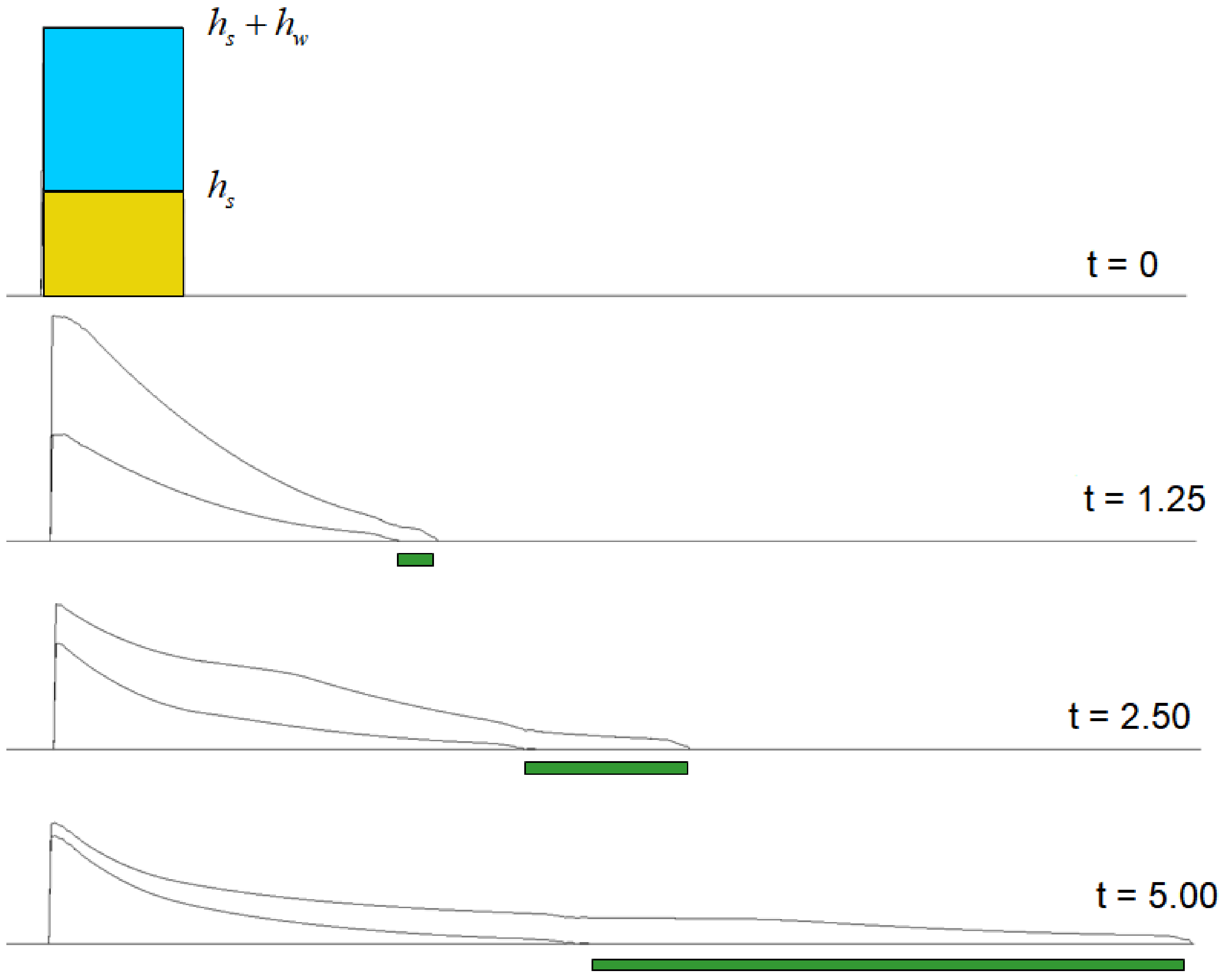


Breaking of a dam over a dry, horizontal basal surface (no interaction between phases)

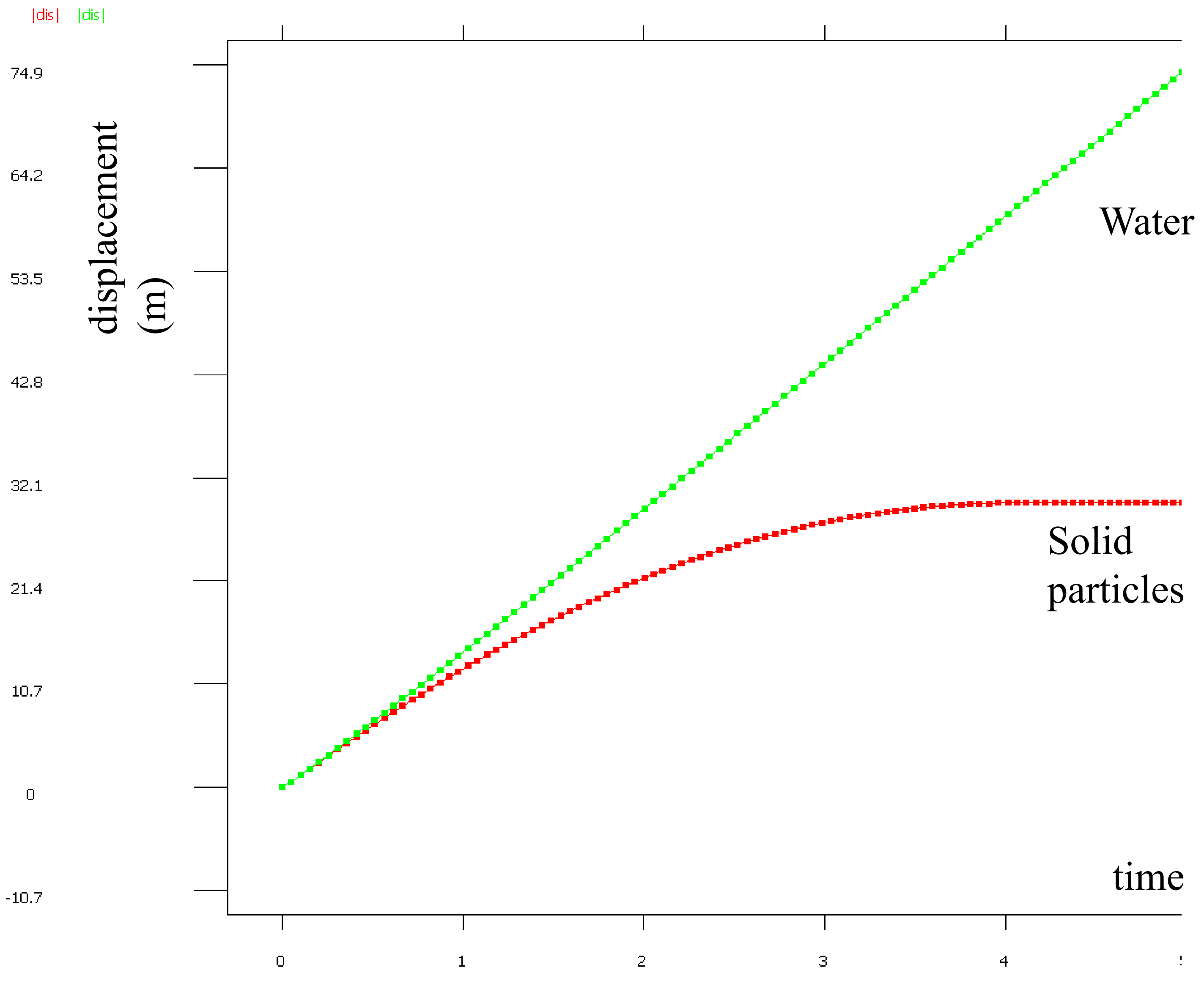




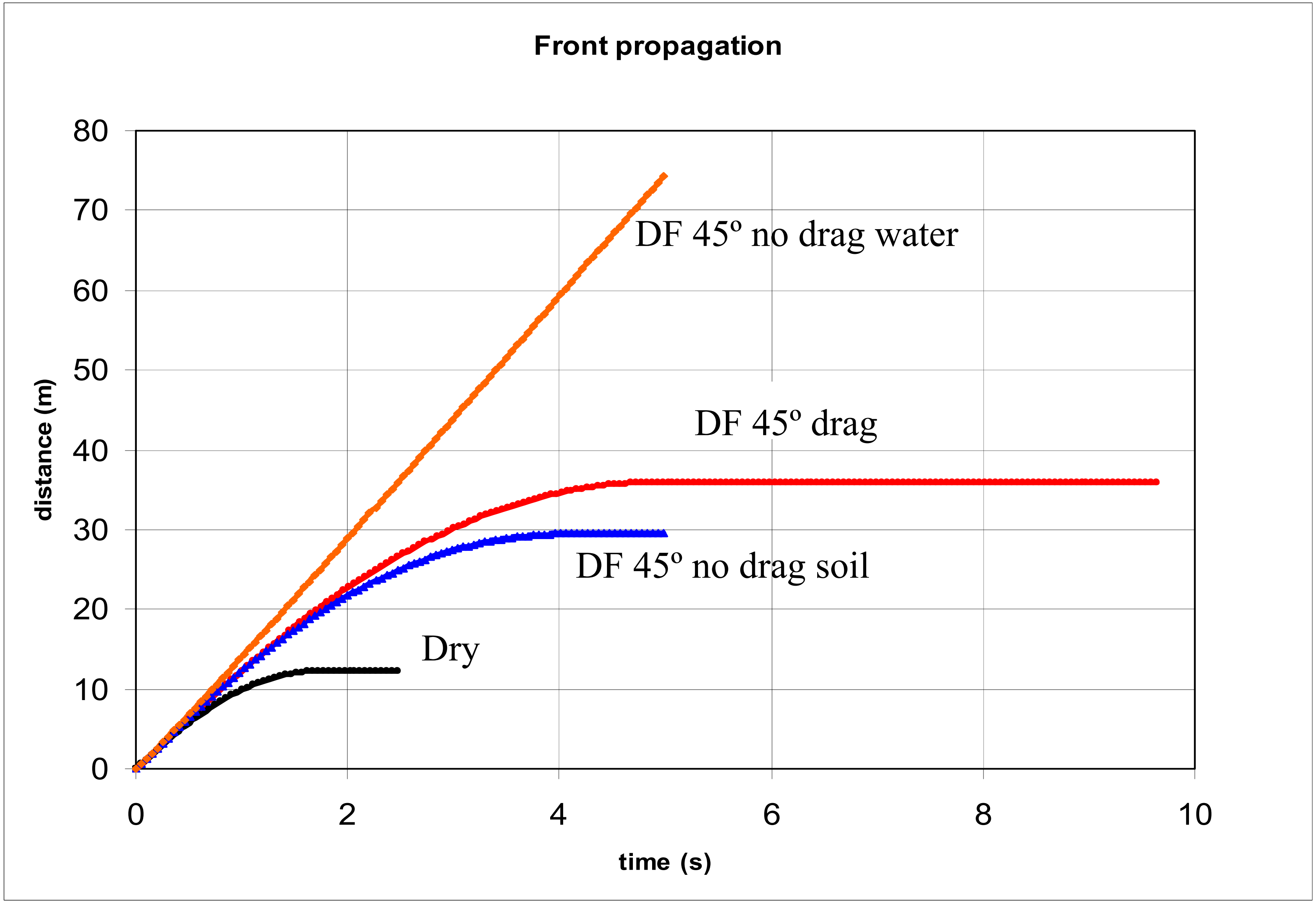
Breaking of a dam over a dry, horizontal basal surface (no interaction between phases)



Breaking of a dam over a dry, horizontal basal surface (no interaction between phases)



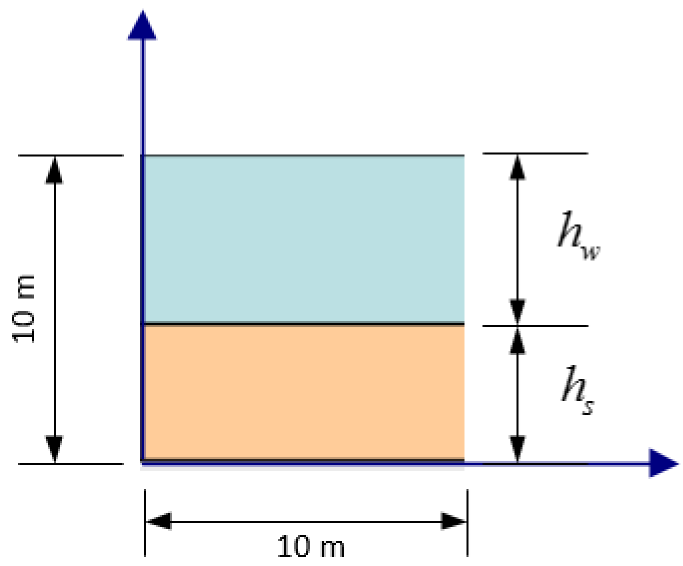
Comparision between both cases



Limit case (I)

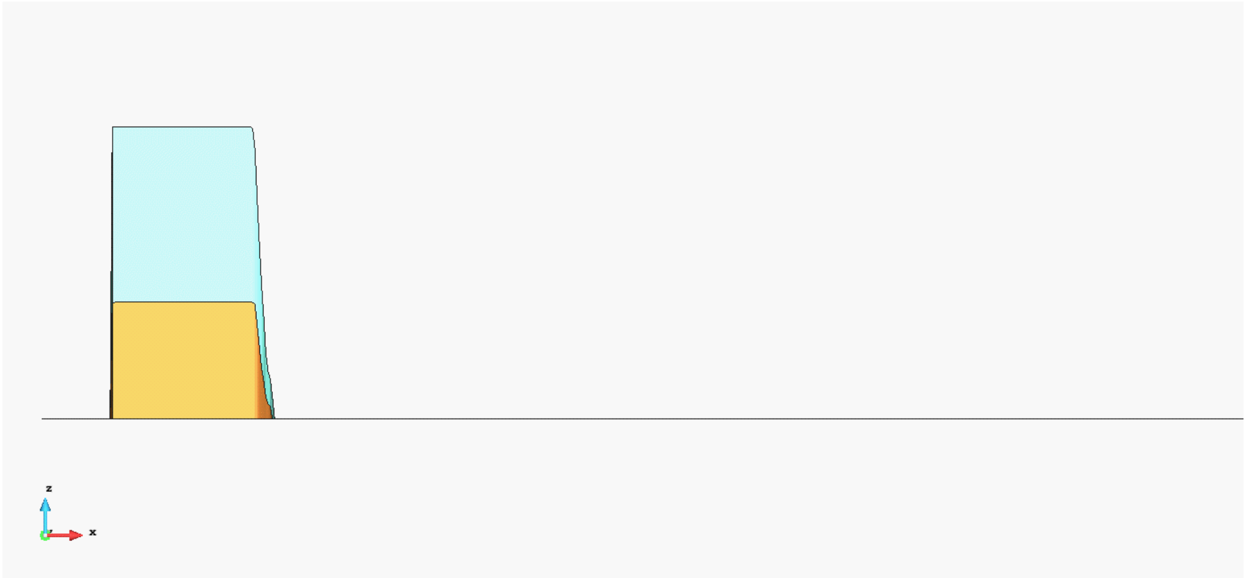
- We will present a **limit case** where we have modelled the solid choosing a tangent of the friction angle equal to 19, which is unrealistic, and we have not considered interaction between phases.

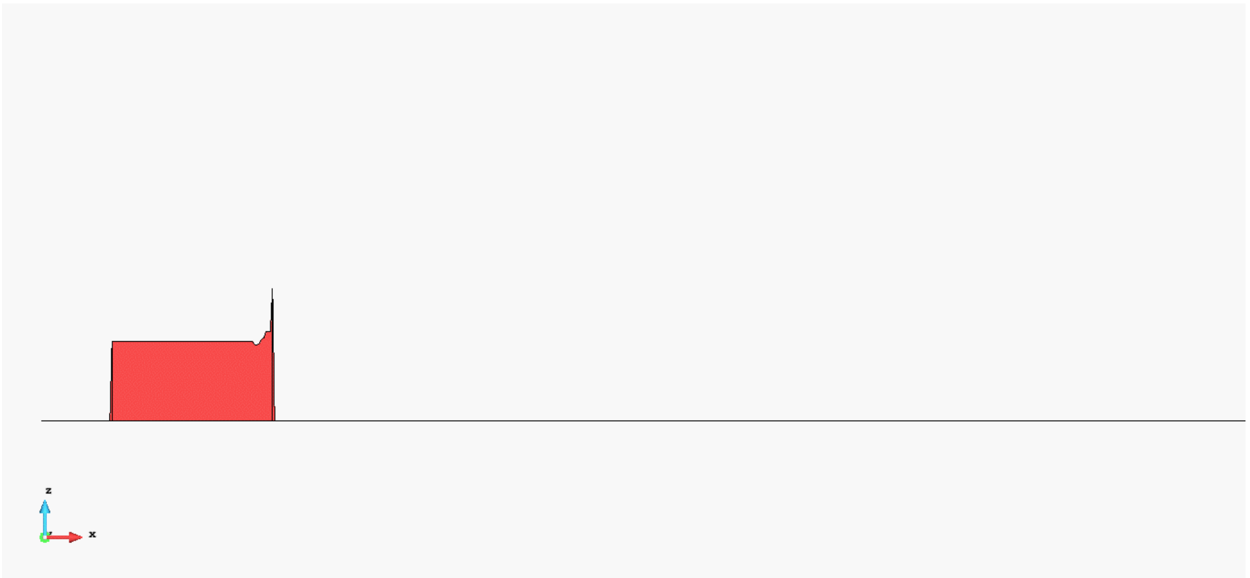
- This limit case represents a debris flow having a very high -unrealistic- friction angle, such that the solid grains stop while water leaves the skeleton, the interaction force is not being able to mobilize the solid.



$\tan\theta = 19$

Limit case (II)





Contents

- Introduction
- Mathematical model
- Numerical models: SPH
- Examples and applications
- Conclusions ⇦

Conclusions

- These mathematical and numerical models can be applied to reproduce the propagation of debris flows, taking into account coupling between solid and fluid phases

- We propose to use a double set of nodes (solid and fluid) which can move relatively to each other. It is possible to find situations where the fluid abandons the body of the mixture using these two set of nodes.

.

ALERT Geomaterials Workshop 2017

ADVANCED NUMERICAL MODELLING OF GEOMATERIALS

Grupo de Modelos Matemáticos en Ingeniería Civil – M2i

POLITÉCNICA

Coupling solid and fluid phases in fast landslide propagation

Manuel Pastor, **Ángel Yagüe**, Miguel Martín Stickle, Diego Manzanal, Pablo Mira, Saeid Mousavvi, Chuan Lin, José Antonio Fernández Merodo and Caitlin Chalk

Alert Geomaterials