Aussois 2012

ELASTIC WAVES

and particulate materials

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References:

Santamarina, J.C., in collaboration with Klein, K. and Fam, M. (2001). Soils and Waves, J. Wiley and Sons, Chichester, UK, 488 pages.

Lee, J. S. and Santamarina, J. C. (2005a). "Bender Elements." ASCE Journal of Geotechnical and Geoenvironmental Engineering, Vol. 131, No. 9, pp. 1063-1070.
Lee, J. S. and Santamarina, J. C. (2005b). "P-wave Reflection Imaging." ASTM Geotechnical Testing Journal, Vol. 28, pp. 197-206.

Wang, Y. H., Santamarina, J. C., and Cascante, G. (2003). "Counter EMF effects in Resonant Column Testing." ASTM Geotechnical Testing Journal, Vol. 26, No. 4, pp. 410-420.

Cascante, G., Santamarina, J. C., and Yassir, N. (1998). "Flexural Excitation in a Standard Torsional-Resonant Column Device." Canadian Geotechnical Journal, Vol. 35, No. 3, pp. 478-490.

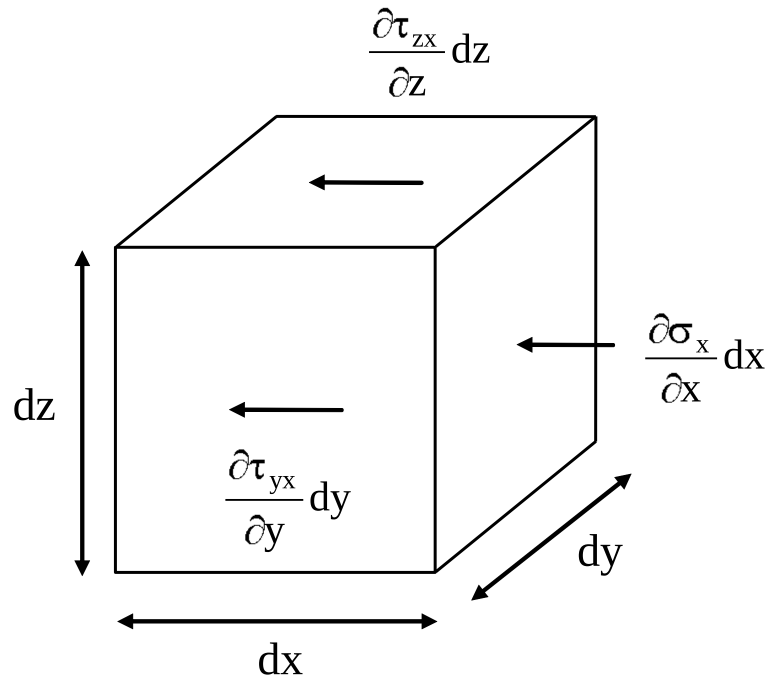
Some pdfs (these and related papers) available at http://pmrl.ce.gatech.edu under "Publications"

ELASTIC WAVES

Let's assume…

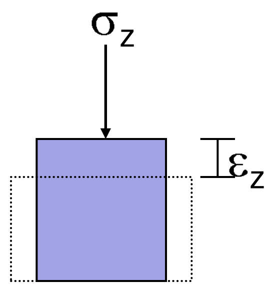
infinite
homogeneous
isotropic
single-phase
linear
elastic
continuum

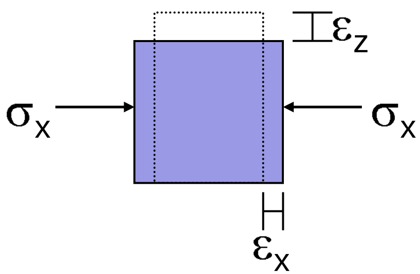
Mechanics - 1: Equilibrium

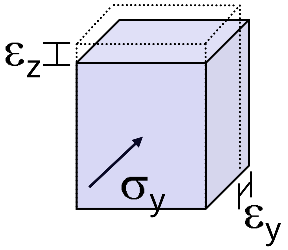


$$\rho \frac{\partial^2 u_x}{\partial t^2} = \frac{\partial \sigma_x}{\partial x} + \frac{\partial \tau_{yx}}{\partial y} + \frac{\partial \tau_{zx}}{\partial z}$$

Mechanics - 2: Constitutive Equations







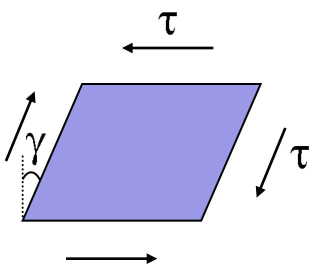
$$\varepsilon_z = \frac{\sigma_z}{E}$$

$$\varepsilon_z = -\nu\varepsilon_x = -\nu\frac{\sigma_x}{E}$$

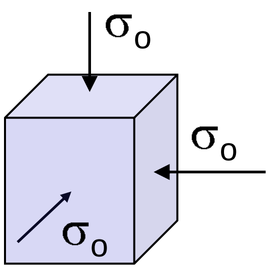
$$\varepsilon_z = -\nu\varepsilon_y = -\nu\frac{\sigma_y}{E}$$

$$\boxed{\varepsilon_z = \frac{1}{E}\left[\sigma_z - \nu\left(\sigma_x + \sigma_y\right)\right]}$$

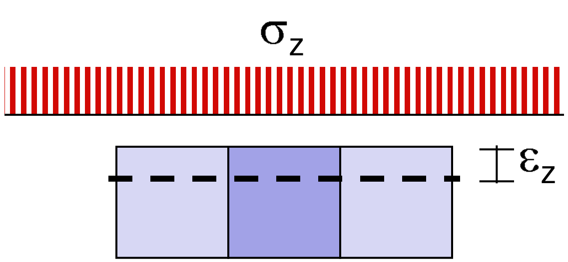
Mechanics - 2: Constitutive Equations



$$G = \frac{\tau}{\gamma} = \frac{E}{2(1+\nu)}$$

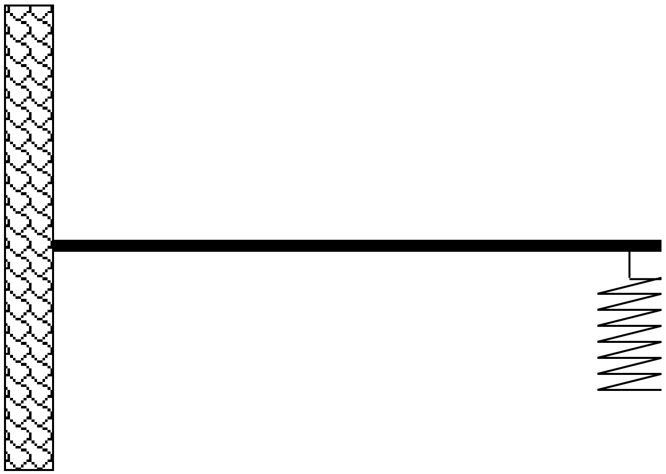


$$B = \frac{\sigma_o}{\varepsilon_{vol}} = \frac{E}{3(1-2\nu)}$$



$$M = \left.\frac{\sigma_z}{\varepsilon_z}\right|_{\varepsilon_x=\varepsilon_y} = \frac{E(1-\nu)}{(1+\nu)(1-2\nu)} = B + \frac{4}{3}G$$

Mechanics - 3: Compatibility



$$\boxed{u_{spring} = u_{beam}}$$

in the continuum

$$\boxed{\varepsilon_x = \frac{\partial u_x}{\partial x}}$$

$$\boxed{\gamma_{xy} = \frac{\partial u_x}{\partial y} + \frac{\partial u_y}{\partial x}}$$

Wave Equation

Equilibrium

$$\rho \frac{\partial^2 u_x}{\partial t^2} = \frac{\partial \sigma_x}{\partial x} + \frac{\partial \tau_{yx}}{\partial y} + \frac{\partial \tau_{zx}}{\partial z}$$

Constitutive

$$\sigma_x = M\varepsilon_{vol} - 2G(\varepsilon_y + \varepsilon_z) \qquad \tau_{xy} = G\gamma_{xy}$$

Compatibility

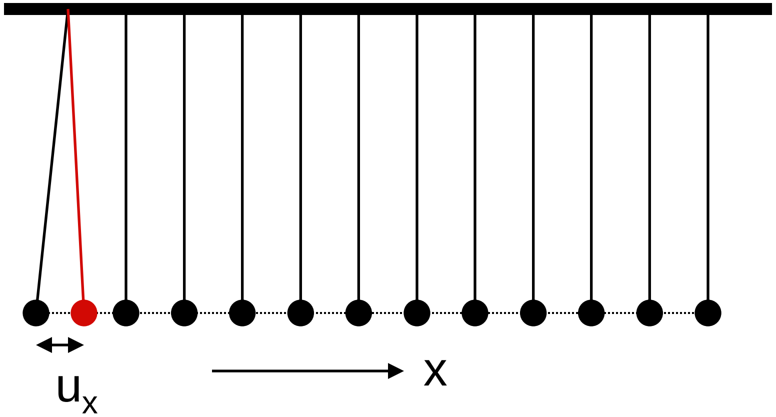
$$\varepsilon_x = \frac{\partial u_x}{\partial x} \qquad \gamma_{xy} = \frac{\partial u_x}{\partial y} + \frac{\partial u_y}{\partial x}$$

Wave Equation

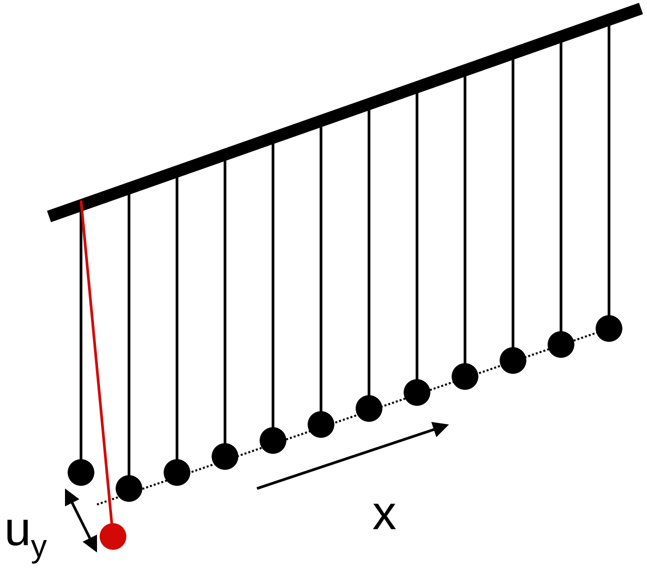
$$\boxed{\rho \frac{\partial^2 u_x}{\partial t^2} = (M - G)\left(\frac{\partial^2 u_x}{\partial x^2} + \frac{\partial^2 u_y}{\partial x \partial y} + \frac{\partial^2 u_z}{\partial x \partial z}\right) + G\left(\frac{\partial^2 u_x}{\partial x^2} + \frac{\partial^2 u_x}{\partial y^2} + \frac{\partial^2 u_x}{\partial z^2}\right)}$$

Two Propagation Modes

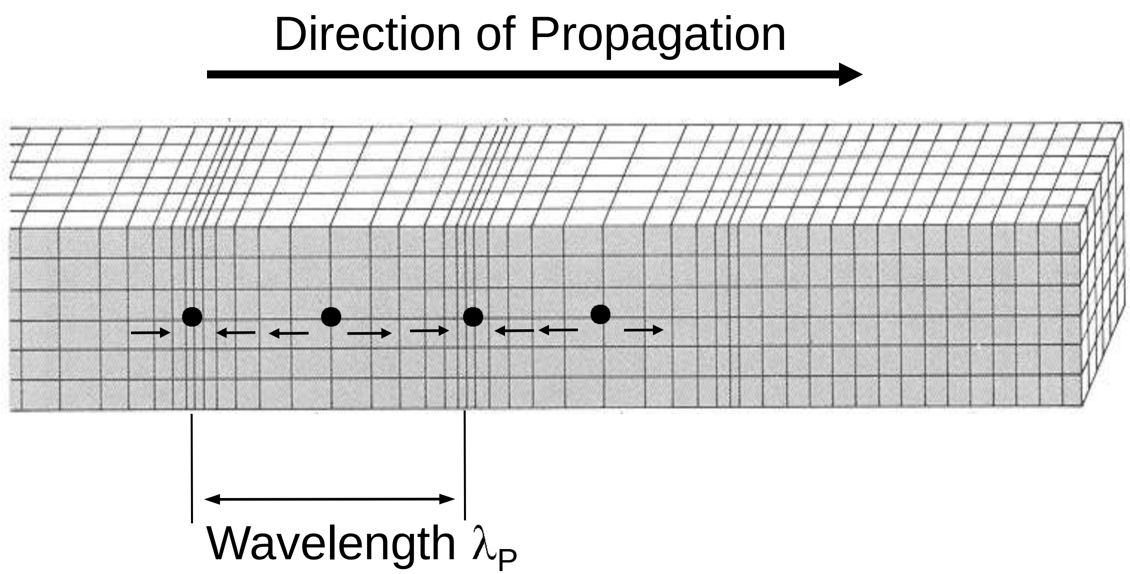
Longitudinal



Transverse



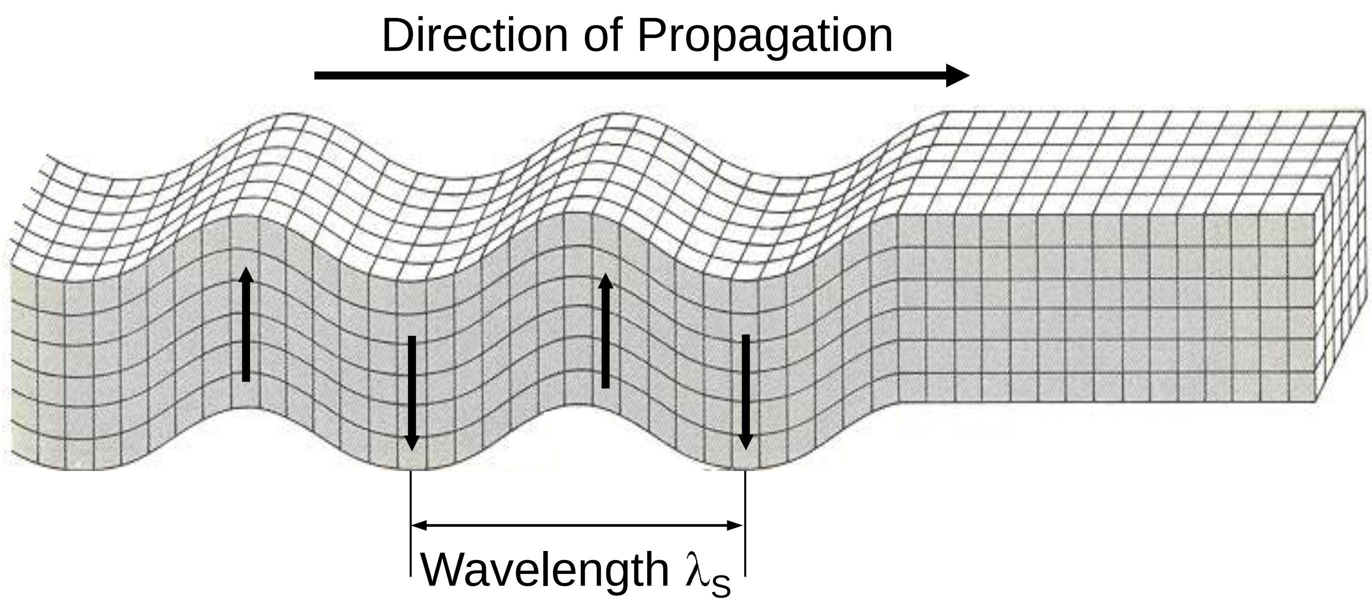
Compression P-wave



(Bolt)

$$\frac{\partial^2 u_x}{\partial t^2} = \frac{M}{\rho}\frac{\partial^2 u_x}{\partial x^2}$$

Shear S-wave



(Bolt)

$$\frac{\partial^2 u_y}{\partial t^2} = \frac{G}{\rho}\frac{\partial^2 u_y}{\partial x^2}$$

Solution of the Wave Equation

$$u = Ae^{(i\omega t - \kappa x)}$$

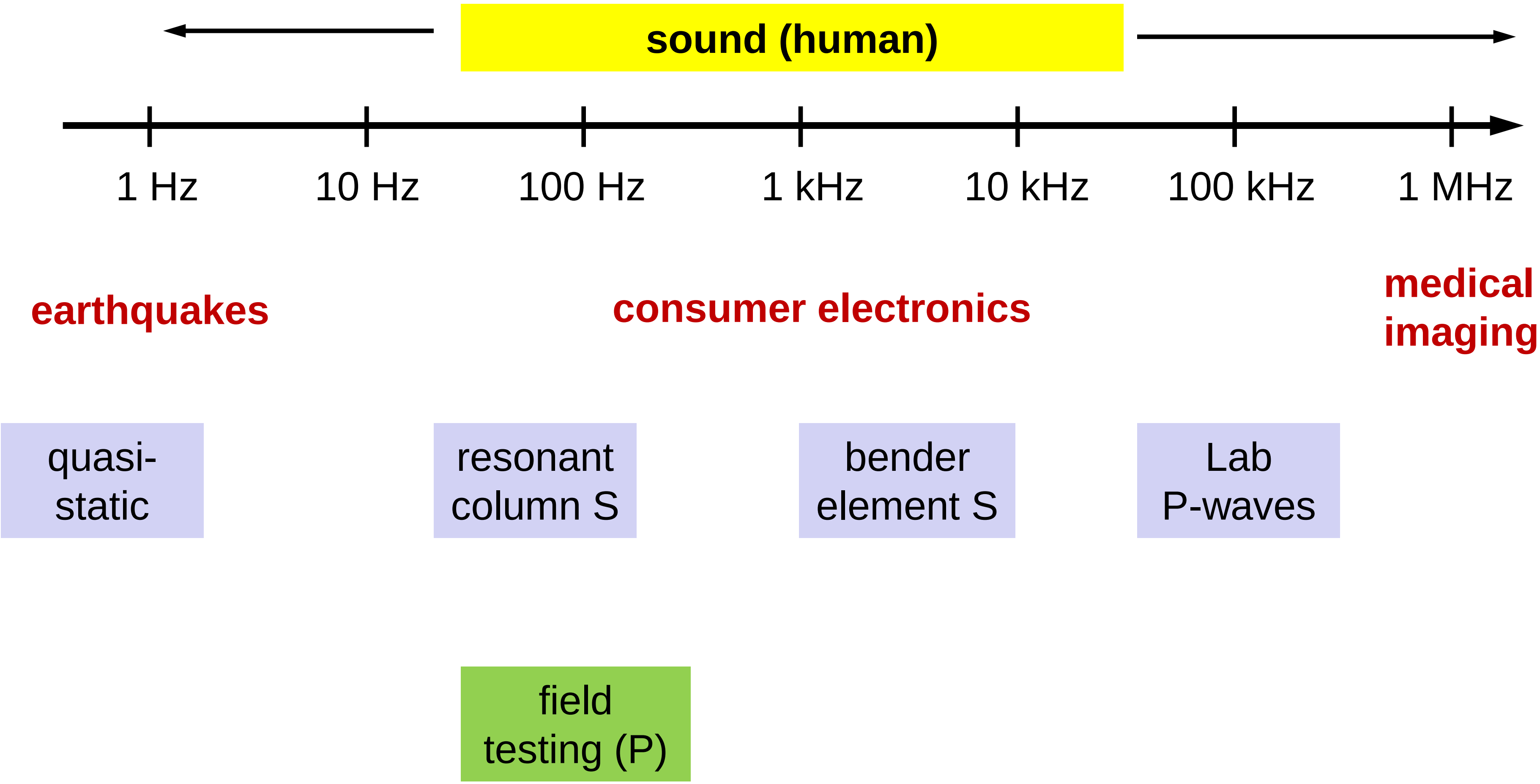
$$\boxed{\frac{\partial^2 u_x}{\partial t^2} = \frac{M}{\rho}\frac{\partial^2 u_x}{\partial x^2}}$$

$$\frac{\omega}{\kappa} = \sqrt{\frac{M}{\rho}} = V_P$$

$$\boxed{\frac{\partial^2 u_y}{\partial t^2} = \frac{G}{\rho}\frac{\partial^2 u_y}{\partial x^2}}$$

$$\frac{\omega}{\kappa} = \sqrt{\frac{G}{\rho}} = V_S$$

Spectrum



sound (human)
1 Hz
10 Hz
100 Hz
1 kHz
10 kHz
100 kHz
1 MHz
earthquakes
consumer electronics
medical imaging
quasi-static
resonant column S
bender element S
Lab P-waves
field testing (P)

So far…

infinite
homogeneous
isotropic
single-phase
linear
elastic
continuum

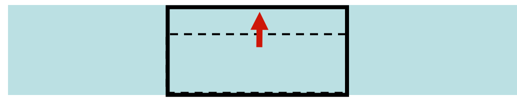
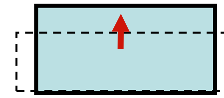
$\boxed{\text{P \& S}}$

Finite:

Geometry dispersion

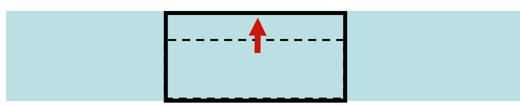
Other propagation modes

Reflection & refraction

	infinite medium	*rod*
		
Longitudinal	Stiffness: M $\boxed{V_P = \sqrt{\frac{M}{\rho}}}$	Stiffness: E $\boxed{V_P = \sqrt{\frac{E}{\rho}}}$
Transverse	$\boxed{V_S = \sqrt{\frac{G}{\rho}}}$	$\boxed{V_S = \sqrt{\frac{G}{\rho}}}$ *torsional*

Geometry Dispersion

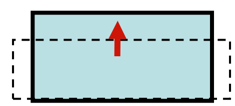
infinite medium



$$V_P = \sqrt{\frac{M}{\rho}}$$

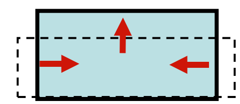
rod

low f or high λ/d



$$V_P = \sqrt{\frac{E}{\rho}}$$

high f or low λ/d



$$V \rightarrow V_R$$

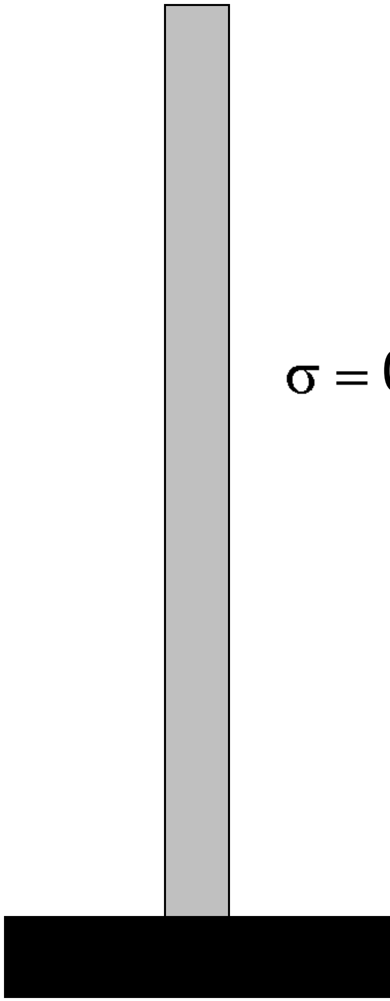
S-Wave: NO geometry dispersion

$$V_S = \sqrt{\frac{G}{\rho}}$$

Finite:

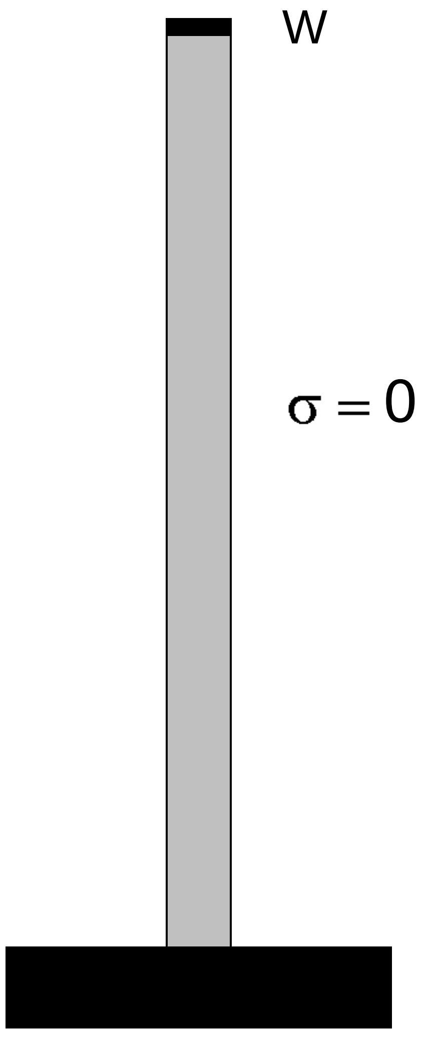
Reflection ***…. resonance***

Reflection:

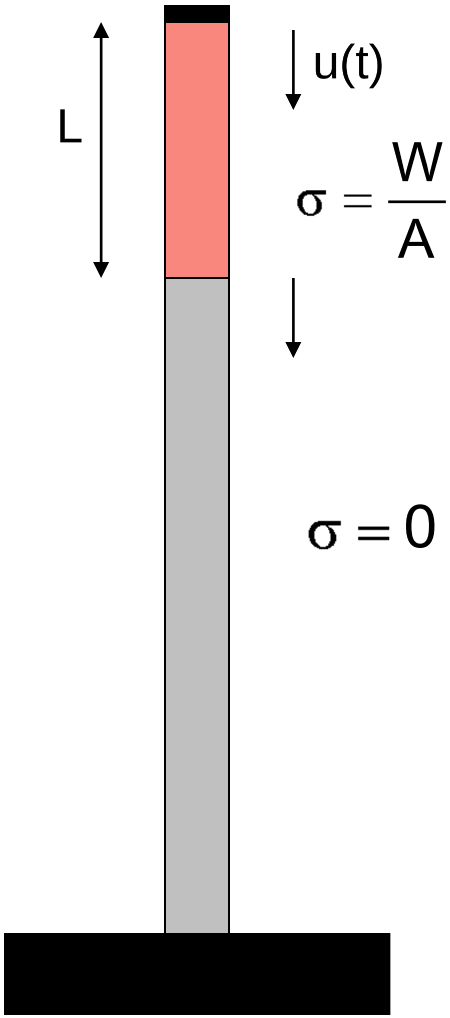


$\sigma = 0$

Add weight W at time t=0



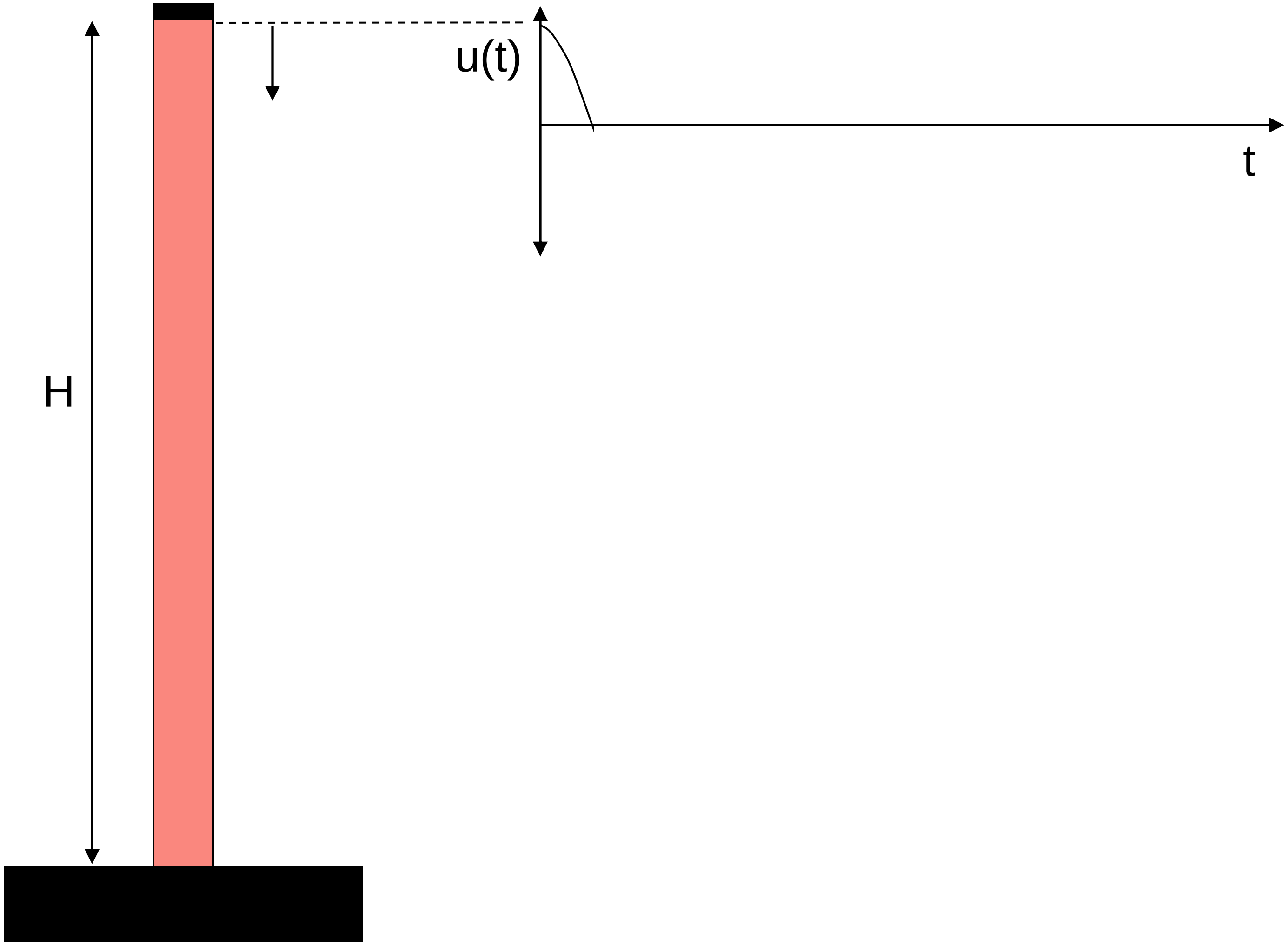
At time t>0



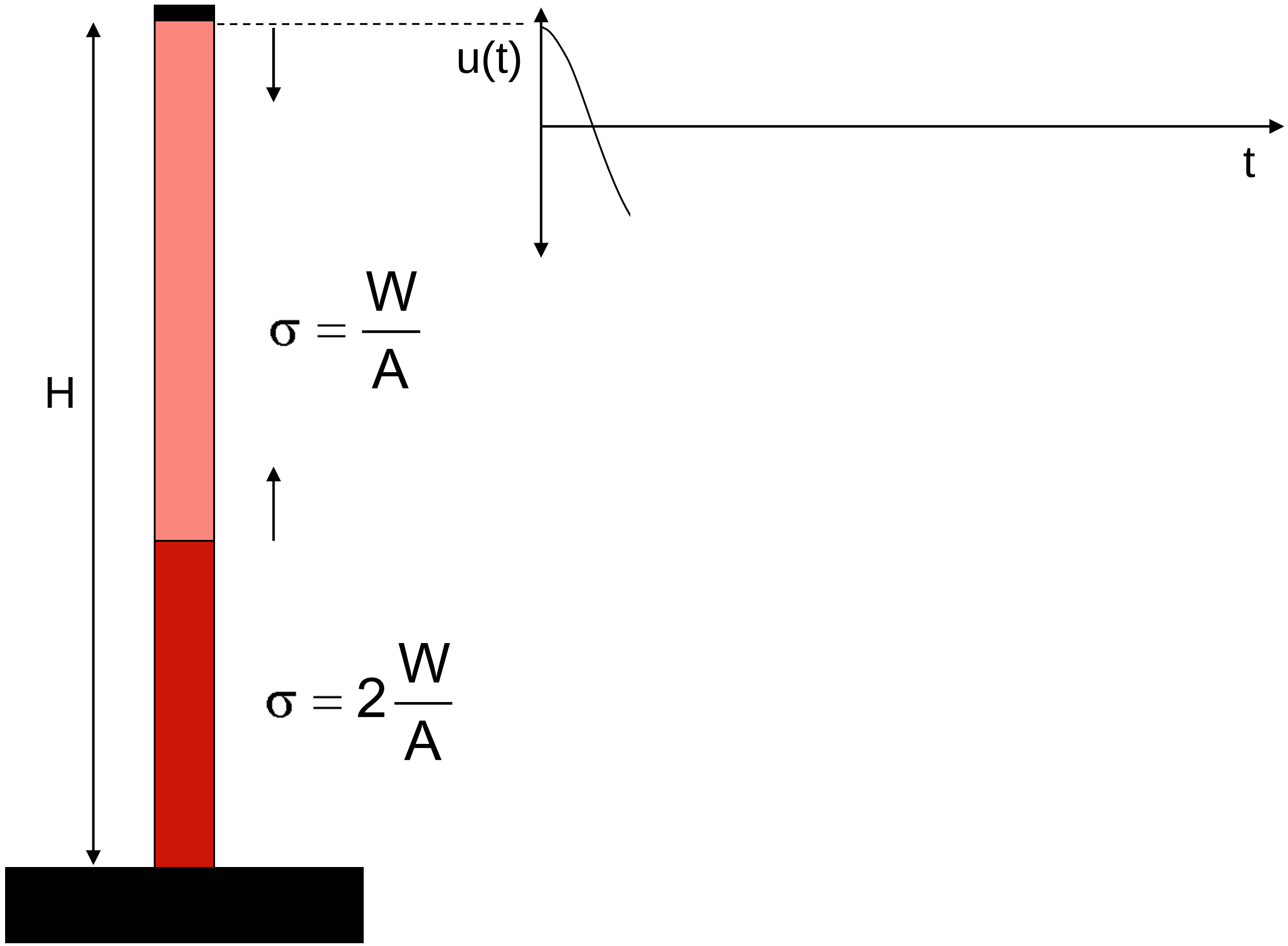
$$u(t) = L(t)\,\varepsilon = tV\frac{\sigma}{E}$$

$$\boxed{\dot{u} = V\frac{\sigma}{E}}$$

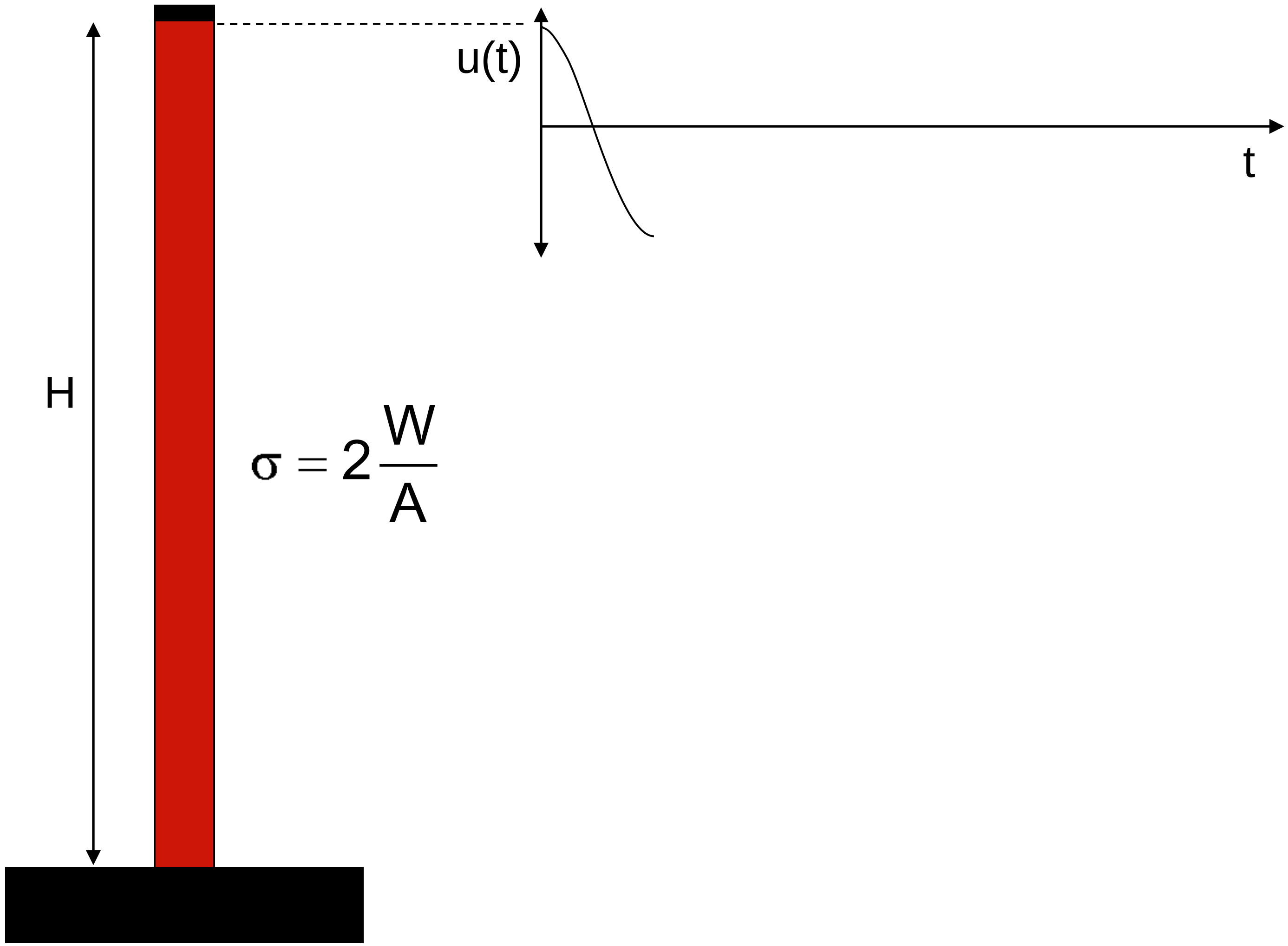
At time t=H/ V



At time t>H/ V

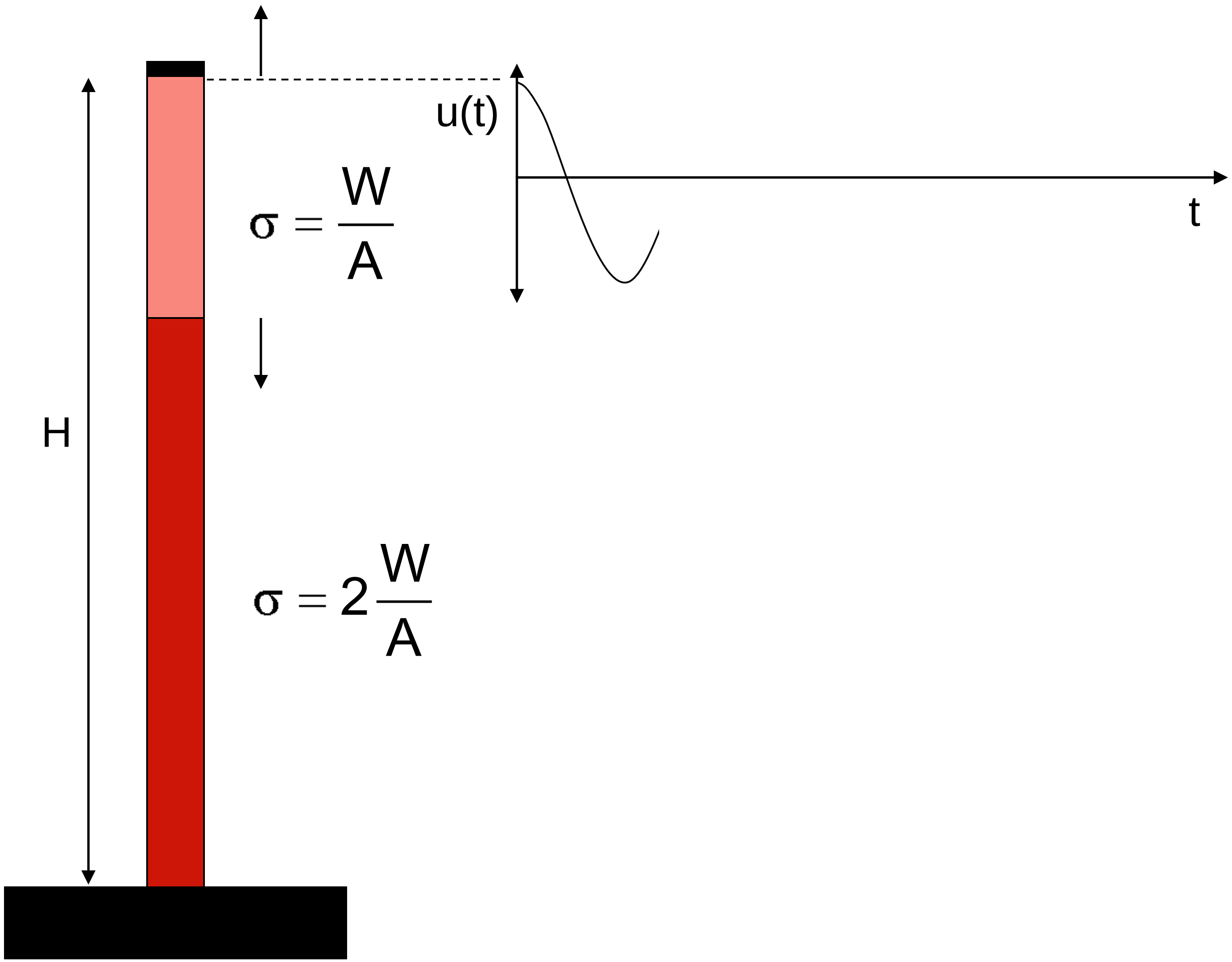


At time t=2H/ V

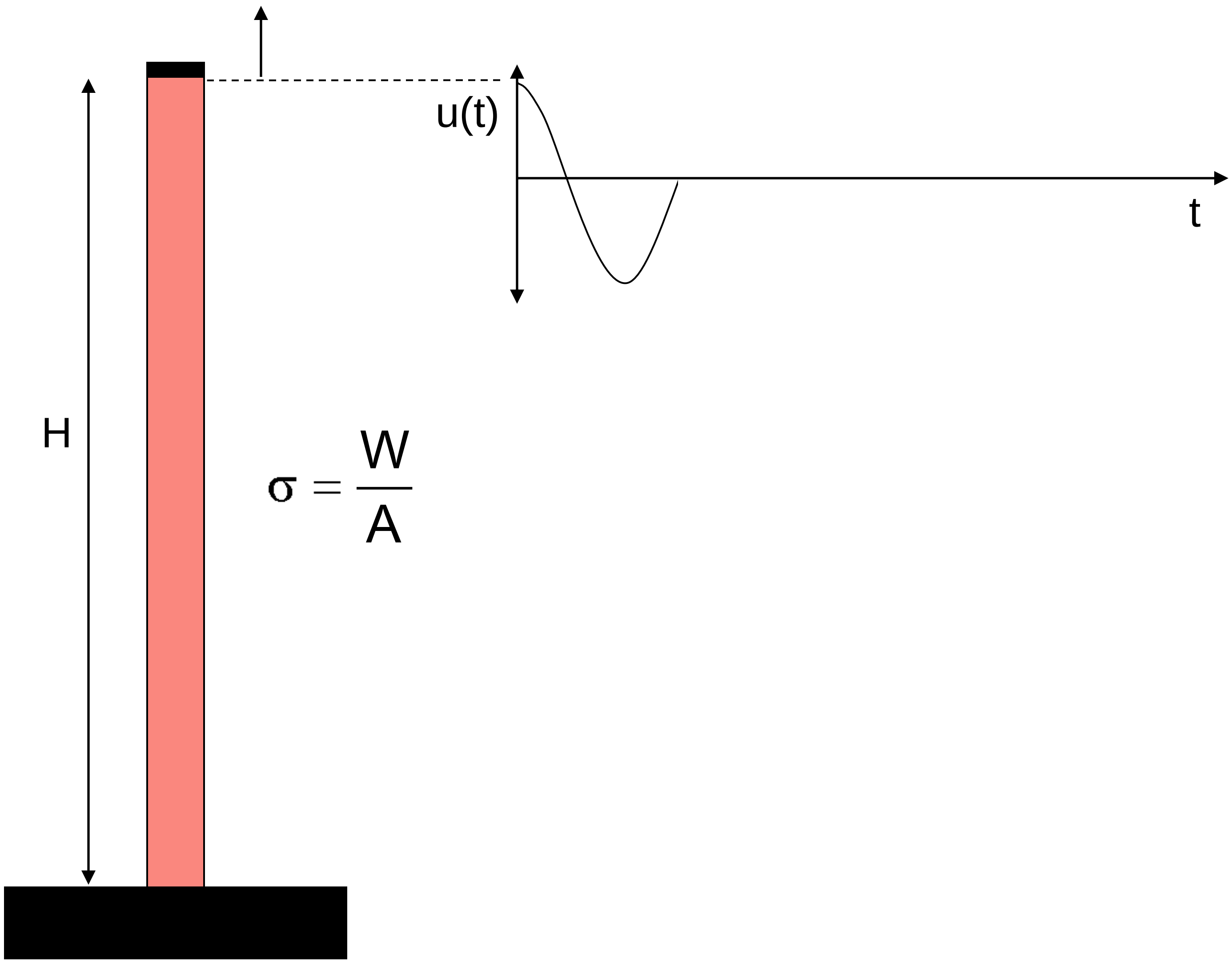


$$\sigma = 2\frac{W}{A}$$

At time t>2H/ V

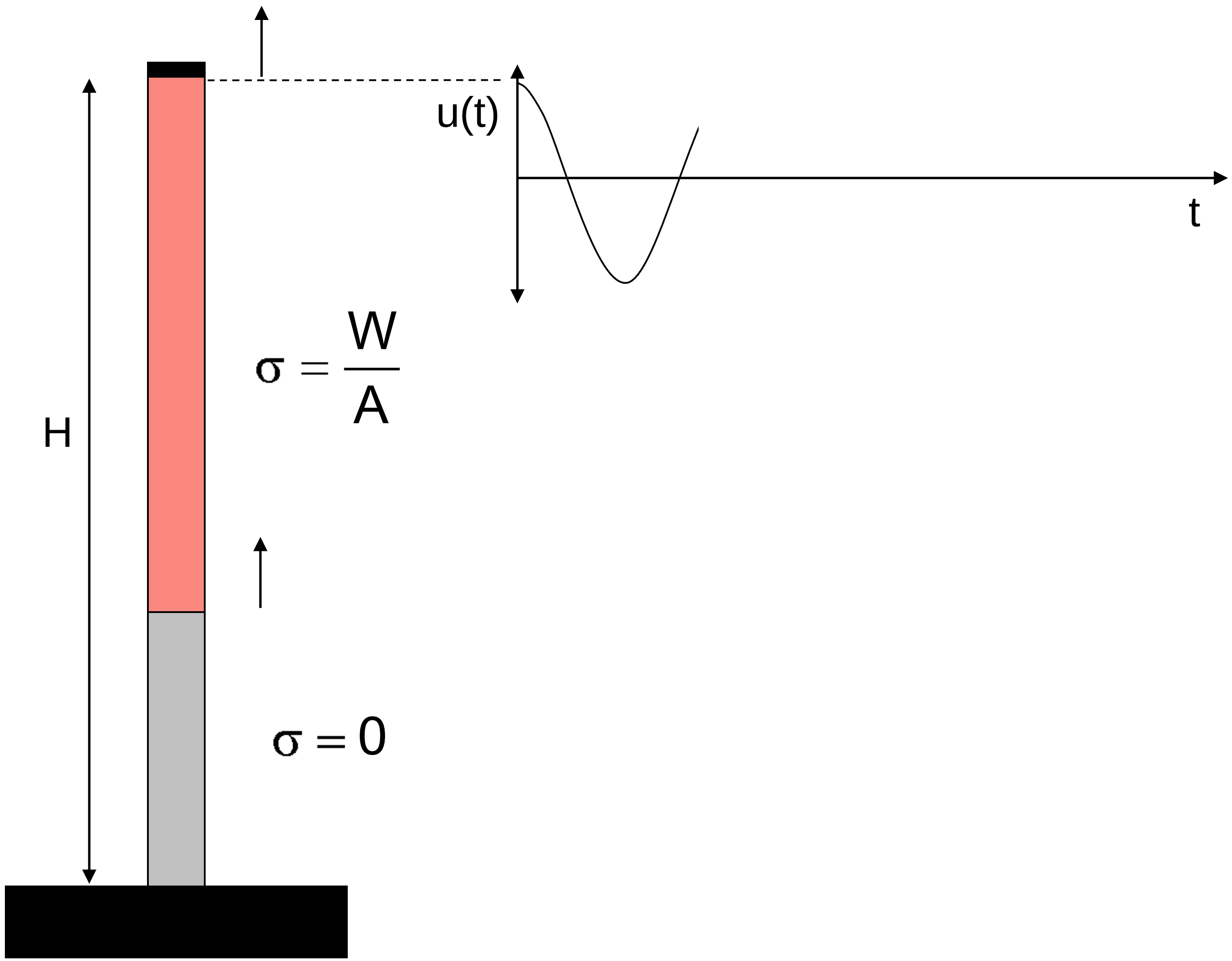


At time t=3H/ V

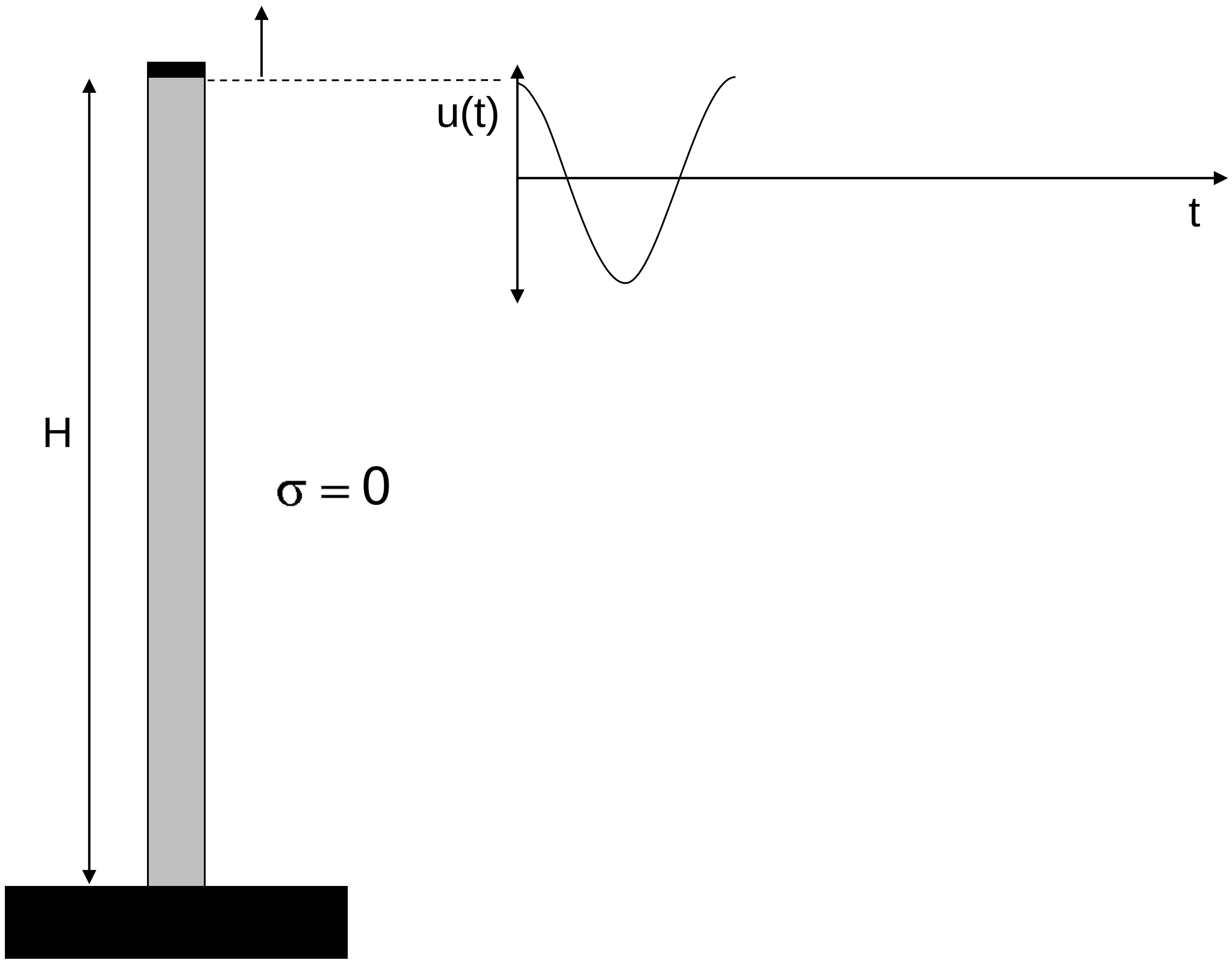


$$\sigma = \frac{W}{A}$$

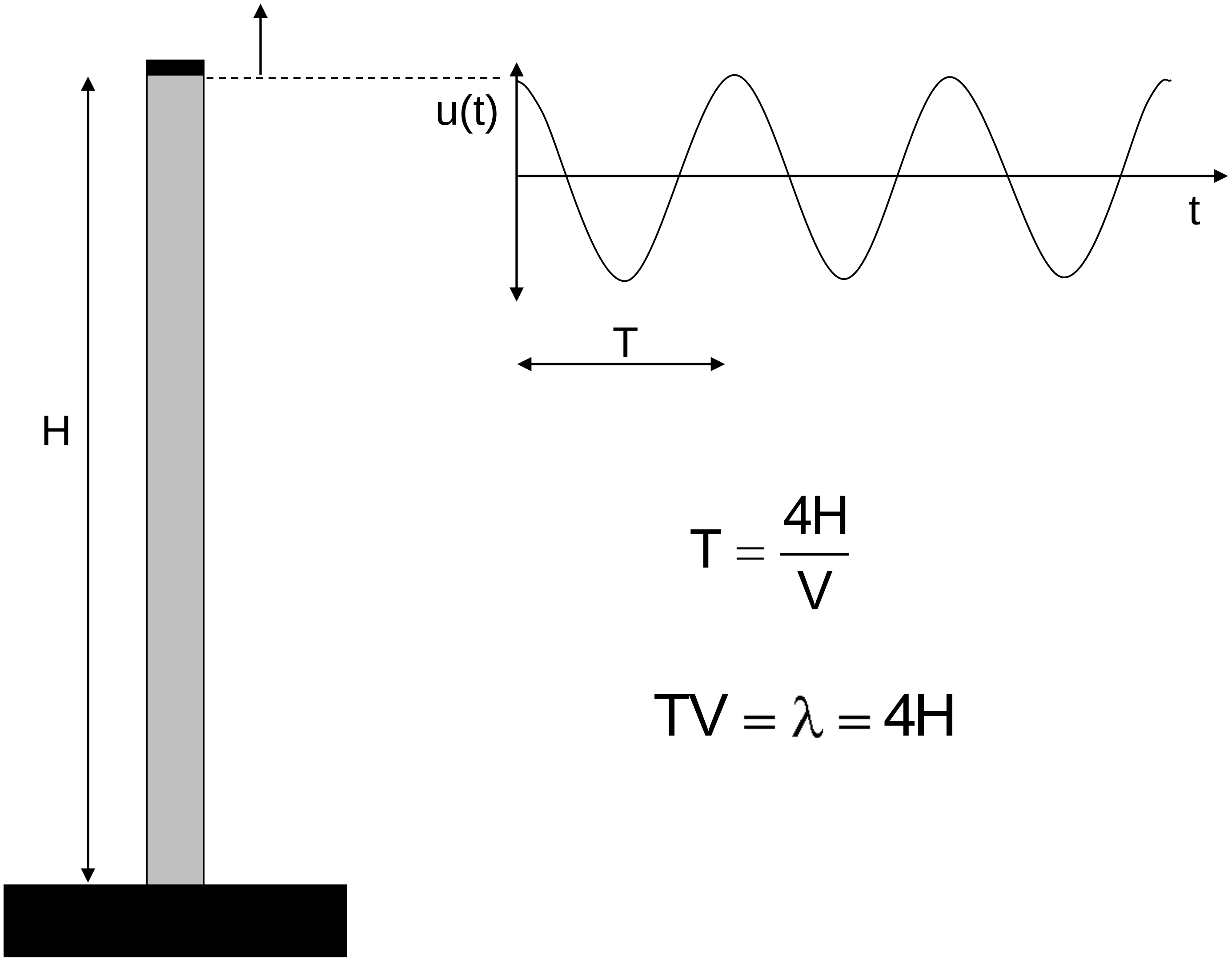
At time t>3H/ V



At time t=4H/ V



Resonance

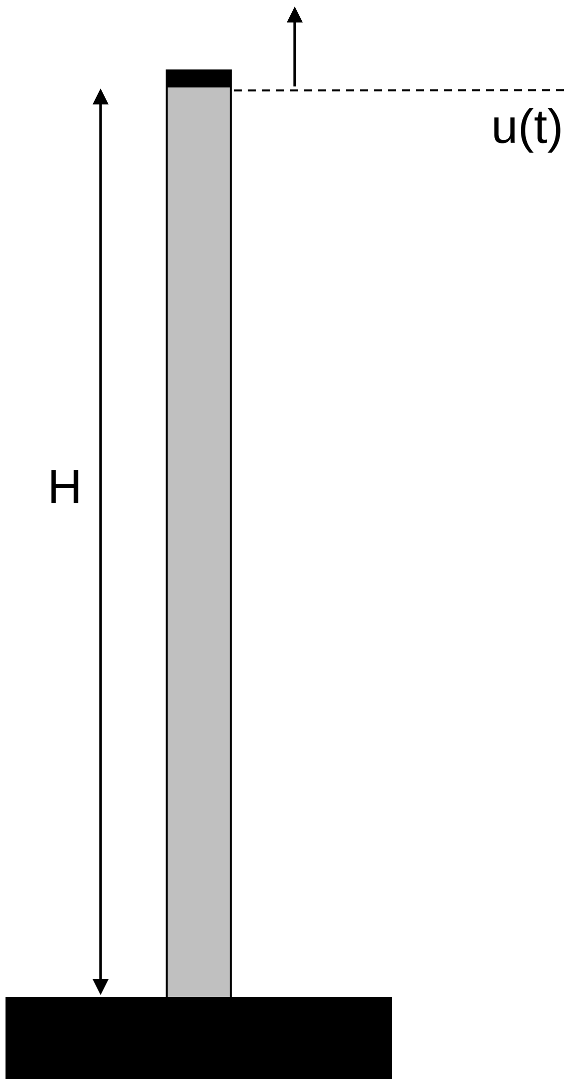


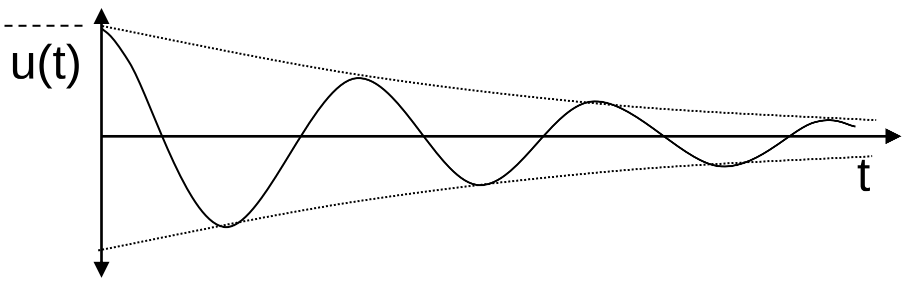
$$T = \frac{4H}{V}$$

$$TV = \lambda = 4H$$

Non-Elastic: Lossy

Lossy

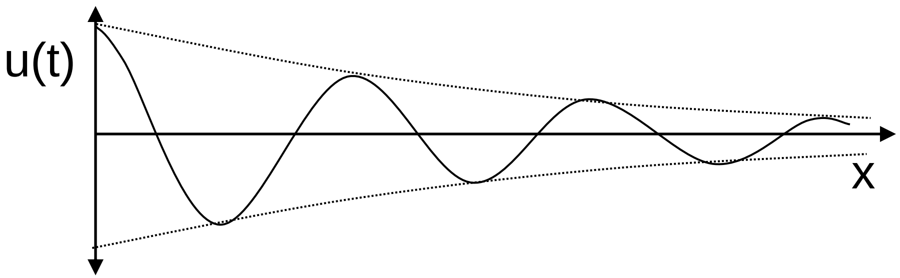




$$u(t) = u_o e^{-\alpha t}$$

$$u(t = \infty) = \frac{LW}{EA}$$

Lossy



$$u(t) = A\,e^{-\alpha x}\,e^{i(\omega t - \kappa x)}$$

$$\boxed{\frac{\partial^2 u_x}{\partial t^2} = \frac{M}{\rho}\frac{\partial^2 u_x}{\partial x^2}}$$

$$V_P^* = \frac{\omega}{\kappa - i\alpha} = \sqrt{\frac{M^*}{\rho}}$$

Lossy

$$V_P^* = \frac{\omega}{\kappa - i\alpha} = \sqrt{\frac{M^*}{\rho}} = \sqrt{\frac{M' + iM''}{\rho}} = \sqrt{\frac{M'}{\rho}\ 1 + i\tan\delta}$$

$$V_P = \frac{\omega}{\kappa} \approx \sqrt{\frac{M'}{\rho}}$$

$$\alpha \approx \frac{\omega\ \tan\delta}{2V_P}$$

Lossy

$$\alpha = \frac{1}{S_d} = \frac{2\pi D}{\lambda} = \frac{\pi f}{V}\tan\delta$$

$$\frac{1}{Q} = 2D = \frac{2V\alpha}{\omega} = \frac{V\alpha}{\pi f} = \tan\delta$$

Total Attenuation

$$\frac{A_1}{A_2} = \left(\frac{r_2}{r_1}\right)^{\varsigma} e^{\alpha \; r_2 - r_1} \; T^{-1}$$

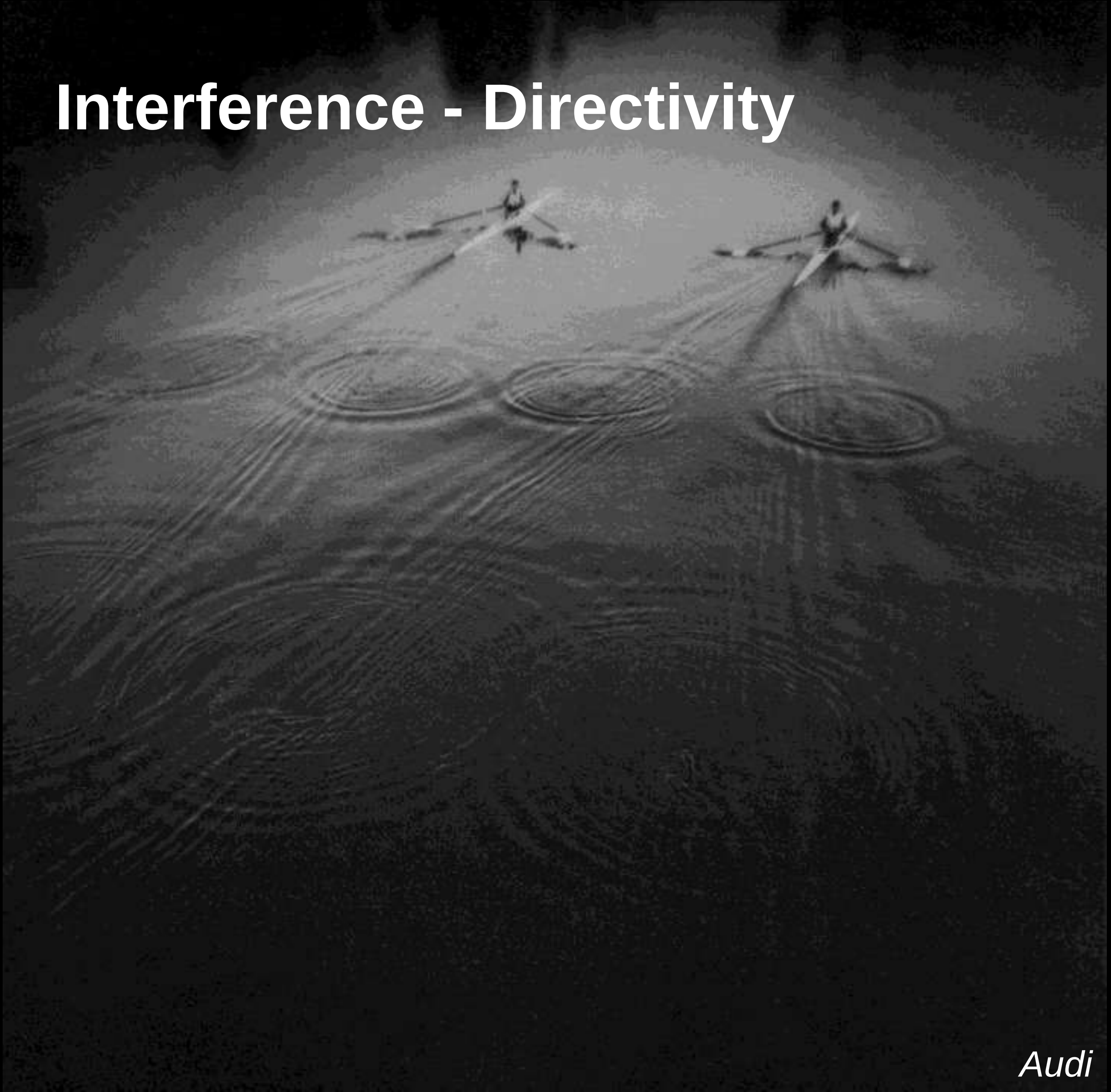
Material Attenuation $\leftrightarrow$ Dispersion

Kramers–Kronig

$$\frac{V_b - V_a}{V_b} \approx 1.5D \qquad \text{for} \quad \omega_b = 10\,\omega_a$$

Multiple reflections
Interference
Diffraction

Interference - Directivity

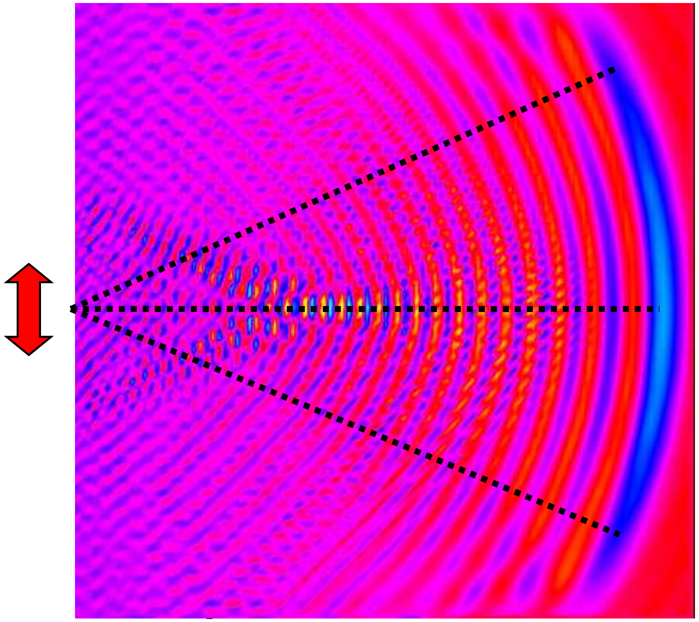


Audi

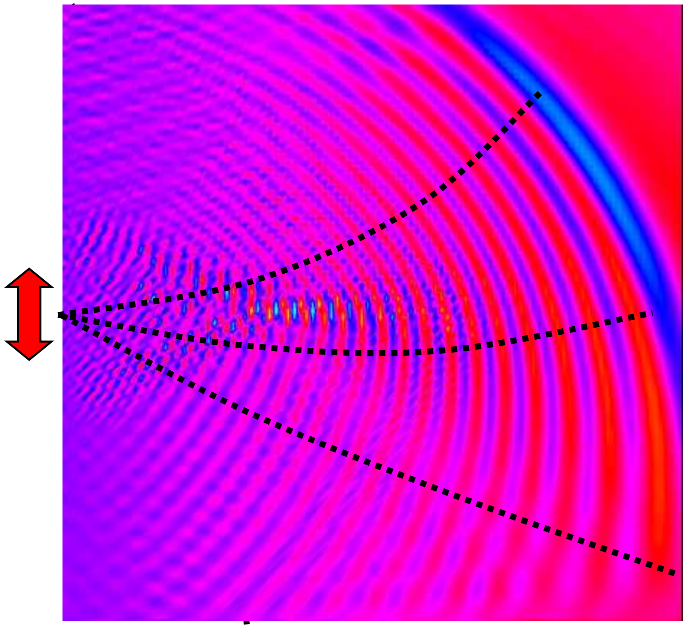
Diffraction Healing

Gradual Heterogeneity
Non-linear: Shock

Ray Bending: Fermat



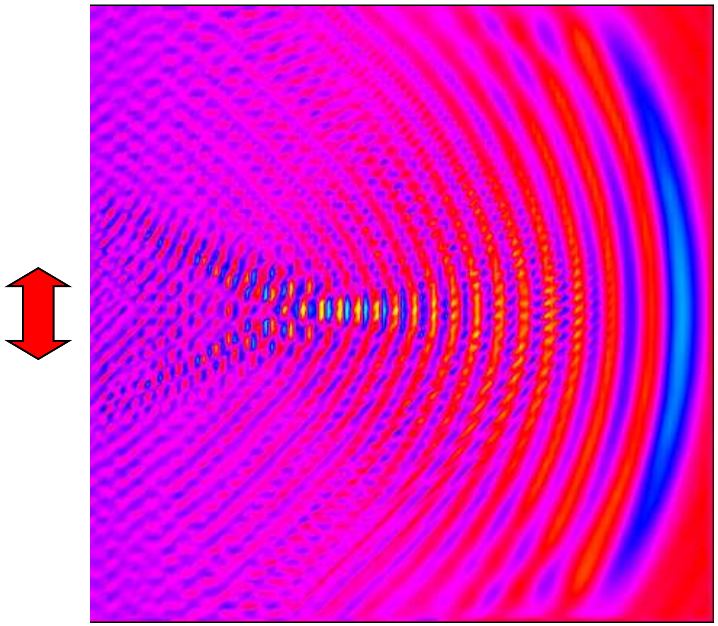
Homogeneous
Isotropic
Linear Elastic



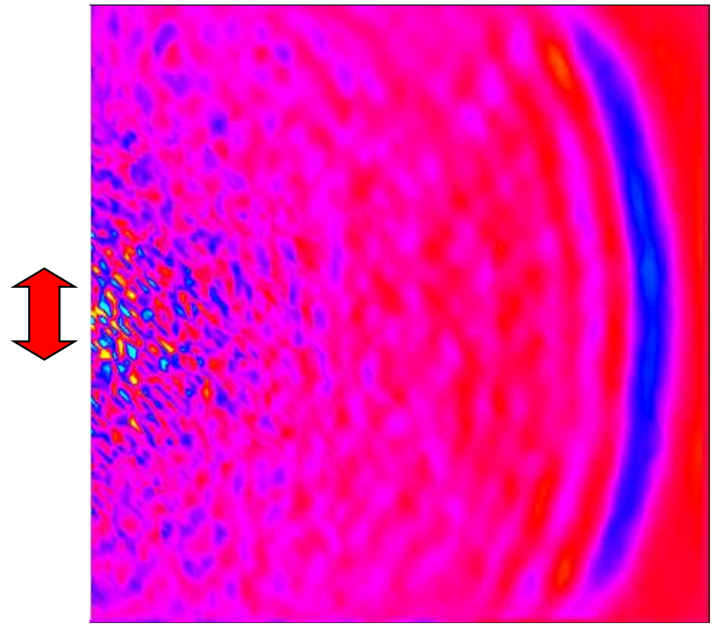
Vertically heterogeneous
Cross-anisotropic
Linear Elastic

HK Kim

Random Heterogeneity



Homogeneous
Isotropic
Linear Elastic



Correlated Random heterogeneous
Isotropic
Linear Elastic

HK Kim

Shock waves

Wave Phenomena: Complexity ⇔ Richness

Infinite, homogeneous, isotropic, single-phase, linear elastic, continuum	P, S
Finite medium	R – L – Rod - Tube
Interfaces	Reflection - Transmission,
	Refraction - Mode conversion
Gradually Heterogeneous	Curved rays (Fermat)
Anomalies	Diffraction - Scattering (Huygens)
Anisotropic	Quasi-propagation - Splitting
Multiphase (poroelastic)	Slow-P (Biot)
Visco-elastic	Attenuation & dispersion - Relaxation
Non-linear	Shock waves - Non-Lin. coupling
Discrete	Dispersion - Low-pass filtering

Measurement

Laboratory Testing

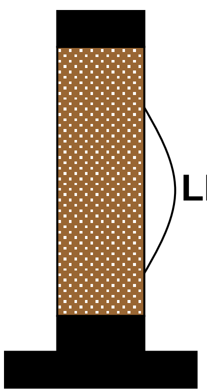
Quasi static — $\lambda >>$ cell
Standing wave — $\lambda \sim$ cell
Wave propagation — $\lambda <<$ cell

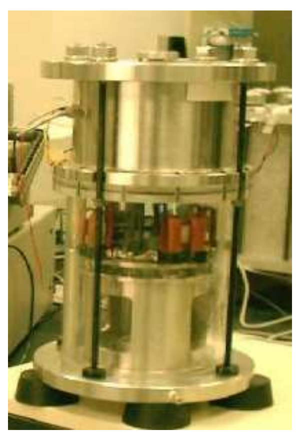
f_{res} — f

E or G
hyteresis

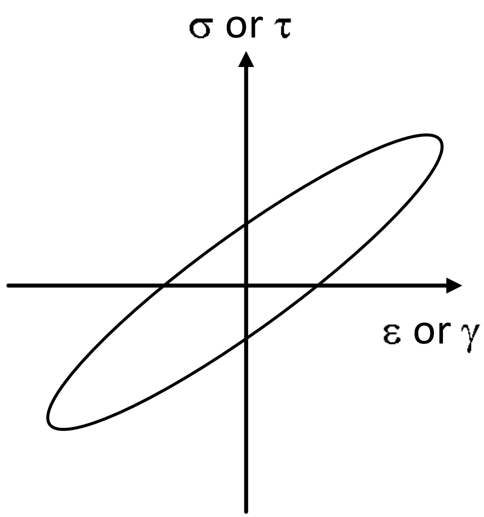
E or G
D

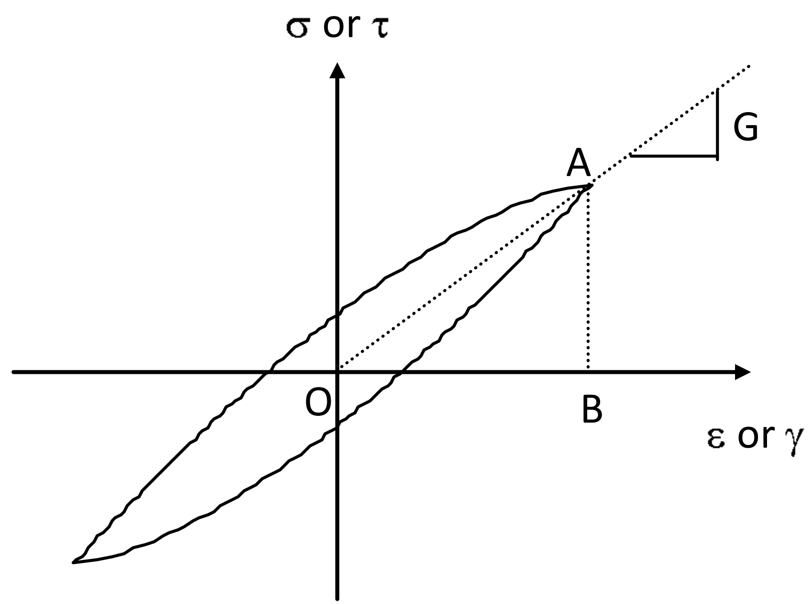
V_P or V_S
α


LDT



Quasi-static





$$E \text{ or } G = \text{slope}$$

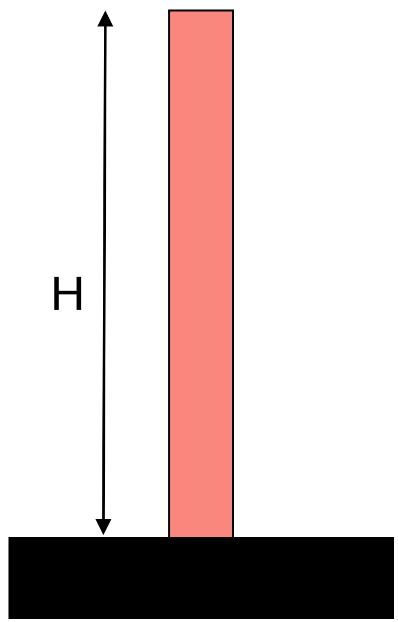
$$D = \frac{\text{area inside loop}}{4\pi \text{ area ABO}}$$

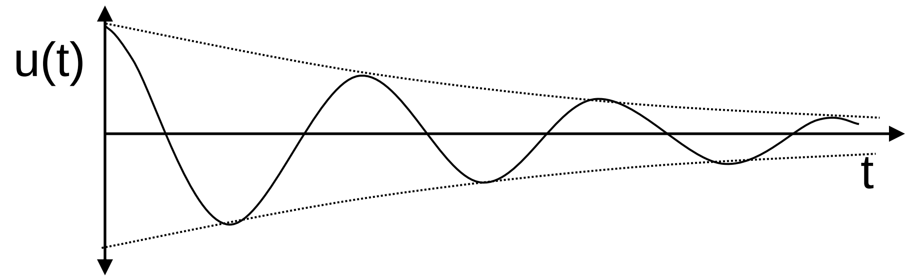
STANDING WAVE:

Step response

Resonant Column

Step Response





$$u(t) = A\,e^{-\alpha_t t}\cos\left(2\pi f_r t\right)$$

$$D = \frac{\alpha_t}{2\pi}$$

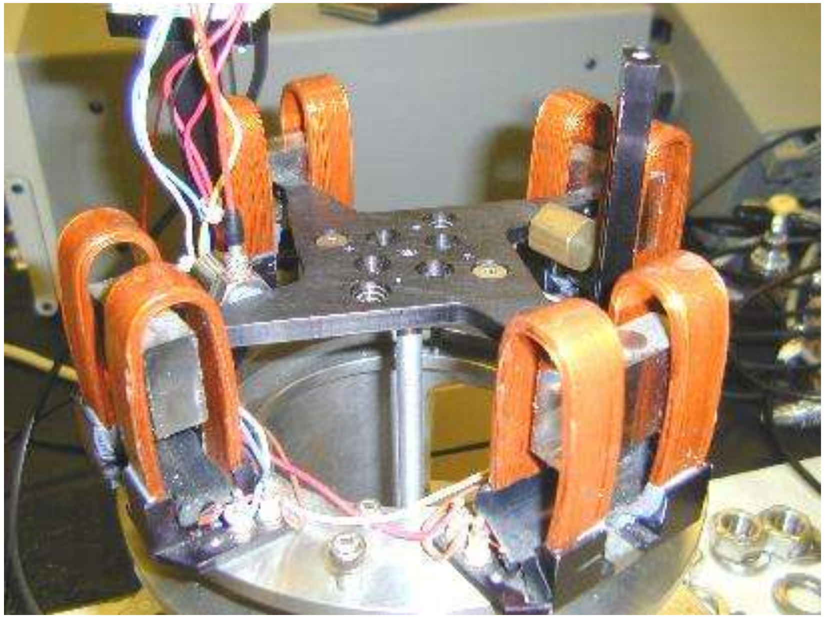
$$V = \frac{\lambda}{T} = \lambda f_r$$

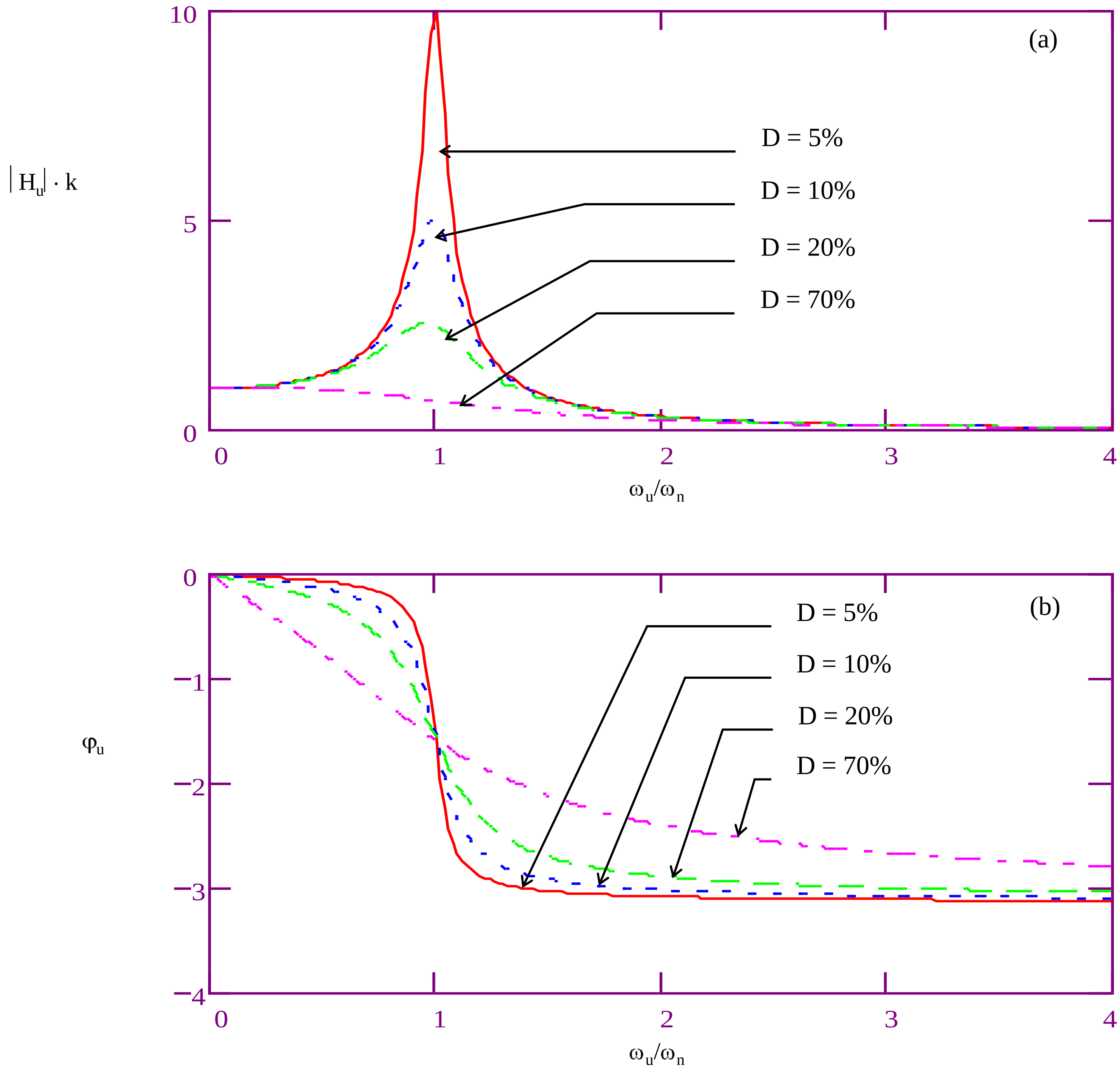
Wavelength?
Fixed-Fixed or Free-Free λ=2H
Fixed-Free λ=4H

Resonant Column

Resonant column

Driving head

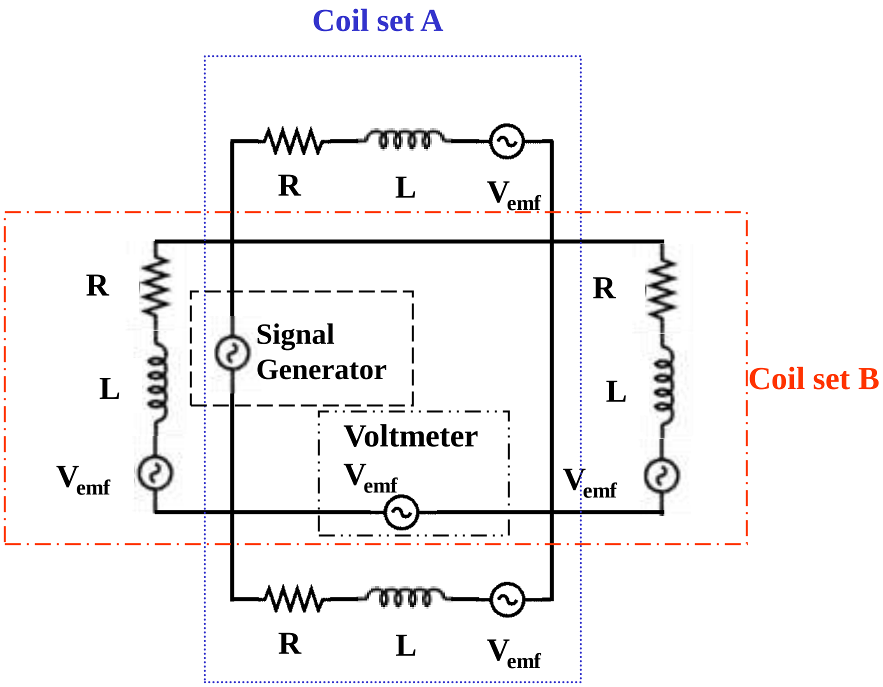




10
5
0
(a)
$|H_u| \cdot k$
D = 5%
D = 10%
D = 20%
D = 70%
0
1
2
3
4
ω_u/ω_n
0
−1
−2
−3
−4
(b)
φ_u
D = 5%
D = 10%
D = 20%
D = 70%
0
1
2
3
4
ω_u/ω_n

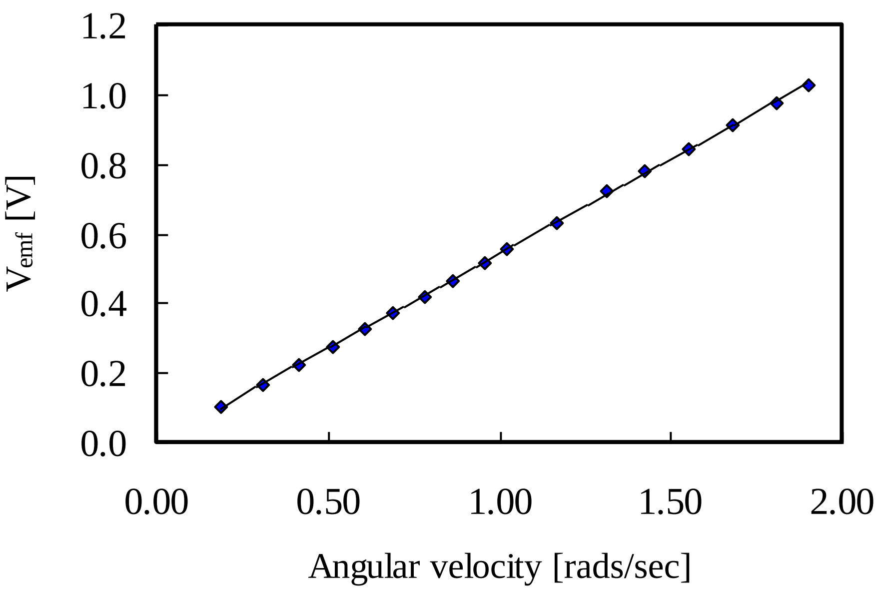
Induced Counter emf

- Equivalent circuit



- Induced counter emf

$$V_{emf} = \alpha \dot{\phi}, \alpha = 0.543$$

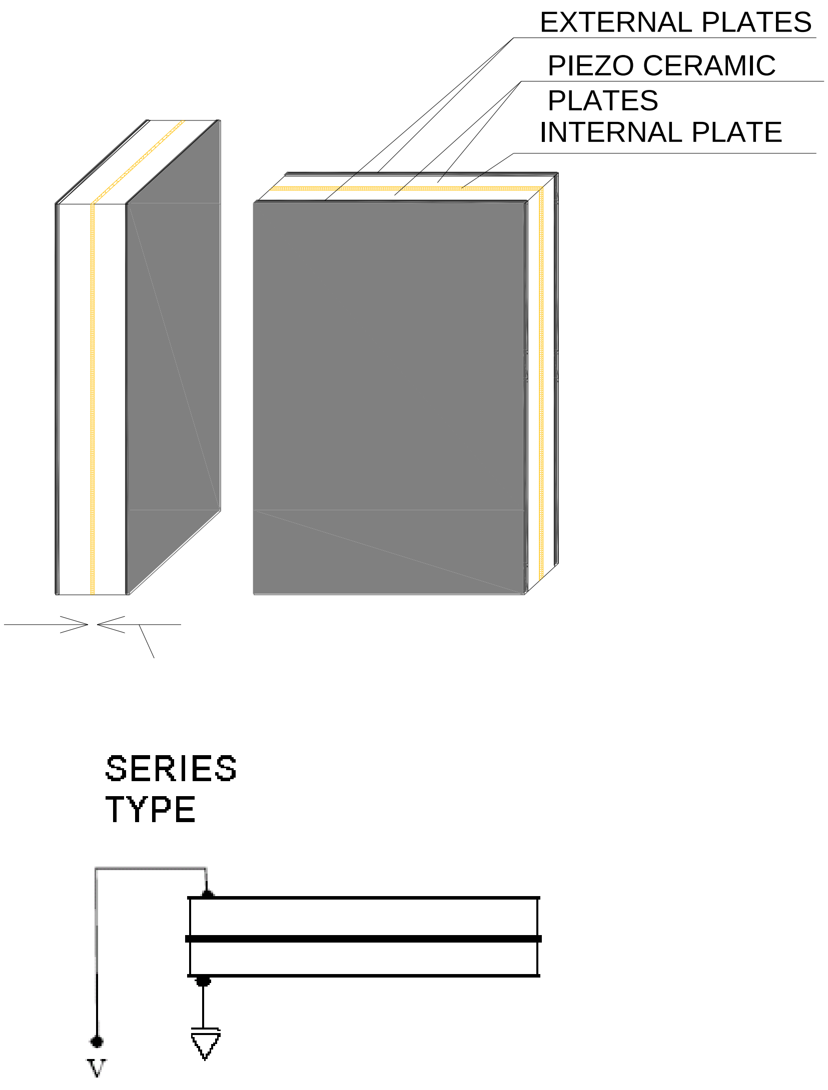


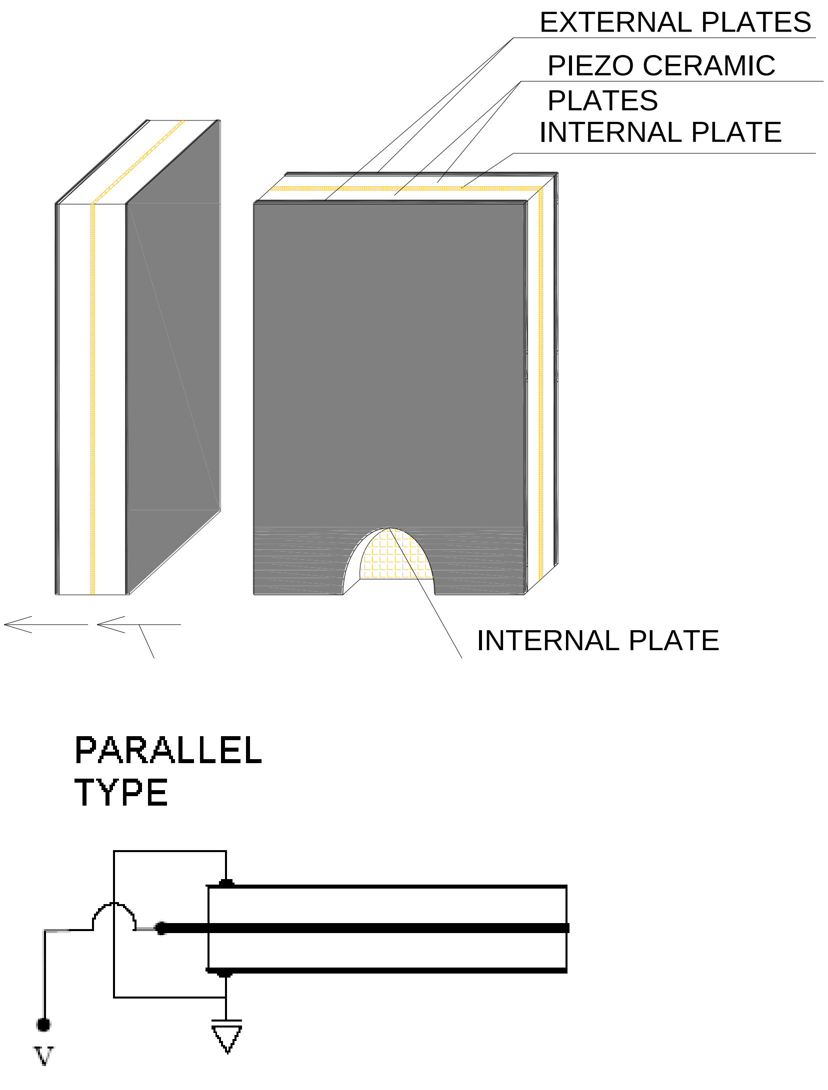
WAVE PROPAGATION

S: bender elements

P: standard piezo-elements

Bender element types



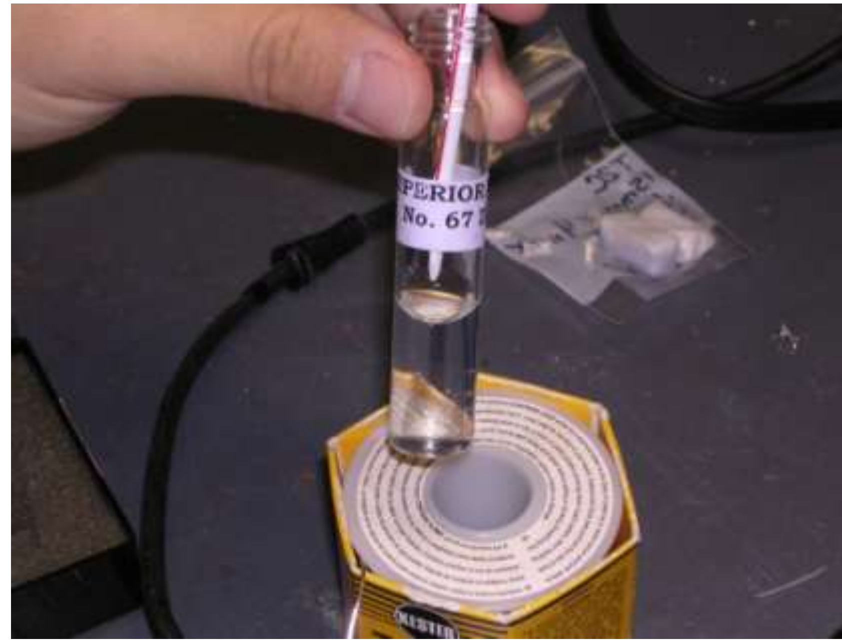


Devices and materials

1. Soldering iron and accessories
2. Soldering flux
3. Coaxial cable
4. Epoxy
5. Multimeter
6. Silver conductive paint
7. Heat-shrink tubing
8. Nylon flat point socket screw
9. Polyurethane

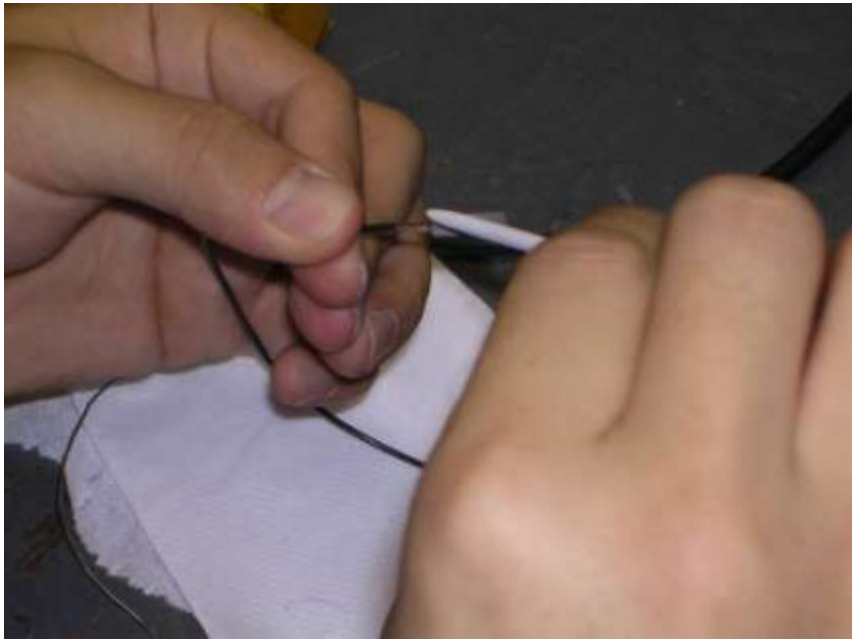
Preparation

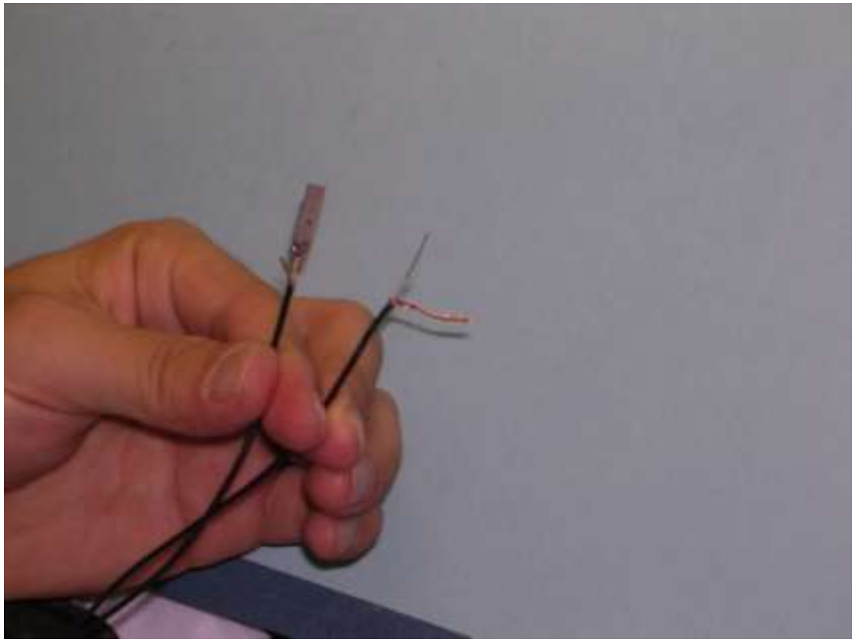
Remove outer shield from one end of coaxial cable. Separate the inner core from the copper mesh. Remove the end of inner core shield. If making a parallel BE, divide the copper mesh into two branches.



Coat the ends of the cable and the BE with soldering flux.

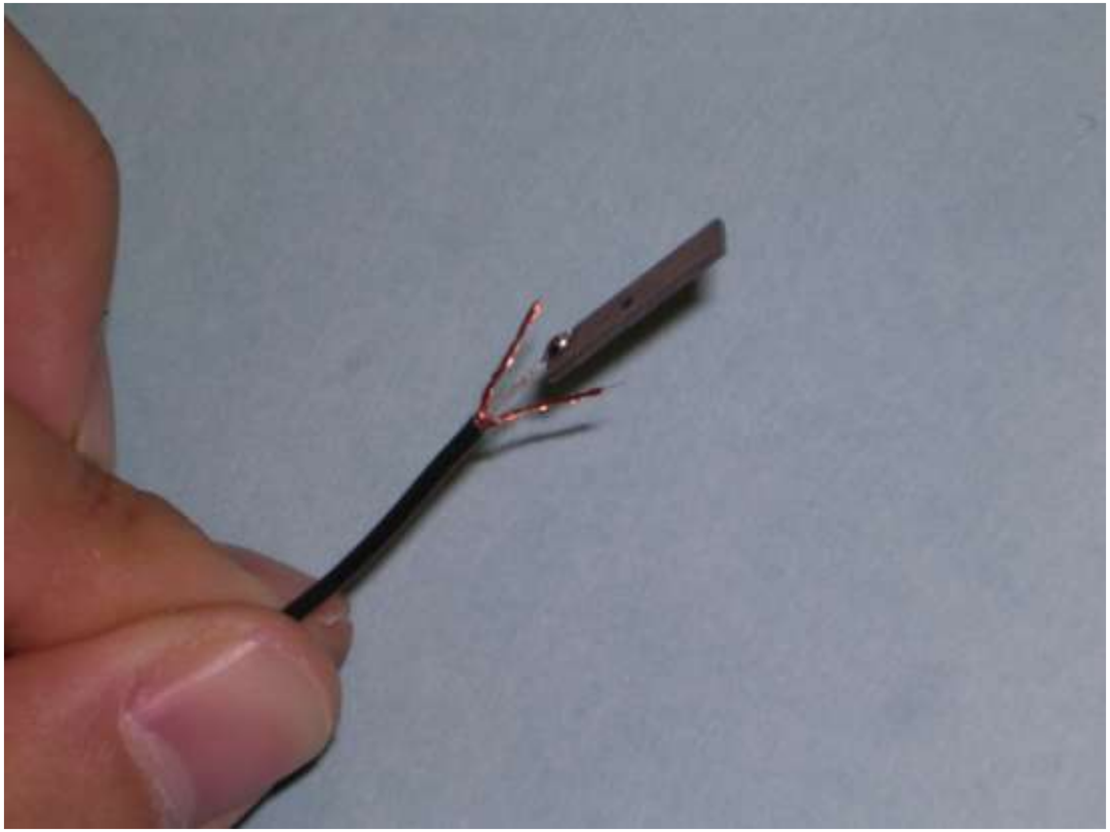
Preparation





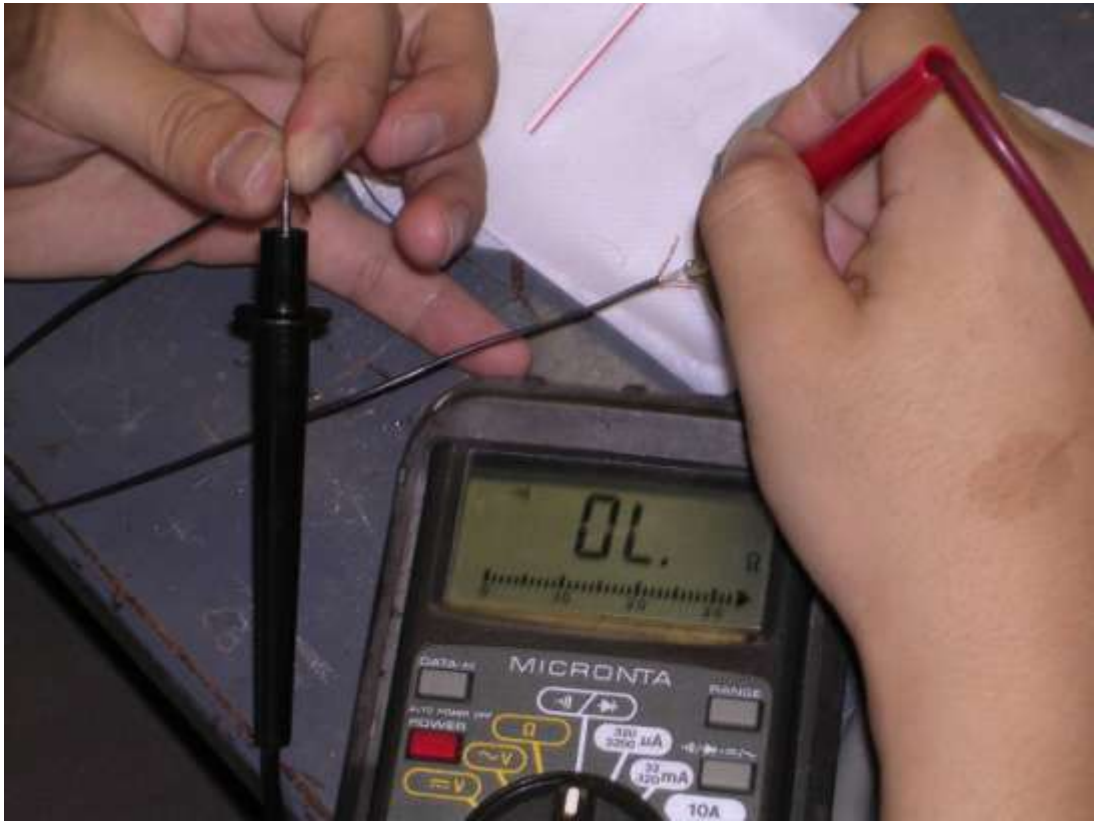
If making a series type BE, solder the core to one external plate and the copper mesh to the other one.

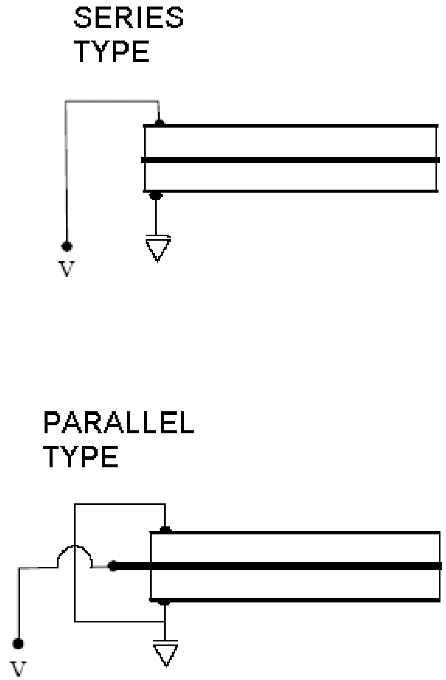
Preparation



If making a parallel BE, solder the core of the cable to the BE internal plate.
Caution: The core/soldering metal should not touch the external plates. Solder the two copper branches to the external plates.

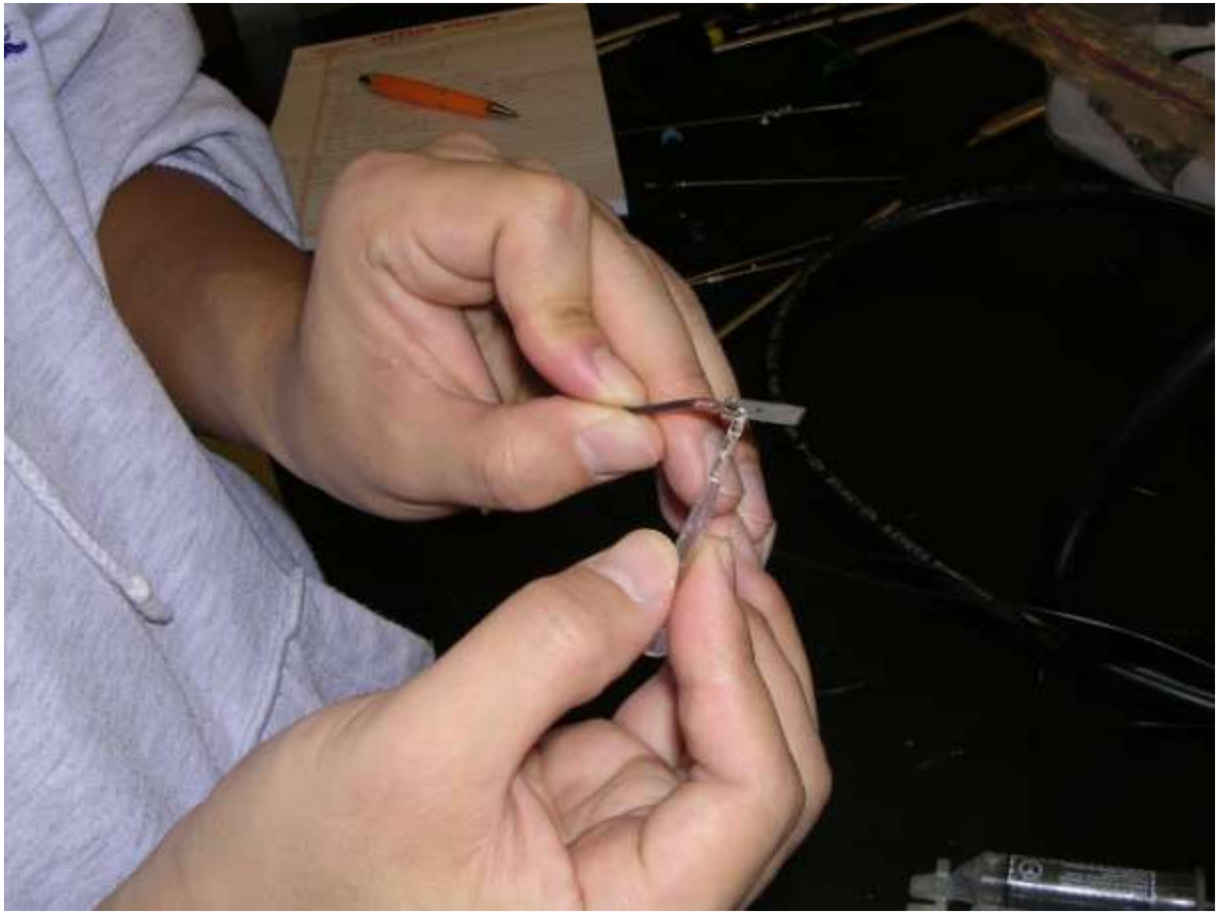
Check connections





Check the circuits with a multimeter.
The core-to-shield resistance must be infinite (open circuit).

Coating

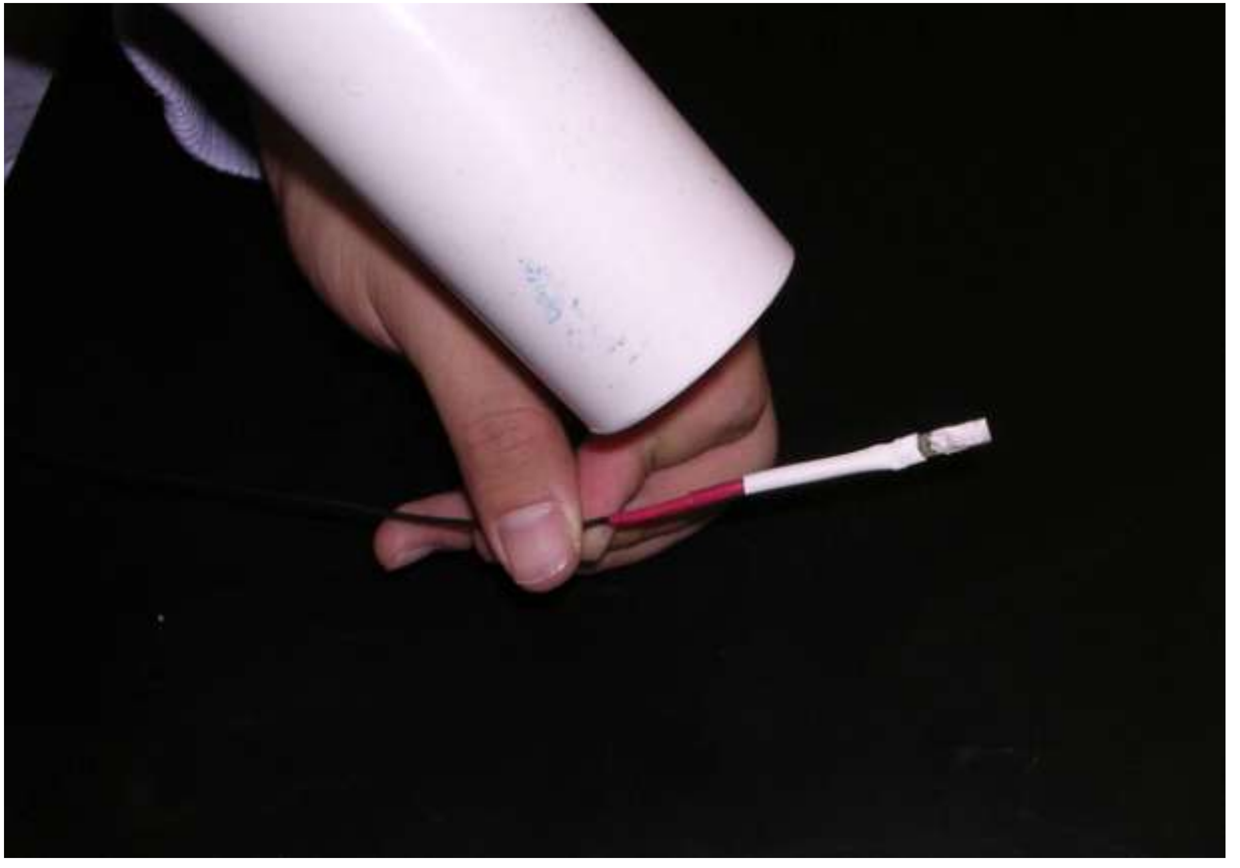


Water-proof the BE by coating the BE and the exposed portion of the cables with low viscosity polyurethane. Be sure to coat all BE faces, including the edges. Allow the polyurethane to dry with the BE in the upright position. A second coat may be applied if needed.

Electric shield

An electric shield is needed to prevent cross talk phenomena
(critical in wet soils – Parallel bender elements are “self-grounded”).
Spread a layer of silver conductive paint over the surfaces of the coated bender element.
The conductive paint must contact the shield in the coaxial cable, i.e., ground.

Cable reinforcement



Reinforce the connections using heat-shrink tubing. Shrink the tube using a hair dryer.
May use more than one shrink-tube layers.

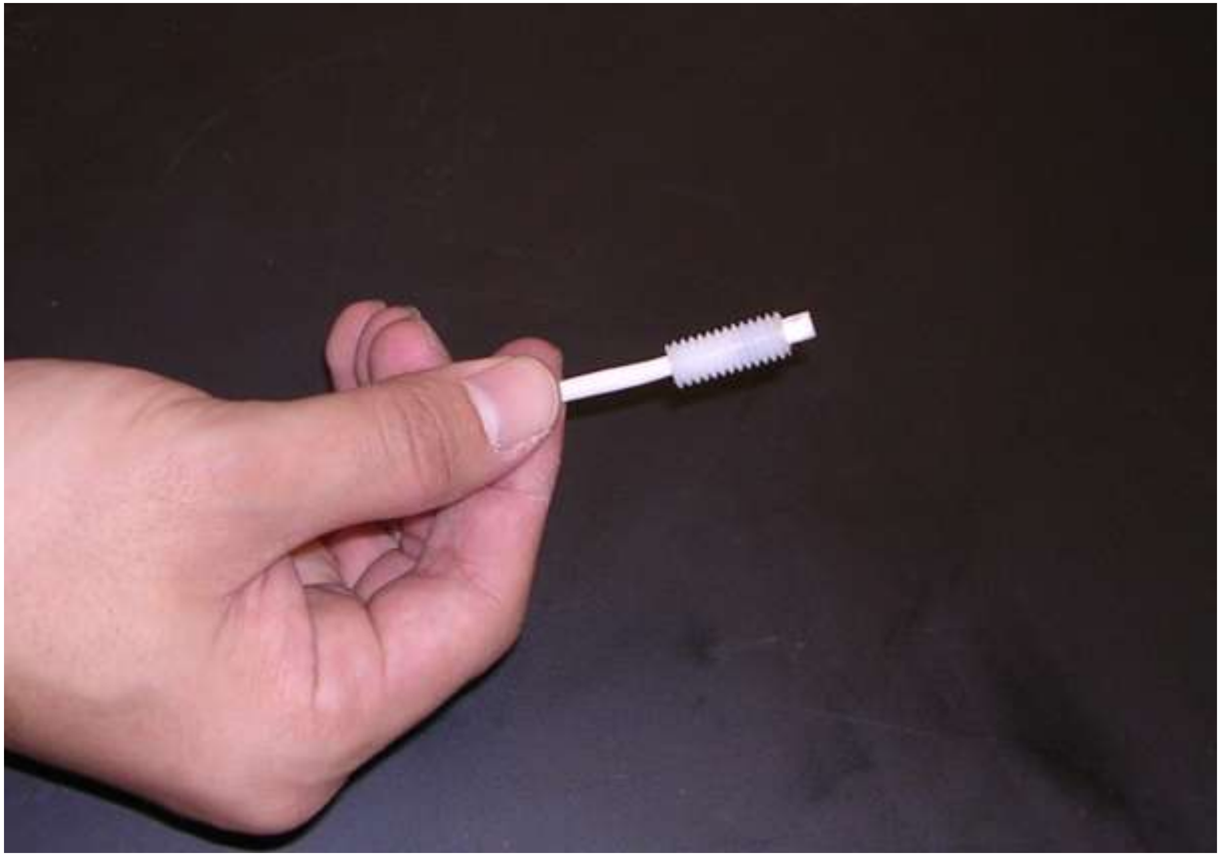
Housing in nylon socket screw: 1-drill

Nylon screw

Drilled Nylon screw

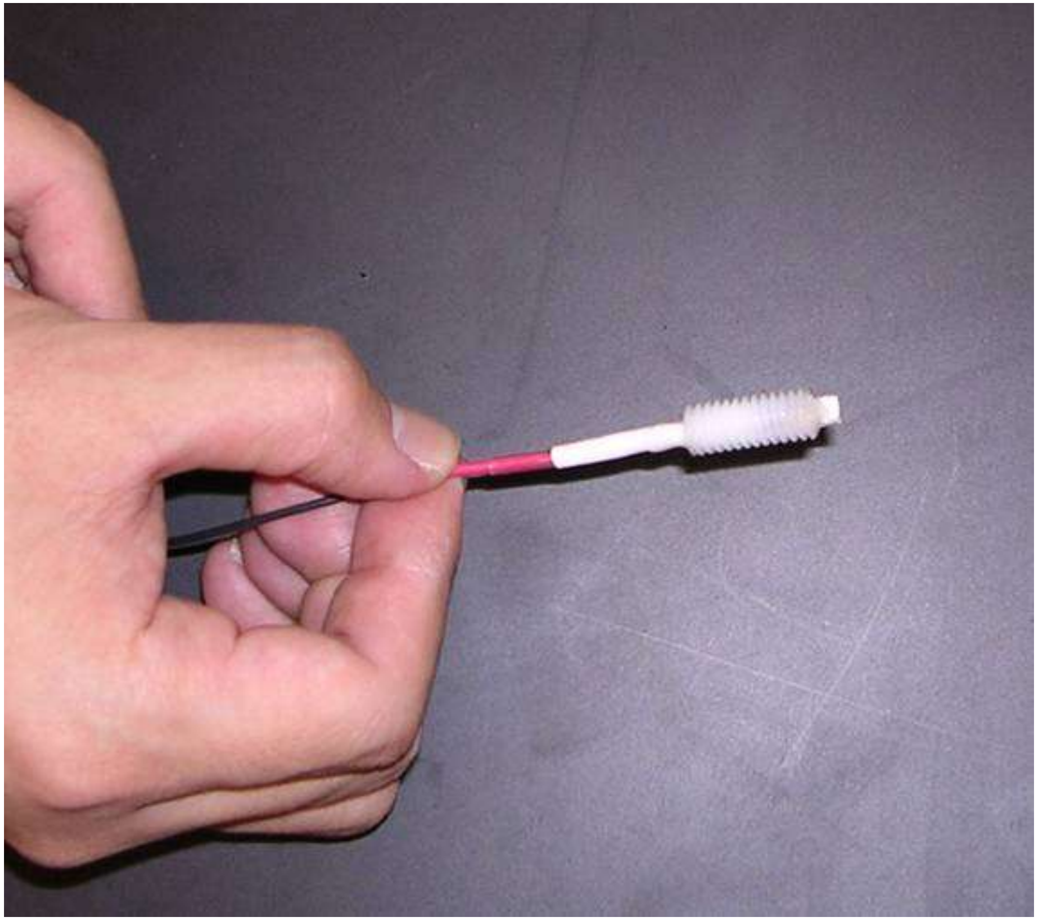
Take a nylon socket screw and make a hole through its center with a drill

Housing in nylon socket screw: 2-fix



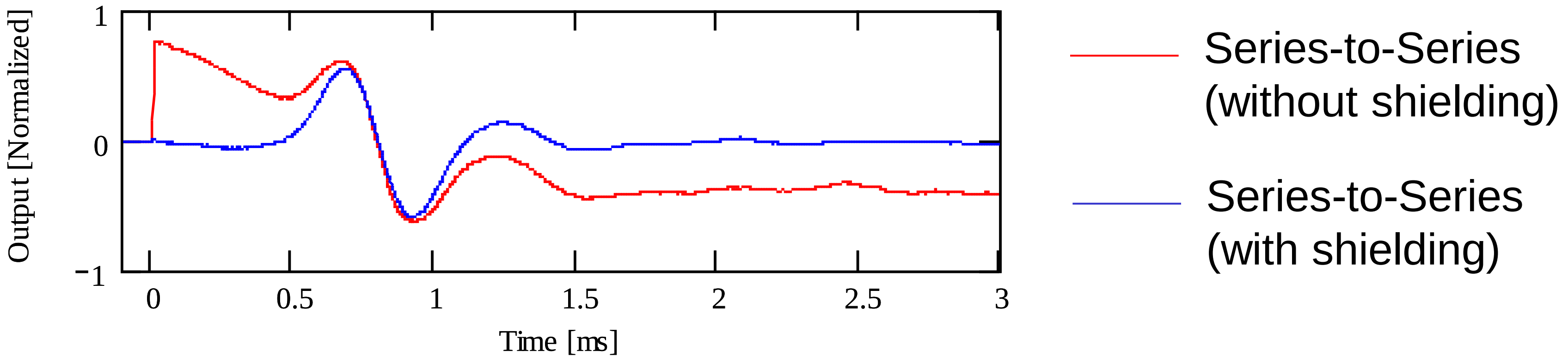
Slide the BE into the hole inside the nylon screw.
Fill in the air gap between the BE assembly and the screw with epoxy.

Done !

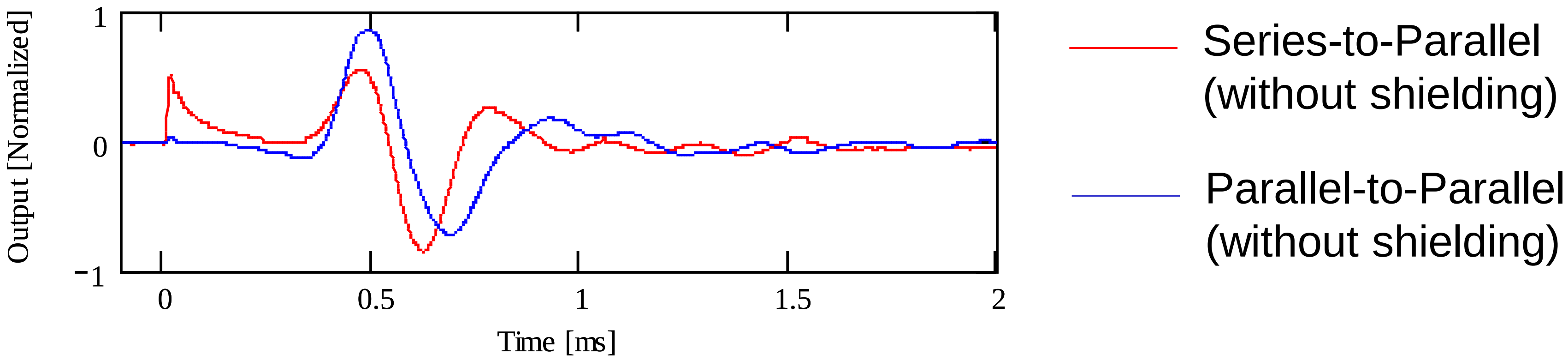


The BE assembly is ready for use once the epoxy has cured.
The threaded nylon screw housing can be conveniently installed in any geotechnical cell, and easily replaced in case of malfunction.

Cross-talk



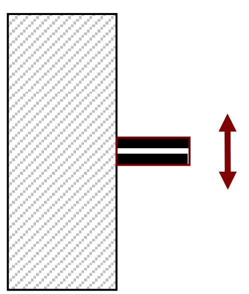
1
0
−1
0
0.5
1
1.5
2
2.5
3
Time [ms]
Output [Normalized]
Series-to-Series
(without shielding)
Series-to-Series
(with shielding)



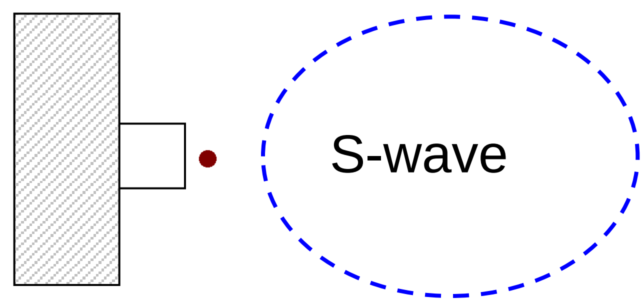
1
0
−1
0
0.5
1
1.5
2
Time [ms]
Output [Normalized]
Series-to-Parallel
(without shielding)
Parallel-to-Parallel
(without shielding)

Directivity

Transverse

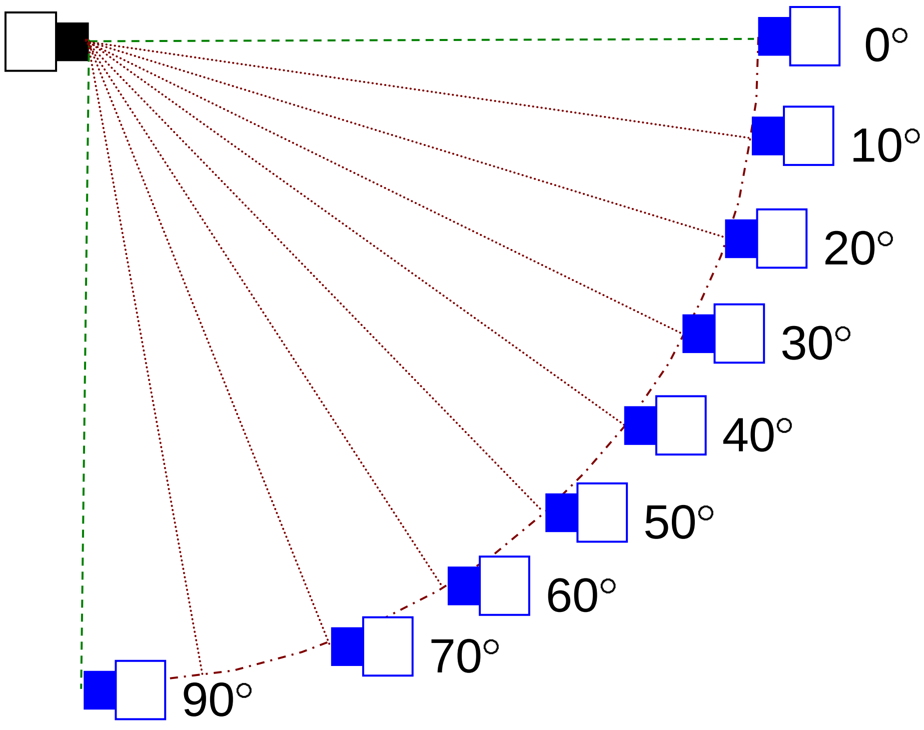


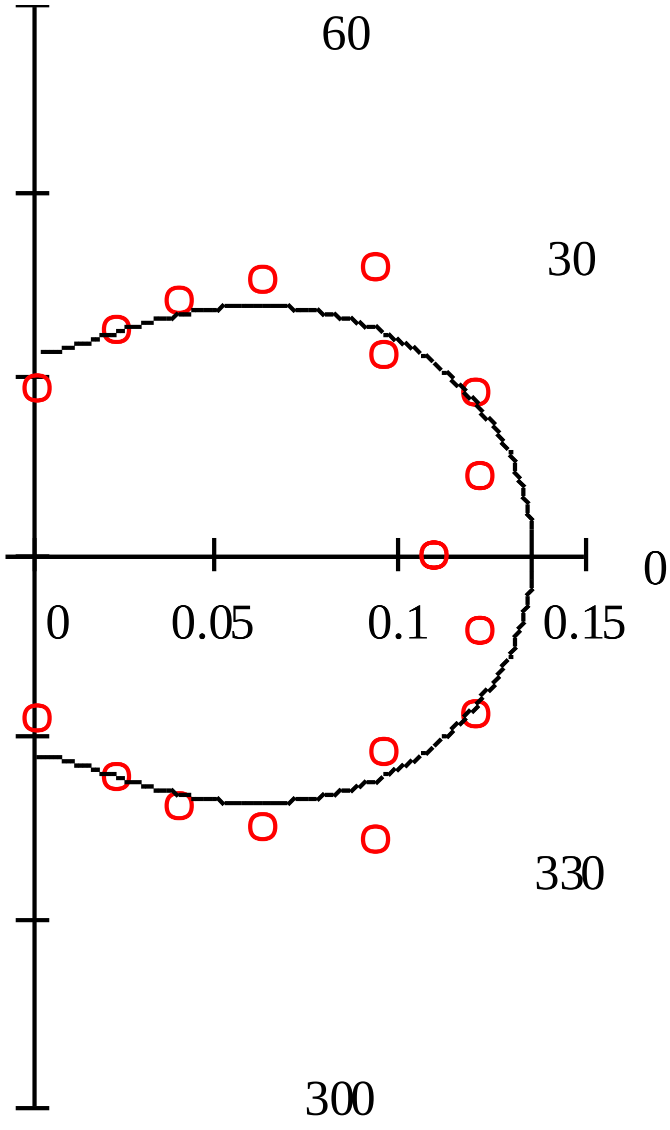
In-plane



In-plane directivity

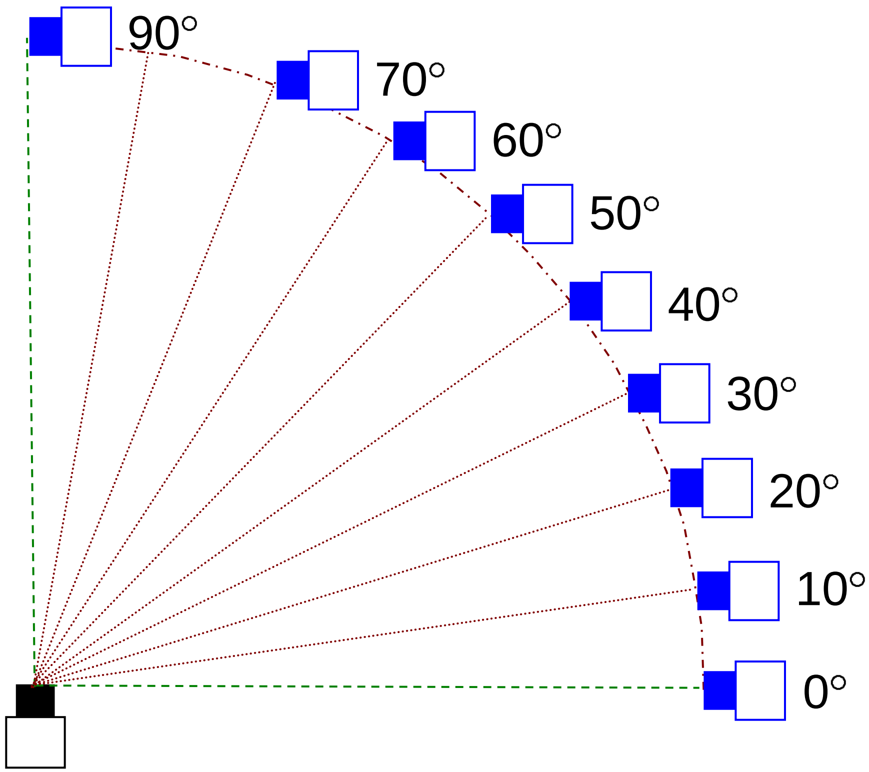
Crosshole
tomographic configuration

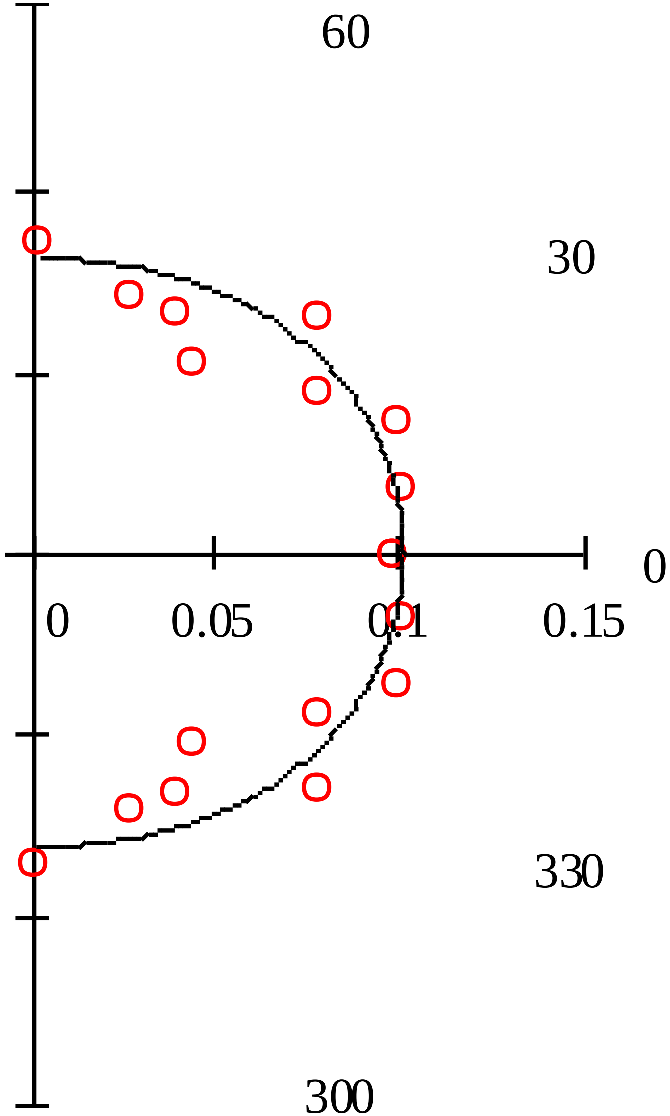




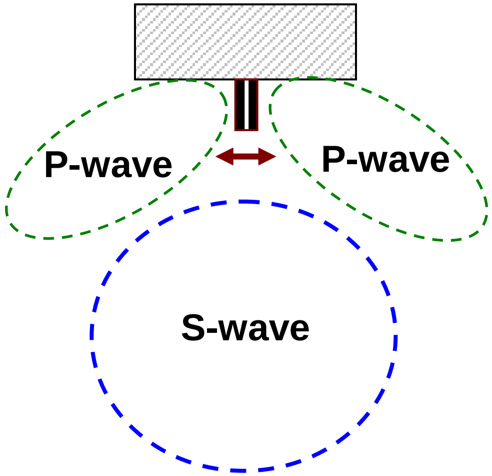
Directivity

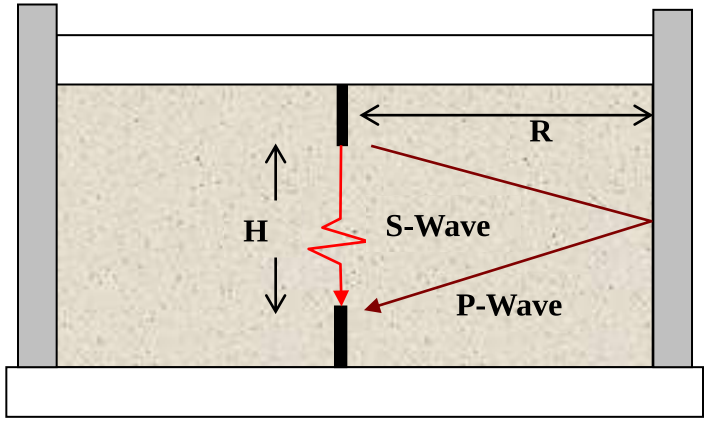
Base-to-borehole
tomographic configuration

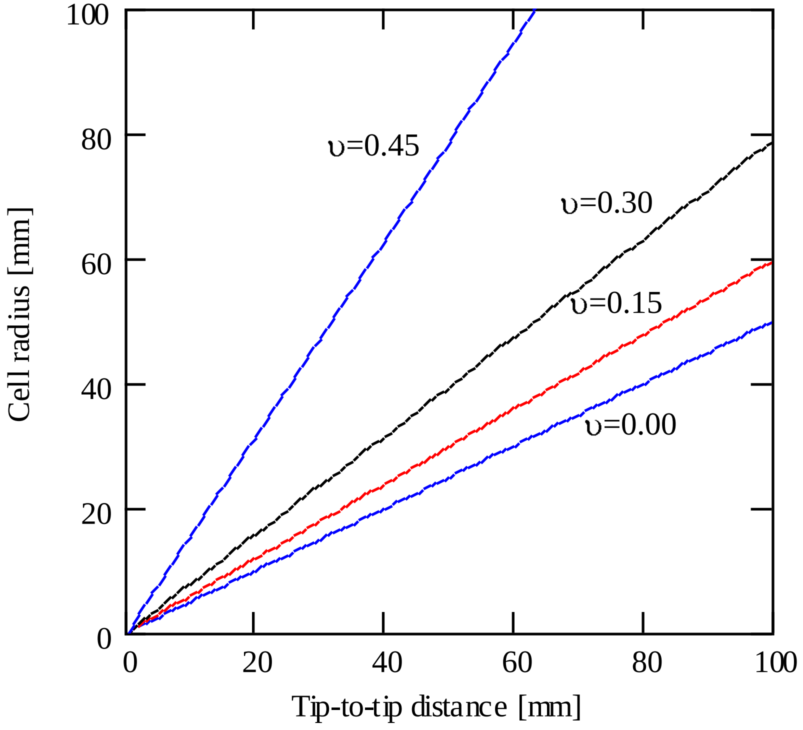




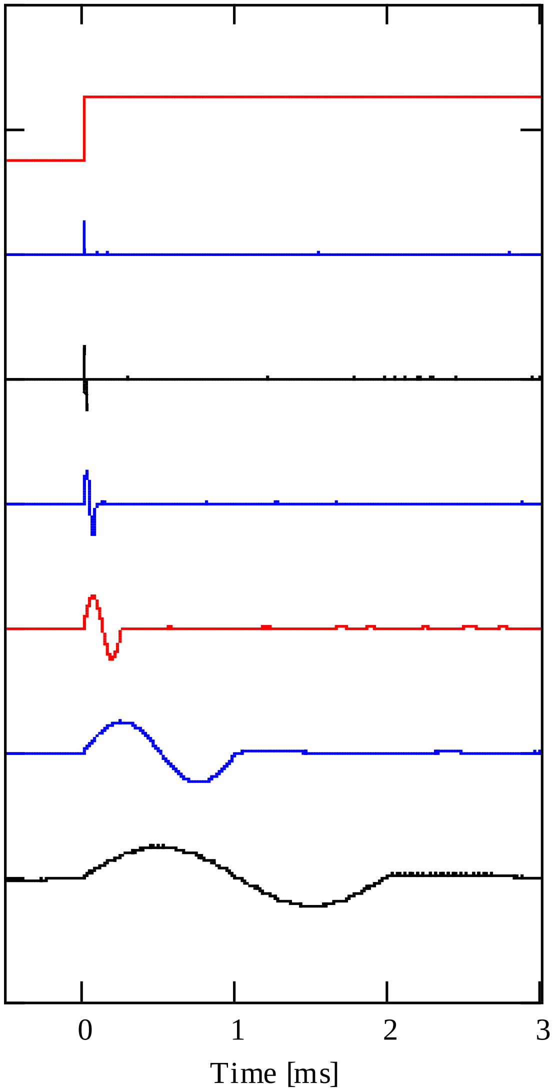
Transverse directivity: Side lobe P-wave (specimen size)

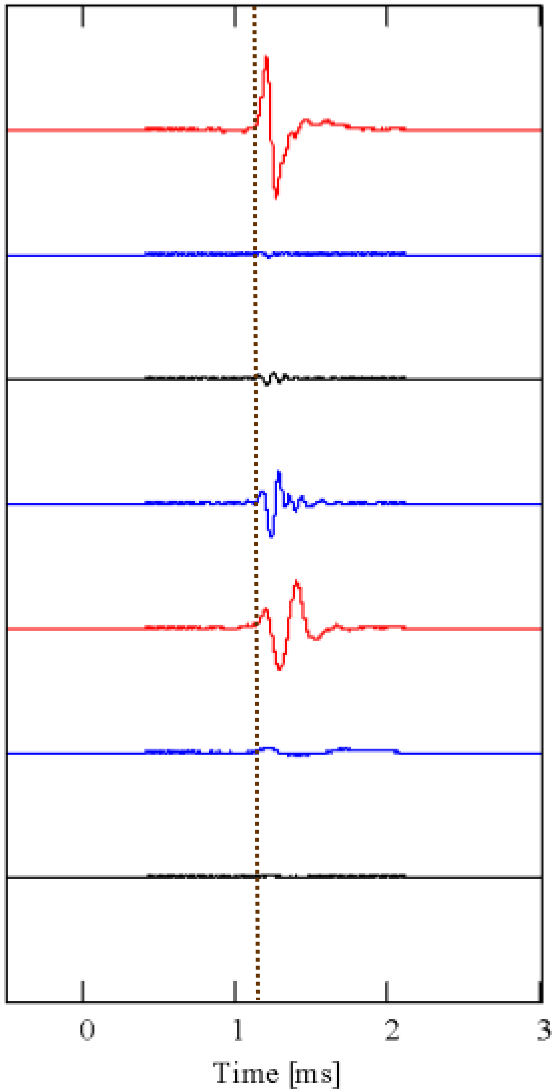






Input and output - Convolution





Square, $f_r \approx$ 4kHz

Impulse

Sine: f = 40kHz

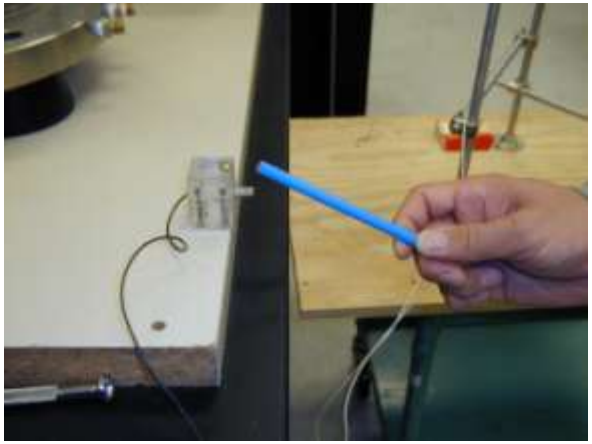
Sine: f = 12kHz

Sine: f = 4kHz

Sine: f = 1kHz

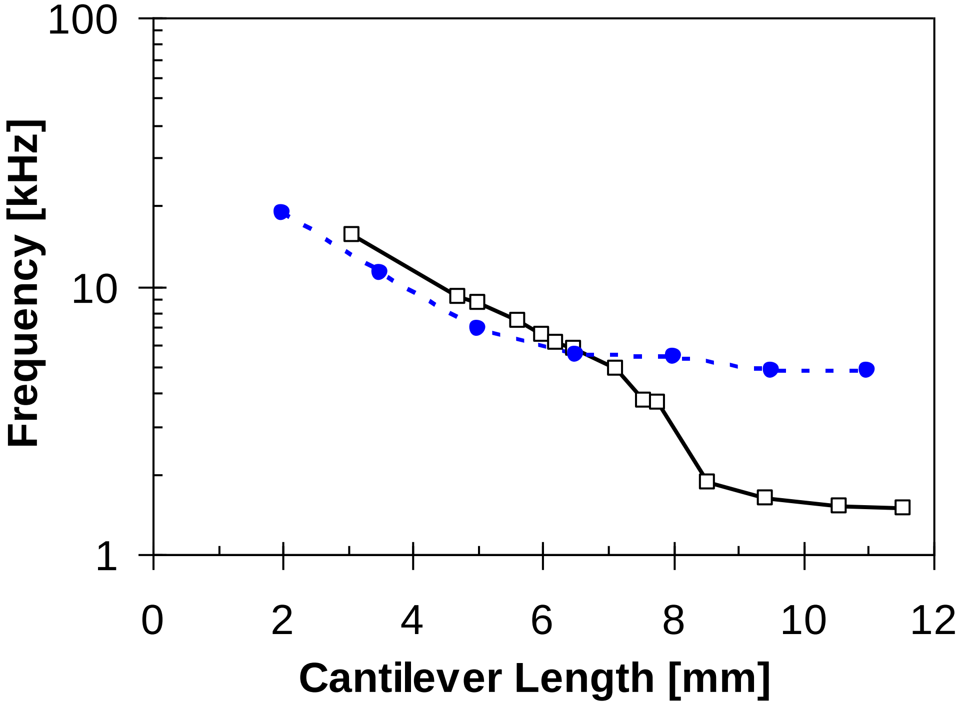
Sine: f = 0.5kHz

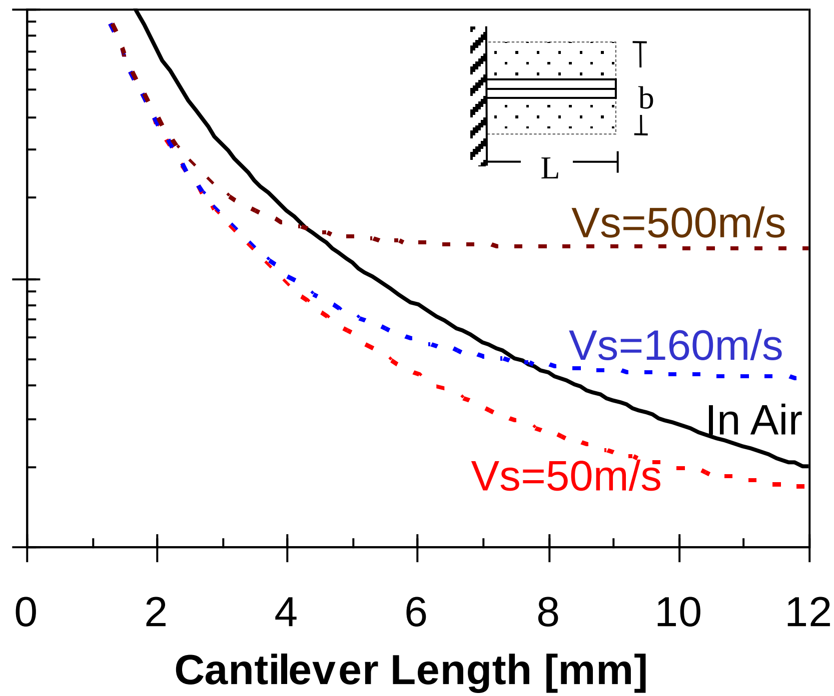
Resonant frequency

	Experimental study	Analytical formulation
In Air		$f = \frac{1.875^2}{2\pi}\frac{t}{L^2}\sqrt{\frac{E}{12\rho_{be}}}$
In Soil		$f = \frac{1}{2\pi}\left[\frac{1.875^4 \frac{EI}{L^3} + 2V_s^{\,2}\rho_{sl}(1+\upsilon)L}{\rho_{be}btL + \rho_{sl}b^2L}\right]^{\frac{1}{2}}$

Operating Frequency - Comparison

- Experimental Results
- Analytical Results



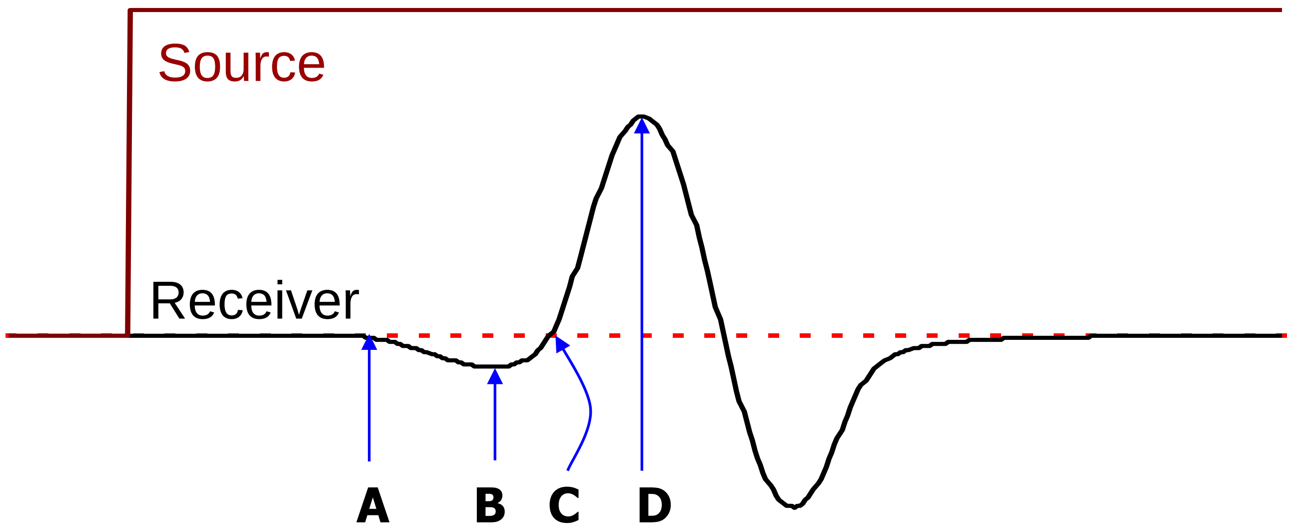


Controlling parameter:

Short cantilever length → Bender element

Long cantilever length → Soil properties

First arrival?



A: First deflection

B: First inflection

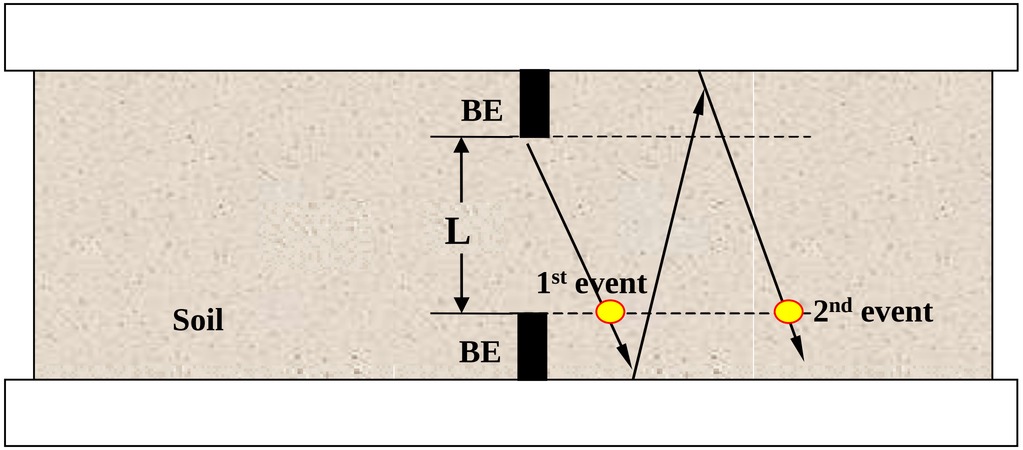
C: Zero after first inflection

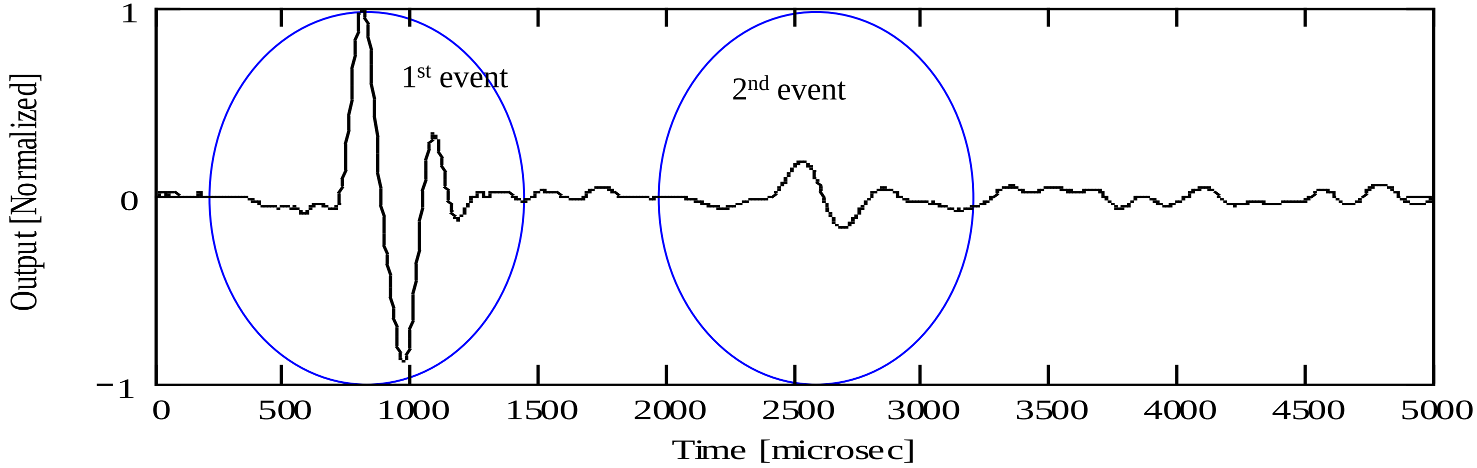
D: Second inflection

Multiple Reflection

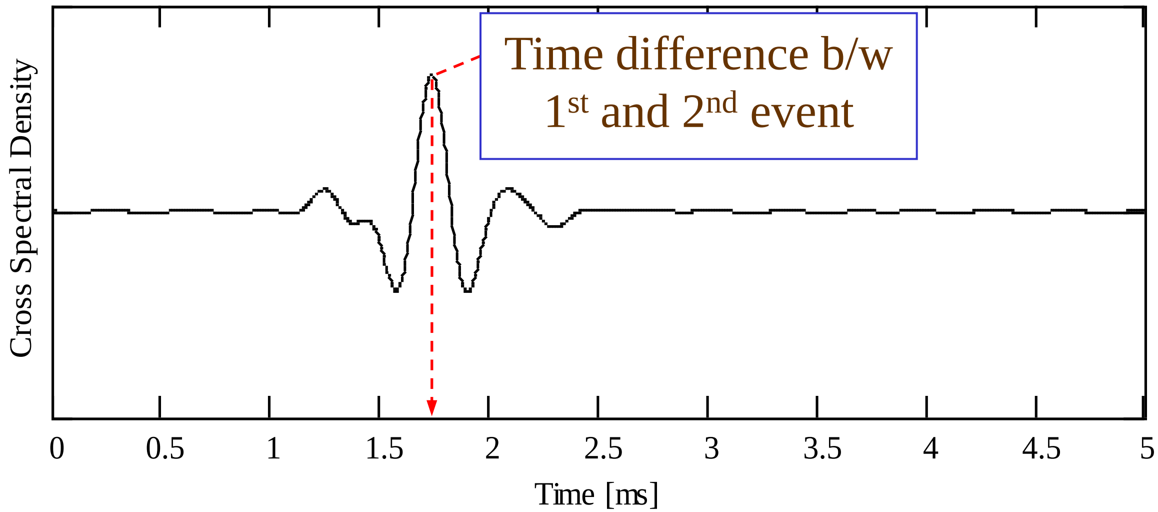
Goals:

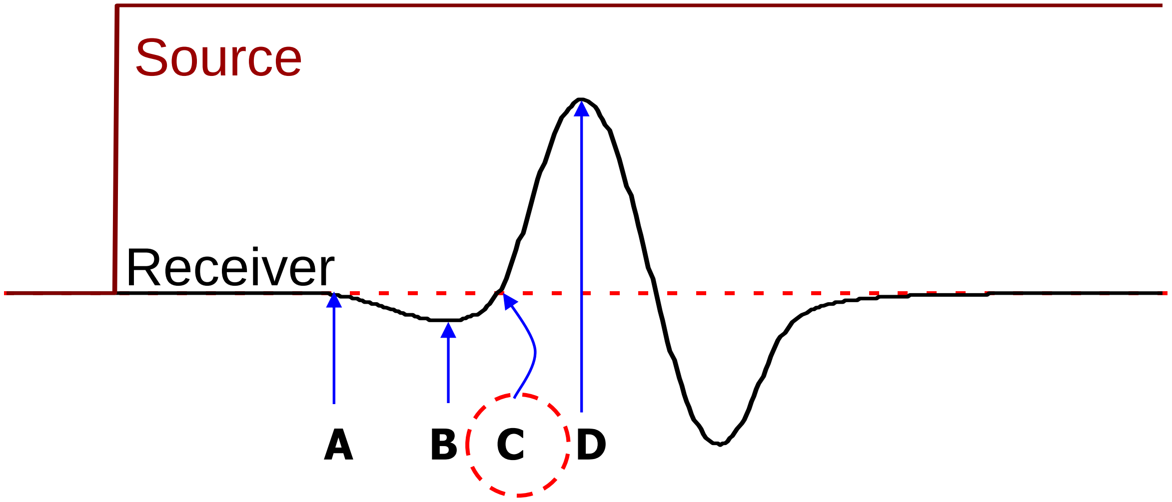
- High R-boundaries
- No P-wave from side walls
- No uncertainty in length
- No uncertainty in time





Experimental Study - Results





Near Field: Signal matching

Mathematical Solution

Cruse and Rizzo (1968)

Stokoe and Sanchez-Salinero (1987)

Procedure: Signal Matching

For given values L and μ

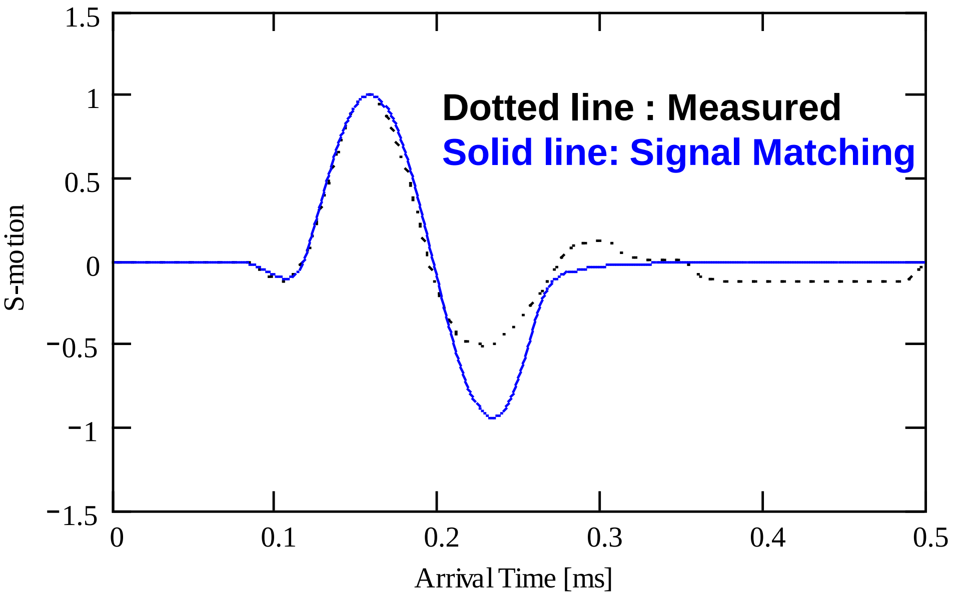
1: Measure the signal S_m

2: Estimate f_r and V_s

3: Compute predicted signal

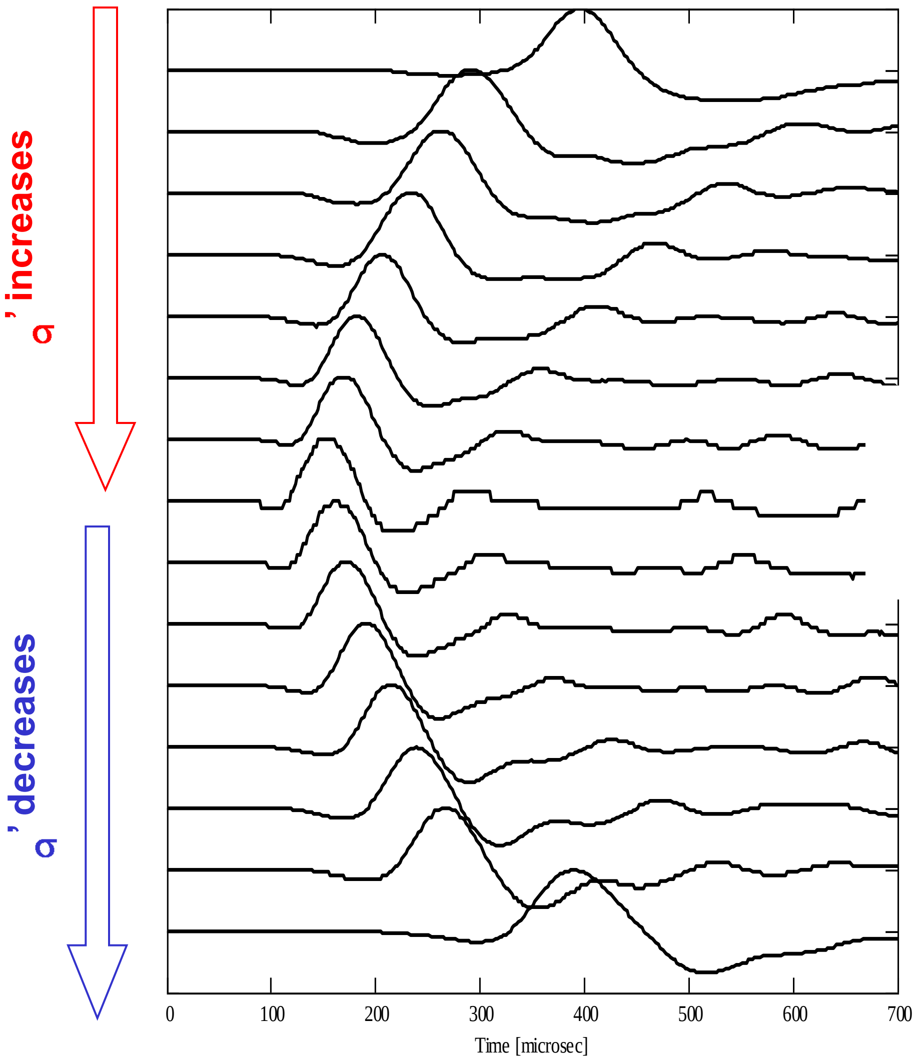
$S_p = f(V_s, f_r)$

4: Change f_r and V_s until $S_p \sim S_m$

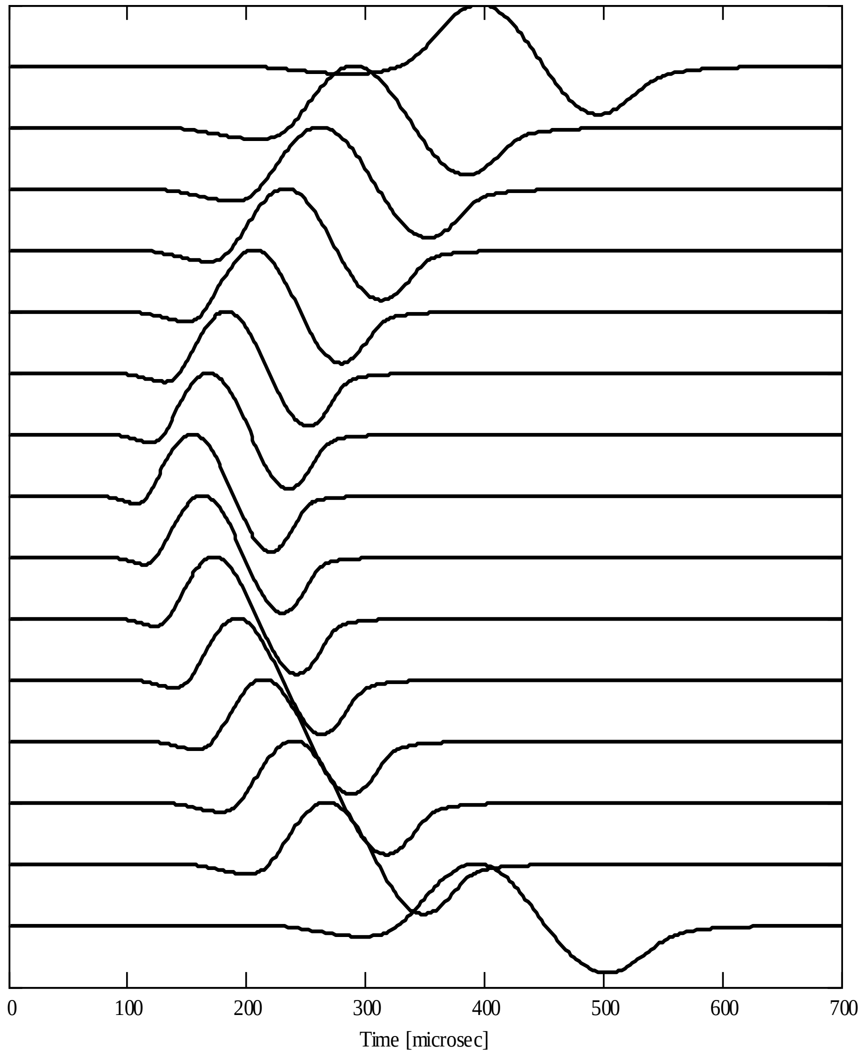


Analytical Approach

Measured signals

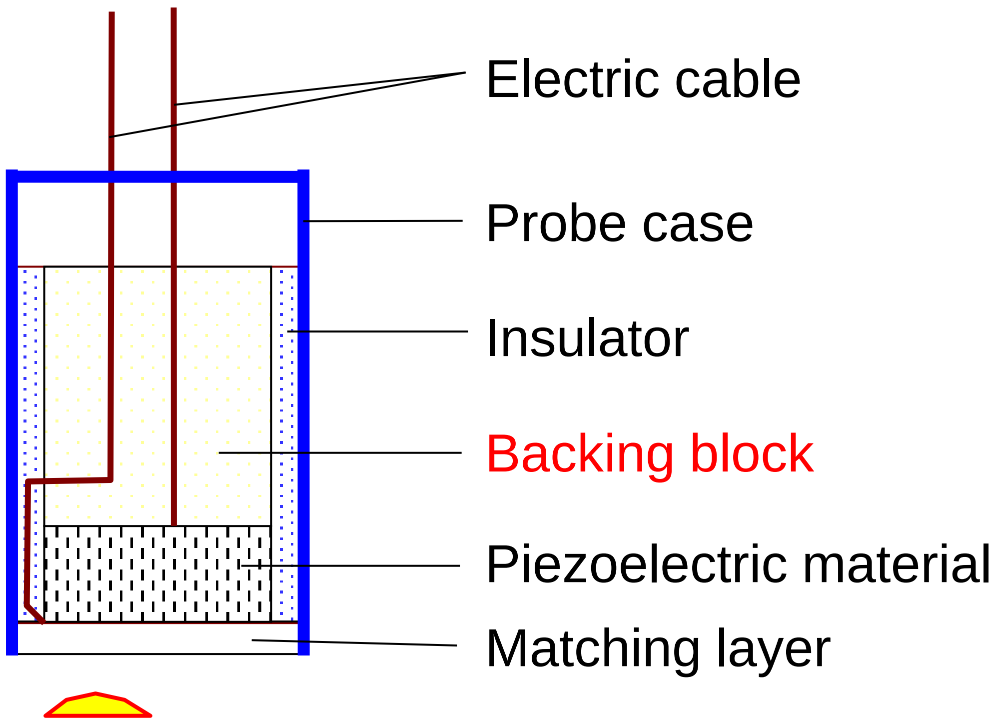


Predicted signals

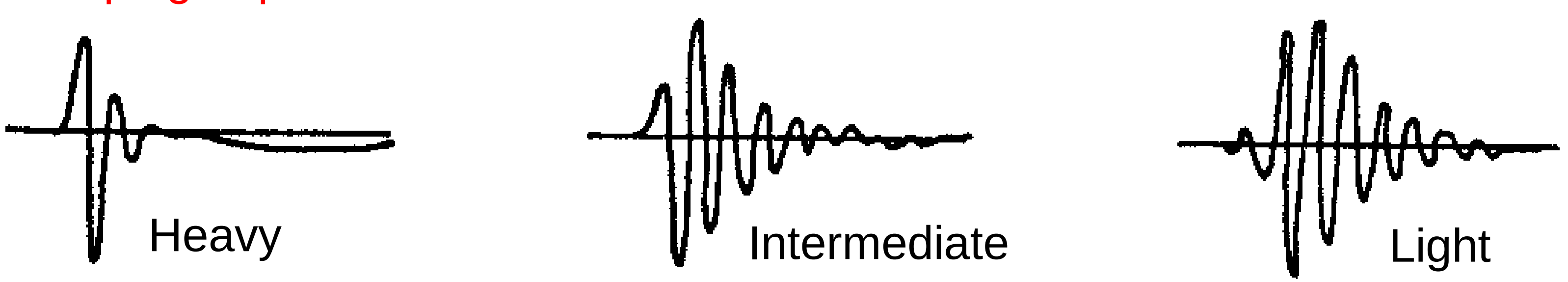


P-Transducer

Ultrasound Transducer



Damping dependent



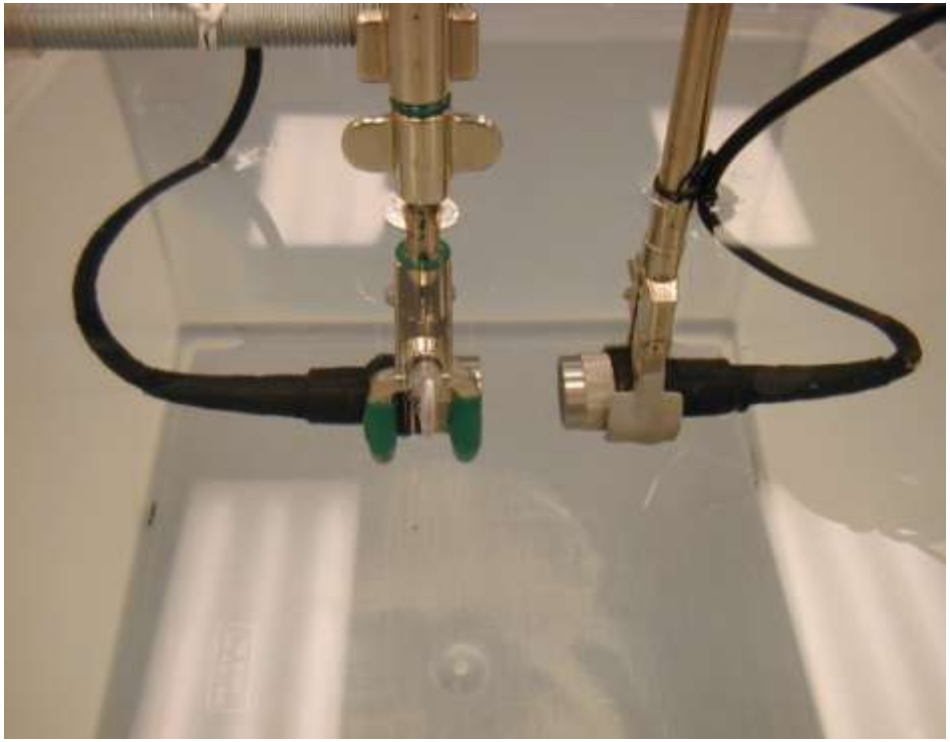
Ultrasound Transducer

- Transducer (A3441):
  - GE Panametrics
  - Immersion type

    To avoid z mismatch with water.
  - High frequency ($f_r \approx 500$kHz)
- Goals:
  - Assess homogeneity

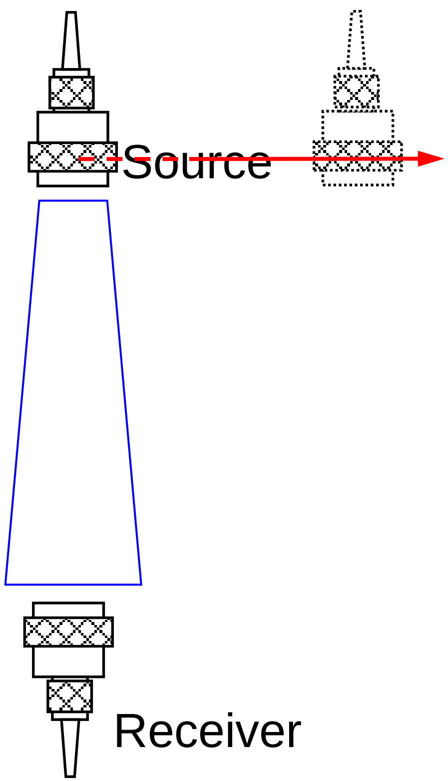
    Layer detection
  - Position objectives

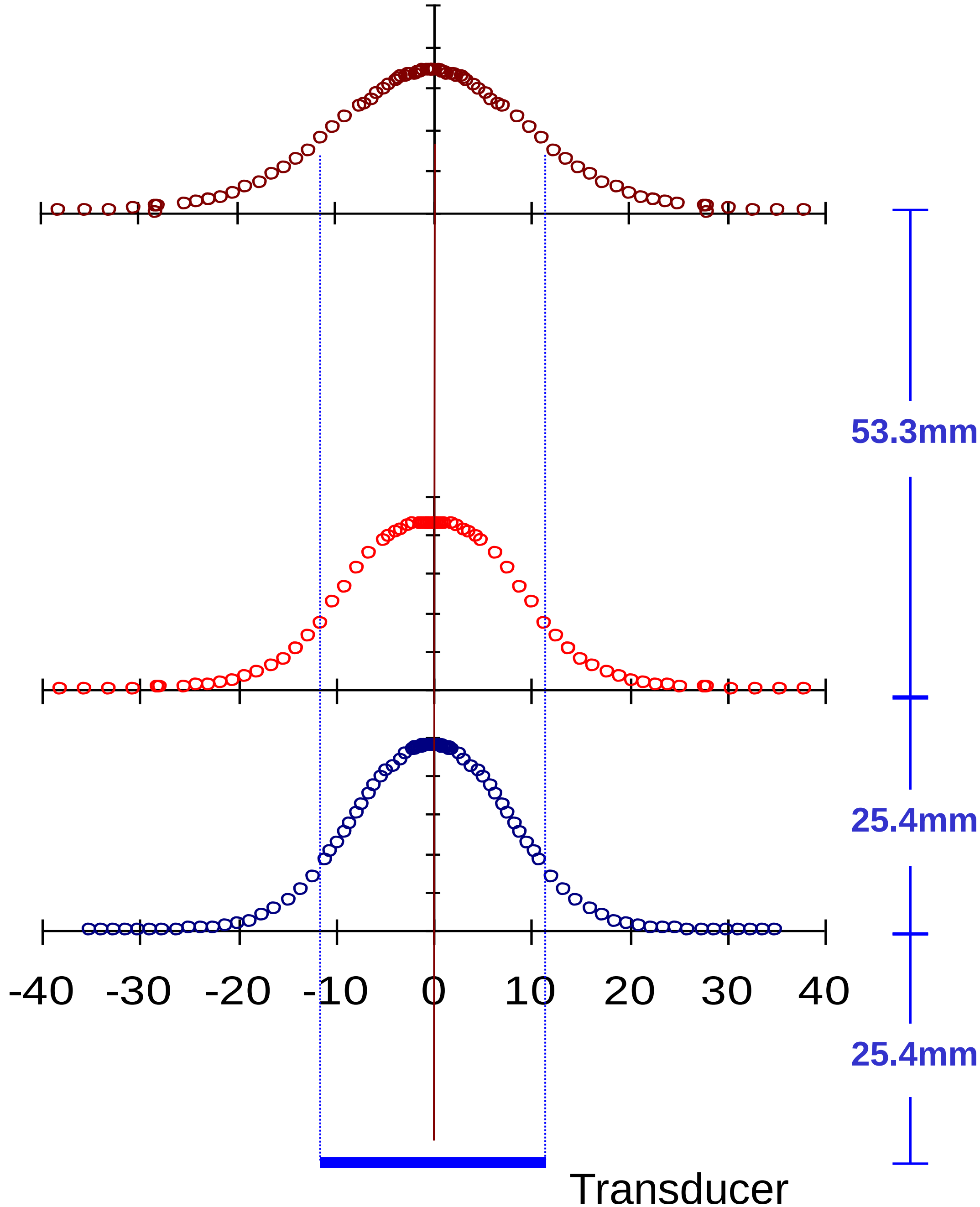
    (e.g., Transducers)



Directivity

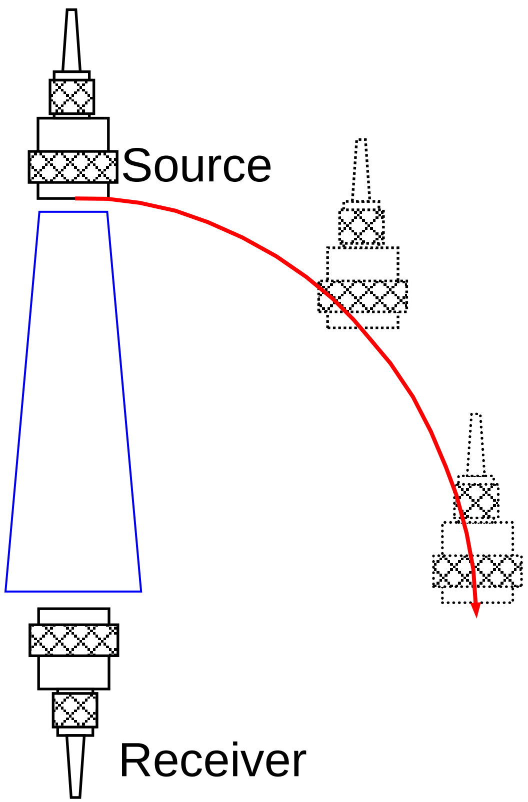
Fixed axial distance

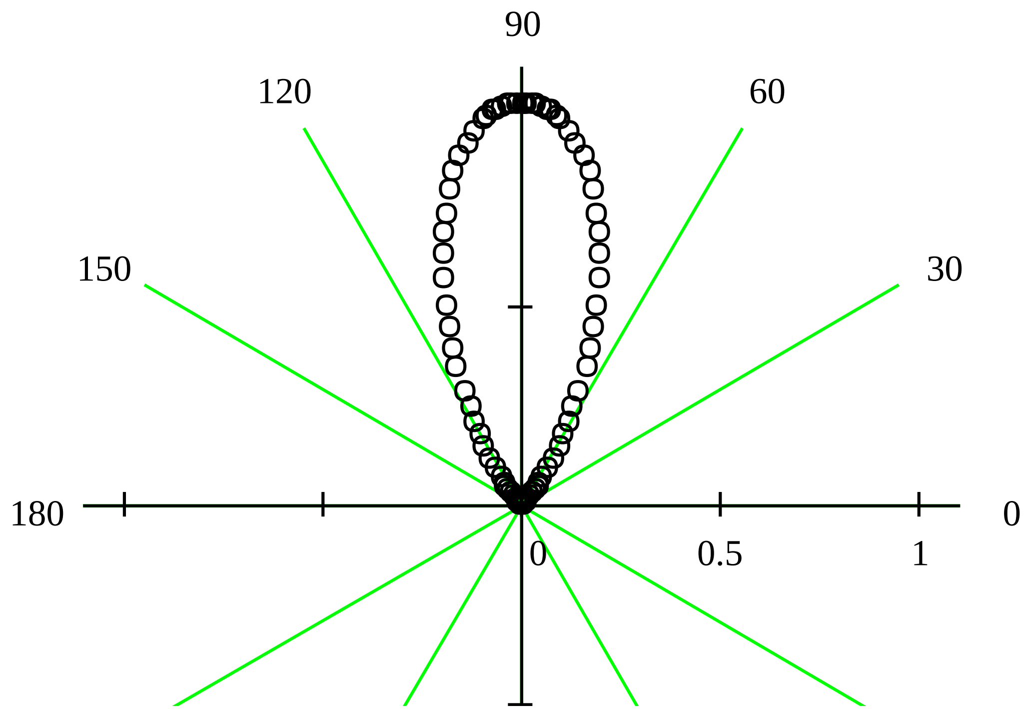




Directivity

Fixed center-to-center distance (=25mm)





Wave Parameters

Velocity and Attenuation

$$\frac{A_2}{A_1} = e^{-\alpha \Delta x}$$

$$V_S = \sqrt{\frac{G}{\rho}}$$

$$V_P = \sqrt{\frac{M}{\rho}} = \sqrt{\frac{B + \frac{4}{3}G}{\rho}}$$

Mechanical Waves

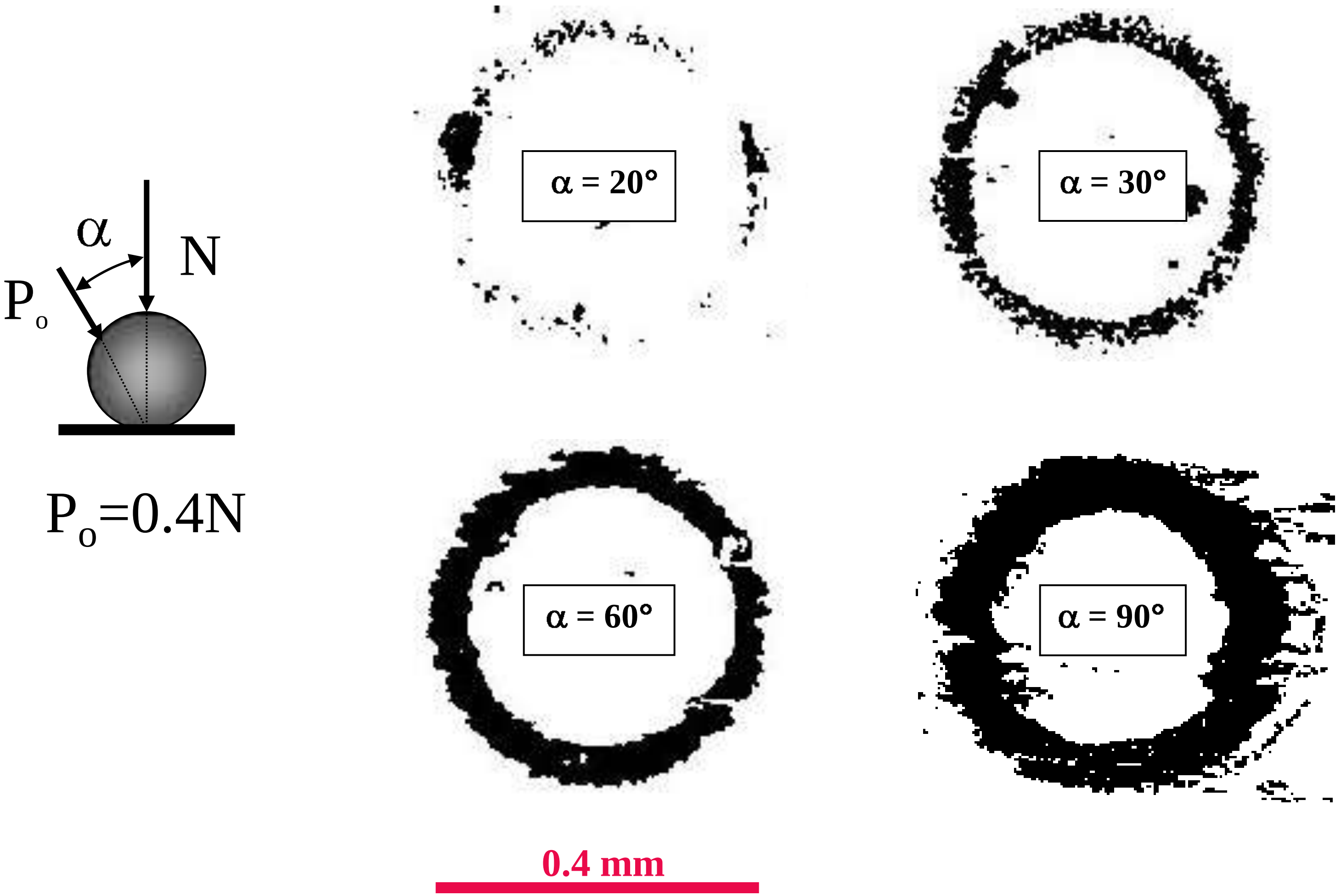
attenuation

S-waves

P-waves

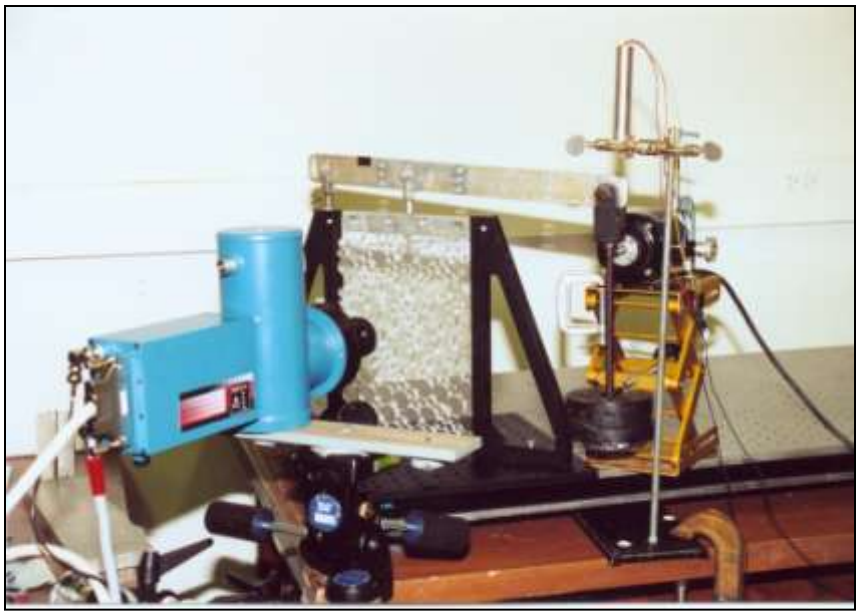
Mindlin contact: Inherently non-elastic

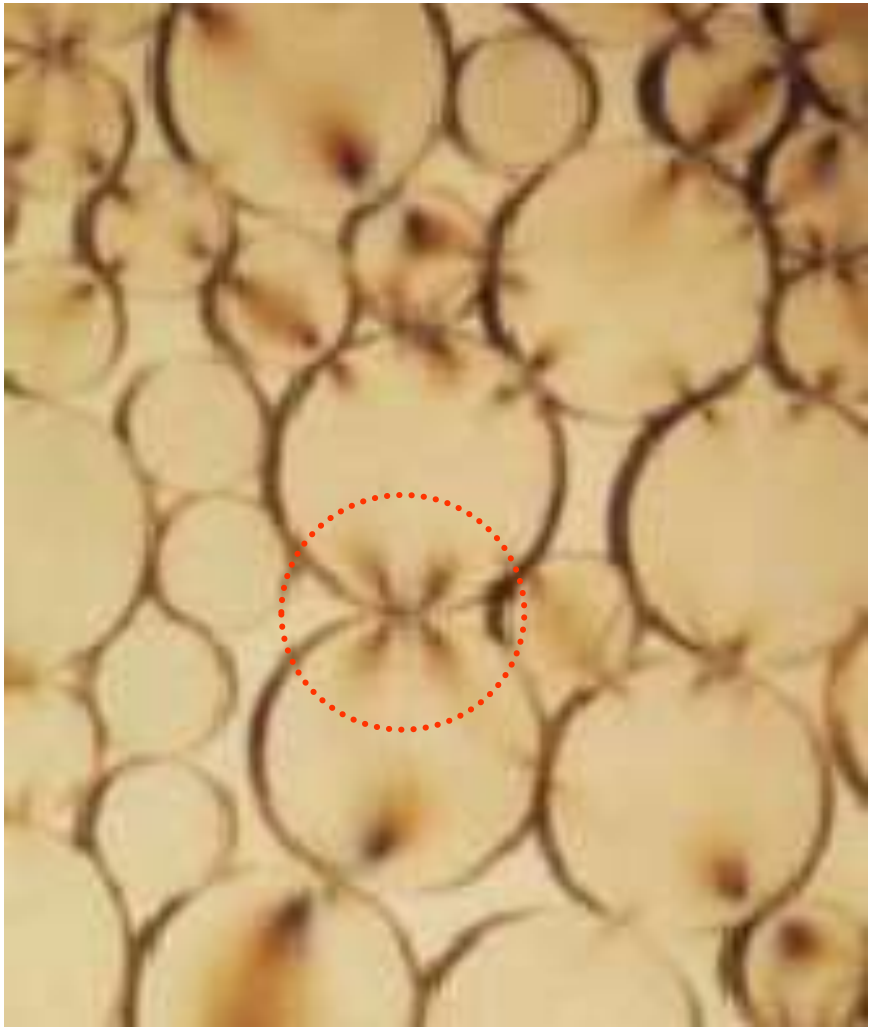
(Fretting damage after 10000 cycles - steel)



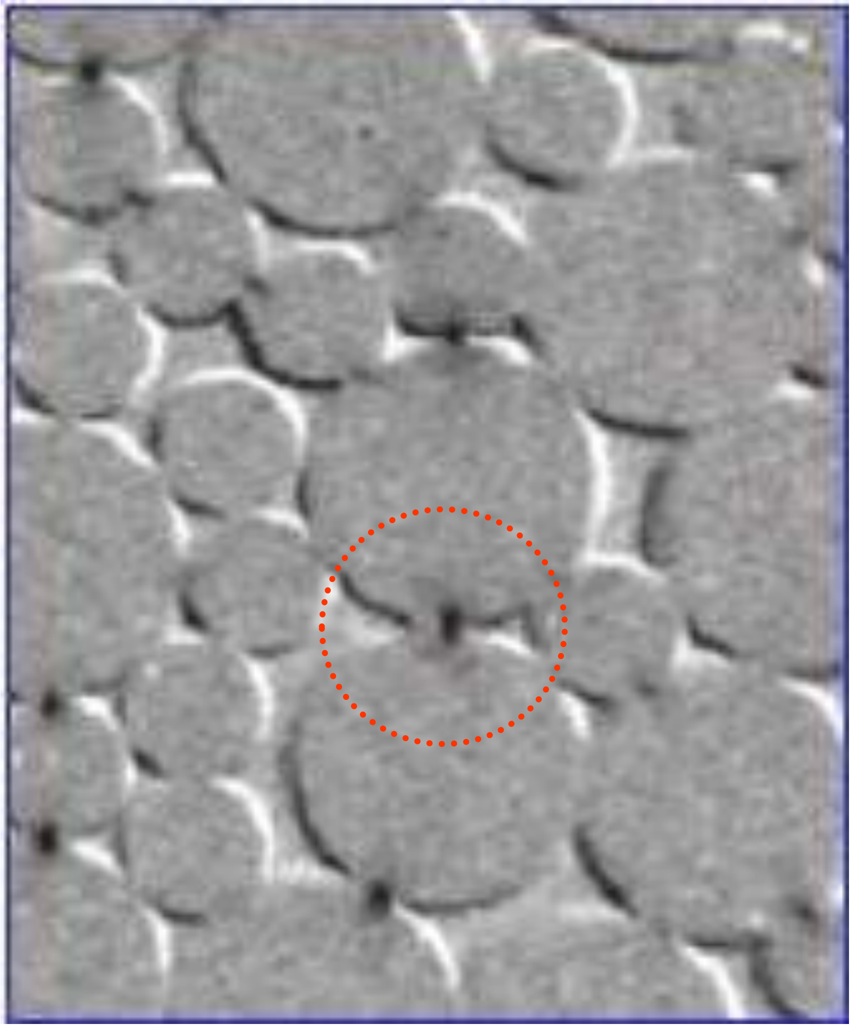
(Johnson, 1961)

Photoelasticity and Thermal IR Imaging

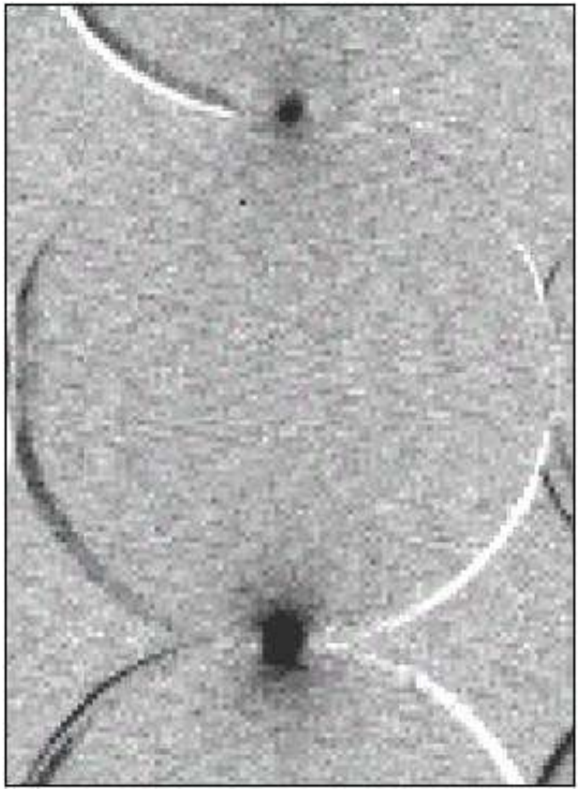




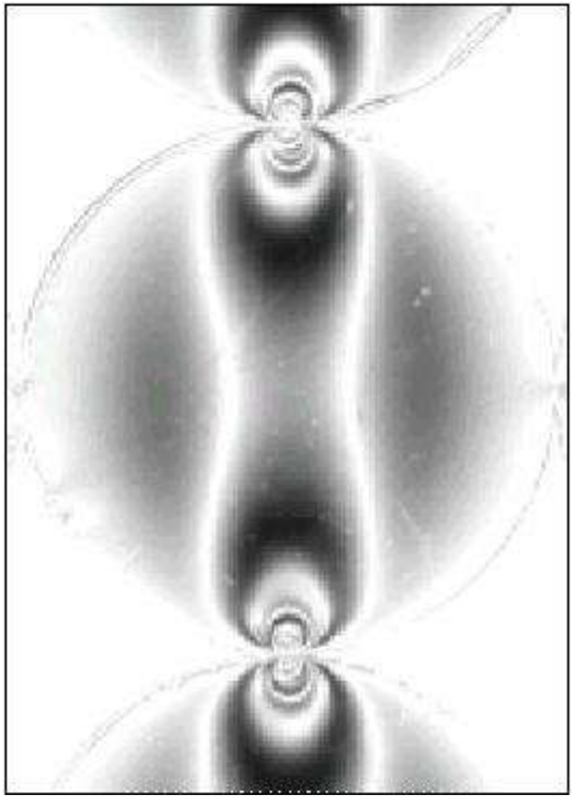
Photoelasticity and Thermal IR Imaging



Thermo-mechanical coupling



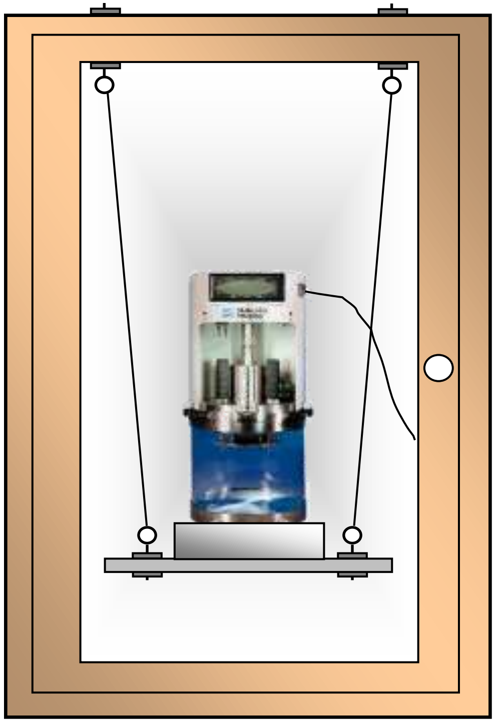
IR image



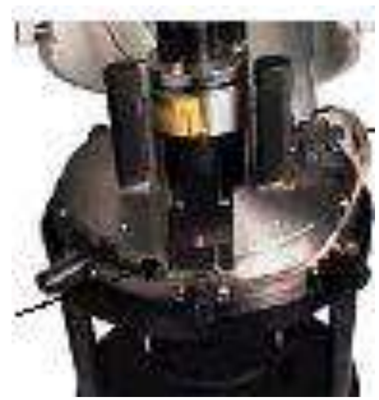
Photoelastic image

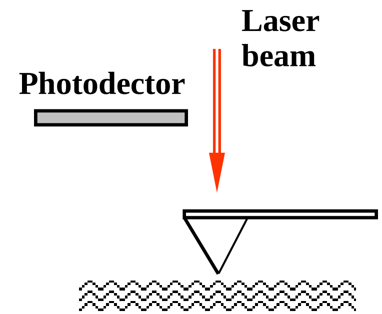
Atomic Force Microscopy (AFM)

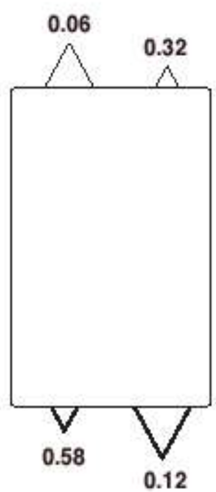
- Surface topography
- Surface properties
- Forces at nano-scale
- Atomic-scale experiments

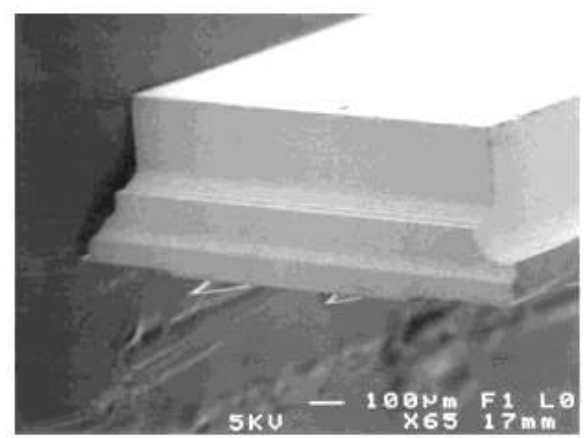


Environmental chamber (A) and Isolation box







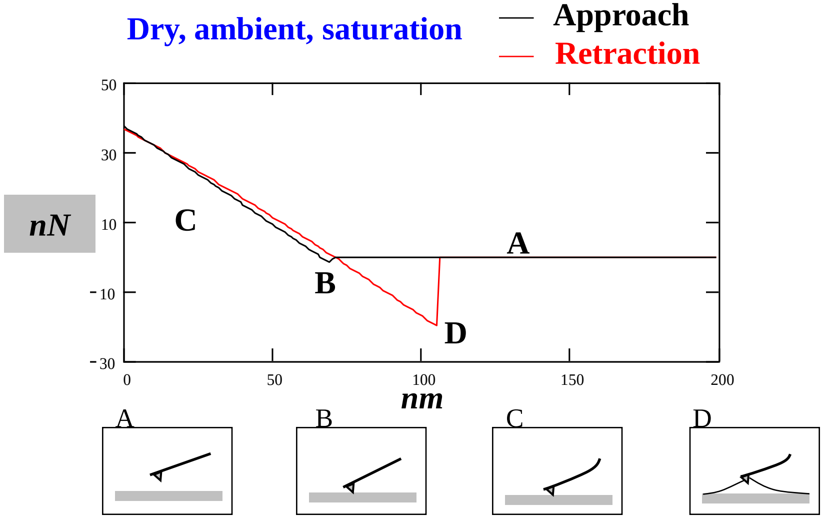


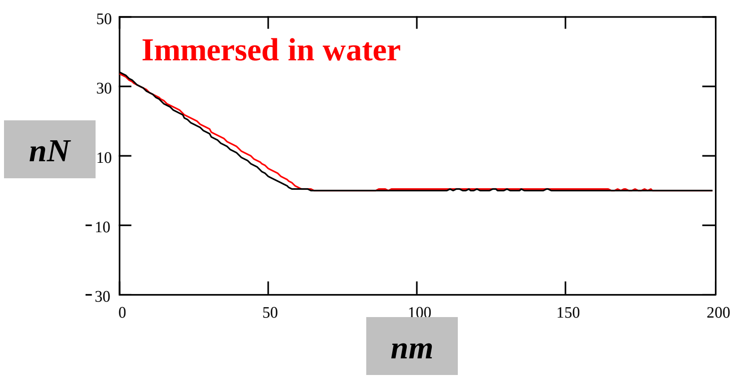
Tip radius: 20 nm

Stiffness :0.58 N/m

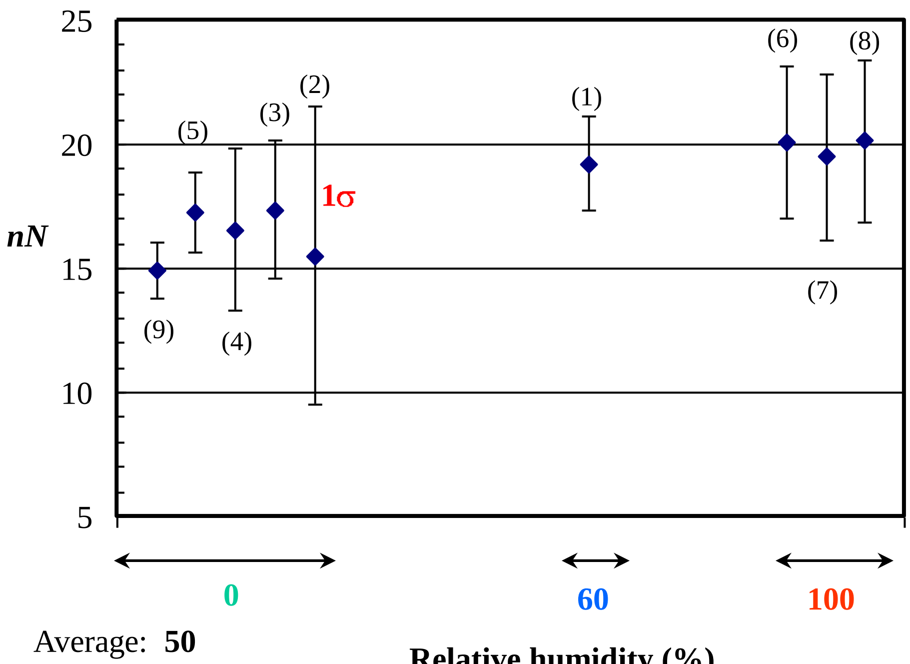
Results of AFM Test

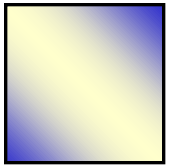
- Force curve





- Pull-out force



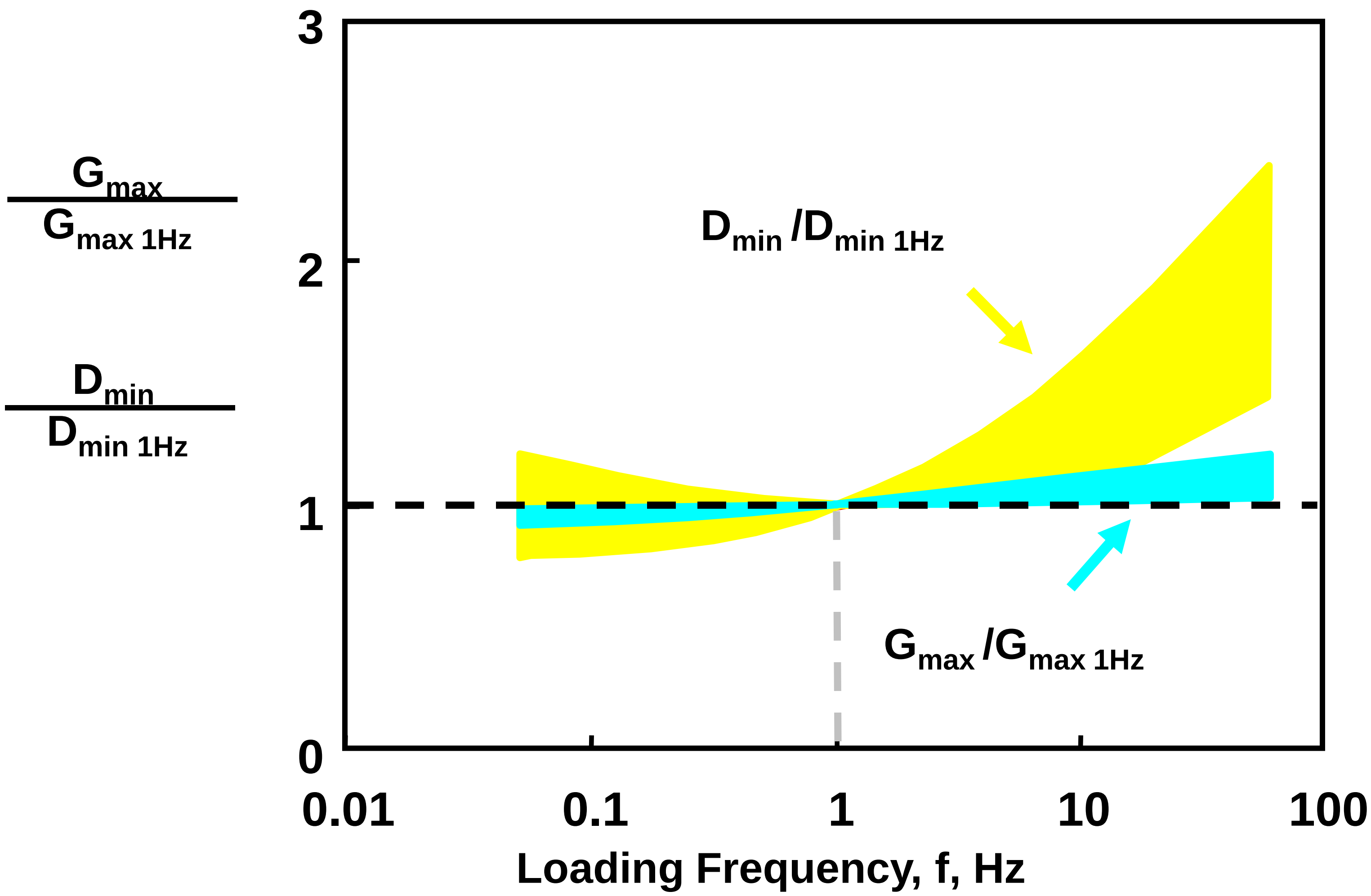


500×500 nm

Summary

Gravelly Soils		D = 0.008 – 0.018
Sand	*Air-dry*	D = 0.002 – 0.01
	Saturated	D = 0.003 – 0.021
Clayey soils		D = 0.01 – 0.052
Residual soils		D = 0.009 – 0.054
Peat ($w_g \approx$ 200%)		D $\approx$ 0.025

The effect of frequency



(Stokoe et al.1999)

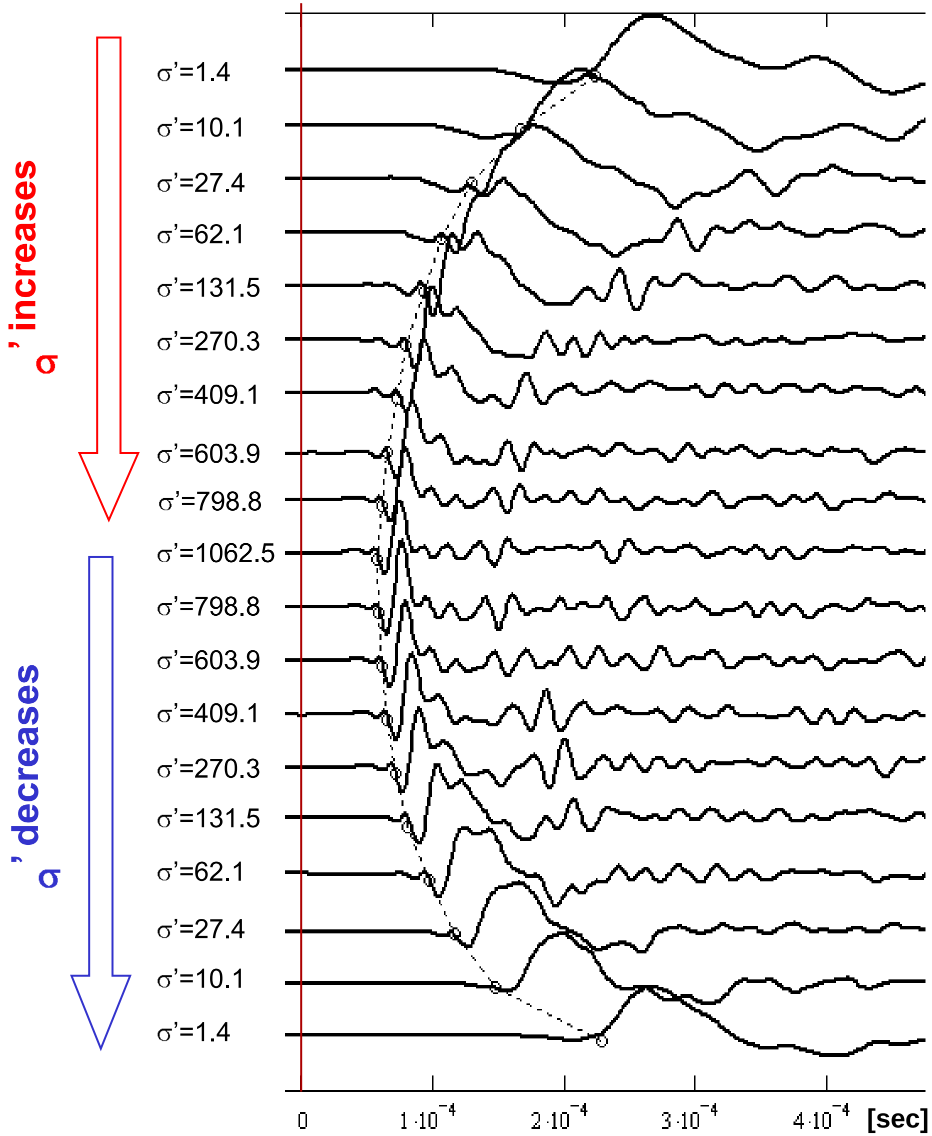
Mechanical Waves

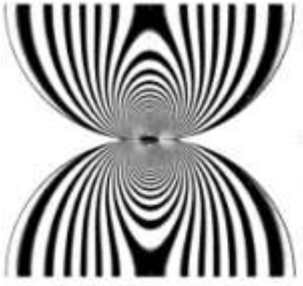
attenuation

S-waves

P-waves

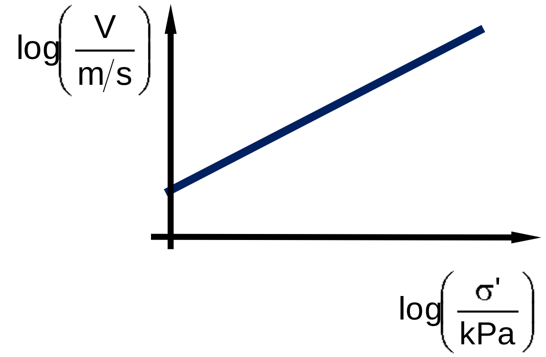
1: Effective Stress

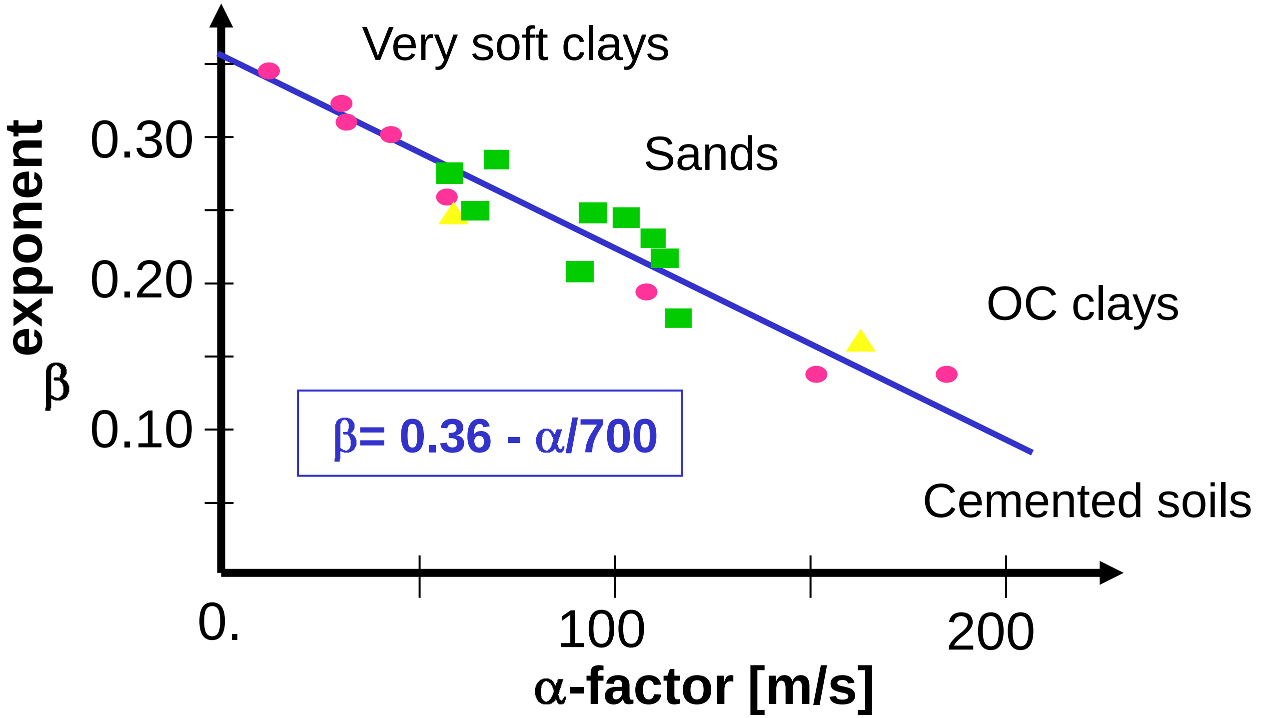




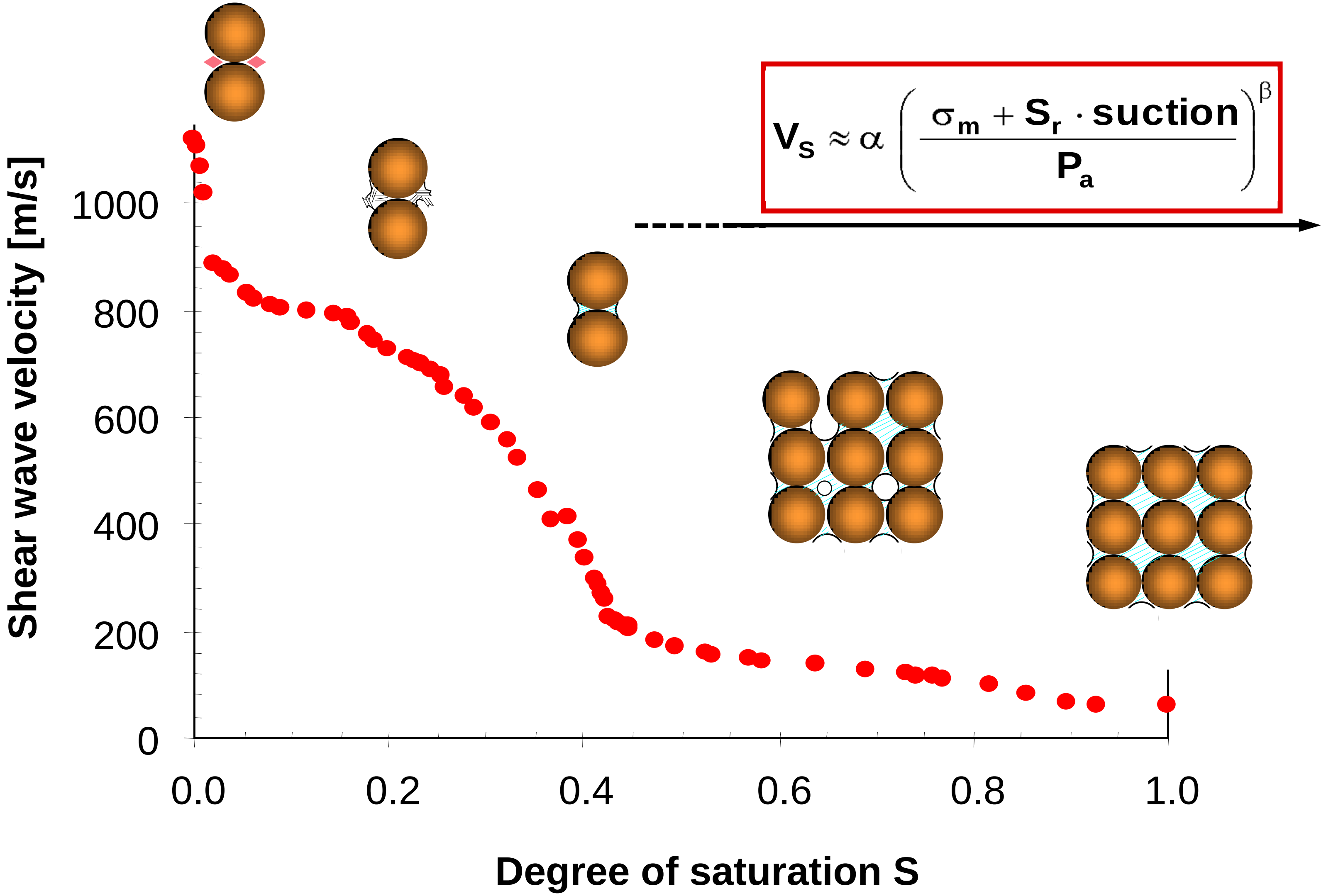
1: Effective Stress

$$V_S = \alpha \left(\frac{\sigma'_x + \sigma'_y}{2 P_a} \right)^{\beta}$$



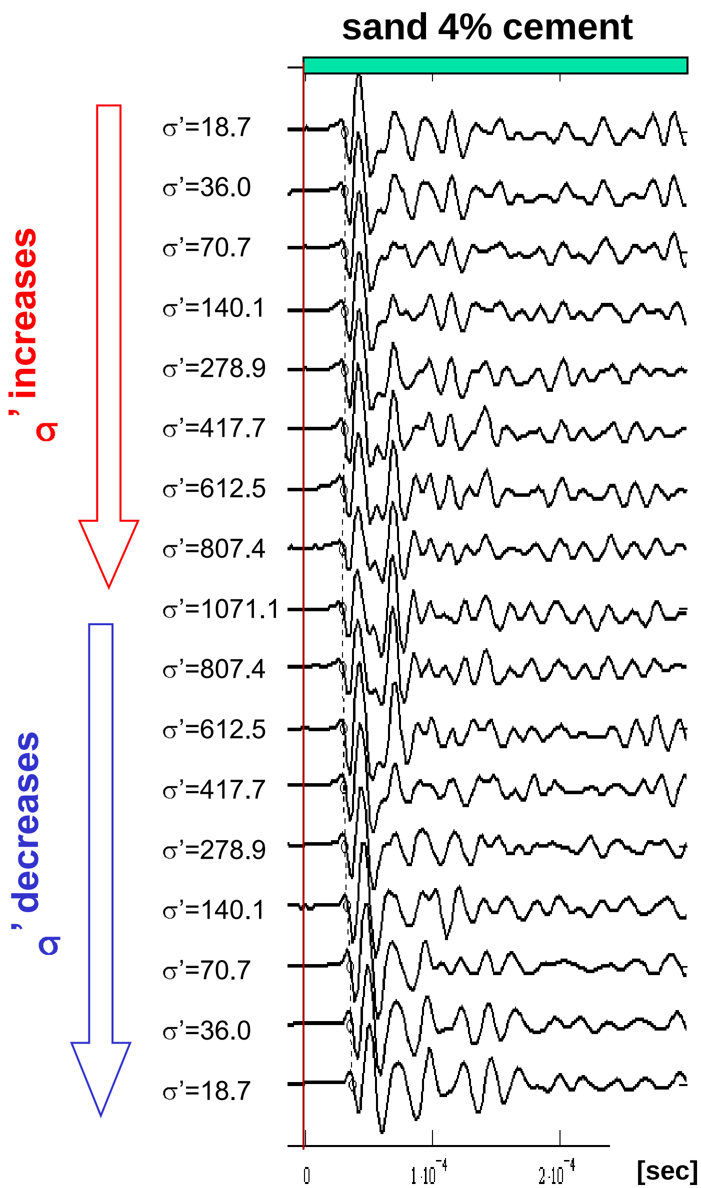


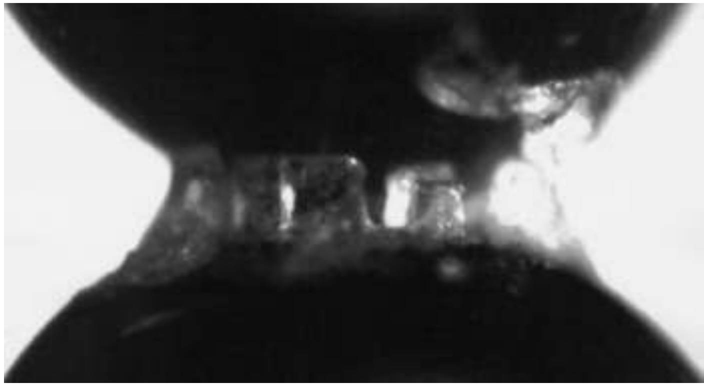
2: Suction - Unsaturated Soils

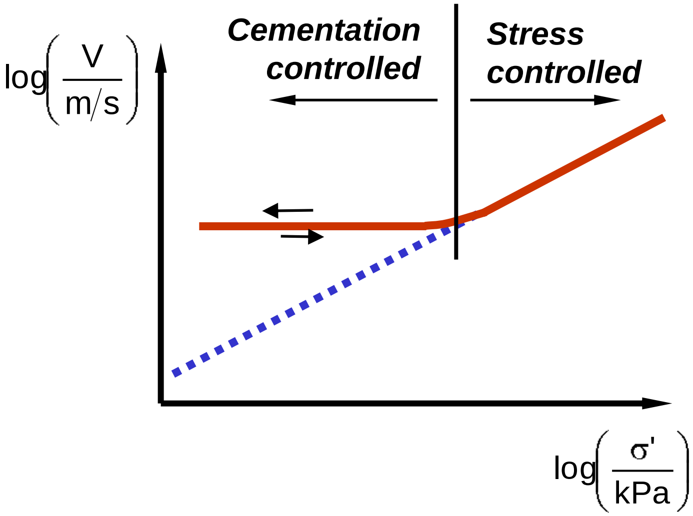


$$V_s \approx \alpha \left(\frac{\sigma_m + S_r \cdot \text{suction}}{P_a} \right)^{\beta}$$

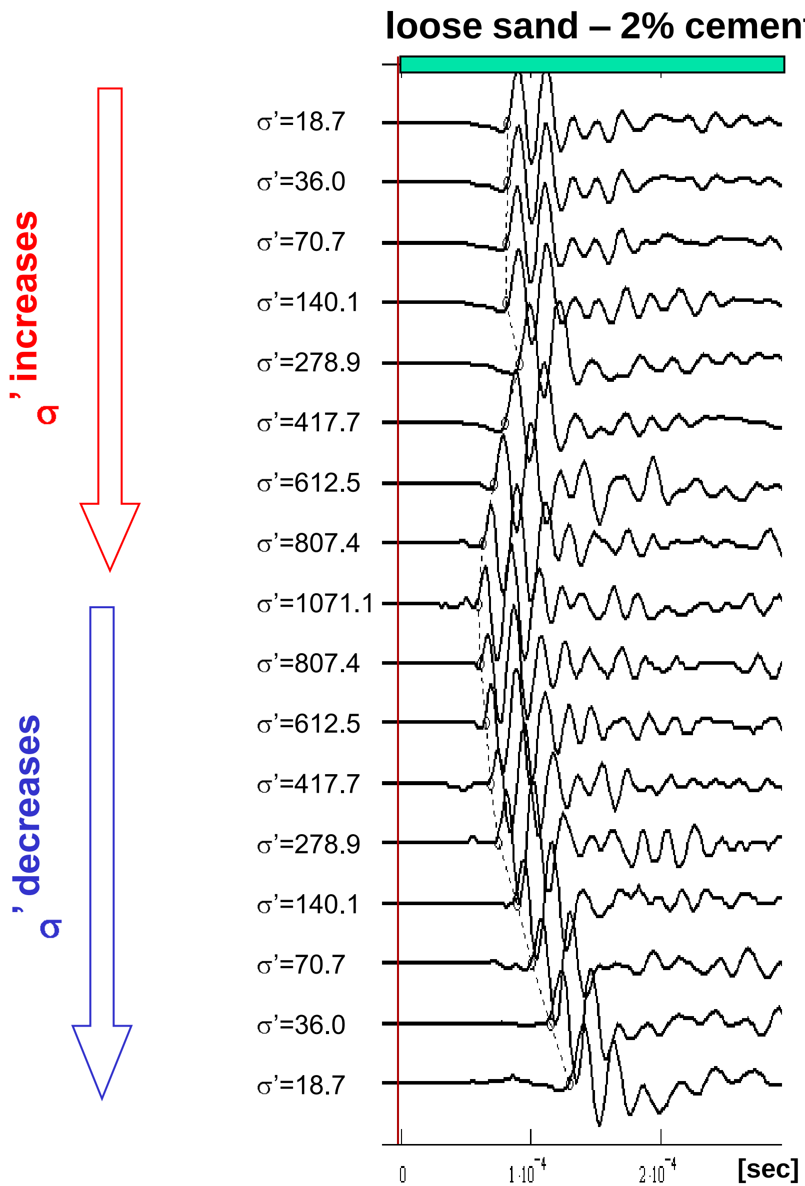
3: Cementation

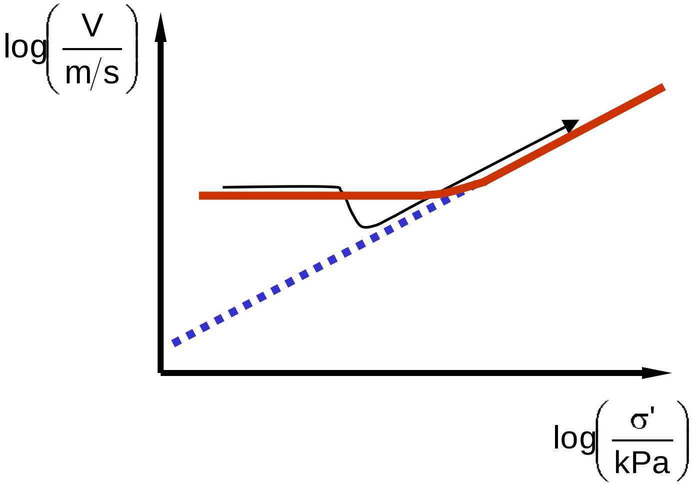






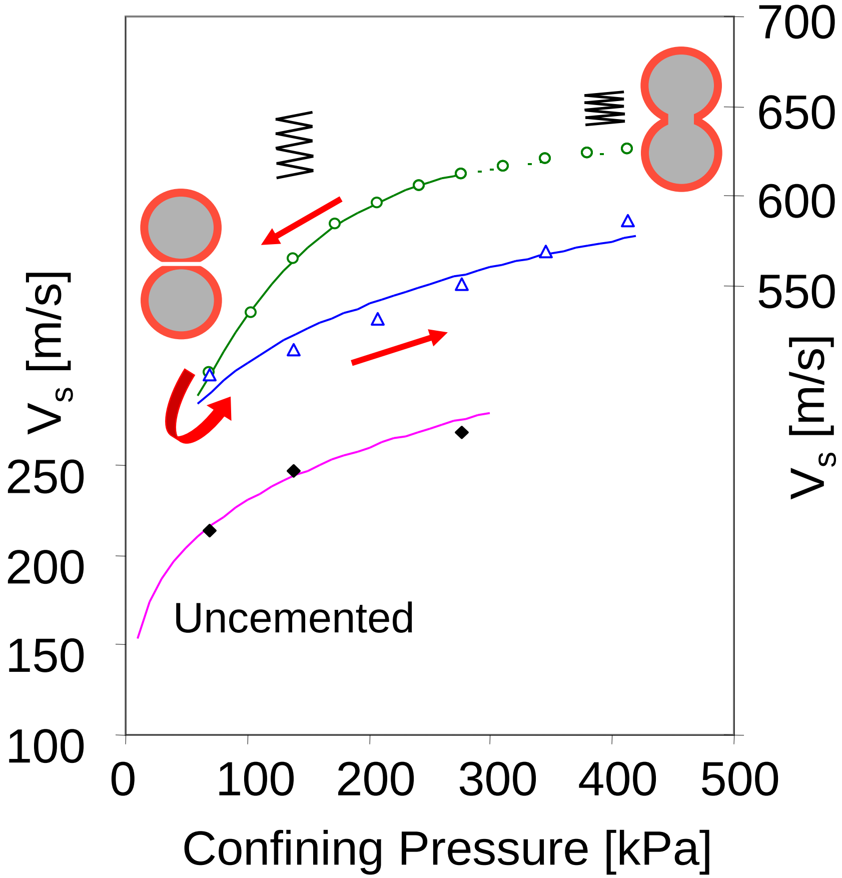
3: Cementation - Loading

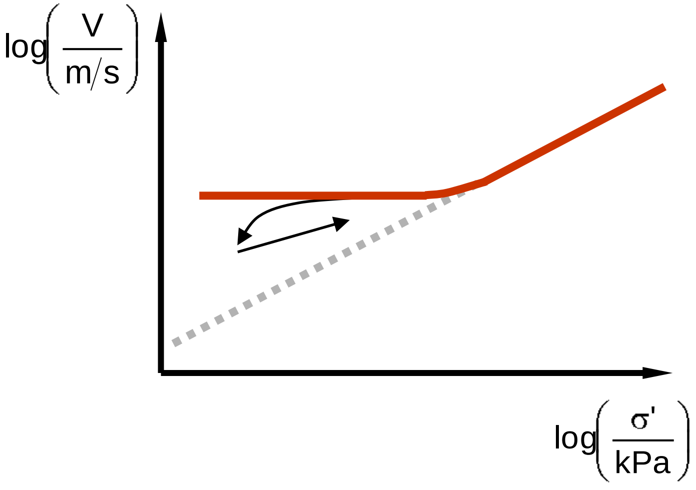




T.Y. Yun

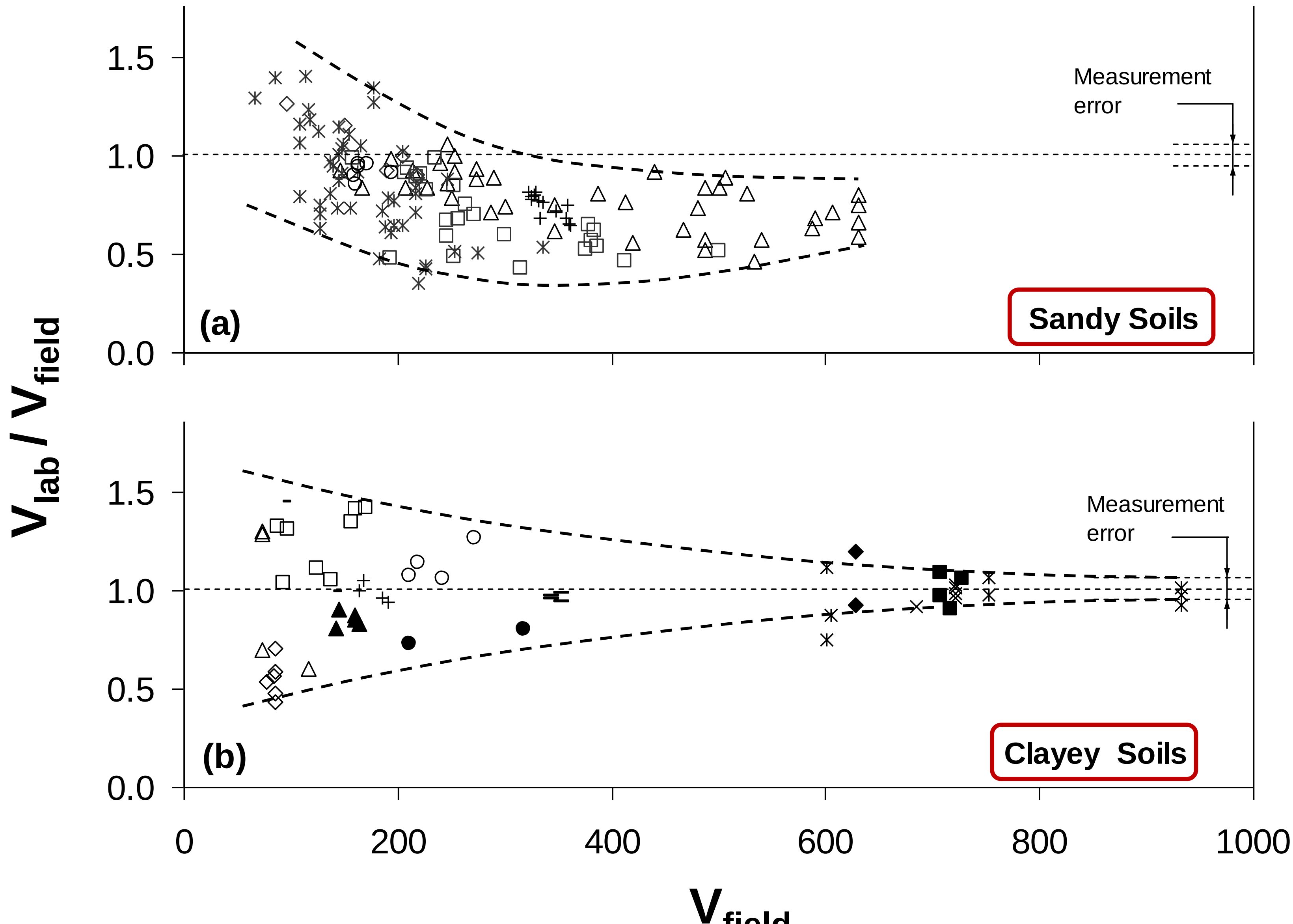
3: Cementation - Unloading





A. Fernandez

3: ➔ Sampling effects



V. Rinaldi

Mechanical Waves

attenuation

S-waves

P-waves

Bulk Stiffness

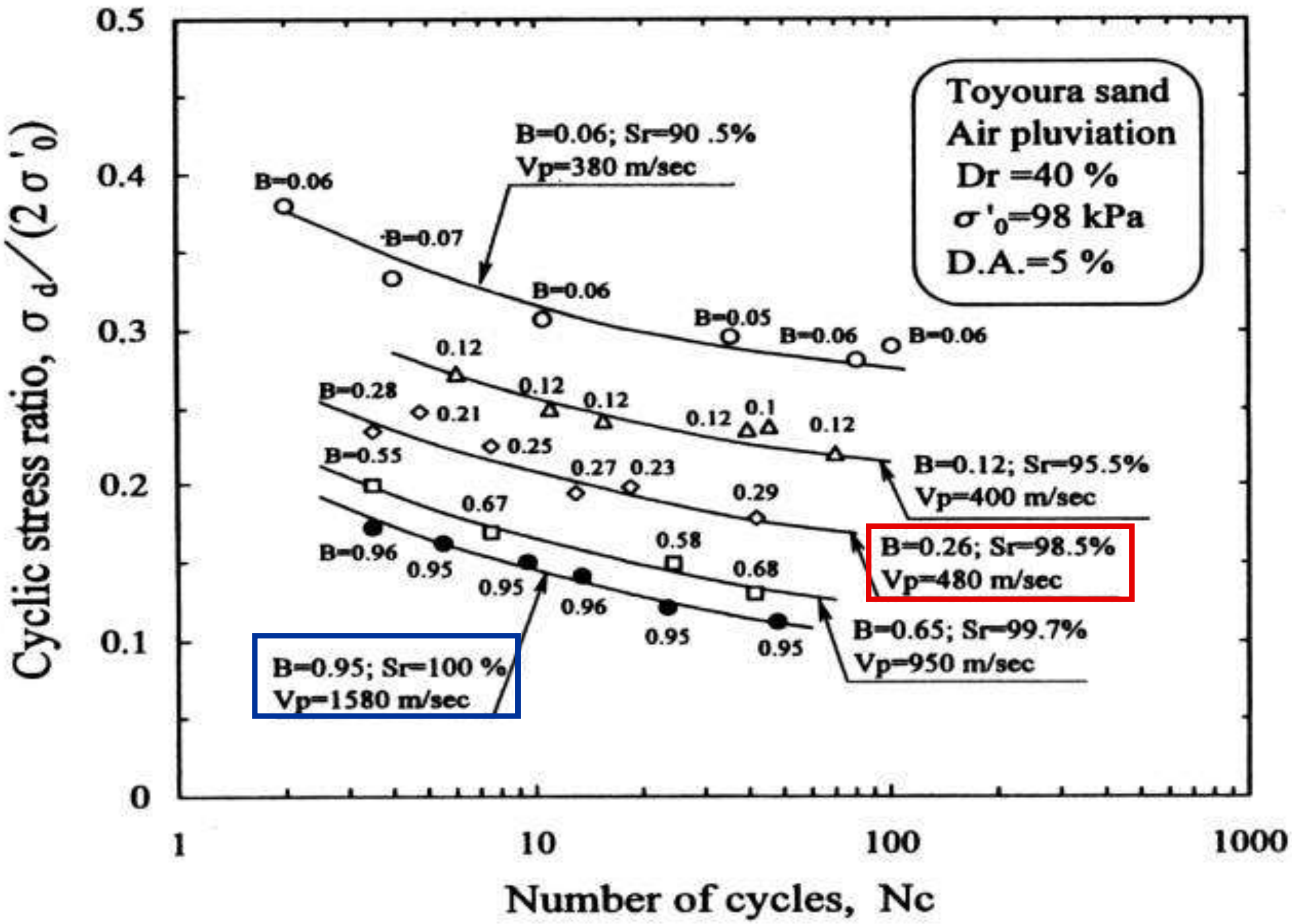
$$V_P = \sqrt{\frac{M}{\rho}} = \sqrt{\frac{B + \frac{4}{3}G}{\rho}}$$

Fluid Mixture	$B_{fl} = \left(\frac{S_r}{B_w} + \frac{1-S_r}{B_a} \right)^{-1}$	$\rho_{fl} = (1-S_r)\,\rho_a + S_r\rho_w$
Suspension	$B_{sus} = \left(\frac{1-n}{B_g} + \frac{n}{B_{fl}} \right)^{-1}$	$\rho_{sus} = (1-n)\,\rho_g + n\rho_{fl}$
Soil (fluid + skeleton)	$B_{soil} = B_{sus} + B_{sk}$	$\rho_{soil} = \rho_{sus} = (1-n)\,\rho_g + n\rho_{fl}$

from $G = V_s^2\,\rho$

Saturation

$$V_P = \sqrt{\frac{\left(B_{sk} + \frac{4}{3}G_{sk}\right) + \left[n\left(\frac{S}{B_w} + \frac{1-S}{B_a}\right) + \frac{1-n}{B_g}\right]^{-1}}{1-n\ \rho_g + nS\rho_w}}$$

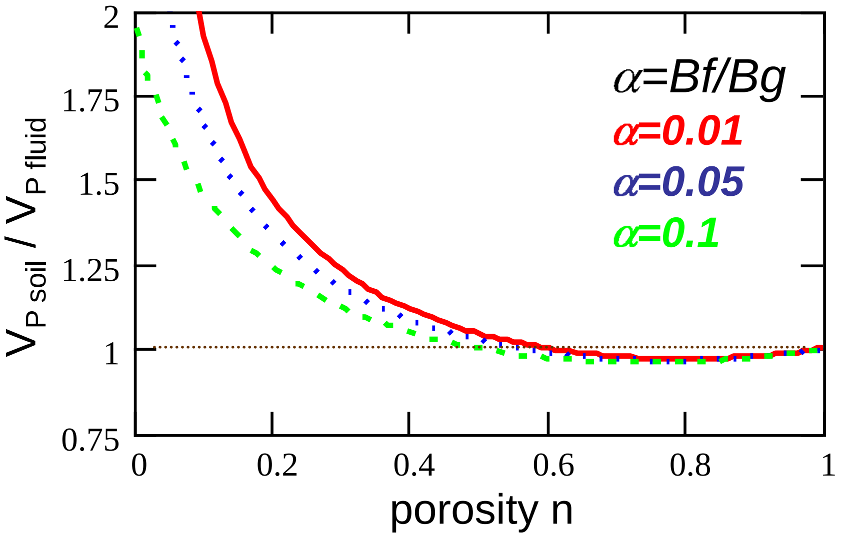


K. Ishihara

Velocity and Impedance (S=100%)

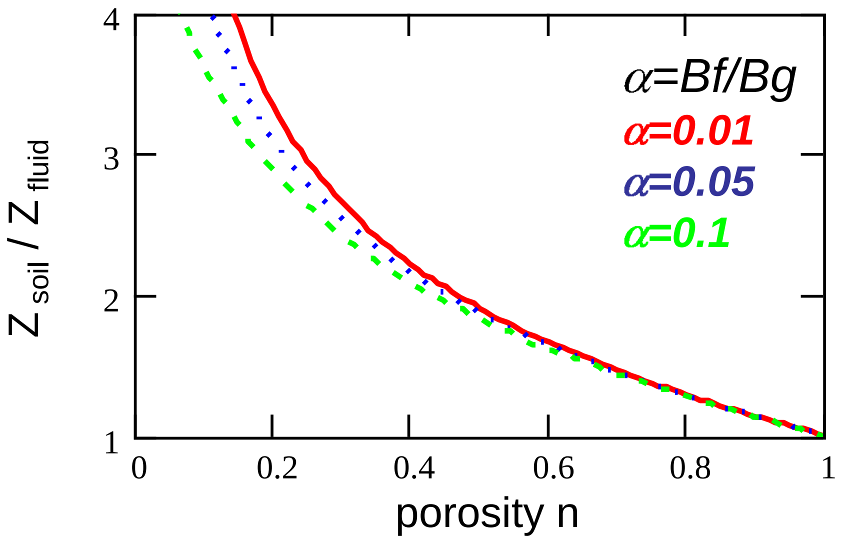
V_P -vs- n

$$\frac{V_{P-soil}}{V_{P-fluid}} = \sqrt{\frac{1}{\left[n+\alpha(1-n)\right]\left[(1-n)G_s+n\right]}}$$



Impedance -vs- n

$$\frac{z_{P-mix}}{z_{P-fluid}} = \sqrt{\frac{(1-n)G_s+n}{n+\alpha(1-n)}}$$



Mechanical Waves

closing….

Summary: P- and S-waves

Waves	Small-strain phenomena
	May be used to monitor large-strain processes
V_s	Skeletal stiffness: G ➔ Geo-mechanical design
	Effective stress, suction, cementation
	Sampling: pronounced effect ➔ measure in situ !
	Simple lab & field devices and methods
V_P	Fluid & skeletal stiffness: B & G
	Proximity to full saturation
V_P &Vs:	Dry ➔ skeletal Poisson's ratio
	Saturated ➔ porosity
	Spatial variability

Summary

V_P in water	1482

V_P in air	343

V_P in saturated soils	1450-1900
V_P in unsaturated soils	<100-800
V_P in lightly cemented soils	400-1000

V_S in saturated soils	<50-400
V_S in unsaturated clayey soils	<100-500
V_S in lightly cemented soils	250-700

V_P and V_S

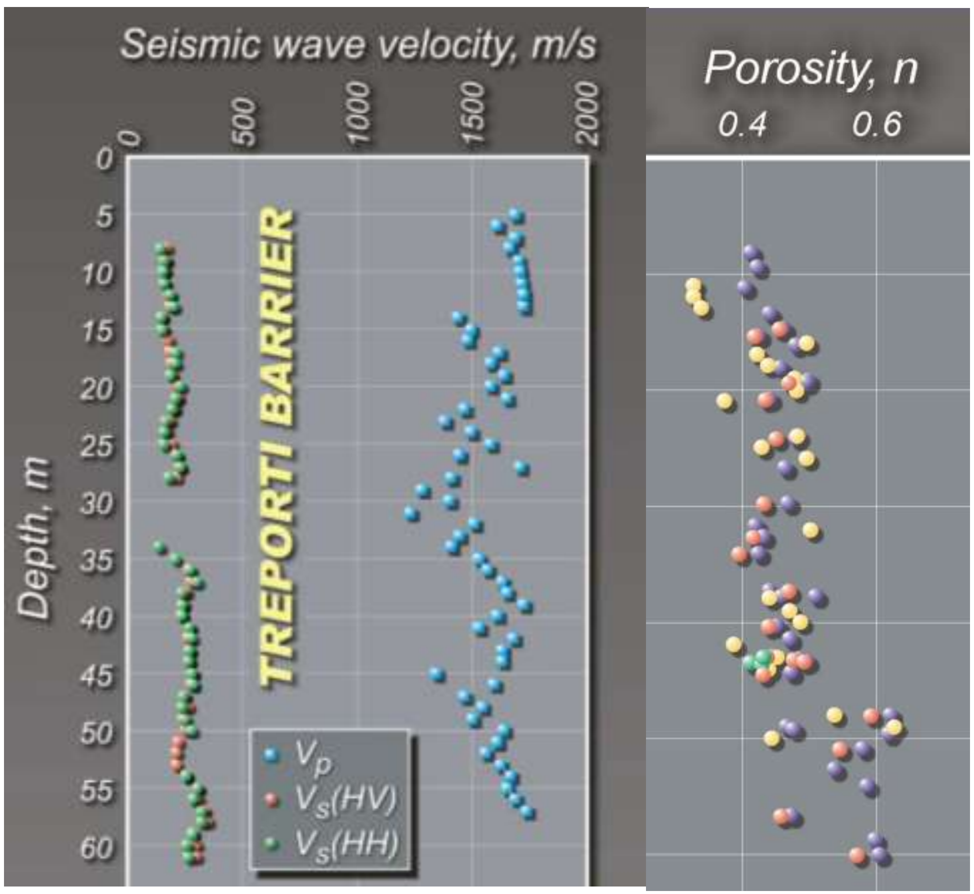
Poisson's ratio (~dry)

$$\nu = \frac{\frac{1}{2}\left(\frac{V_P}{V_S}\right)^2 - 1}{\left(\frac{V_P}{V_S}\right)^2 - 1}$$

Porosity (S=100%)

$$n = \frac{\rho_g - \sqrt{\rho_g^2 - \dfrac{4\ \rho_g - \rho_{fl}\ B_{fl}}{V_P^2 - 2\left(\dfrac{1-\nu_{sk}}{1-2\nu_{sk}}\right)V_S^2}}}{2\ \rho_g - \rho_{fl}}$$

see Foti & Lancelotta



Venice (M. Jamiolkowski)

Some Applications

P-waves

Data Fusion

Navigational

Homing in

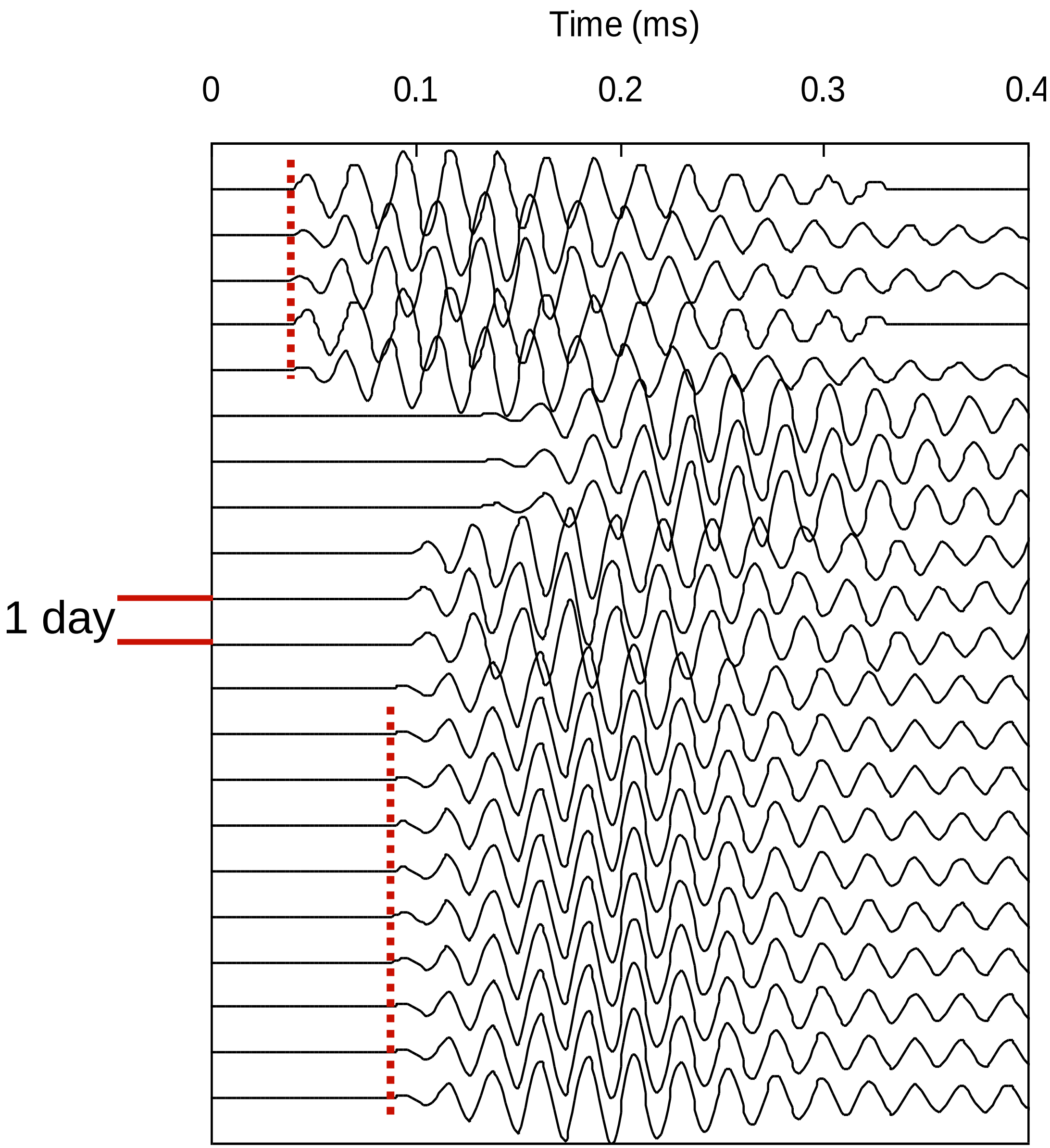
http://sunsite.tus.ac.jp/multimed/pics/animals/bat.jpg

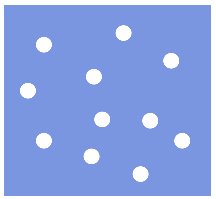
http://www.moorhen.demon.co.uk

Massive data → Processing → Information

Mat. Eval. 1999

P-monitoring: Bio-gas

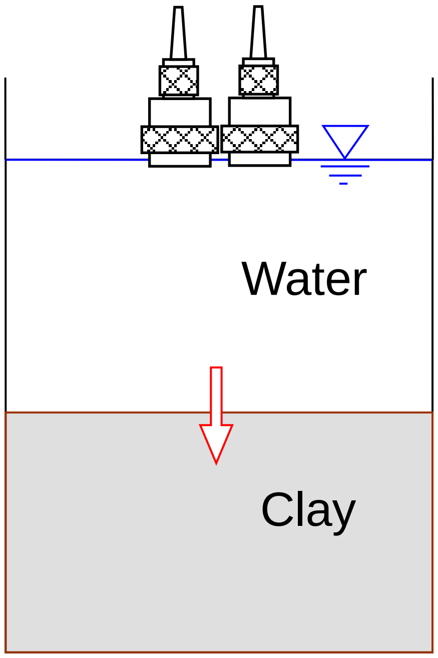


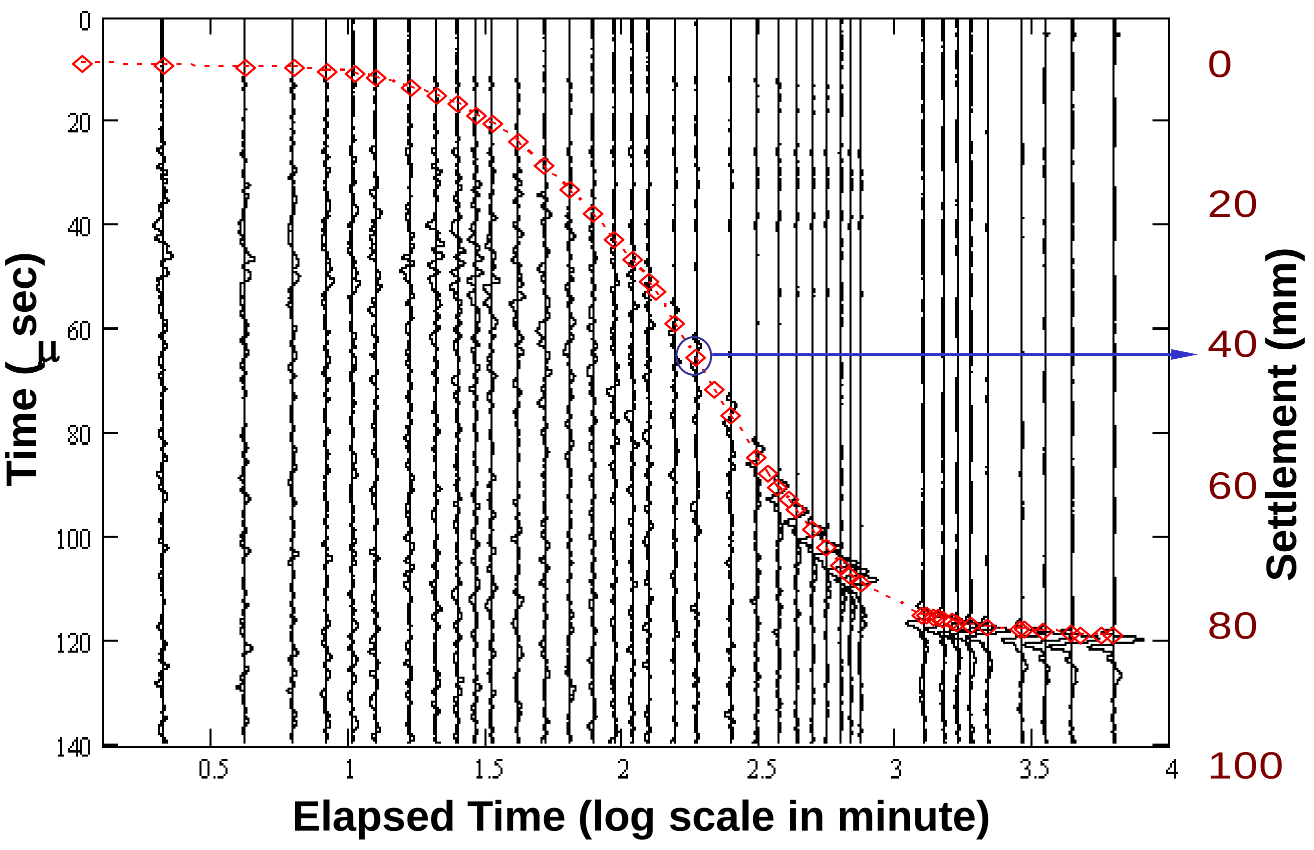


Paracoccus denitrificans
Nitrate broth
F110 + 3%Kaolin

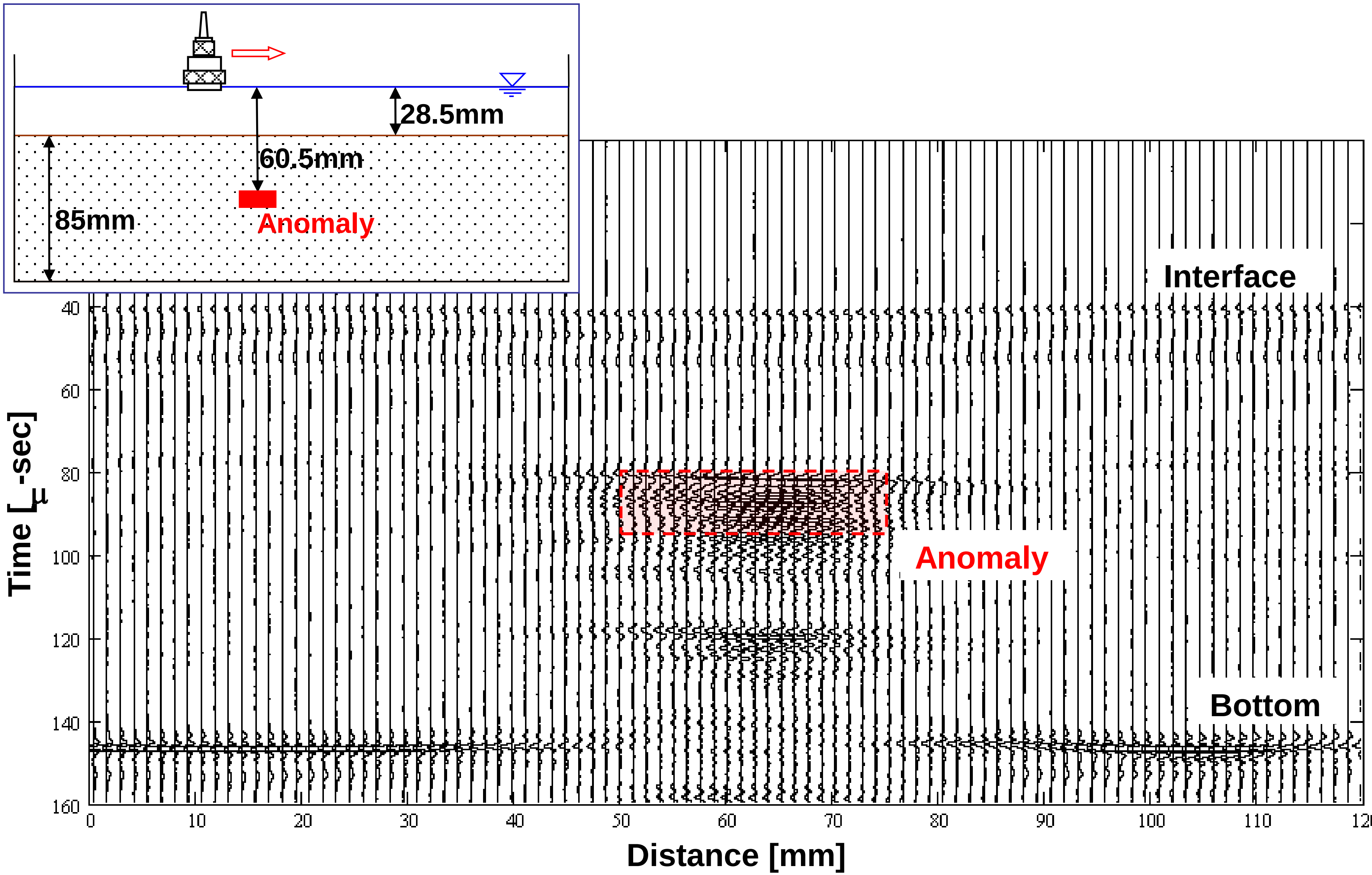
V. Rebata

Laboratory: Sedimentation

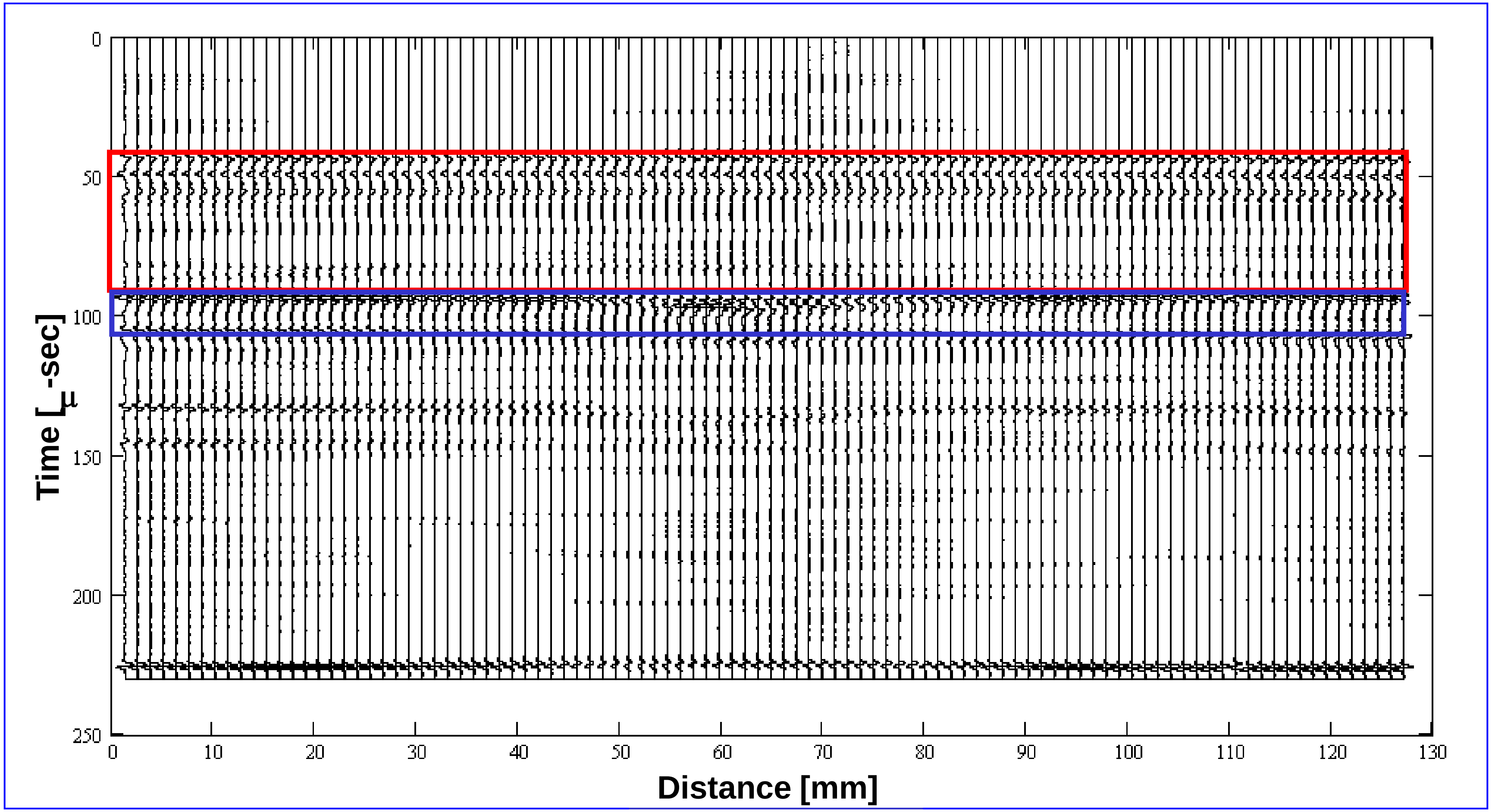




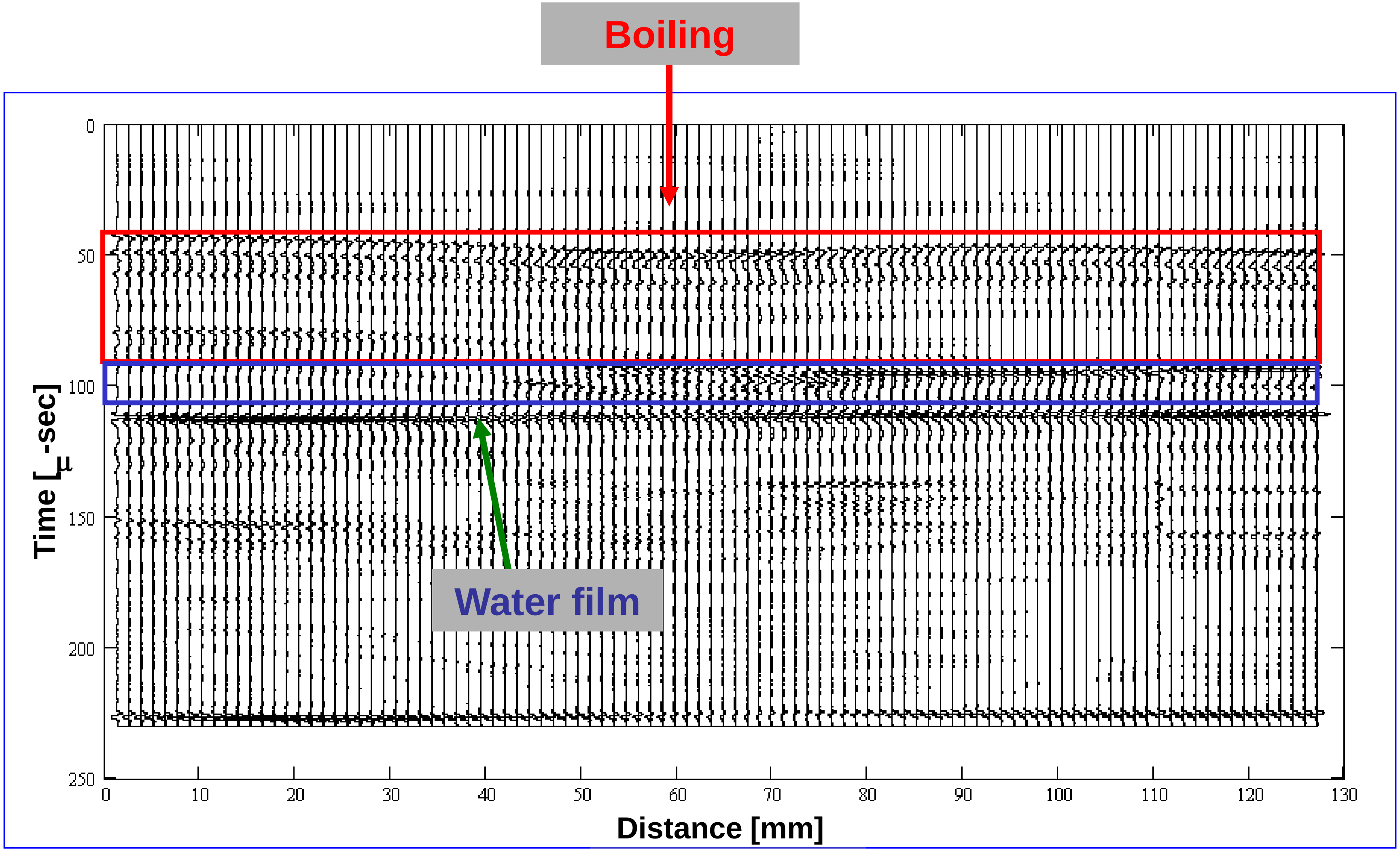
Anomaly Detection



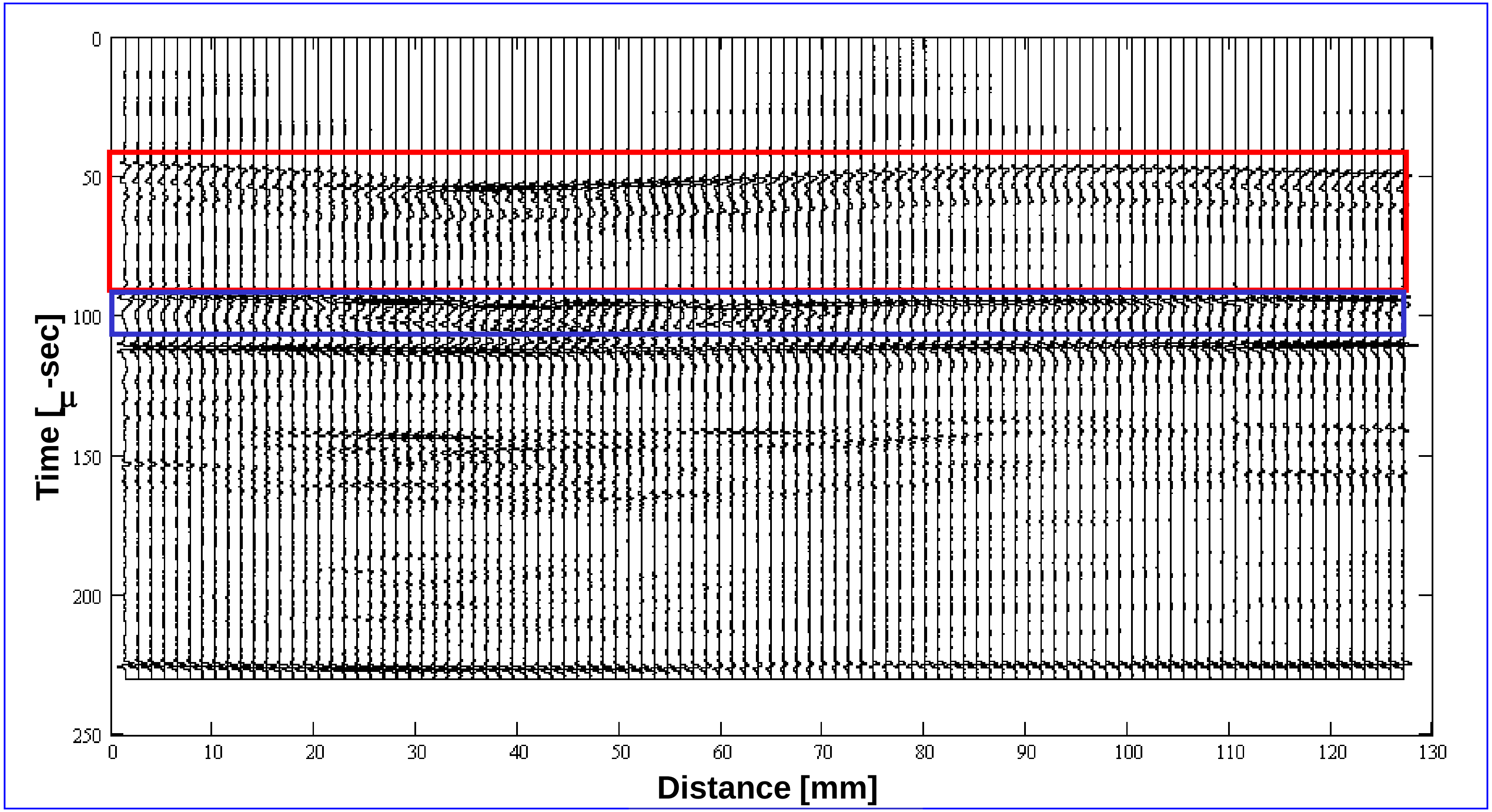
P-wave scanning – Before Liquefaction

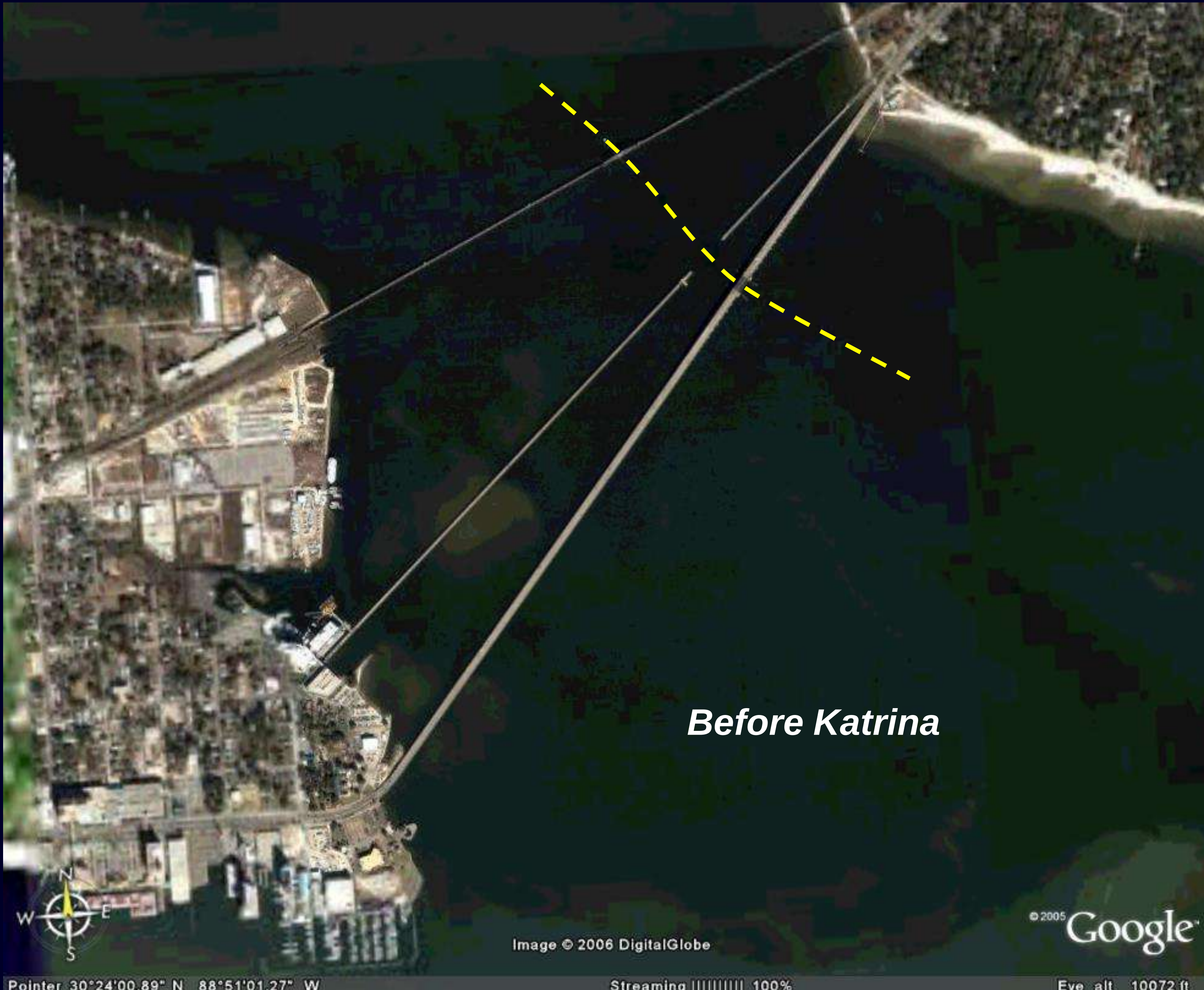


After Liquefaction (~2 hr)

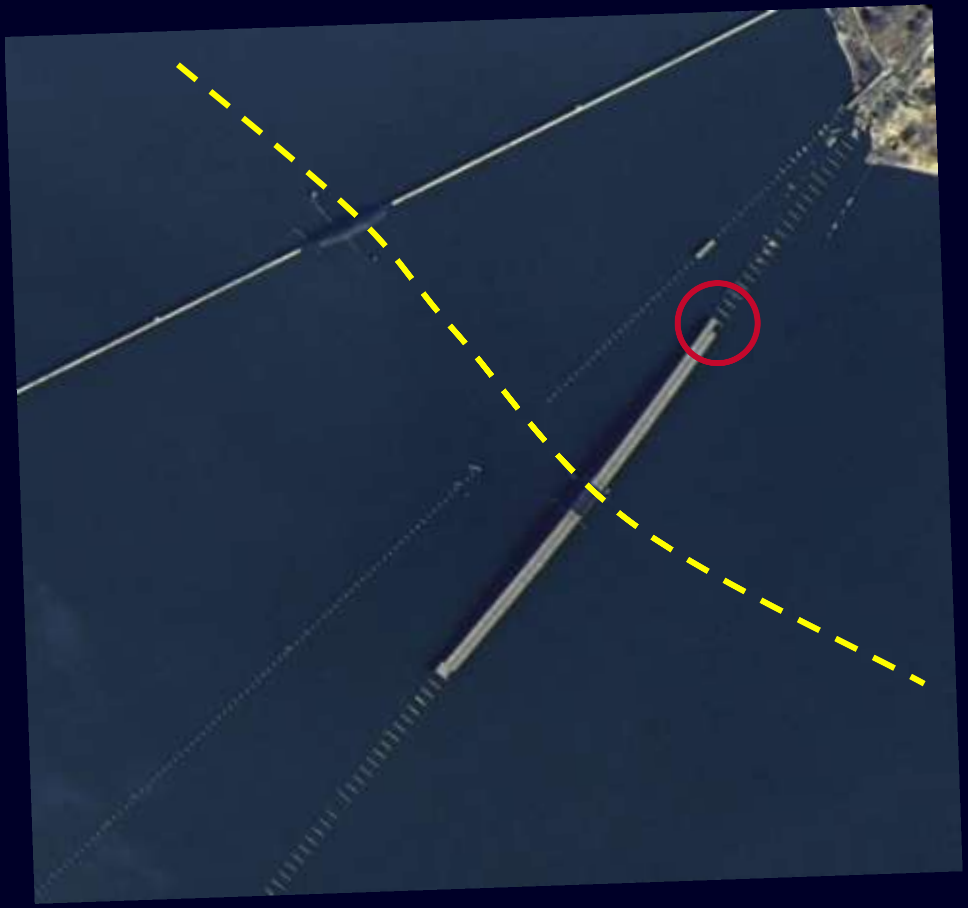


After Liquefaction (2 days)




Before Katrina
Image © 2006 DigitalGlobe
© 2005 Google
N
W
E
S
Pointer 30°24'00.89" N 88°51'01.27" W
Streaming |||||||| 100%
Eye alt 10072 ft

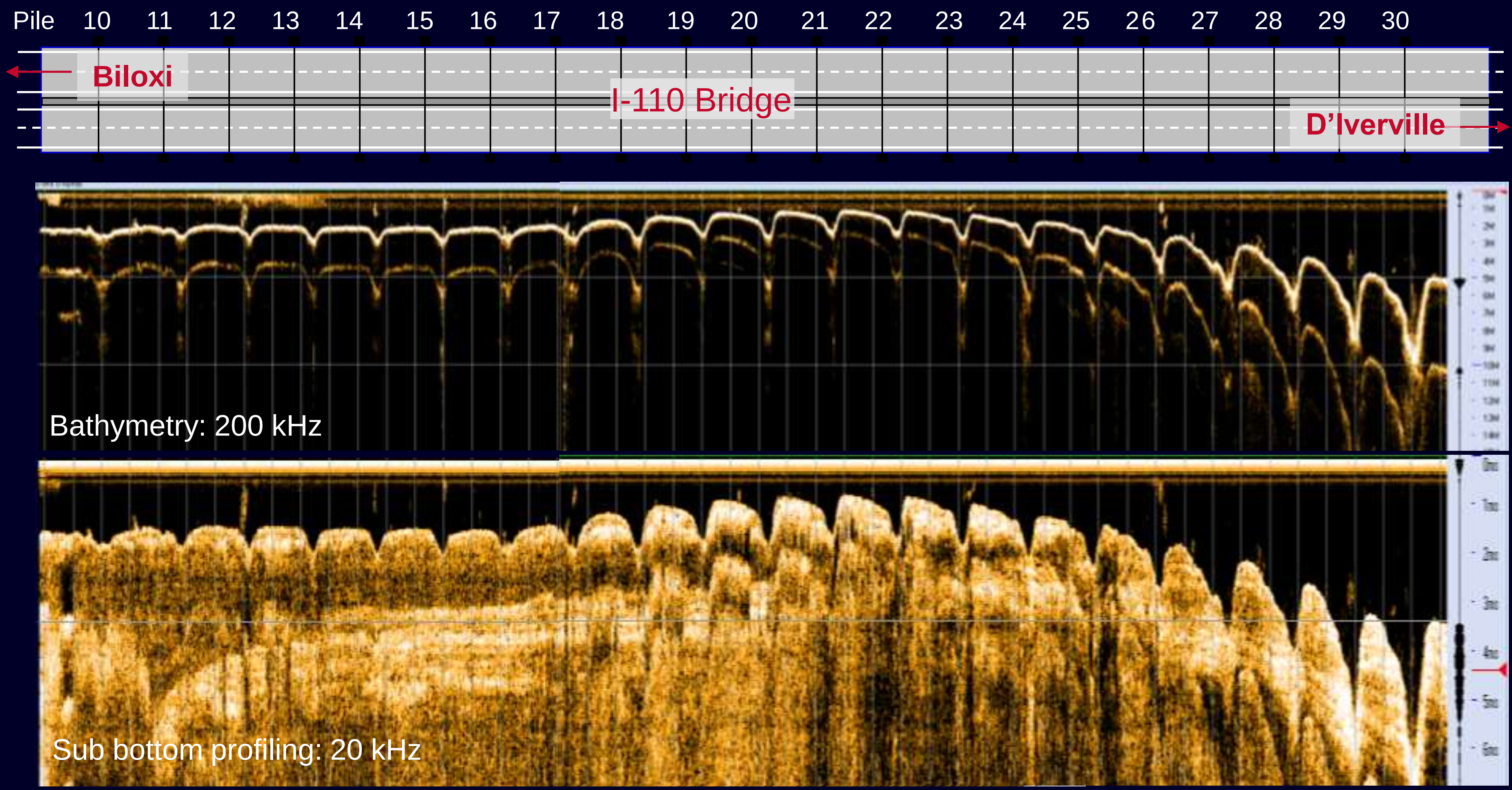

Before Katrina



After Katrina

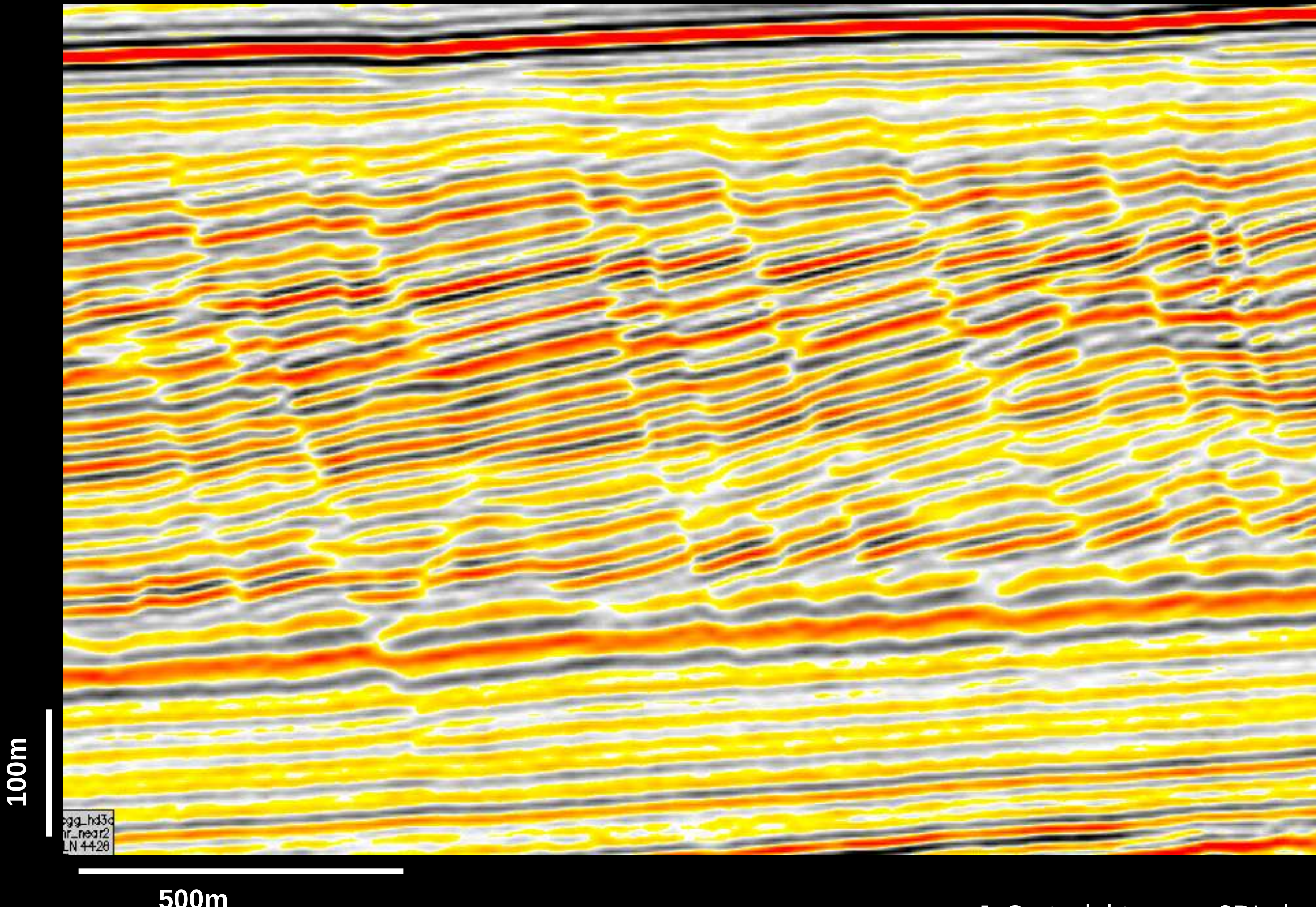
NSF - D. Fratta

Massive data → Display → Information



NSF - D. Fratta

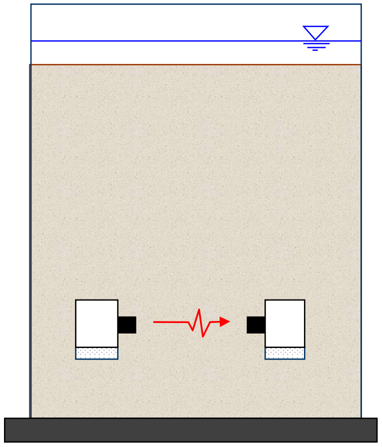
New Phenomena: Polygonal Faults

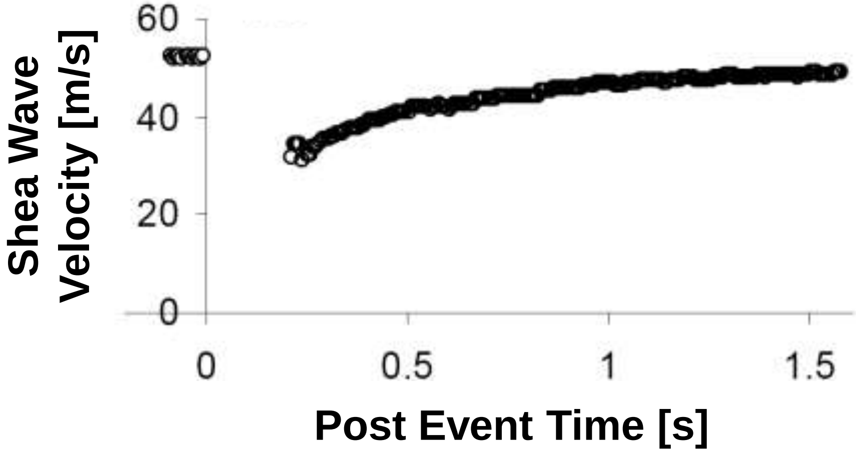


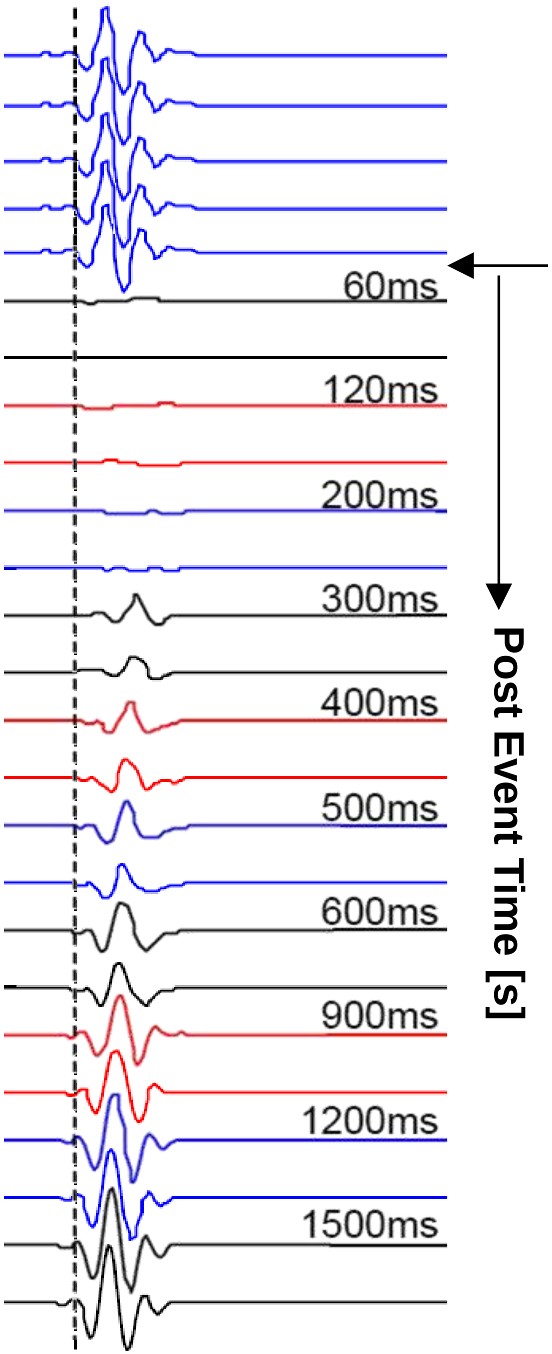
J. Cartwright - www.3DLab.org.uk

S-waves

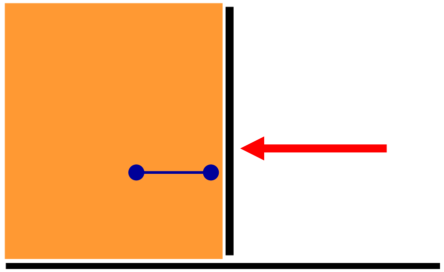
S-monitoring: Liquefaction

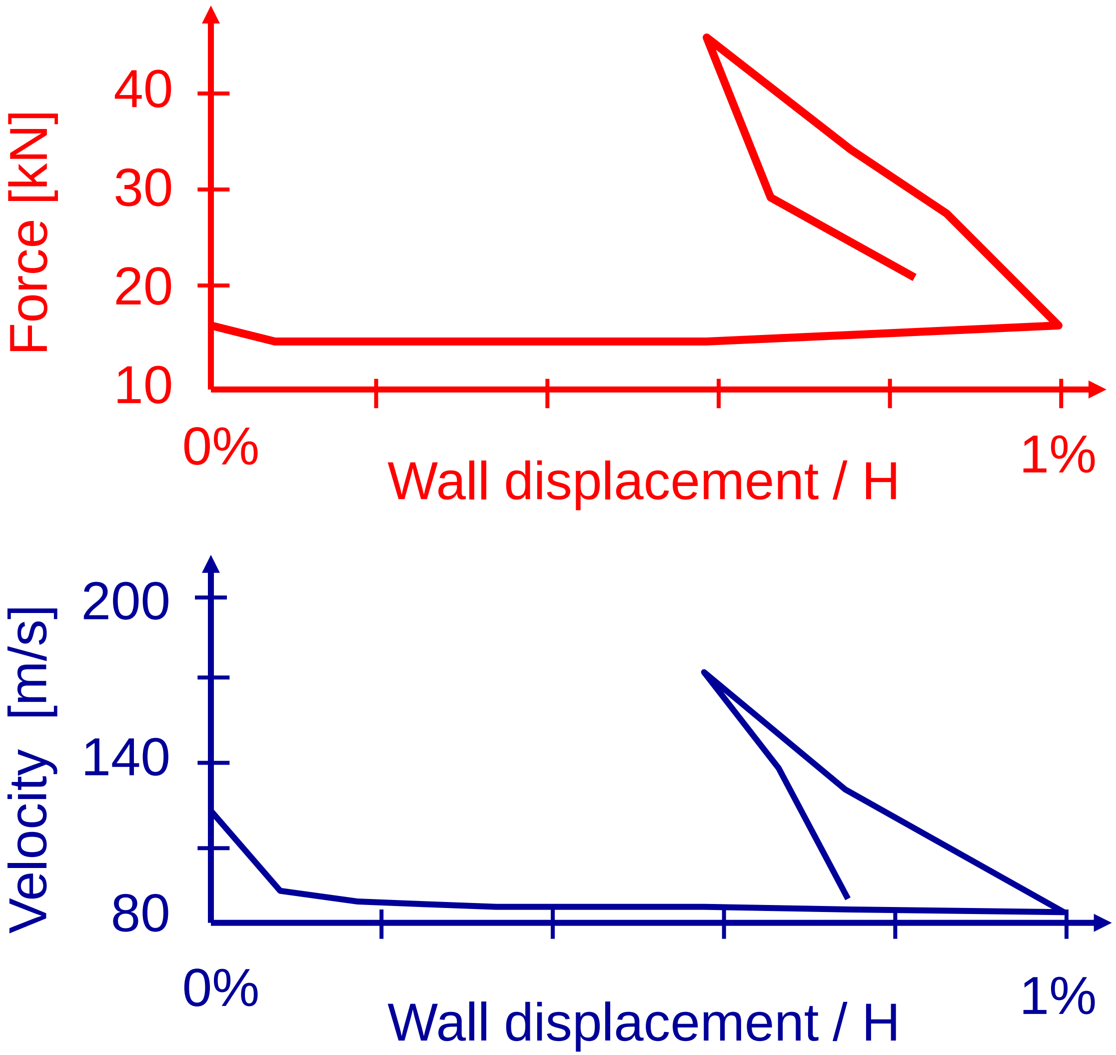


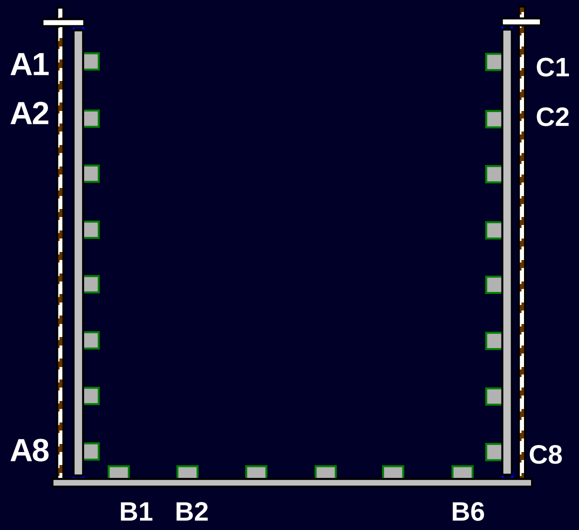


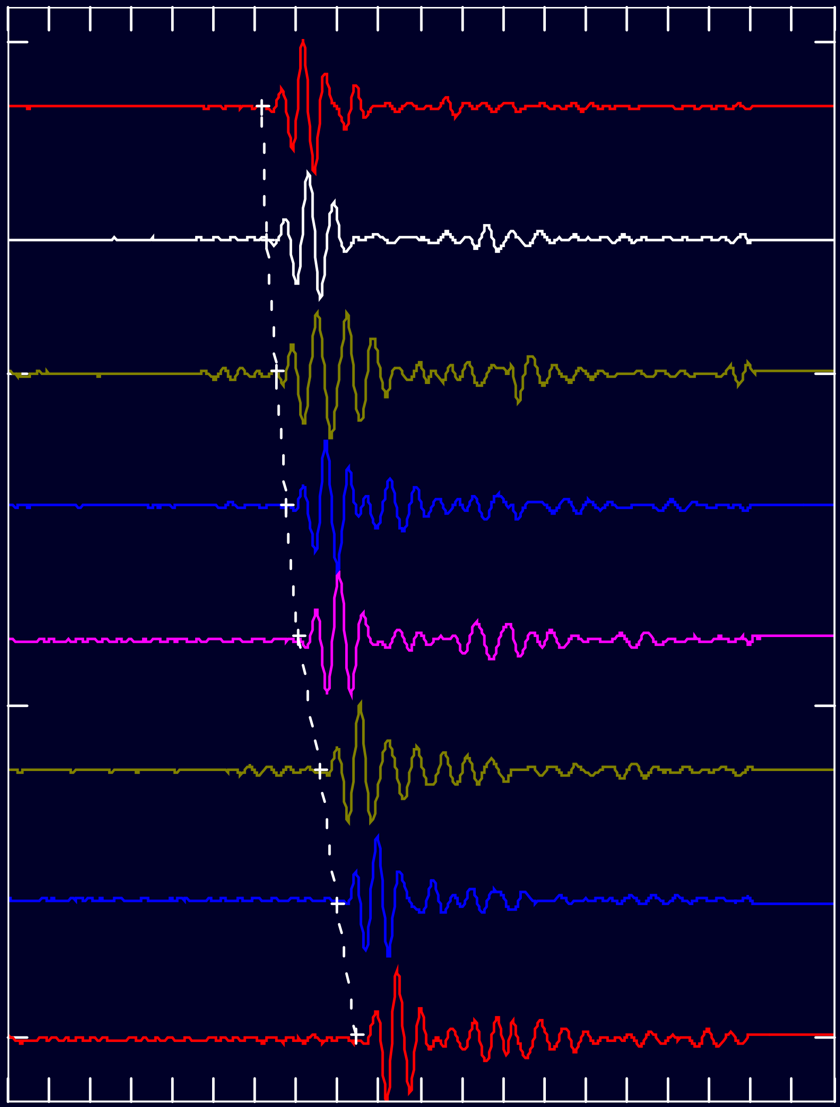


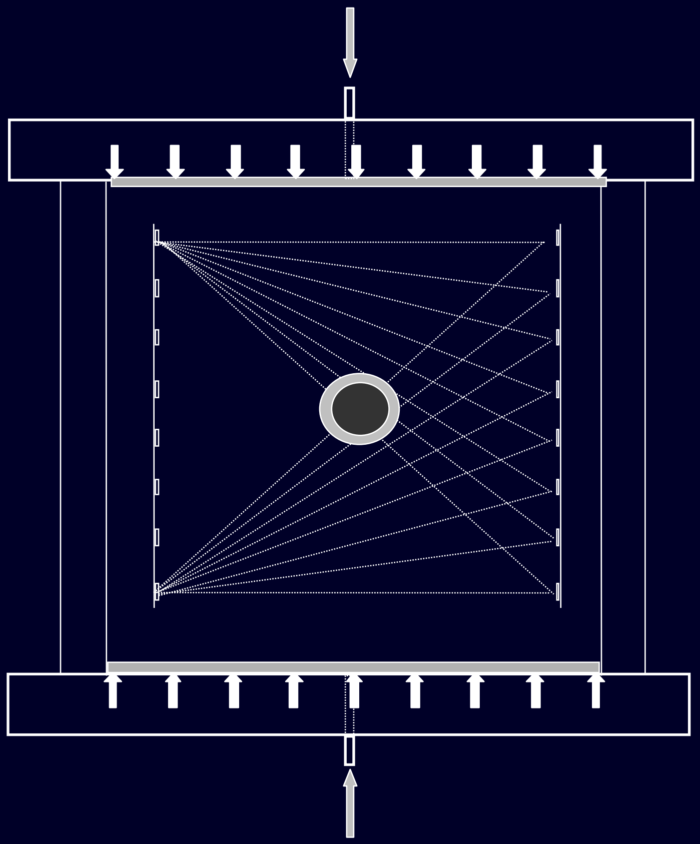
S-monitoring: Excavation & Retaining Walls

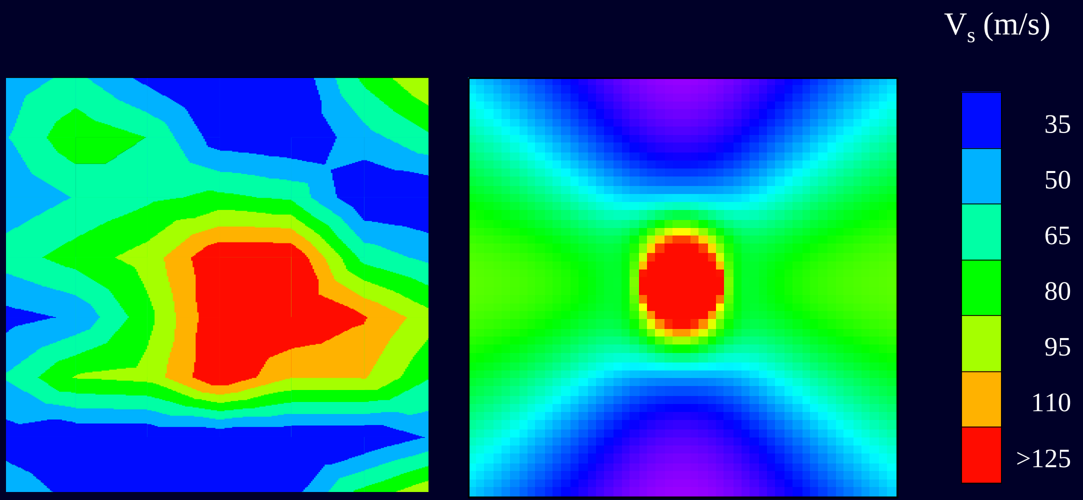





A1
A2
A8
C1
C2
C8
B1
B2
B6







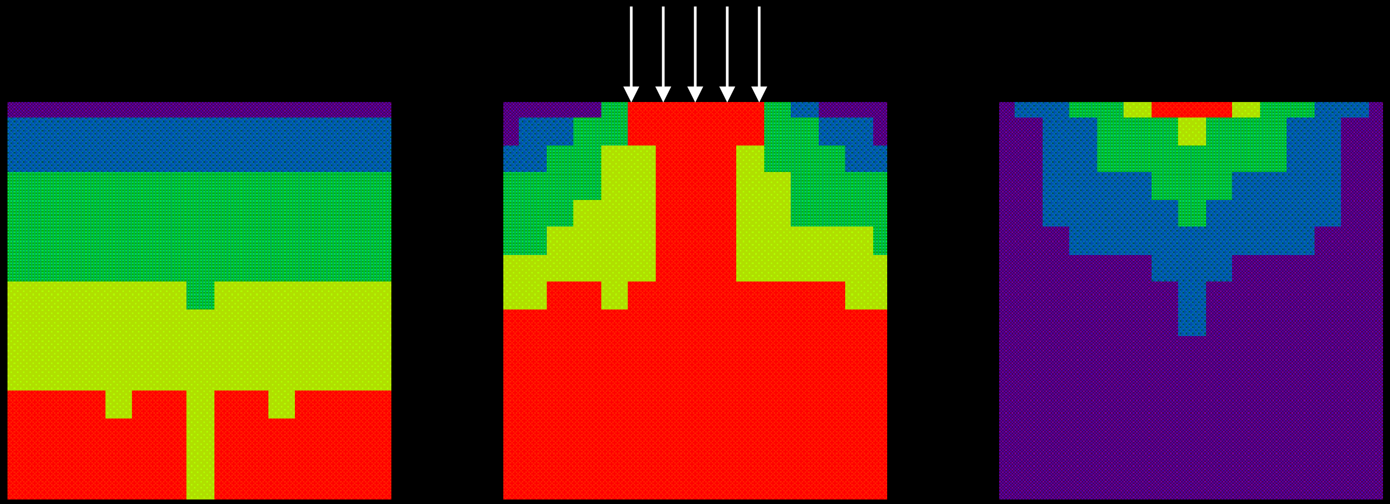
Pixel
(RLSS)

Parametric
(L-norms)

Fernandez, Lee

Imaging the mean stress

$$V_S = \alpha \left(\frac{\sigma'_x + \sigma'_y}{2P_a} \right)^{\beta}$$

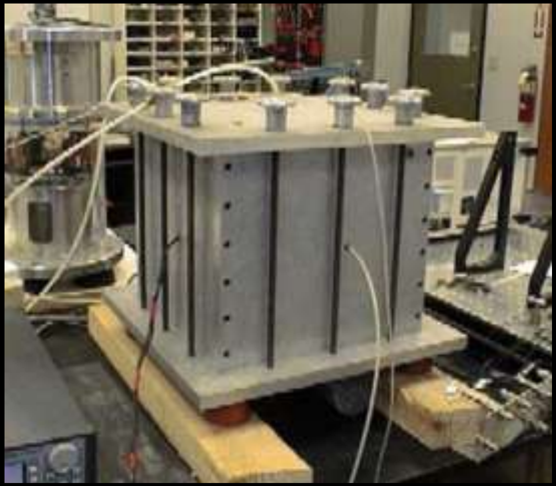


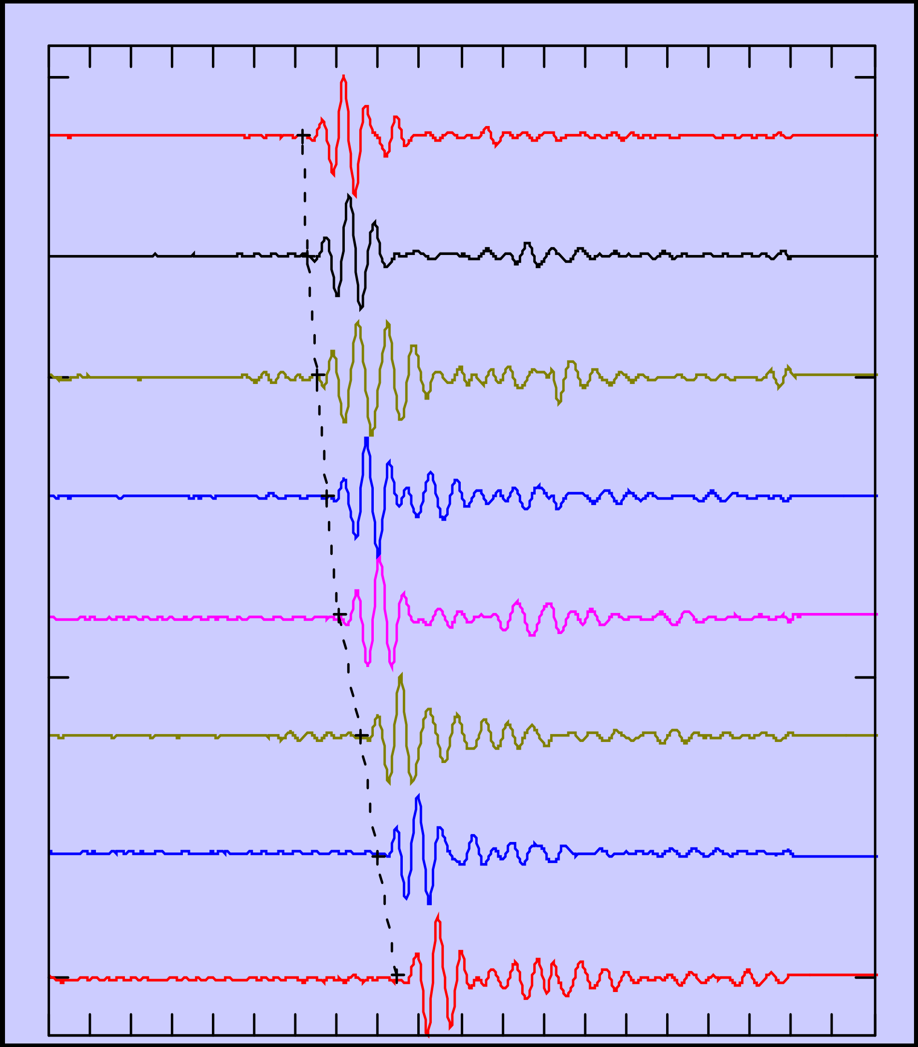
Before Loading

With Loading

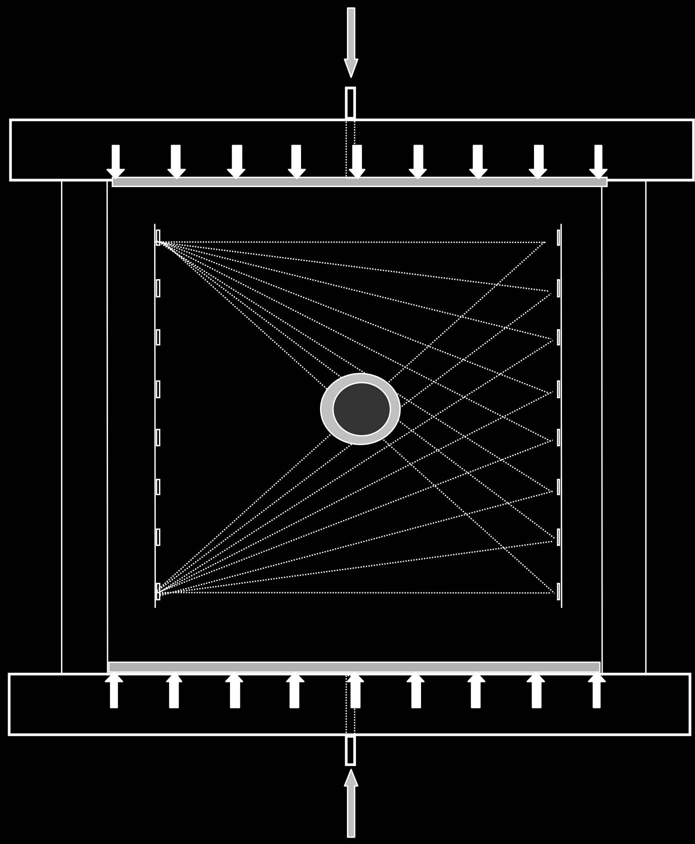
Difference

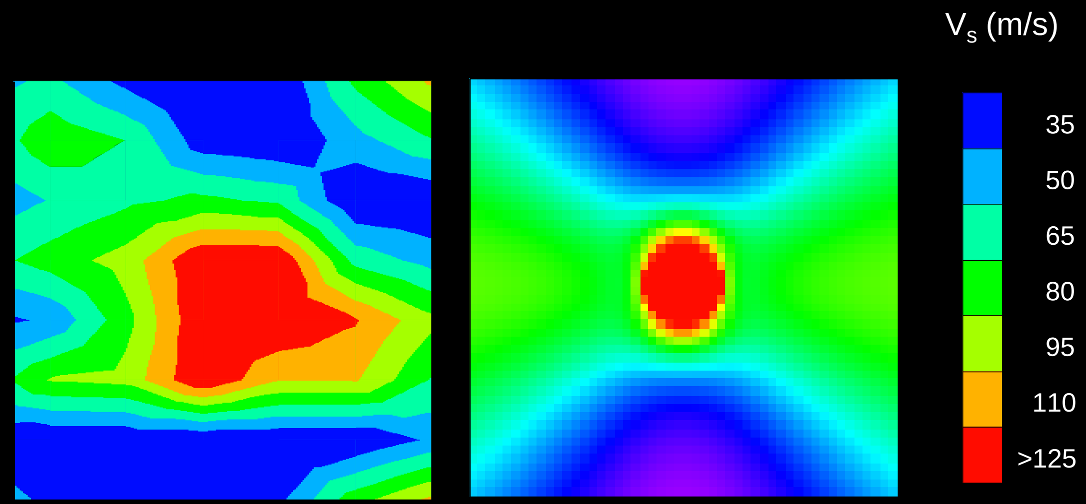
Around tunnels





Around tunnels: velocity tomograms

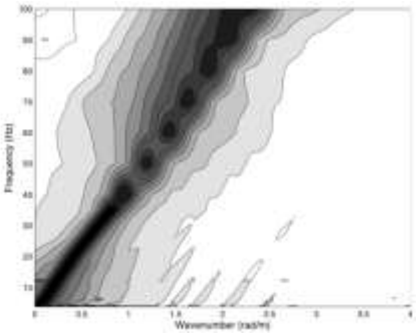
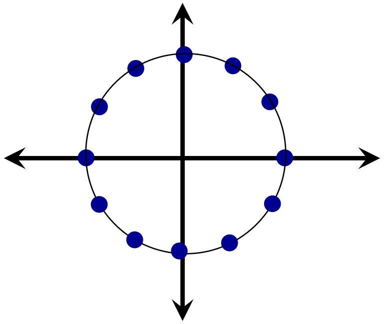
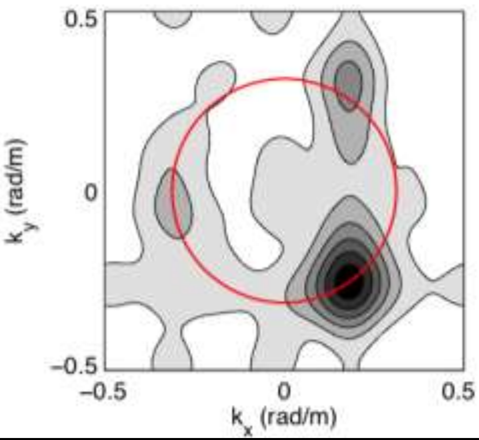




Pixel Parametric

Field: Surface Waves
(*non-invasive*)

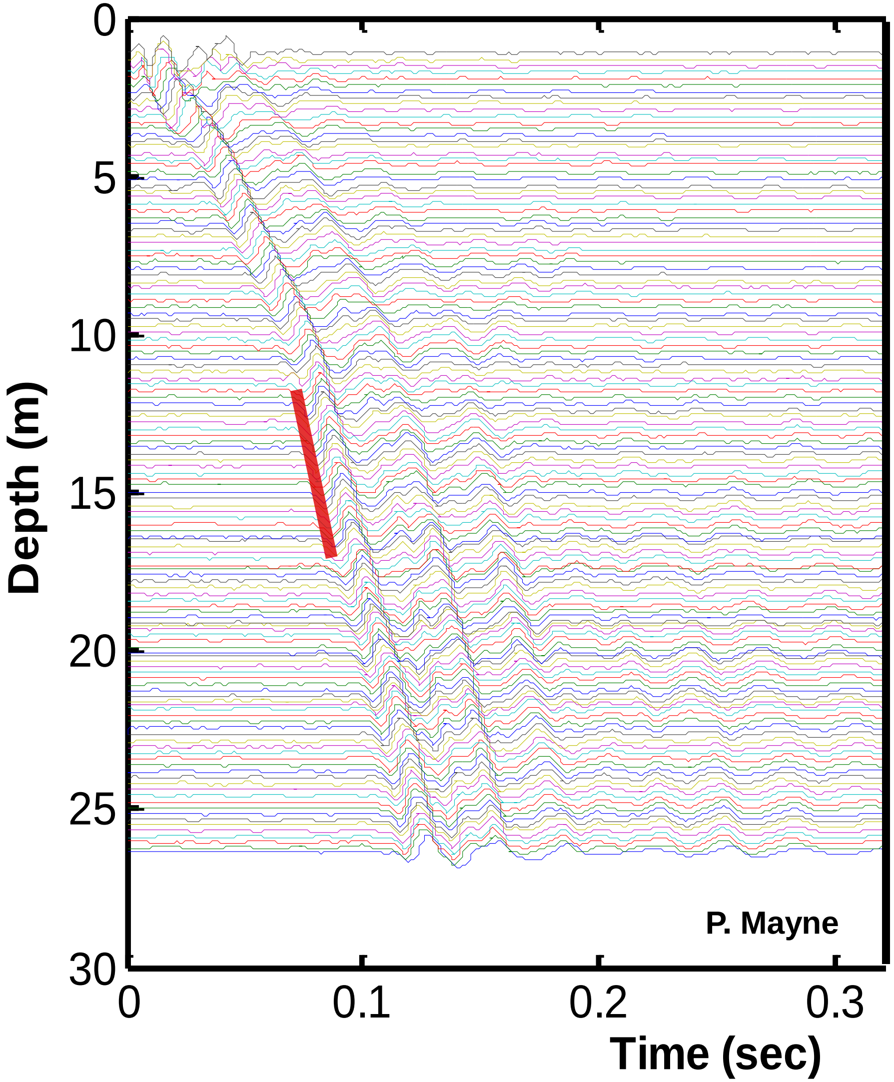
Sensor Arrays

Active		
Pasive		

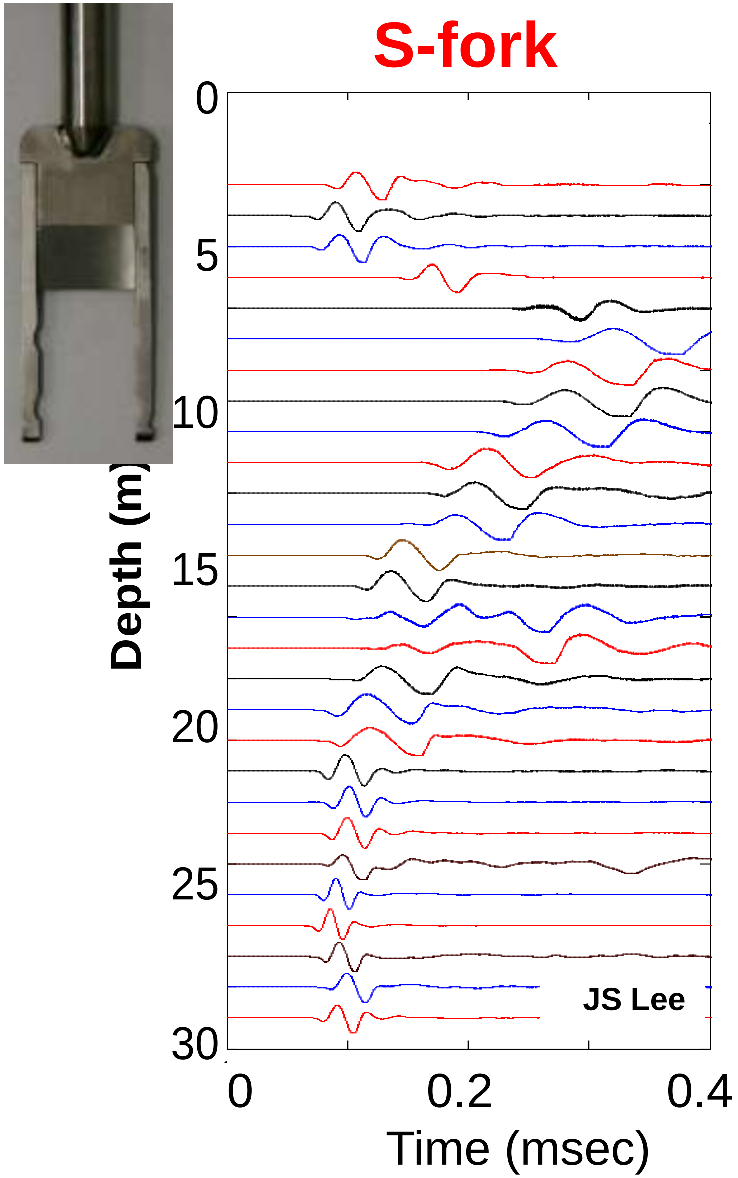
G. Rix

Field: Penetration-based (*invasive*)

S-CPTU



S-fork



Under dams